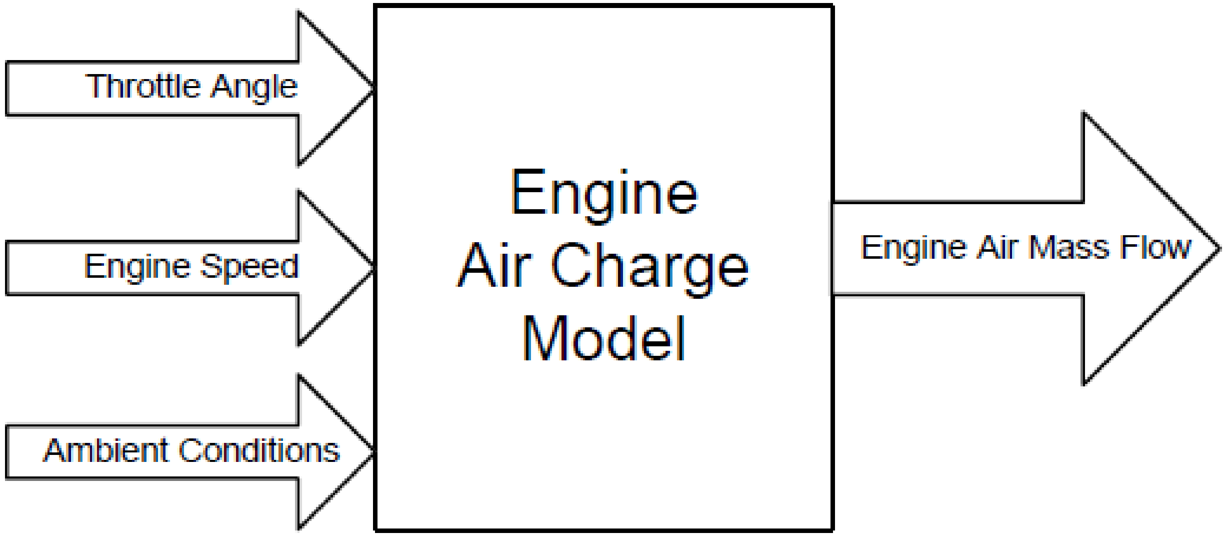
Vehicle Dynamics and Simulation

Dr B Mason

Loughborough University

Mean value model creation

- System representation (naturally aspirated/gasoline)



$$\dot{m} \propto torque$$

Note: Boosted engines will also have wastegate position / duty cycle

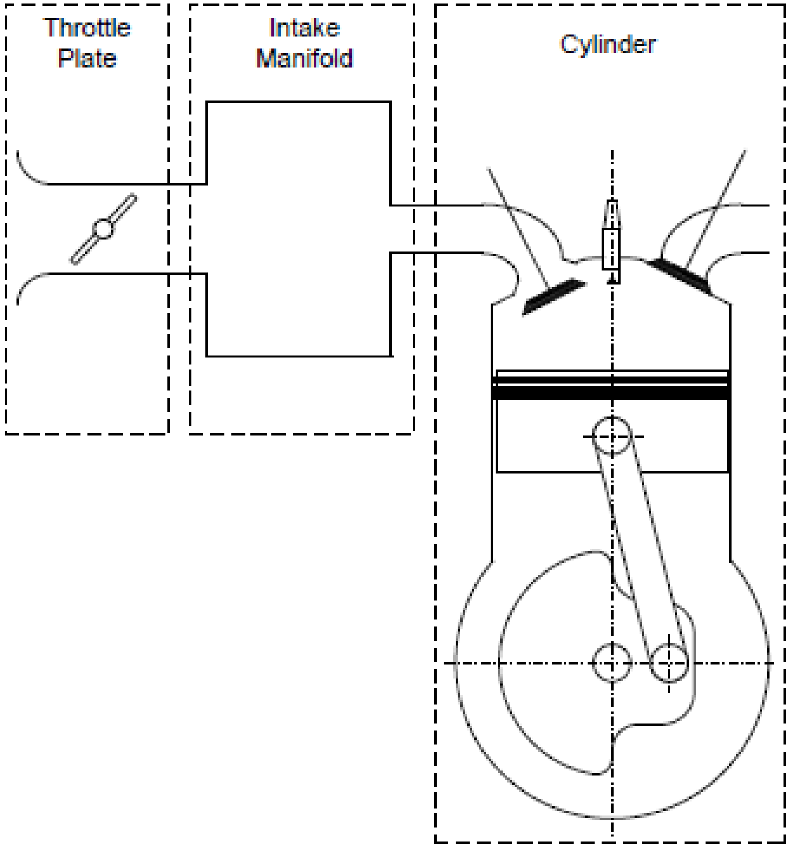
Mean value engine model

- Origin of air flow is induction into cylinder.
- Airflow is throttled.
- Volumetric flow is given;

$$\dot{V} = V_{disp}\frac{N_{eng}}{120}$$

- Mass flow (speed density equation);

$$\dot{m} = \frac{P_{man}}{RT_{man}}\frac{V_{disp}}{120}N_{eng}$$



Easy so far? Things get worse from here!

