

# Cyclus As a Synthetic Testbed of Systems-Level Diversion Signatures

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## Abstract

Already the dominant source of clean energy, nuclear power is growing at a rapid pace. While beneficial to a world confronting climate change, the nuclear security and non-proliferation impacts of expanding nuclear power will become more consequential. As a result, it is imperative to develop credible methods to verify compliance with treaties that control fissile material production, such as the Non-Proliferation Treaty or a potential Fissile Material Cutoff Treaty. As part of the Consortium for Verification Technology, the Cyclus fuel cycle simulator is being used as a testbed for the development of new technologies and analysis approaches to treaty verification. Cyclus is an agent-based, systems-level simulator that tracks discrete material flow through the entire fuel cycle, from mining through burnup in reactors to a repository, or alternatively through one or more iterations of reprocessing. A systems-level view facilitates the study of correlated signals from different facilities that combine to form identifiable signatures of clandestine activity. Cyclus also includes a region/institution/facility hierarchy that can incorporate the effects of tariffs and sanctions in regional or global contexts. Cyclus enables social- behavioral modeling of the interactions between individual facilities or regions. This paper presents the first use of Cyclus to simulate nuclear material diversion from the fuel cycle using a variety of correlated signals: material flow, facility power consumption, effluent emissions (including geospatial distribution), event-logs. Multiple signal modalities can be analyzed in concert using anomaly detection techniques to identify signatures of material diversion or other signatures of clandestine nuclear weapons development. The Cyclus testbed can then be used to examine treaty verification techniques and inspection regimens to to inform their sensitivity and limitations.

## 1 Introduction

Nuclear expertise is rapidly expanding around the world as demand for energy increases steadily. Because nuclear energy is clean and carbon-neutral, climate change concerns further tilt the scales making nuclear power appealing to a growing number of countries [1]. China is already investing heavily in nuclear power, planning to triple its generating capacity from 19 gigawatt electrical (GWe) to 58 GWe by 2020 [2]. As climate change becomes increasingly important with respect to national security, the perception of the risk inherent to nuclear energy is decreasing and states are embracing nuclear energy as a reliable large-scale source of carbon-neutral energy. However, the expansion of nuclear power amplifies nuclear security concerns, because the same technologies used to produce nuclear fuel can also be exploited in the pursuit of nuclear weapons. Moreover, in the 70 years since nuclear bombs were dropped on Hiroshima and Nagasaki, the knowledge and technology required to make these weapons has proliferated around the globe [3]. There are now nine states that have developed their own nuclear weapons either through indigenous research or transfer of knowledge from existing programs. As nuclear power becomes more ubiquitous, it becomes

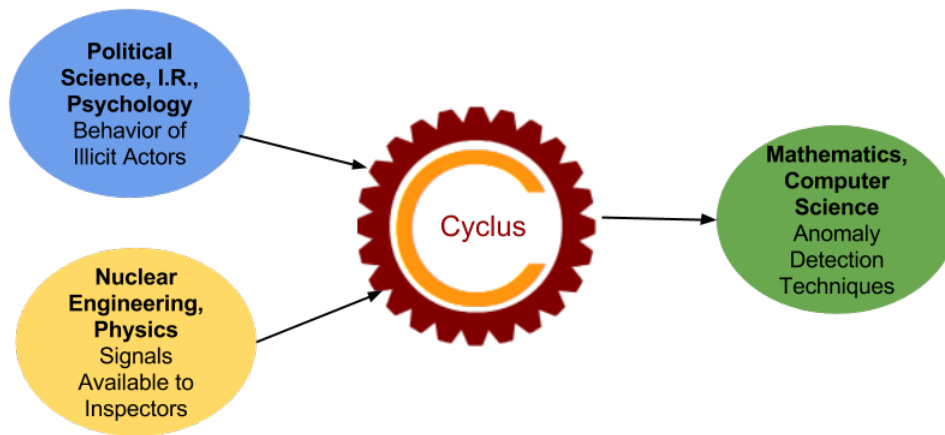


Fig. 1: The CYCLUS nuclear fuel cycle simulator provides a testbed to integrate innovations in treaty verification across many disciplines.

ever more important to meaningfully decouple the nuclear expertise required for the pursuit of energy from that of nuclear weapons.

## 1.1 Use of Treaties to Curtail Proliferation

While it has not proven possible to prevent the spread of nuclear knowledge entirely, international treaties have been used in an attempt to minimize it. The Nuclear Nonproliferation Treaty (NPT), which has been signed by 190 states including the original five nuclear weapons states, has codified a set of rules and norms for allowing the peaceful pursuit of nuclear energy [4]. The NPT created the International Atomic Energy Agency (IAEA), whose role is to verify compliance with the treaty by periodically inspecting facilities related to nuclear technology. Other relevant treaties include Comprehensive Nuclear-Test-Ban Treaty (CTBT), which placed a moratorium on testing nuclear weapons, and the Strategic Arms Reduction Treaties (START) in which the United States (US) and Russia agreed to nuclear arms reductions [5, 6]. (The CTBT has been signed by 164 states but has not yet entered into force).

