

Vehicle Dynamics and Simulation

Drivetrain Dynamics

Dr B Mason

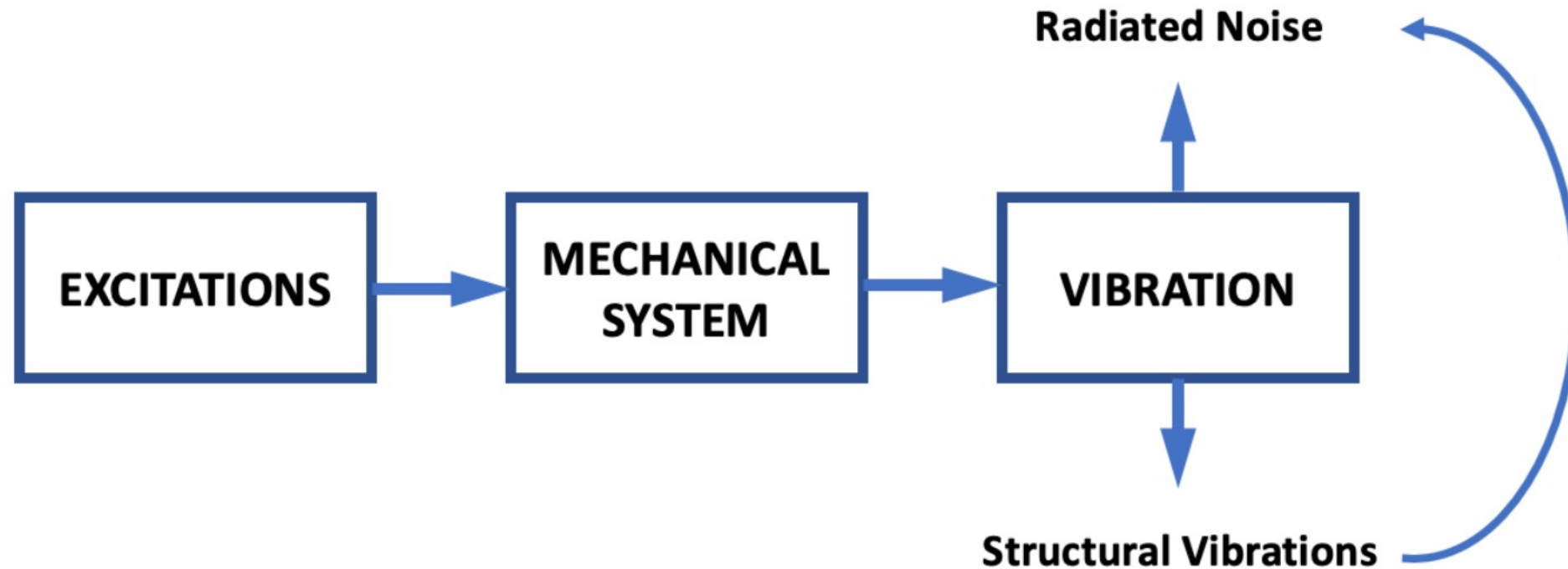
Note

- The test is on Tuesday at 2pm
- Location is SM109 computer lab
- There will be no online session (contact me before 9am on the day if you are unable to attend)
- Main focus;
 - Application of fundamentals to Drivetrain Dynamics
 - Drivetrain dynamic model use.

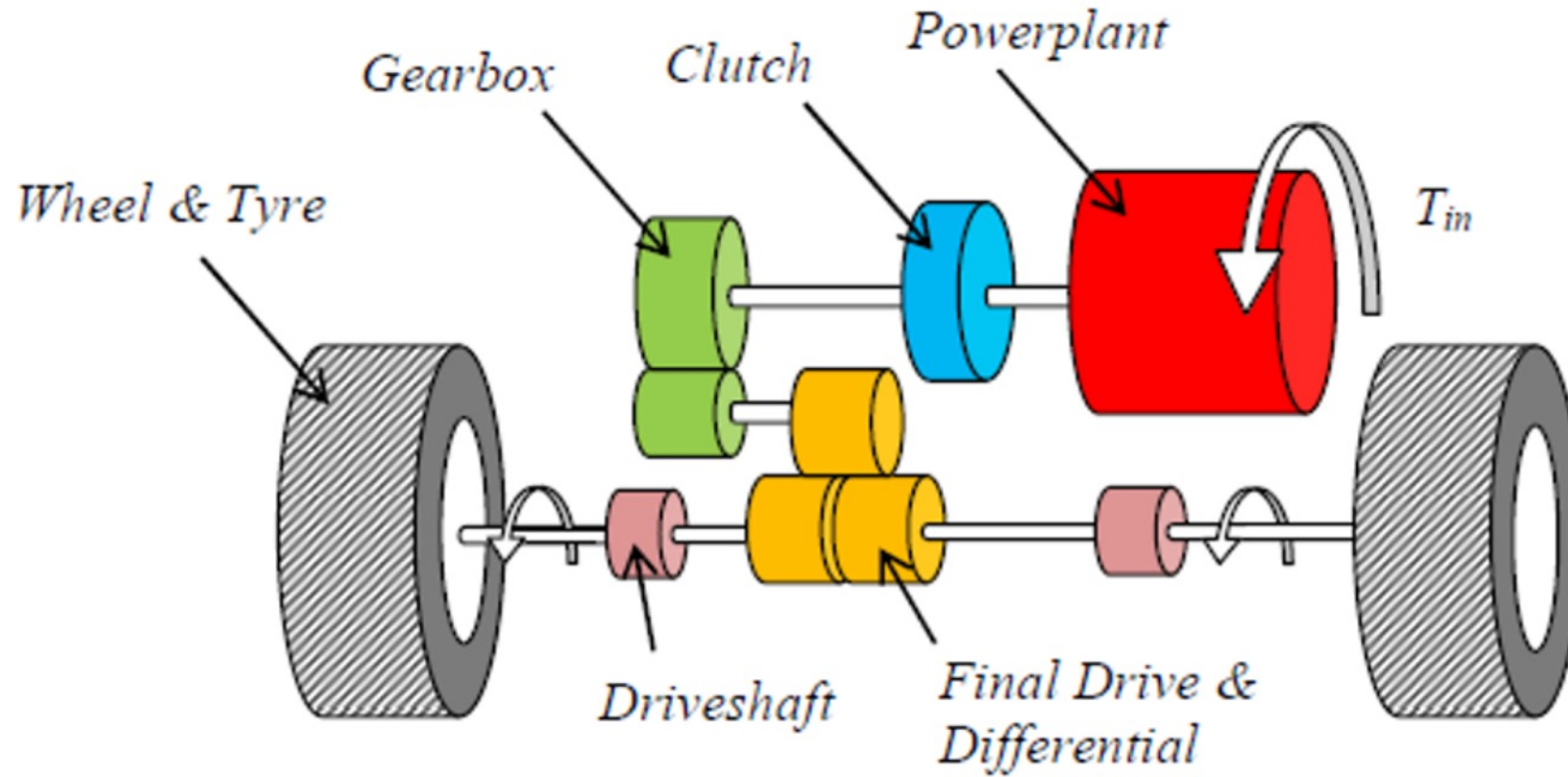
Lecture overview

- Drivetrain as a vibrational system
- Torsional drivetrain model
- Excitation sources
- Driveline Components
- Vibration analysis