Mean value engine model – volumetric efficiency

- Volumetric efficiency, η

$$\dot{m} = \eta \frac{P_{man}}{RT_{man}} \frac{V_{disp}}{120} N_{eng}$$

$0.5 < \eta < 1.2$

Max is ≈ 1 for Naturally aspirated

- Modifies the speed density equation
- Depends on;
  - Intake and exhaust geometry
  - Intake and exhaust manifold pressure
  - Engine speed
  - Valve timing
  - Acoustic and inertial air effects
  - etc
- Perhaps the most important parameter in all of the mean value models!

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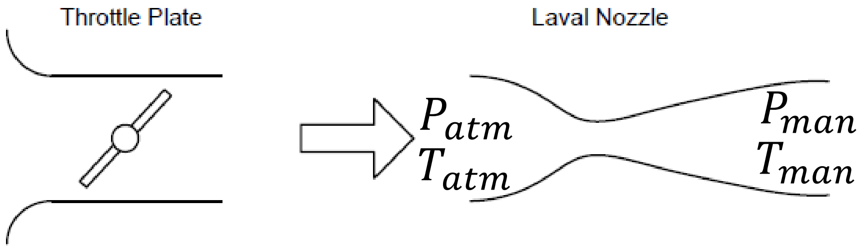
Throttle

- Can be modelled as Laval nozzle of variable throat area (projected cross sectional area).
- For $\frac{P_{man}}{P_{atm}} > 0.528$ mass flow depends on P_{atm} and P_{man};

$$\dot{m} = \frac{C_d A_{th} P_{atm}}{\sqrt{RT_{atm}}} \left(\frac{P_{man}}{P_{atm}}\right)^{\frac{1}{\gamma}} \left\{\frac{2\gamma}{\gamma - 1}\left[1 - \left(\frac{P_{man}}{P_{atm}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}}$$

- For $\frac{P_{man}}{P_{atm}} \leq 0.528$ flow depends on P_{in} alone (sonic/choked flow);

$$\dot{m} = \frac{C_d A_{th} P_{atm}}{\sqrt{RT_{atm}}} \sqrt{\gamma} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$



- At max throat flow velocity;

$$\frac{P_{man}}{P_{atm}} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

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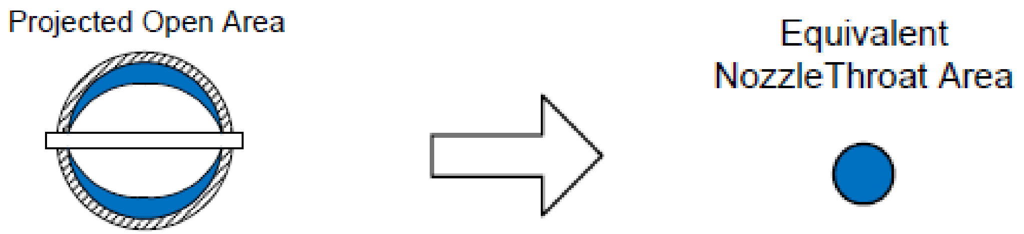
Throttle



- Throttle effective area

$$A_{th} = \frac{\pi D^2}{4}\left\{\left(1 - \frac{\cos\psi}{\cos\psi_0}\right) + \frac{2}{\pi}\left[\frac{a}{\cos\psi}\left(\cos^2\psi - a^2\cos^2\psi_0\right)^{\frac{1}{2}} - \frac{\cos\psi}{\cos\psi_0}\sin^{-1}\left(\frac{a\cos\psi_0}{\cos\psi}\right) - a\left(1-a^2\right)^{\frac{1}{2}} + \sin^{-1}a\right]\right\}$$