These treaties have done much to prevent the spread of nuclear weapons knowledge, but they do not address the weapons production capabilities of states that already possess nuclear weapons. A potential Fissile Material Cutoff Treaty (FMCT) would place limits on the amount of weapons-grade fissile material that each signatory state could stockpile, possibly including current stockpiles in the case of weapons states. However, a major unresolved issue is the difficulty of developing verification techniques to ensure compliance [7]. Furthermore, measuring nuclear material for treaty verification is itself a sensitive issue, as even collecting the spectra of a material to confirm its authenticity can potentially expose sensitive information to the inspecting party [8]. Particularly if non-weapon states are to contribute to treaty verification, it is important to prevent the further dissemination of nuclear weapons knowledge.

An effective treaty verification regime must synthesize knowledge from the realms of political science, international relations, nuclear physics and engineering, and even behavioral psychology. Figure 1 illustrates the role of a fuel cycle simulator such as CYCLUS in bringing together these disparate fields to provide insights into proposed verification technologies. A fuel cycle simulator tracks the flow of nuclear material through the facilities in a fuel cycle[9]. Uranium enrichment and spent fuel reprocessing are two particularly

sensitive parts of the fuel cycle, but correlated signatures of illicit activity are likely to be present across the fuel cycle. A fuel cycle simulator creates synthetic data, such as what would be available to an inspector, for many different facilities simultaneously while incorporating a system-level perspective of proliferation scenarios. This synthetic data can then be used as a testbed to investigate the efficacy of new detection and analysis techniques. In this way, simulators can be used to to illucidate the strengths and weaknesses of various verification strategies.

## 2 The Cyclus Fuel Cycle Simulator

The Computational Nuclear Engineering Research Group (CNERG)<sup>1</sup> group at the University of Wisconsin has developed the CYCLUS<sup>2</sup> nuclear fuel cycle simulator to model all aspects of the nuclear fuel cycle in a flexible way [10, 11, 12]. CYCLUS produces a database file containing information on the flow of nuclear material through the fuel cycle at each timestep. The database provides information on facility inventories, material composition, transactions between facilities, and facility build and decommissioning histories, among others.

CYCLUS is designed using an agent-based framework, meaning that each actor in a fuel cycle (such as a mine, a nuclear reactor, or even a governing body) is modeled as an independent agent [13, 14]. Each agent in the simulation is self-contained and may include physics, economics, or behavioral components [15, 16, 17]. The agents interact with one another through the dynamic resource exchange (DRE), which facilitates the trading of resources and commodities [18]. At each timestep, agents can choose to request resources. Resources are defined using both a quantity (e.g. 1 metric ton), and a quality, such as having a composition of 99.7%  $^{238}\text{U}$  and 0.3%  $^{235}\text{U}$ . The DRE then solicits bids from any facilities that are interested in offering those resources. Resources can be offered as bids even if they do not exactly match the requested material. For example, a reactor might request a commodity called “fuel”, which it has defined as being uranium oxide (UOX). It may receive two bids for “fuel” that are specified as UOX and mixed oxide (MOX), having two distinct isotopic compositions. After the bids are received, the requestor is able to state a preference for one bid over another. Finally, once the preferences have been applied, the DRE calculates all potential trades across all agents, then executes an optimization algorithm to find the solution that most closely fulfills a maximum of requests. It is possible that as a result of the preference adjustment, no trades are executed for some facilities in a given time step. Once this solution is found, material is transfered across the facilities and the timestep is concluded.

Although CYCLUS was designed to assess the transition from once-through fuel cycles to alternative next-generation scenarios including technologies such as spent fuel recycling, it is also an excellent tool for examining proliferation issues. Other fuel cycle simulators designed to study energy issues are typically too inflexible to be used for other purposes. For example, they may have extremely accurate physics models of reactor burnup, but no ability to vary or modify other facilities in the fuel cycle. CYCLUS has three key features that make it well-suited to non-proliferation studies: it is *agent-based*, it tracks *discrete materials*, and it incorporates *social and behavioral interaction models*. Agent-based design allows for modular simulations where individual facilities can be swapped compared in otherwise identical simulations. CYCLUS tracks discrete material flow through the simulation, and uses data from PyNE,<sup>3</sup> to track decay and transmutation data at all timesteps [19, 20]. Finally, behavioral modeling allows facilities and institutions to engage in dynamic decision-making based on their preferences, needs, or political constraints. A specific

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<sup>1</sup> <http://cnerg.github.io/>

<sup>2</sup> <http://fuelcycle.org/>

<sup>3</sup> <http://pyne.io/>

agent might have preferences based on material composition, physical proximity between facilities, or allowed and disallowed trading partners, which are implemented in a region-institution-facility hierarchy that enables economic modeling [21].