The Drivetrain System

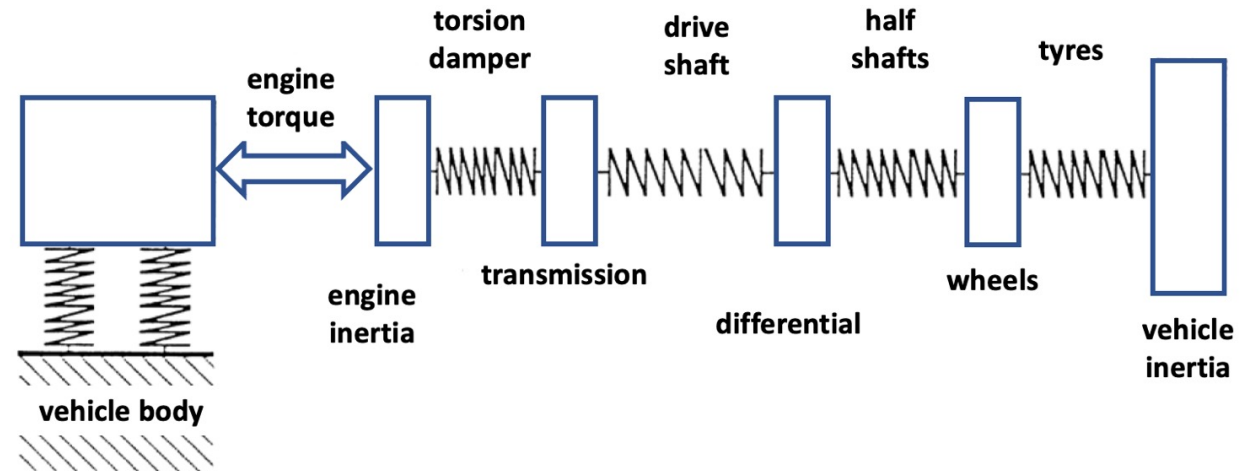


The Drivetrain System



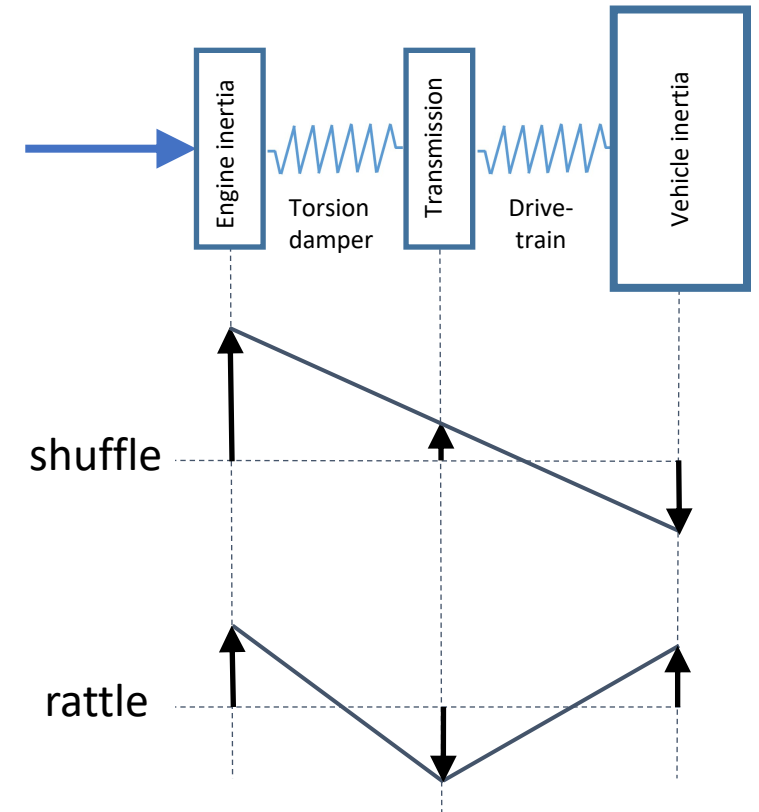
The Drivetrain System

- The drivetrain can be represented as a number of springs and masses
- Each mass is isolated as a point-mass
- These types of models are known as lumped parameter models
- Control of vibration is achieved by;
 - Reducing excitation
 - Changing stiffness and damping (resonance frequencies)

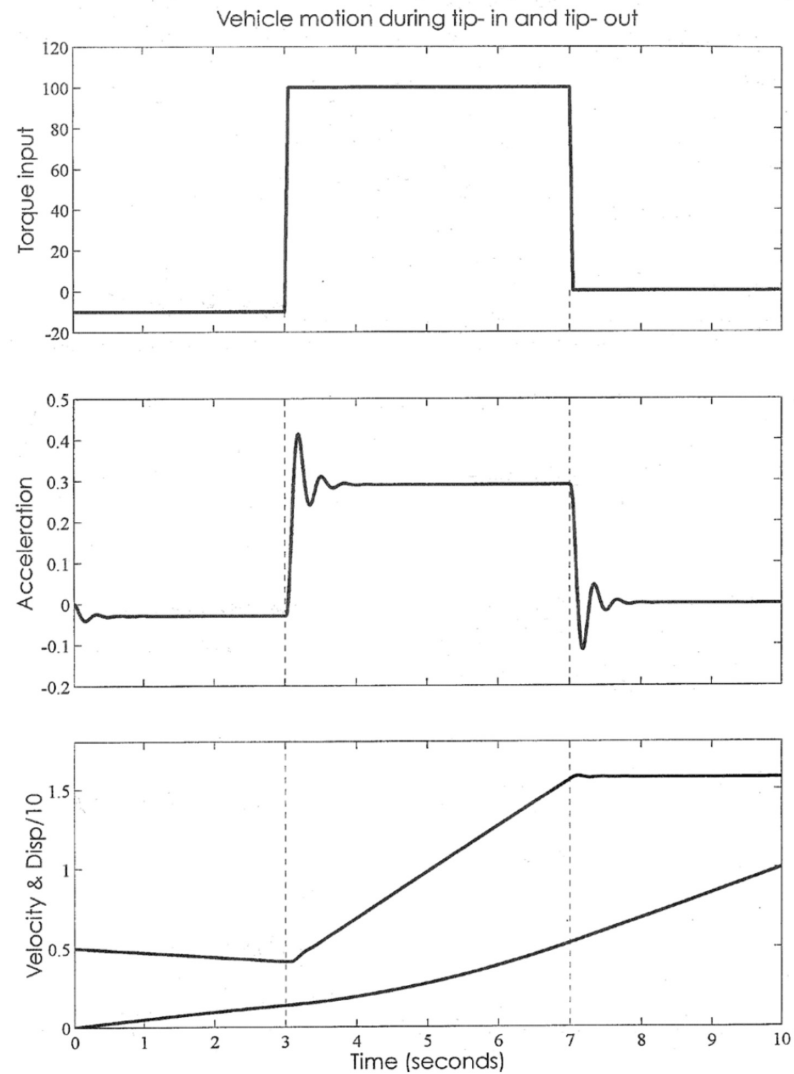


The simplest useful model

- Three main modes;
 - Shuffle (4-12 Hz)
 - Rattle (40 – 80 Hz)
 - Rigid body rotation
- And others;
 - Boom (interior compartment)
 - Judder (low frequency on clutch engagement)
 - Clonk (lash in driveline)

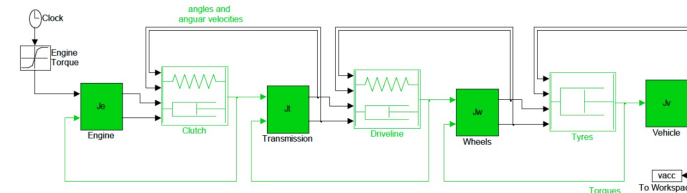
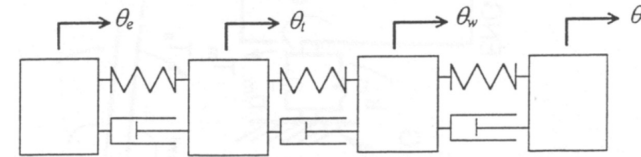
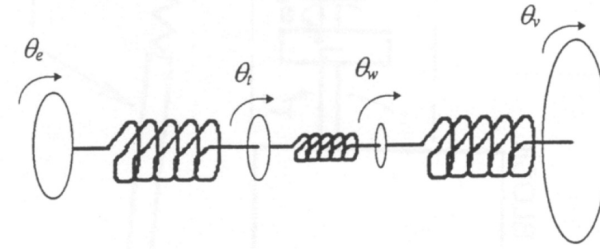


