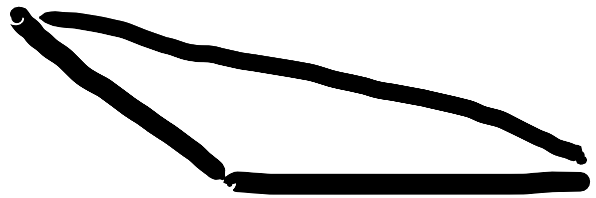
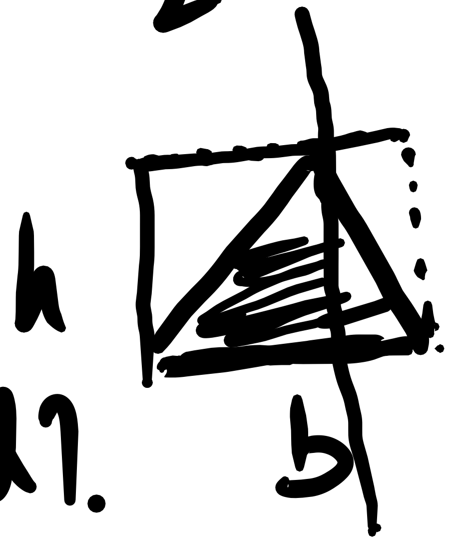
$1+2+\dots+n = \frac{(n+1)\cdot n}{2}$ (why?)



↑ what about this?

Area of triangle $= \frac{1}{2} bh$

Is this picture general?



Area of shaded $= \frac{1}{2} \cdot$ Area of Rectangle

why?

- Understand statements
  - Understanding definitions.

Ex. $(n+1)^2 \underset{\in}{=} O(n^2)$ → fn?.
→ set?. ✓

$$(n+1)^2 = n^2 + \underbrace{2n+1}$$

These are lower-order terms

$$= O(n^2)$$

Proof uses the definition of $O(n^2)$

$O(n^2) = \{ f: \mathbb{N} \to \mathbb{N}:$

there exists c, n_0

Pay attention to quantifiers.

s.t forall $n \geq n_0$

$f(n) \leq c \cdot n^2 \}$

$$\exists c, n_0 > 0 \; \forall n \left[n \geq n_0 \Rightarrow f(n) \leq c \cdot n^2 \right]$$

To prove $f \in O(n^2)$. Produce c, n_0 s.t the condition $\underline{\forall n \geq n_0 \, (f(n) \leq c \cdot n^2)}$

for all

Prove by picking $c = 2, n_0 = 3$

$(n+1)^2 \leq 2 \cdot n^2$ is true when

$2n+1 \leq n^2$ for all $n \geq 3$

You can prove existential statements by picking specific values for the existentially quantified variables.

Structure of math Statements.

atomic

$2+3=5$ ← True

7 is a prime ← True

57 is a Prime ← False

non-atomic (A and B) ← What is the truth value of this Stmt?.

It only depends on truth of A, B.

Truth table

A	B	A and B
F	F	F
F	T	F
T	F	F
T	T	T

$A \Rightarrow B$

the statement $A \Rightarrow B$ is not making any claim so it is vacuously true.

A	B	$A \Rightarrow B$
F	F	T
F	T	T
T	F	F
T	T	T

The original claim is false.

Not(A)

A	Not(A)
T	F
F	T

$\exists x \, A(x)$ $\qquad$ $A(x) = x$ is a prime.

$\forall x \, A(x)$

$\exists x \, A(x) \rightsquigarrow$ True

$\forall x \, A(x) \rightsquigarrow$ False

Let p be $\overset{a}{\text{any}}$ prime. If $p \mid ab$ then $p \mid a$ or $p \mid b$.

$$\forall p \left[p \text{ prime} \Rightarrow \left(\forall a,b \left(p \mid ab \Rightarrow p \mid a \text{ or } p \mid b \right) \right) \right]$$

don't care about p not prime.

How do you Prove $(A \rightarrow B)$ is true?

Assume A. (why?. Because if A is false.) Claim is already true

Conclude using logical arguments that B is true. (once you assume A, you have to establish that B is true)

How to conclude

$\forall a, b \left(P \mid ab \Rightarrow P \mid a \text{ or } p \mid b \right)$

A is true $\equiv$ $\neg A$ is false

negating with quantifiers

$\forall p, a, b\ (p \mid ab \Rightarrow p \mid a \text{ or } p \mid b)$

is false

$\equiv \neg \forall p, a, b\ (p \mid ab \Rightarrow p \mid a \text{ or } p \mid b)$

$\neg \forall_x A(x) \equiv \exists x\ \neg A(x)$

equivalent to

$$\neg \forall p,a,b\ (p \mid ab \Rightarrow p \mid a \text{ or } p \mid b)$$

$$\equiv \exists (p,a,b)\ \neg (p \mid ab \Rightarrow p \mid a \text{ or } p \mid b)$$

$$\neg (p \nmid ab \text{ or } (p \mid a \text{ or } p \mid b))$$

$$\equiv p \mid ab \text{ and } \neg (p \mid a \text{ or } p \mid b)$$

$$= \exists (P, a, b)\left[P \mid \underline{ab} \text{ and } \underline{P \nmid a} \text{ and } \underline{P \nmid b}\right]$$

$$\begin{array}{l|l} P = 6 & 6 \mid 2 \cdot 3 \\ a = 2 & 6 \nmid 2 \\ b = 3 & 6 \nmid 3 \end{array}$$

$$A \Rightarrow B \equiv \neg A \text{ or } B$$

"inclusive-or" is the default or in math.

A	B	$\neg A$ or B
F	F	T
F	T	T
T	F	F
T	T	T

De-Morgan's laws

$\neg (A \text{ or } B) \equiv \neg A \text{ and } \neg B$

$\neg (A \text{ and } B) \equiv \neg A \text{ or } \neg B$

The contra-positive

$p \mid ab \Rightarrow p \mid a$ or $p \mid b$

Say $p \mid ab$ $\quad p \nmid a \quad p \nmid b$

$a = q_a \cdot p + r_a \qquad 0 < r_a, r_b < p$

$b = q_b \cdot p + r_b$

$ab = (q_a p + r_a)(q_b p + r_b) = \underbrace{p(\ldots)} + \underbrace{r_a r_b}$

$0 < r_a, r_b < p$

$$\underbrace{p \nmid r_a \text{ and } p \nmid r_b \;\Rightarrow\; p \nmid r_a r_b}$$ $\square$

This is a special case of the question that haven't been proved.

$$A \Rightarrow B \equiv \neg B \Rightarrow \neg A$$

A	B	$\neg B \Rightarrow \neg A$
F	F	T
F	T	T
T	F	F
T	T	T

$A \Rightarrow B \not\equiv B \Rightarrow A$

$$\begin{cases} \text{If } f(x) = \sin x, \text{ then } |f(x)| \le 1 \\ \text{If } |f(x)| \le 1, \text{ then } f(x) = \sin x \end{cases}$$

Assume $\neg A \Rightarrow \neg B$ after Proving $A \to B$

$$\frac{A \text{ or } B}{}$$

Assume $\neg A$.

Logically derive B.

Proof by cases

Case A is trivial

or

$$A \text{ or } B \equiv \neg A \Rightarrow B$$

usage of this equivalence.

A	B	A or B
F	F	F
F	T	T
T	F	T
T	T	T