Behavior is a critical aspect of non-proliferation modeling. For example, an enrichment facility receiving illicit requests for highly enriched uranium (HEU) may define its own criteria for whether or not to fulfill this order. It may disallow production of enrichments above a certain level, choose to trade only with specific facilities, or choose to preferentially fulfill requests at one enrichment level over another. Likewise, a requestor of HEU can make requests at regular or random intervals, and may request randomly or gaussian distributions of material quantity. At the institution level, CYCLUS allows trading decisions between facilities to be controlled by the owner of those facilities, which may be a commercial entity or a nation-state. This facilitates modeling of the interactions between multiple states within a region.

### 3 Signatures and Observables

Figure 2 is a table-in-progress to identify potential signatures and observables of illicit activity at all major facilities in the nuclear fuel cycle. Each potential signature is color coded to represent the accessibility of the data. Green signatures are available through open, independent sources such as satellite imagery. Blue signatures may not be publicly available but could be attained through official channels such as IAEA inspections. Yellow signatures may or may not be made available but must be considered unreliable or unverifiable due to physical or political constraints, e.g. state declarations. Not all of these signatures are currently collected, and it would be resource intensive to maintain comprehensive surveillance of all signatures. For example, a truck that leaves a fuel fabrication plant should arrive at a reactor (or train station) with the fuel shipment, but it is infeasible to comprehensively track individual truck movement. However, correlations in these individual signatures can be leveraged to overcome sparse datasets or resource constraints.

CYCLUS is being used to produce a variety of synthetic signals spanning a range of modalities. We are currently developing signatures that would be available either publicly or via official inspections (green or blue). As can be seen in Section 4, all facilities in CYCLUS automatically produce time-series data of material inventory. Additionally, the enrichment facility reports its SWU consumption as a time series that can be used as a rough proxy for facility power consumption. As seen in Figure 3, CYCLUS can combine atmospheric transport models with facility effluent concentration ( $I$ ) to track geographic dispersion. In this example, a simple atmospheric diffusion model of wind from the left is used to illustrate how a clandestine reprocessing facility (middle) could be hidden in close proximity to two declared facilities (top and bottom)[?]. In addition to regularly sampled time-series data, CYCLUS also models sparse, discrete-event data such as declared truck shipments from a facility.

CYCLUS is also being used to model inspections at an enrichment facility that test for the presence of HEU. IAEA inspections typically involve multiple swipe samples per location, with some likelihood of false-positive or false-negative results.[?]. Figure 4 pairs inspections with undeclared truck shipments of HEU. In this example, it is assumed that the likelihood of detecting HEU in the enrichment facility increases with each shipment (because contamination is possible when HEU is removed from cascades and bottled for shipping). The undeclared HEU shipments are shown as black bars, where amplitude incorporates the gross quantity HEU that has been produced at the facility. The colored dots scale from yellow to red, indicating the fraction of positive swipes at an IAEA inspector visit (assuming a total of 10 swipes are taken per inspection, and an average of two inspections per year). It is assumed that both false-positive and false-negative results are possible.