Response to step input

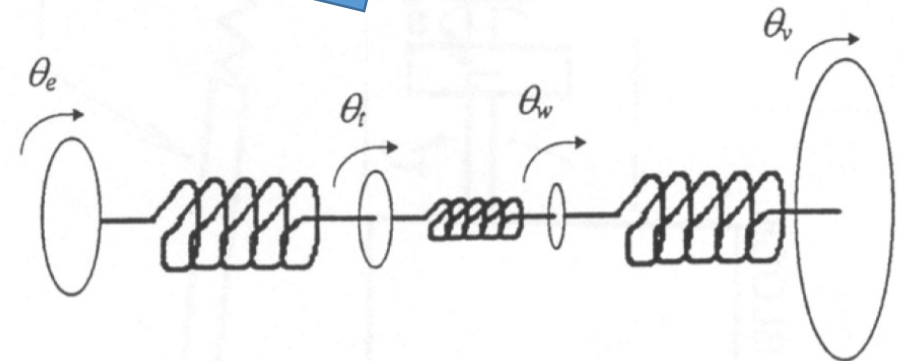
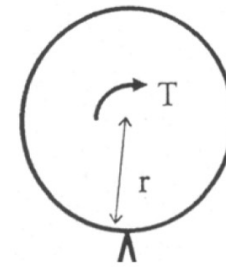
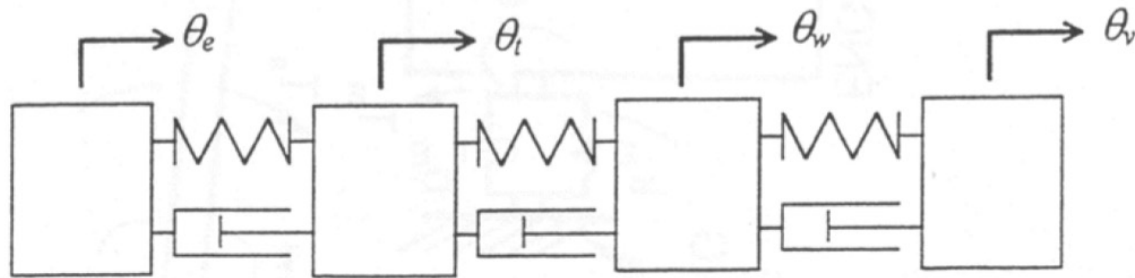
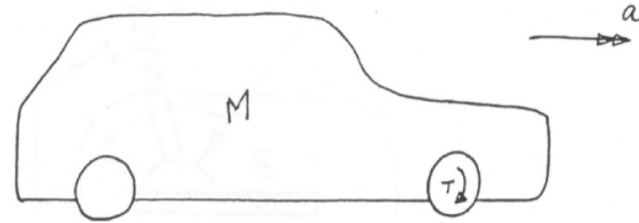


More advanced models

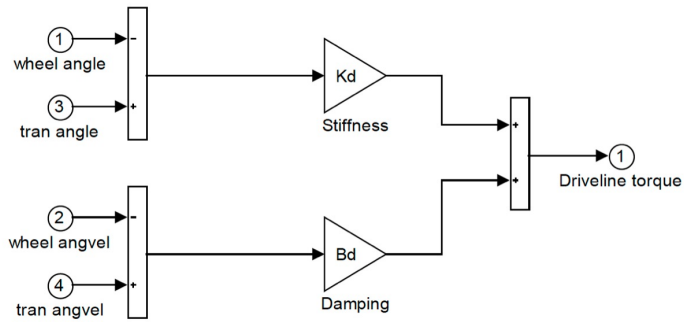
- Three, four, six, ..., twenty! mass models
 - Complexity driven by requirements
- Can add masses to suit
- Other components in the drivetrain are important
 - Clutch
 - Differential
 - Actuators and related controls



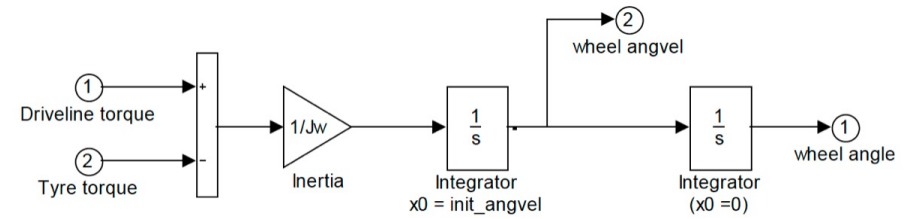
Rotation or Translation?



Standardised (Simulink) components



Spring-damper



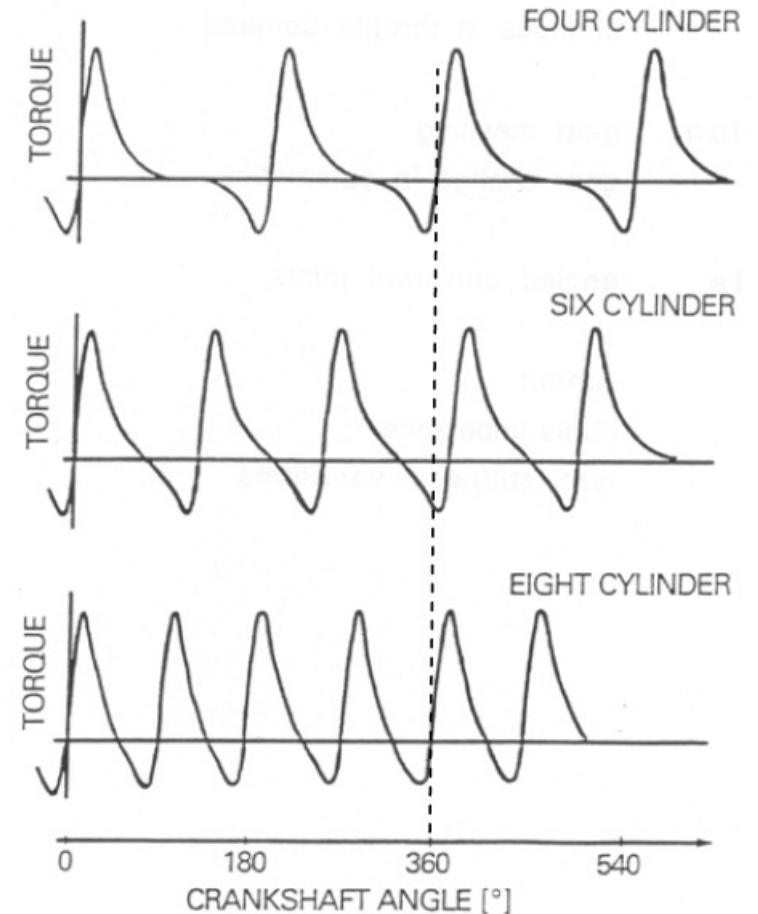
Mass

Sources of excitation

- Main sources of excitation
 - Combustion
 - Step-in and step-out
 - Electric machine
 - HEV transitions
 - Cylinder deactivation
- Transmission
 - Gear meshing
 - Ratio changes
- Driveshaft
 - Universal joints
- Tyres and wheels
 - Runout
 - Mass imbalance
 - Stiffness variations

Excitation - Combustion

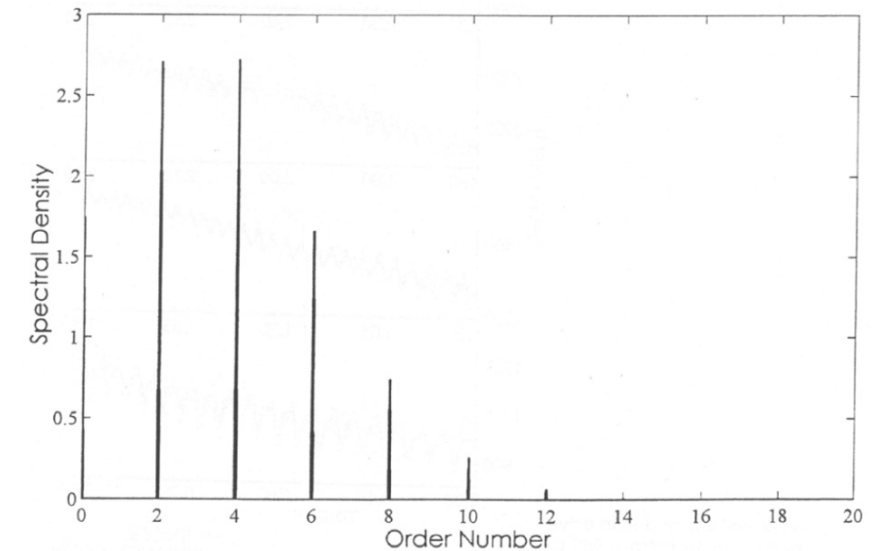
- Firing of engine causes torque pulses
- Varies in frequency
 - Number of cylinders
 - Engine speed
- Other events
 - Valve opening and closing (x1 per rotation)
 - Injector opening and closing (x1 per cycle)