Signatures	Mining/Milling	Conversion	Enrichment (centrifuge)	Fuel Fabrication (U)	Reactors (Gen)	Separations	Waste Disposal
Nuc Matl In (estimate)		# 55 gall drums	# tanks (150tmes)	#tanks	# assemblies	# casks or assemblies	# casks or barrels
Nuc Matl In (qty, precise)		Mass	Mass, PSI	Mass, PSI	mass	mass	barrels, tanker trucks, casks, truck2 concrete barrels
Nuc Matl In (quality)		assays	IAEA assays	assays	IAEA assays	assays, radiometry	radiometry
Nuc Matl Out (estimate)	Trucks	# tanks	# tanks	# fuel assemblies (decoy rods in assembly?)	pools, dry cask storage	UF6 tanks (UF6), Pu or Pu-FP	
Nuc Matl Out (qty, precise)		mass, psi	mass, psi (~3000)	mass	mass	mass? (total mass can be tracked stream fraction depends on burnup etc)	
Nuc Matl Out (quality)	radiometry	assays	assays	assays	assays	assays	
Supplemental Material In	Sulfuric acid (20kg/tonne), magnesium chloride, lime, manganese, ammonia, or caustic soda	nitric acid, HF, tributyl phosphate		zircaloy(tin, niobium,iron,chromium,nickel), NSF cladding (zircaloy+iron), H2, ammonia or ammonium carbonate, argon, erbium, organic binding agent (polyvinyl alcohol), He, HNO3-HF	(heavy water, gas cooling, etc) ?	nitric acid, kerosene or dodecane, Na2CO3, NaOH, HNO3	entombment matls (concrete, filler matl), vitrification material (borosilicate glass)
Supplemental Material Out		Chem waste, silicon flouride (commercial) from DU		fluorine, hafnium (natural Zr byproduct)	(sodium? other chemicals)	Chem waste, Alternative isotope streams (Ti, I, Tc, Pt, Np,Am), fuel casing metals	
Power		Trans line/substation	Trans line/substation	Trans line/substation	produce vs consumed	Trans line/substation	(vitrification)
Nuc Waste Quantity	Tailings pond (satellite)		Trucks	"lost fuel", grinding, scraps (diff. to measure)	low-level (trucks)	Containers (borosilicate glass, high level waste)	
Nuc Waste Quality	radiometry		Tails Enrichment	assay	radiometry	radiometry	
Effluent	tailpipe emissions, radon	Ammonia gas leaks	uranyl, HF		fuel leak (Kr, Xe)	Kr85, Xenon, tritium, I, CO2	
Environmental Signatures	dynamite blast signatures	soil contam.	trace enriched U?	soil contamination	soil contam, fuel leak in coolant water (LiCs)	soil contam	soil contam (barren)
Declarations to IAEA	Out Qty/Quality	In/out Qty/Quality	In/out Qty/Quality	In/Out Qty/Quality	In/Out Qty/Quality	In/Out Qty/Quality	In Qty/Quality
Shipping In		Satellite (# trucks), freq	Satellite (# trucks), freq	Satellite (# trucks), freq	Satellite (# trucks, rail), freq	Satellite (# trucks), freq	Satellite (# trucks),
Shipping Out	Satellite (# trucks), freq	Satellite (# trucks), freq	Satellite (# trucks), freq	Satellite (# trucks), freq	Satellite (# trucks, rail), freq	Satellite (# trucks), freq	freq
Specialized Equipment	bulldozers, dynamite	centrifuge	Forklift rigs	Forklift rigs	radioactive handling	acid storage/handling, radioactive handling	radioactive handling
Heat Processes/Signatures		furnace (800C)		furnace (1750C)	cooling towers/ponds (temp, vol, fan speed)	furnace (1200C - pyroprocessing)	vitrification (1100C)
Connections outside facility	where do trucks go	where do trucks go	where do trucks go	where do trucks go	trans lines to correct location	where do trucks go	where do trucks come from
Special Handling Protocols/Rates	acoustic signatures of dynamite	florine handling, heavy tanks	florine handling, heavy tanks, building below atmospheric pressure	flouride handling, heavy shipping (0.5tons/assembly), high temp vacuum oven	refueling frequency, transition to offsite waste storage	refresh acid supply, toxic waste, venting (operated at negative pressure), radioactive handling	
Configuration Change (staging materials etc)	addition of mill to mine site	combined w/ enrichment?	piping			special construction matls (ceramic vats, piping)	

Fig. 2: Table of potential signatures across the fuel cycle: measureable through open, independent sources such as satellite imagery (green), available through official inspections (blue), or potentially unreliable due to physical or political constraints (yellow).

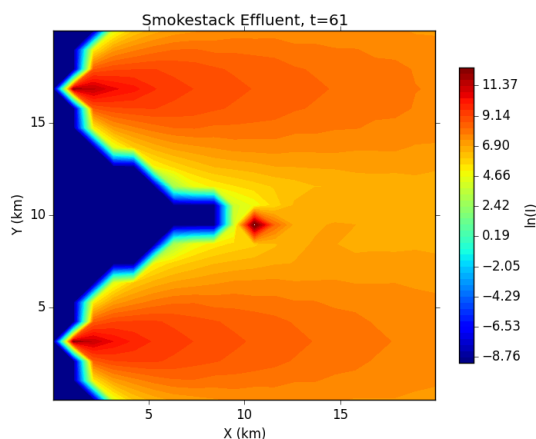


Fig. 3: Effluent transport with wind from left, hides a clandestine reprocessing facility  $x,y=(11,9)$  in close proximity to two declared facilities  $x,y=(1,3), (1,17)$

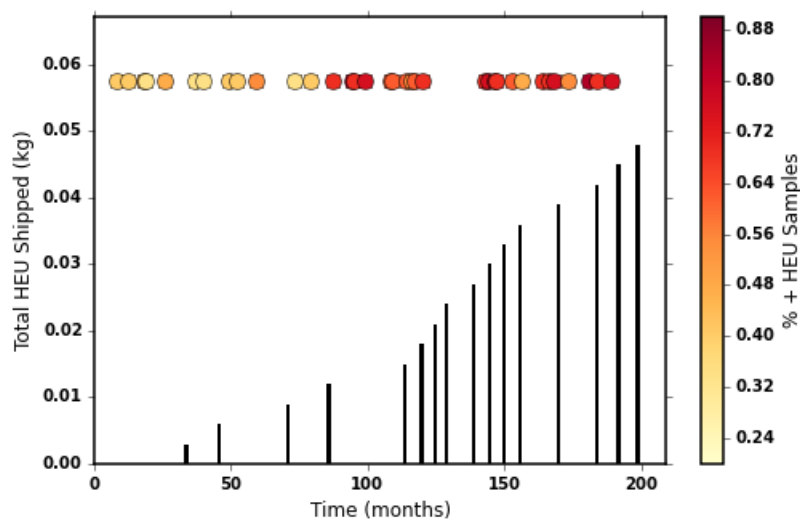


Fig. 4: Black bars indicate clandestine shipments of HEU, amplitudes show gross HEU production at the enrichment facility. Colored circles are fraction of swipes (out of 10) testing positive for HEU during an inspection. Both false positives and false negatives may be present, introducing uncertainty into the data. As more HEU is produced, the detection rate increases.

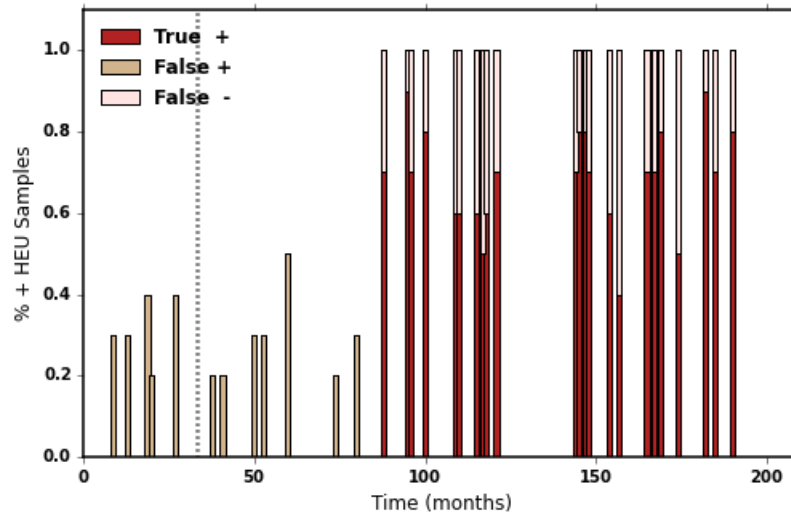


Fig. 5: The scenario in Figure 2 had a 30% rate for both false-positives and false-negatives. Before month 30, no HEU has been produced so all detections are false positives (tan). Once HEU contamination is present (near month 80), true detections (red) combine with false-negatives (pink), resulting in an effective 70% detection rate.

Figure ?? shows the breakdown of true positive, false positive, and false negative results for each inspection (for illustrative purposes only, in practice the true accuracy of a particular set of field data is unknowable). Before month 30, no HEU has been produced in the facility, so any “positive” swipes are false-positives (tan). Once the facility begins producing HEU, likelihood of contamination increases until it is detected near month 80. If the swipe tests were perfect, there would be a 100% detection rate for all inspections after that time. However, the possibility of false-negatives (pink), make it so that the measured inspection data appears to be the tan before month 80 and the red after month 80. In this scenario, it would be difficult for an inspector to conclude that HEU was being produced with this dataset alone.

## 4 A Diversion Scenario: Highly Enriched Uranium

As a part of the Consortium for Verification Technology (CVT)<sup>4</sup>, CYCLUS is being used to generate multi-modal datasets with signatures of diversion with the goal of improving anomaly detection techniques. In the simplest implementation, HEU is clandestinely produced and then diverted from an enrichment facility. Figure 6 illustrates a toy model of this portion of the fuel cycle. A facility such as a mine supplies natural uranium (0.7%  $^{235}\text{U}$ ) to an enrichment facility. The enrichment facility in turn receives requests for low enriched uranium (LEU) one of five enrichment levels (3-5%) from several declared light-water reactors (ignoring the fuel fabrication facility for simplicity). The enrichment facility also receives requests for 90% enriched HEU from an undeclared actor seeking to build a nuclear weapon. Material production for each facility is calculated once each month for a total of 200 months. At each timestep, the enrichment facility fulfills an order for one LEU enrichment level, and sometimes produces small quantities of HEU request.