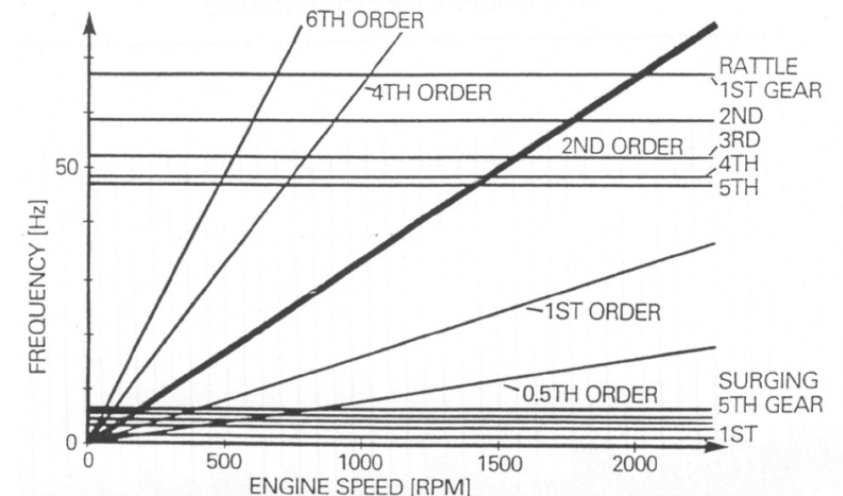
Excitation - Combustion

- Spectral analysis (FFT) to look at frequency content of measured response
- Can normalise spectral analysis result with rotational speed (at fixed speed)
 - Shows how response relates to engine speed
- Can also create interaction / Campbell diagram to show response relative to excitation (engine).

Engine order vibration

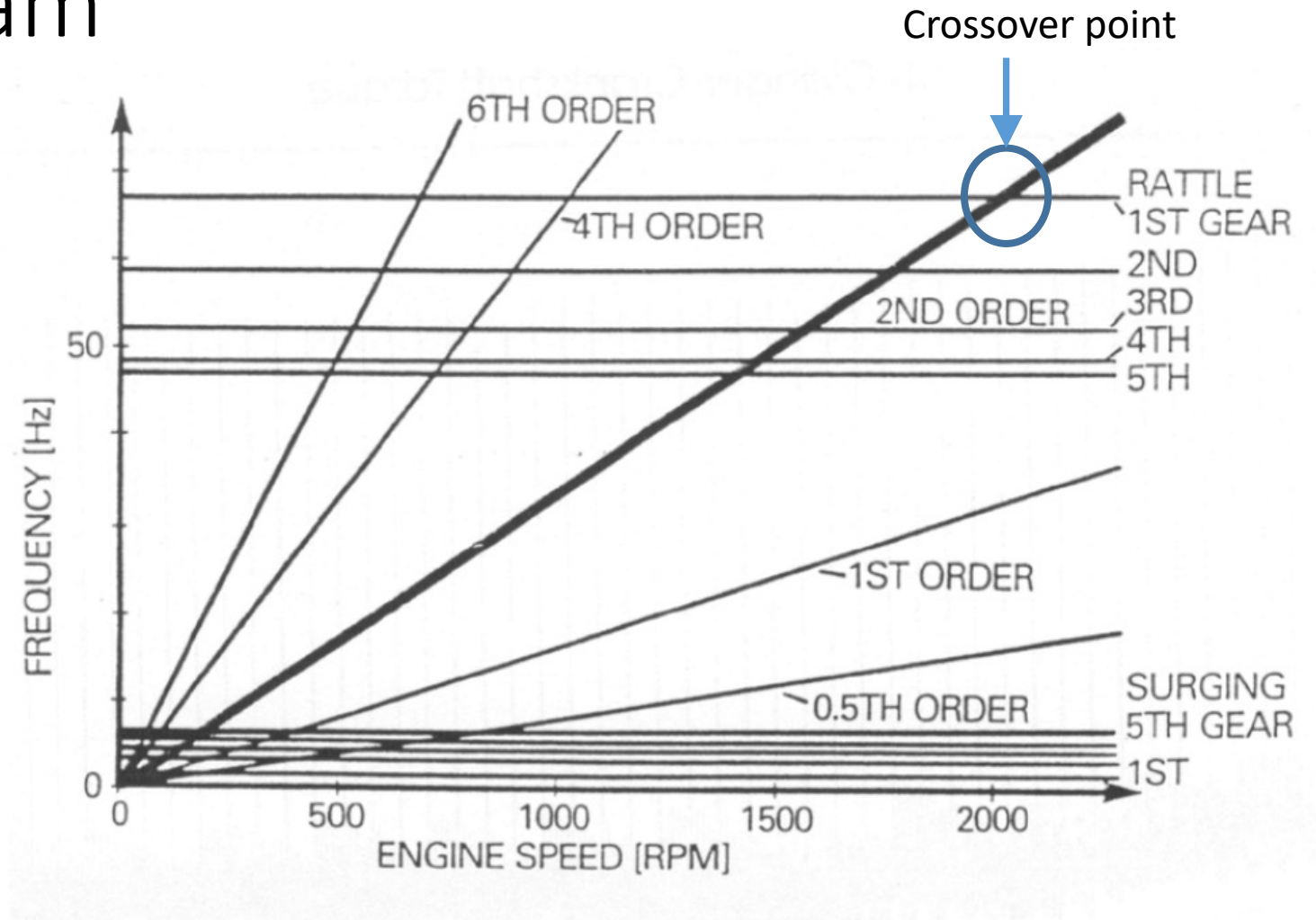


Interaction Plot / Campbell Diagram



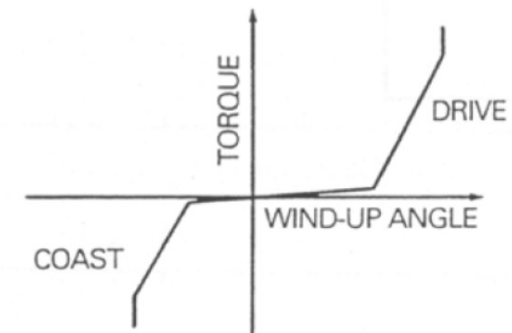
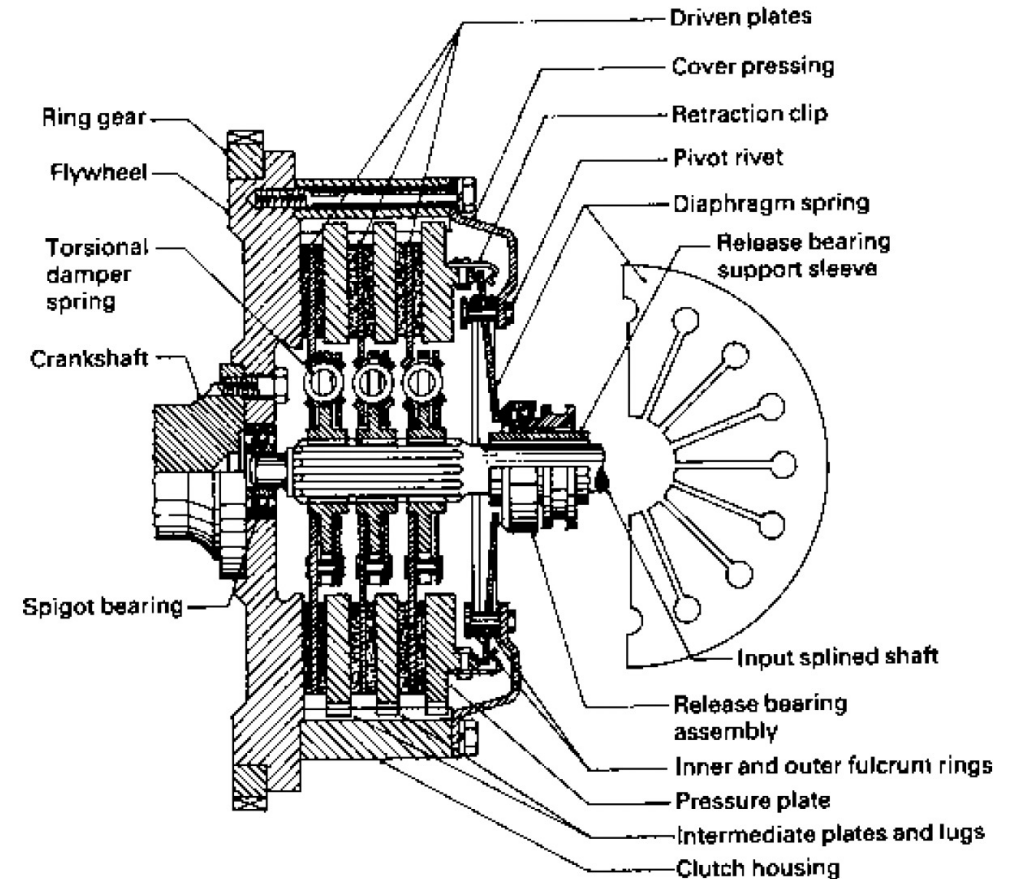
Campbell Diagram

- 1st order is 1:1 mapping e.g. at 1000 rpm = 1000 / 60 cycles per second (Hz).
- Where the 'order' crosses the 'resonance' line is the point of max vibration.



Torsion Damper

- Used as first line of defence against excitation in the driveline.
- Different physical arrangements.
- Nonlinear spring rate with some damping.



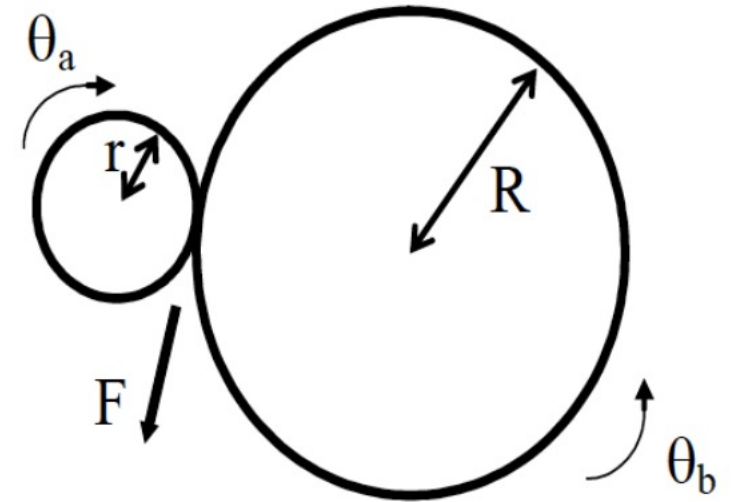
Gearbox

- Can be modelled as a single pair (and 'switched').
- Output torque is calculated (T_a is input torque);

$$T_b = GT_a$$

- With the gear ratio, G given by;

$$G = R/r$$



Simple Drivetrain Model

- Three inertias
- Compliantly connected by rotational springs
- Engine inertia accel;

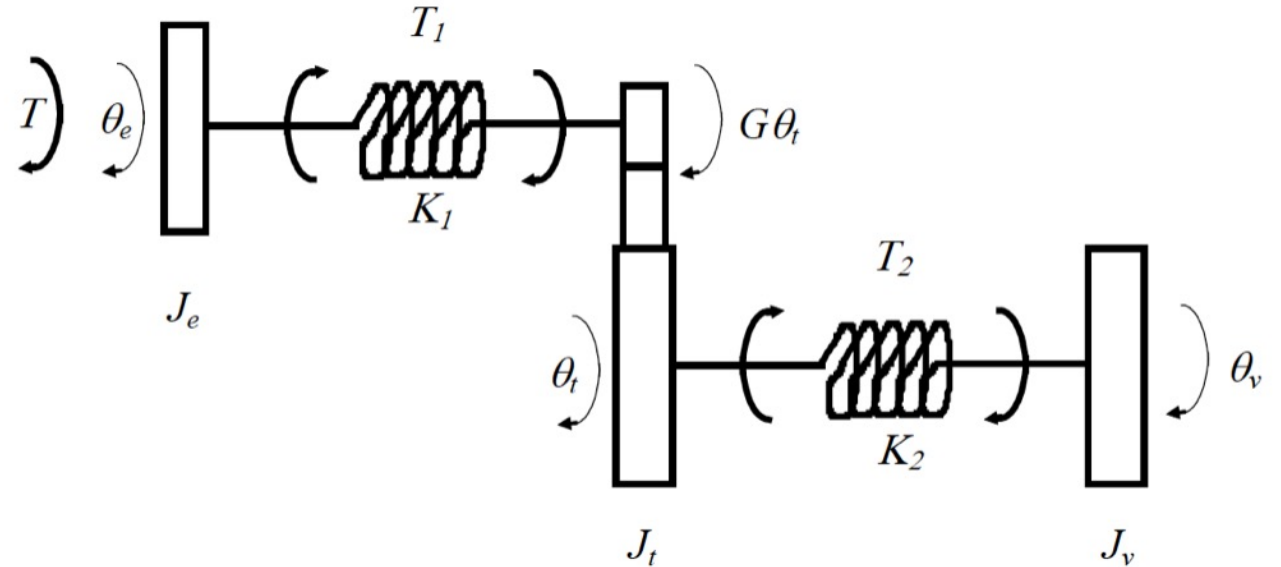
$$J_e \ddot{\theta}_e = T - T_1$$

- Transmission inertia accel;

$$J_t \ddot{\theta}_t = GT_1 - T_2$$

- Vehicle inertia accel;

$$J_v \ddot{\theta}_v = T_2$$



Simple Drivetrain Model

- Torque transmitted between engine and transmission;

$$T_1 = K_1(\theta_e - G\theta_t)$$

- Torque transmitted between transmission and vehicle;

$$T_2 = K_2(\theta_t - \theta_v)$$

- So that;

$$J_e \ddot{\theta}_e = T - K_1 (\theta_e - G\theta_t)$$

$$J_t \ddot{\theta}_t = GK_1 (\theta_e - G\theta_t) - K_2 (\theta_t - \theta_v)$$

$$J_v \ddot{\theta}_v = K_2 (\theta_t - \theta_v)$$

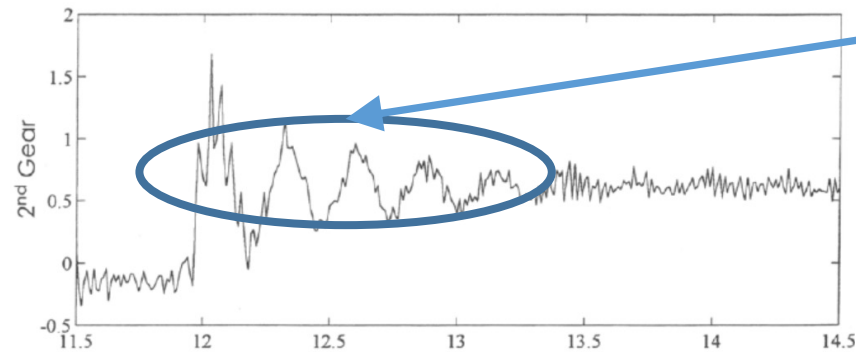
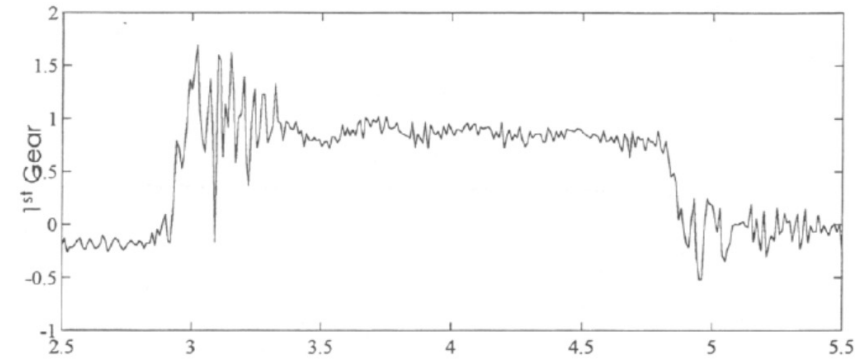
- A less messy formulation is (see notes);