<sup>4</sup> <http://cvt.engin.umich.edu/>

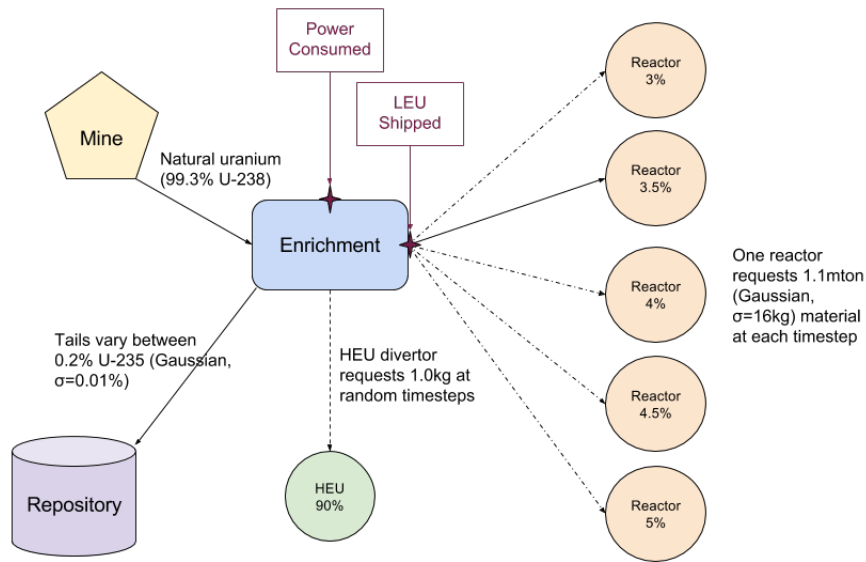


Fig. 6: Diversion scenario: Enrichment facility produced LEU for 5 different customers, as well as secretly making HEU occasionally. Both LEU quantity and tails assays have slight variation to introduce “noise” into the system.

Figure 7 shows the time series data for declared production of LEU (top) and the total separative work unit (SWU) consumed by the enrichment plant (middle) available to the inspector (SWU can be used as a rough proxy for power consumption). Months where HEU is produced are denoted with green on the LEU plot. The HEU signature is hidden in the material flow data because there is a gaussian variation in both the tails assay and the quantity of LEU produced. Therefore it is not possible to detect diversion from the individual time-series data alone. However, the two signals can be combined to highlight the correlated signature of diversion. The bottom plot in Figure 7 shows time-series data for the ratio of SWU-consumption to declared LEU production. When the variation due to tails assay (“noise”) is sufficiently small, the deviations may be seen by eye. In practice, as the noise increases or the HEU quantity reduces in amplitude, this signature quickly becomes difficult to detect by eye.

While this example is clearly a toy model, it illustrates the power of combining multiple signals from a scenario to improve detection capabilities. An important application of CYCLUS is to produce more complex synthetic datasets which are then provided to groups that specialize in developing advanced anomaly detection techniques. An ongoing collaboration with researchers at the Michigan Institute for Data Science<sup>5</sup> seeks to apply innovative anomaly detection techniques to these simulations to investigate detection limits for scenarios with sparse data sets or low signal-to-noise ratios.

## 5 Discussion

CYCLUS is able to model signatures of diversion from a diverse set of facilities in the nuclear fuel cycle and with a variety of data modalities. Table 1 lists a variety of signal modalities and their applications in the fuel cycle paradigm. One modality with diverse set of potential applications is satellite imagery. We are

<sup>5</sup> <http://midas.umich.edu/>



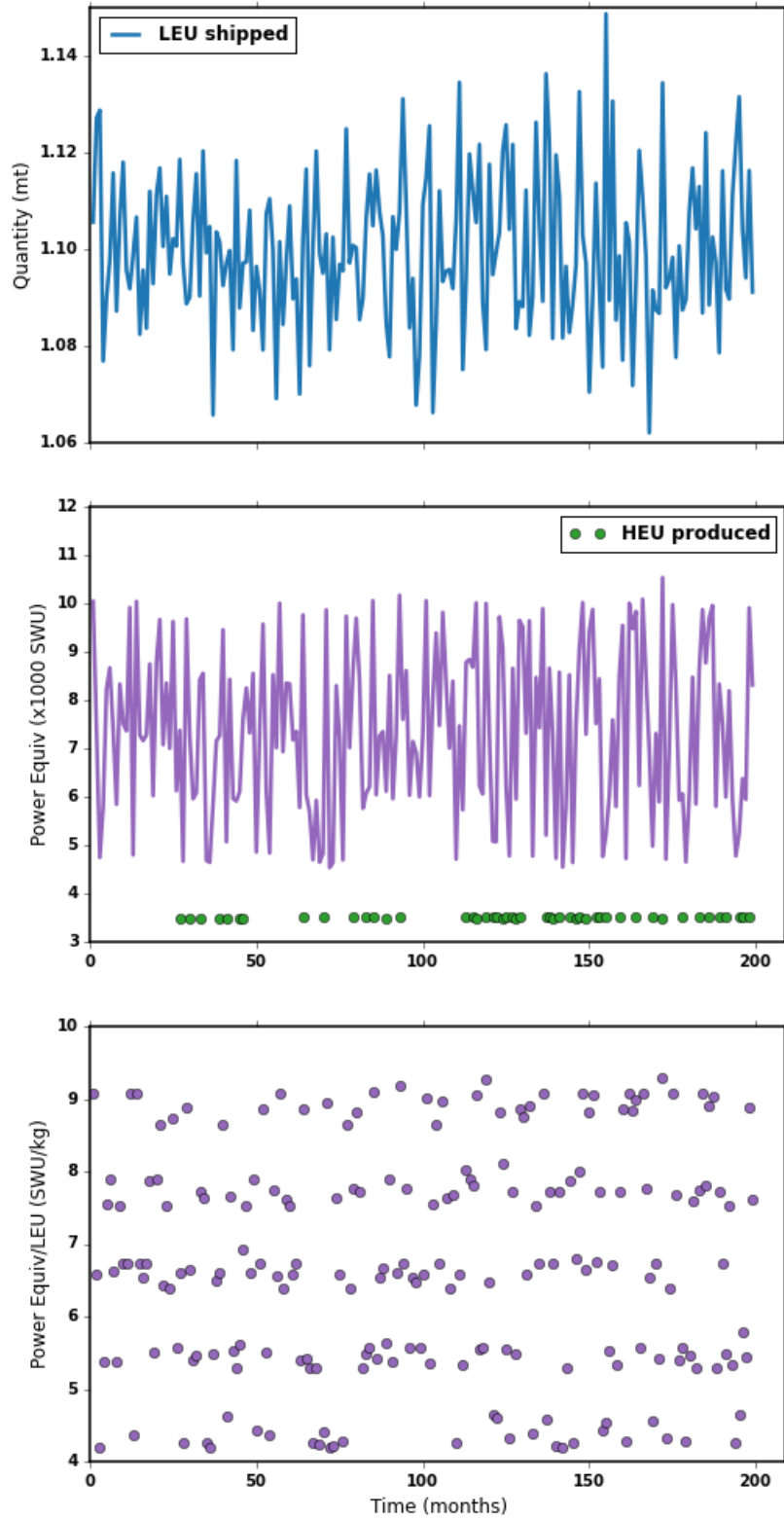


Fig. 7: Time-series data for declared LEU production (top) and facility SWU consumption (middle). Green dots illustrate times at which HEU is also produced. The ratio of power to LEU quantity (bottom) shows deviations from the mean when HEU is produced.

Modality	Physical Signal
Time-series	Material Flow Power Usage
Discrete Event	Shipments
Sparse Data	Inspections
Geospatially Distributed	Effluent Dispersion
Energy-series	Neutron Spectra
Surveillance Images*	Resource tracking Thermal Maps Infrastructure Modifications

Tab. 1: Data Modalities available in CYCLUS (\*in progress) and their fuel cycle applications

now developing the software infrastructure to create synthetic satellite images that may contain signals of diversion. Satellite imagery has a variety of applications: tracking personnel or truck movement patterns, thermal or visible signatures of effluent or heat, or major facility changes such as new or removed buildings.

These diverse datasets can be combined to highlight signatures of diversion that are small enough to be hidden in the noise of individual signals. We have illustrated this technique by combining time-series data for power consumption and declared LEU production for a simple scenario of HEU production in an enrichment facility. More realistic scenarios require advanced anomaly detection techniques such as those being developed at University of Michigan (UM). A collaboration with UM and Sandia National Laboratories (Sandia) will investigate ways to optimize subsets of diverse signal modalities to ensure reliable detection while minimizing resource usage.