$$J_e^* \ddot{\theta}_e^* = GT - K_1^* (\theta_e^* - \theta_t)$$

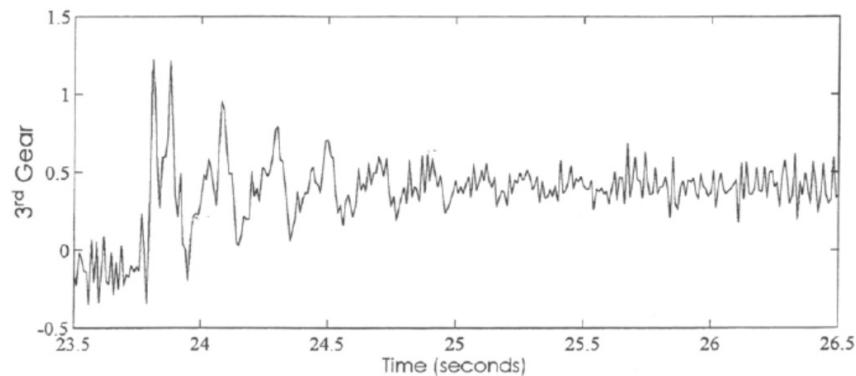
$$J_t \ddot{\theta}_t = K_1 (\theta_e^* - \theta_t) - K_2 (\theta_t - \theta_v)$$

$$J_v \ddot{\theta}_v = K_2 (\theta_t - \theta_v)$$

Measure Response (Mondeo 1.8l)

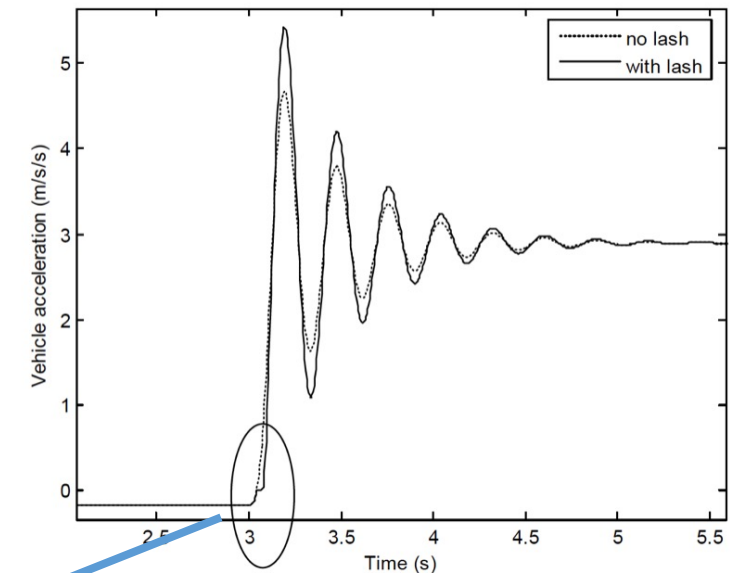
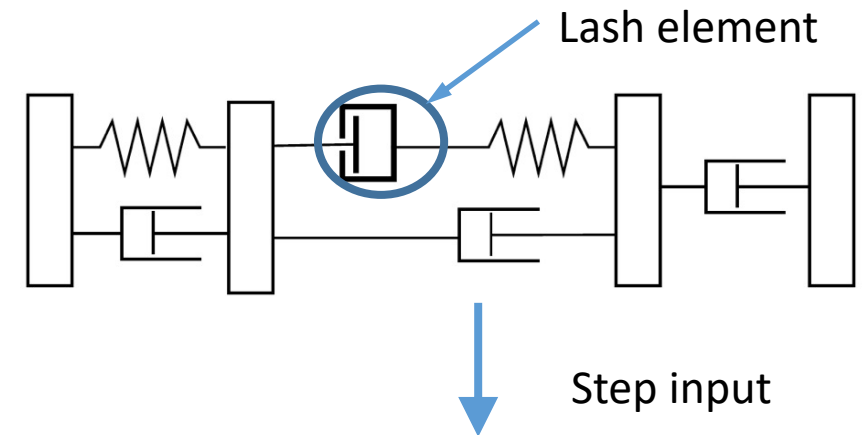
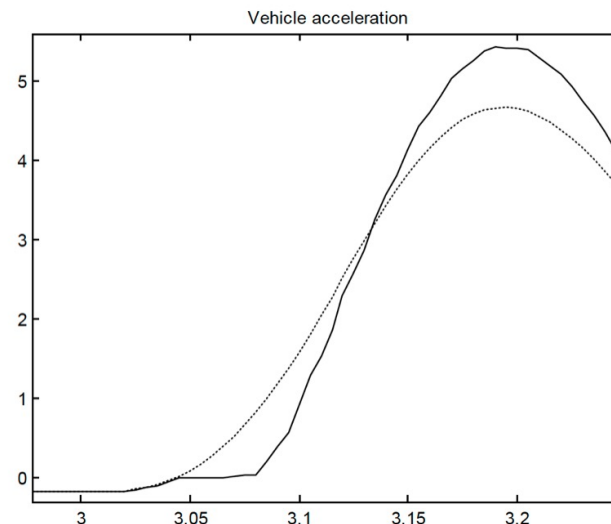
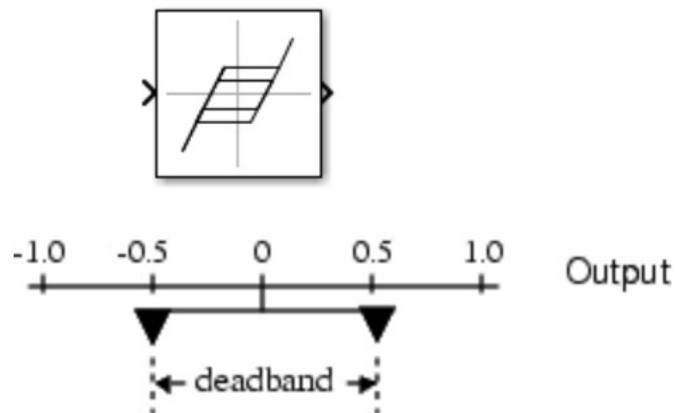