The CYCLUS fuel cycle simulator is being used as a framework for combining techniques and knowledge from a variety of disciplines to support a cohesive approach to treaty verification. Moving forward, CYCLUS will be used to study more complex and realistic diversion scenarios. Additionally, CYCLUS has the capability to produce synthetic signals of inherent physical processes such as neutron spectra of various materials. In this way, CYCLUS simulations can provide theoretical signals to researchers developing experimental detectors in order to test sensitivity and detector response. CYCLUS is also being used to explore behavioral

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## References

- [1] C. MOONEY, “Why climate change is forcing some environmentalists to back nuclear power,” *The Washington Post* (2014). <http://www.washingtonpost.com/news/wonkblog/wp/2014/12/16/why-climate-change-is-forcing-some-environmentalists-to-back-nuclear-power/> (accessed 2015-09-17).
- [2] “China plans for nuclear growth,” 2014. <http://www.world-nuclear-news.org/NP-China-plans-for-nuclear-growth-2011144.html> (accessed 2015-09-17).
- [3] H. A. FEIVESON, A. GLASER, Z. MIAN, and F. N. V. V. HIPPEL, *Unmaking the Bomb: A Fissile Material Approach to Nuclear Disarmament and Nonproliferation*, The MIT Press, Cambridge, Massachusetts (2014).
- [4] “Treaty on the Non-Proliferation of Nuclear Weapons (NPT),”. <http://www.un.org/disarmament/WMD/Nuclear/NPT.shtml> (accessed 2015-09-18).

- [5] “The Treaty: CTBTO Preparatory Commission,”. <https://www.ctbto.org/the-treaty/> (*accessed* 2015-10-12).
- [6] D. OF STATE, “New START,” 2010. <http://www.state.gov/t/avc/newstart/index.htm> (*accessed* 2015-10-12).
- [7] “Fissile Material Cut-off Treaty (FMCT) at a Glance | Arms Control Association,” 2013. <https://www.armscontrol.org/factsheets/fmct> (*accessed* 2015-09-18).
- [8] A. GLASER, B. BARAK, and R. J. GOLDSTON, “A zero-knowledge protocol for nuclear warhead verification,” *Nature*, **510**, 497 (2014). <http://www.nature.com/nature/journal/v510/n7506/full/nature13457.html> (*accessed* 2015-09-23).
- [9] K. D. HUFF et al., “Fundamental concepts in the Cyclus nuclear fuel cycle simulation framework,” *Advances in Engineering Software*, **94**, 46 (2016). <http://www.sciencedirect.com/science/article/pii/S0965997816300229> (*accessed* 2016-05-16).
- [10] R. W. CARLSEN et al., “Cyclus v1.0.0,” (2014). <http://dx.doi.org/10.6084/m9.figshare.1041745>.
- [11] R. CARLSEN, R. FLANAGAN, M. GIDDEN, A. SCOPATZ, and J. D. XIA, “Cyclus v1.2.0,” (2014). <http://dx.doi.org/10.6084/m9.figshare.1256424>.
- [12] R. CARLSEN et al., “Cyclus v1.3.0,” (2015). <http://dx.doi.org/10.6084/m9.figshare.1427429>.
- [13] N. R. JENNINGS, “On agent-based software engineering,” *Artificial Intelligence*, **117**, 277 (2000). <http://www.sciencedirect.com/science/article/pii/S0004370299001071> (*accessed* 2015-09-23).
- [14] S. TAYLOR, *Agent-based Modeling and Simulation*, Palgrave Macmillan (2014).
- [15] K. D. HUFF, P. P. WILSON, and M. J. GIDDEN, “Open Architecture and Modular Paradigm of Cyclus, a Fuel Cycle Simulation Code,” *Proc. Transactions of the American Nuclear Society*, volume 104, page 183, 2011.
- [16] M. GIDDEN and P. WILSON, “An Agent-Based Framework for Fuel Cycle Simulation with Recycling,” *Proc. Proceedings of GLOBAL*, Salt Lake City, UT, United States, 2013.
- [17] M. J. GIDDEN, *An Agent-Based Modeling Framework and Application for the Generic Nuclear Fuel Cycle*, PhD thesis, University of Wisconsin, Madison, WI, United States, 2015.
- [18] M. GIDDEN et al., “Agent-Based Dynamic Resource Exchange in Cyclus,” *Proc. Proceedings of PHYSOR*, Kyoto, Japan, 2014.
- [19] A. SCOPATZ, P. K. ROMANO, P. P. WILSON, and K. D. HUFF, “PyNE: Python for Nuclear Engineering,” *Proc. Am. Nuc. Soc. Winter Meeting 2012*, volume 107, San Diego, CA, USA, 2012.
- [20] K. D. HUFF, *An Integrated Used Fuel Disposition and Generic Repository Model for Fuel Cycle Analysis*, PhD thesis, University of Wisconsin - Madison, 2013.
- [21] K. M. OLIVER, *GENIUSV2: SOFTWARE DESIGN AND MATHEMATICAL FORMULATIONS FOR MULTI-REGION DISCRETE NUCLEAR FUEL CYCLE SIMULATION AND ANALYSIS*, Master of Science, Nuclear Engineering, University of Wisconsin, Madison, 2009.