Shuffle



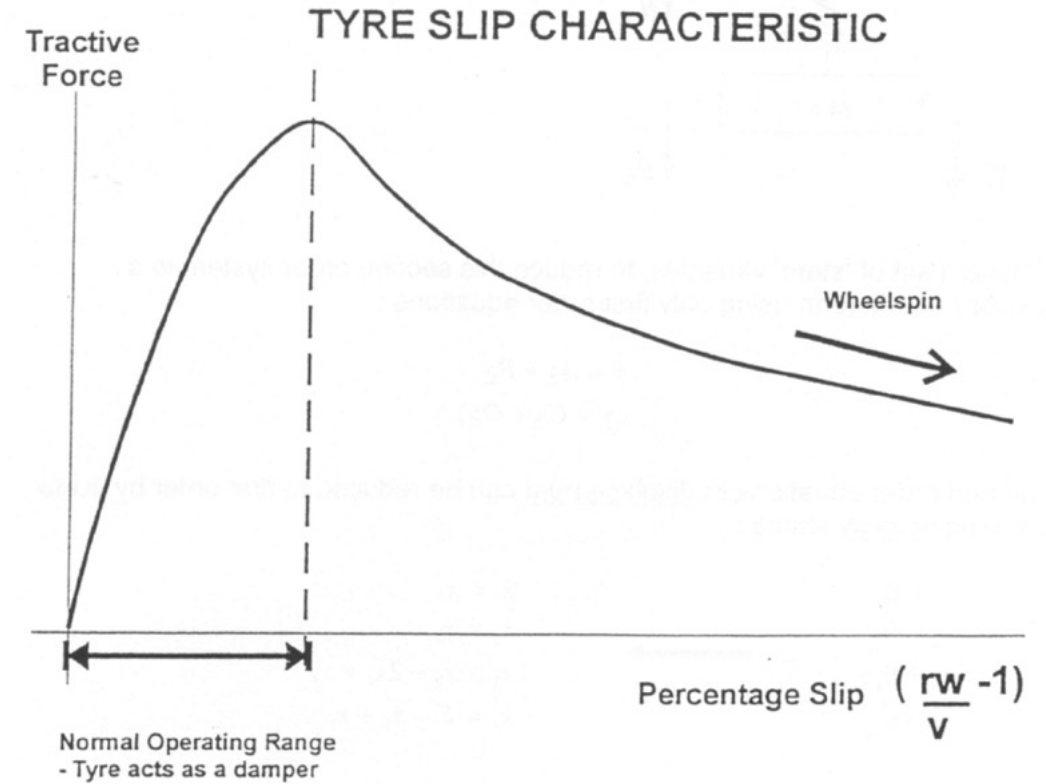
Driveline Nonlinearities

- There are many sources of nonlinearity.
- Typically include 'lash' as most significant.
- Lash element available in Simulink



A Simple Tyre Model

- Longitudinal force in acceleration is generated through contact patch.
- The amount of force generated is proportional to 'slip' i.e. velocity difference between tyre (translation) and vehicle (translation).
- Sharply rises, reaches a peak then falls. Stiction vs Viscous Friction.



Vibration Analysis

Create state matrix

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{F}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

Substitute state variables

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= x_2 - 2x_1 - x_3 \\ \dot{x}_4 &= F - x_2 + x_1 \end{aligned}$$

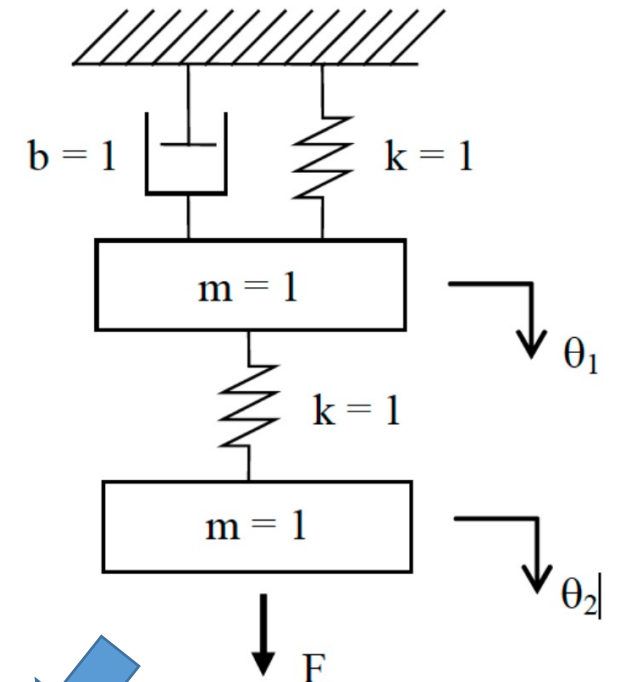
Define states

$$\begin{aligned} x_1 &= \theta_1 \\ x_2 &= \theta_2 \\ x_3 &= \dot{\theta}_1 \\ x_4 &= \dot{\theta}_2 \end{aligned}$$

Create output matrix

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

Draw schematic



Determine system equations

$$\begin{aligned} F - l(\theta_2 - \theta_1) &= l\ddot{\theta}_2 \\ l(\theta_2 - \theta_1) - l\theta_1 - l\dot{\theta}_1 &= l\ddot{\theta}_1 \end{aligned}$$

Vibration Analysis – Modal analysis

- Remember that;

$$\theta(t) = \text{Re} \{ \mathbf{u}_1 e^{\lambda_1 t} + \mathbf{u}_2 e^{\lambda_2 t} + \dots + \mathbf{u}_n e^{\lambda_n t} \}$$

- Where each term is;

$$\mathbf{u}_1 e^{\lambda_1 t} = \mathbf{u}_1 e^{(\sigma + j\omega)t} = \mathbf{u}_1 e^{\sigma t} e^{j\omega t} = \mathbf{u}_1 e^{\sigma t} (\cos(\omega t) + i \sin(\omega t))$$

- For a single component;

$$\theta(t) = \mathbf{u}_1 e^{\lambda_1 t} \longrightarrow \dot{\theta}(t) = \lambda_1 \mathbf{u}_1 e^{\lambda_1 t}$$

- So that;

$$\mathbf{x}(t) = \mathbf{v}_1 e^{\lambda_1 t} \quad \text{where;} \quad \mathbf{v}_1 = \begin{bmatrix} \mathbf{u}_1 \\ \lambda_1 \mathbf{u}_1 \end{bmatrix}$$

Vibration Analysis – Modal analysis

- For free vibration;

$$\lambda_1 \mathbf{v}_1 e^{\lambda_1 t} = A \mathbf{v}_1 e^{\lambda_1 t}$$

- So that;

$$\lambda_1 \mathbf{v}_1 = A \mathbf{v}_1$$

- Which means that λ_1 is a eigenvalue of A and \mathbf{v}_1 is the corresponding eigenvector.

- For the spring-mass-damper system previously (given parameter values);

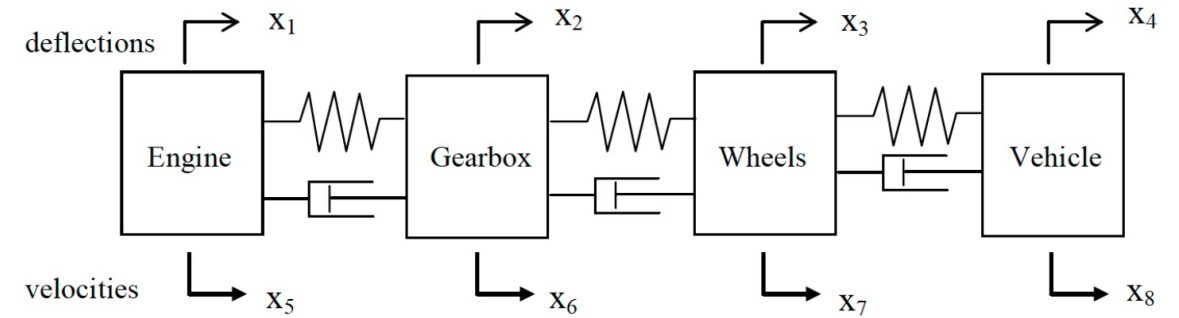
$$\lambda_1 = -0.35 + 1.5j$$

$$\lambda_2 = -0.35 - 1.5j$$

$$\lambda_3 = -0.15 + 0.63j$$

$$\lambda_4 = -0.15 - 0.63j$$

Eigenstructure – Four mass driveline model



	$\lambda_1 =$ 0	$\lambda_2 =$ -779.5	$\lambda_3 =$ -86.9+366.4i	$\lambda_4 =$ -86.9-366.4i
x1	0	0.0000	-0.0000 - 0.0001i	-0.0000 + 0.0001i
x2	0	-0.0000	0.0006 + 0.0026i	0.0006 - 0.0026i
x3	0	-0.0013	0.0000 - 0.0000i	0.0000 + 0.0000i
x4	1.000	0.0000	-0.0000 - 0.0000i	-0.0000 + 0.0000i
x5	0	-0.0000	0.0269 + 0.0007i	0.0269 - 0.0007i
x6	0	0.0129	-0.9996	-0.9996
x7	0	0.9998	0.0059 + 0.0116i	0.0059 - 0.0116i
x8	0	-0.0150	0.0003 - 0.0003i	0.0003 + 0.0003i

	$\lambda_5 =$ -2.40 + 22.26i	$\lambda_6 =$ -2.40 - 22.26i	$\lambda_7 =$ -1.4676e-006	$\lambda_8 =$ 1.4676e-006
x1	-0.0035 - 0.0329i	-0.0035 - 0.0329i	0.5000	-0.5000
x2	-0.0039 - 0.0284i	-0.0039 - 0.0284i	0.5000	-0.5000
x3	-0.0068 + 0.0040i	-0.0068 + 0.0040i	0.5000	-0.5000
x4	0.0005 + 0.0038i	0.0005 + 0.0038i	0.5000	-0.5000
x5	0.7403	0.7403	-0.0000	-0.0000
x6	0.6412 - 0.0186i	0.6412 - 0.0186i	-0.0000	-0.0000
x7	-0.0720 - 0.1618i	-0.0720 - 0.1618i	-0.0000	-0.0000
x8	-0.0848 + 0.0025i	-0.0848 + 0.0025i	-0.0000	-0.0000

...and as if by magic!

```
[A,B,C,D] = linmod('drive4');
[V,D] = eig(A);
```

Eigenstructure – Four mass driveline model

- Normalising (as in previous example)
 - Shuffle;

$$\begin{array}{llll} 0.7403 & / & 0.7403 & = & 1.0000 \\ 0.6412 - 0.0186i & / & 0.7403 & = & 0.8661 - 0.0251i \\ -0.0720 - 0.1618i & / & 0.7403 & = & -0.0972 - 0.2185i \\ -0.0848 + 0.0025i & / & 0.7403 & = & -0.1146 + 0.0034i \end{array}$$

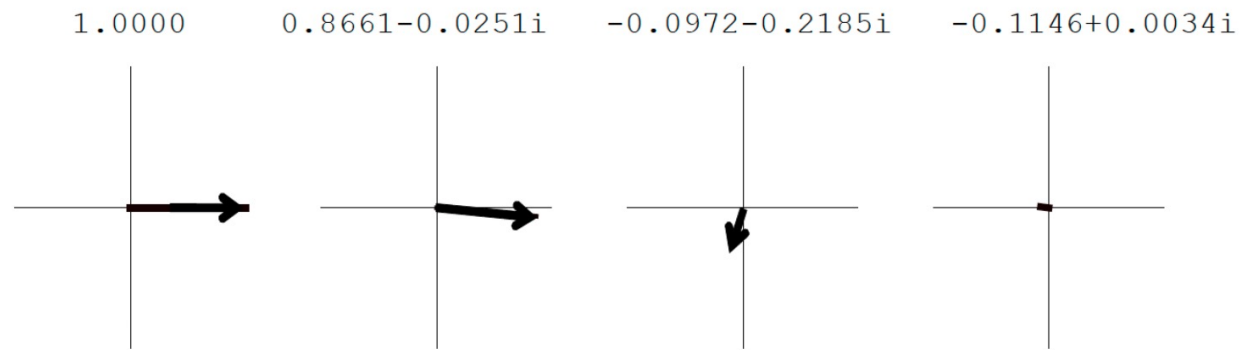
- Rattle:

$$\begin{array}{llll} 0.0269 + 0.0007i & / & -0.9996 & = & -0.0269 - 0.0007i \\ -0.9996 & / & -0.9996 & = & 1.0000 \\ 0.0059 + 0.0116i & / & -0.9996 & = & -0.0059 - 0.0116i \\ 0.0003 - 0.0003i & / & -0.9996 & = & -0.0003 + 0.0003i \end{array}$$

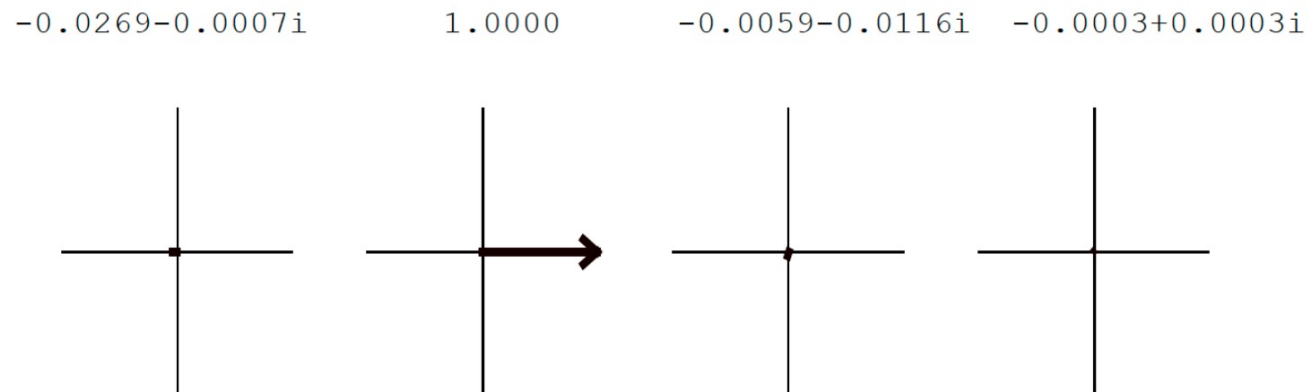
Eigenstructure – Four mass driveline model

- And plotting

- Shuffle;

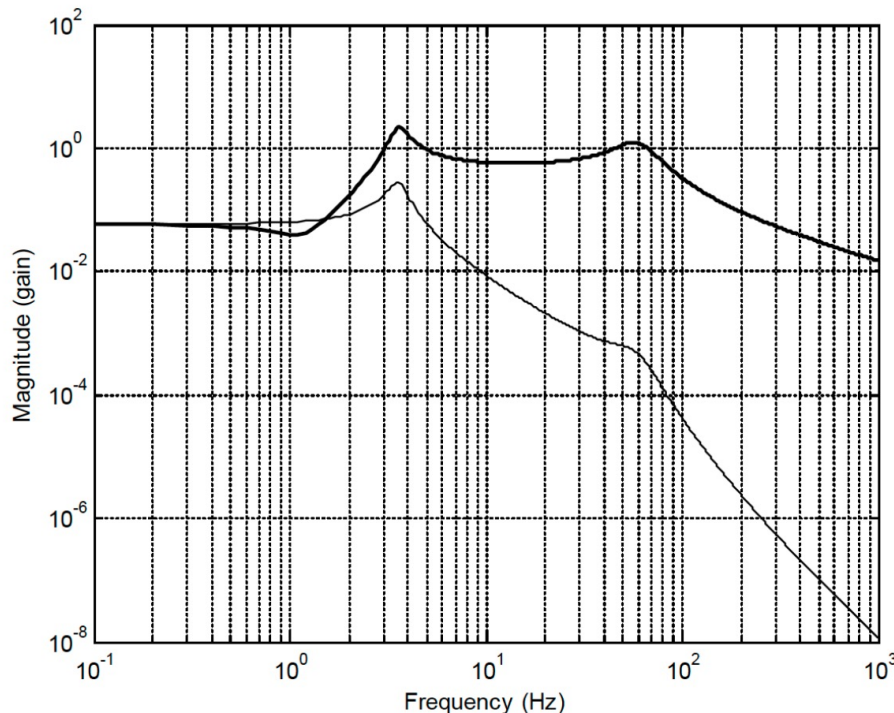


- Rattle;



Eigenstructure – Four mass driveline model

- Bode plot to show frequency response between input torque, vehicle and transmission acceleration.



```
[A,B,C,D] = linmod('drive4');  
sys = ss(A,B,C,D);  
f = [0.1:0.1:1000]';  
[mag,phase] = bode(sys,f*2*pi);  
mag = squeeze(mag)';  
loglog(f,mag);  
grid on;
```