- Where

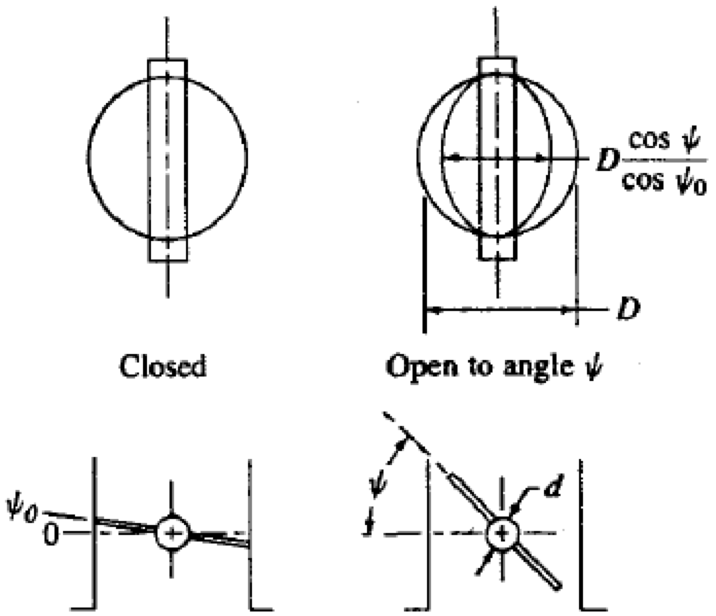
$$a = \frac{d}{D}$$

- Throttle max occurs when

$$\psi = \cos^{-1}(a\cos\psi_0)$$

- So that at ψ_{max}

$$A_{th} \approx \frac{\pi D^2}{4} - dD$$



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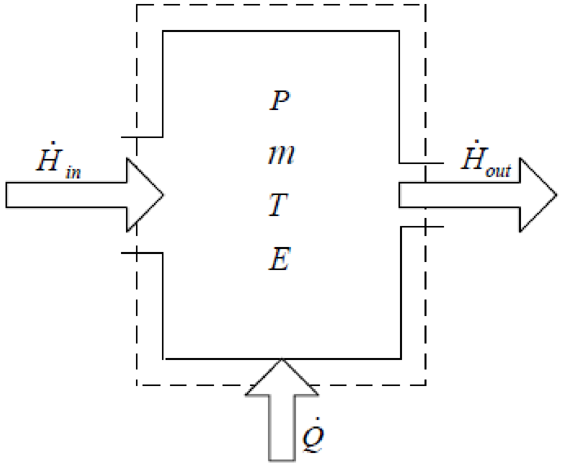
Throttle

- Model is valid for frictionless, adiabatic flow through smoothly convergent-divergent nozzle only!
- Discharge coefficient, C_d is used to 'correct' for reality i.e.
- C_d is not constant it depends on;
  - Throttle position, α
  - Throttle pressure ratio, $\frac{P}{P_{amb}}$
- In reality this tends to be mapped for a specific throttle using a flow bench

Intake manifold



- Can be represented as open system of constant volume.
- System stores mass and energy, represented by state variables P and T.
- Mass balance;

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \tag{1}$$

- Energy balance;

$$\frac{dE}{dt} = \dot{m}_{in} h_{0_{in}} - \dot{m}_{out} h_{0_{out}} + \dot{Q} \tag{2}$$

- Where;

$$h_0 = C_p T + \frac{u^2}{2}$$

And the energy within the volume is;

$$E = mc_v T + \frac{mu^2}{2} + mgz$$

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Intake manifold

- Making some assumptions
  - GPE change is 0
  - KE change is 0
- So that;

$$E = mc_vT$$

$$h_0 = c_pT \qquad (3)$$

- Taking the derivative of $E = mc_vT$ wrt to t;

$$\frac{dE}{dt} = c_vT\frac{dm}{dt} + c_vm\frac{dT}{dt} \qquad (4)$$

- And by substitution (1, 3, 4 into 2);

$$c_vT(\dot{m}_{in} - \dot{m}_{out}) + c_vm\frac{dT}{dt} = \dot{m}_{in}C_pT_{in} - \dot{m}_{out}c_pT_{out} + \dot{Q} \qquad (5)$$

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Intake manifold

- Coupling the energy and mass balances using the ideal gas law;

$$m = \frac{PV}{RT} \qquad (6)$$

- Taking the derivative;

$$\frac{dm}{dt} = \frac{V}{RT}\frac{dP}{dt} - \frac{PV}{RT^2}\frac{dT}{dt} \qquad (7)$$

- Substituting (1, 6 and 7 into 5) and assuming $T_{out} = T$;

$$\boxed{\frac{dT}{dt} = \left[c_p \dot{m}_{in} T_{in} - c_p \dot{m}_{out} T - c_v T \dot{m}_{in} + \frac{dQ}{dt}\right]\frac{RT}{c_v PV}} \qquad (8)$$

$$\boxed{\frac{dP}{dt} = \left[c_p \dot{m}_{in} T_{in} - c_p \dot{m}_{out} T + \frac{dQ}{dt}\right]\frac{R}{c_v V}} \qquad (9)$$

And;

$$\frac{dQ}{dt} = hA_{wall}(T_{wall} - T) \qquad (7)$$

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Torque model

- Torque produced is a function of;
  - Spark advance
  - Inducted air mass flow
  - AFR
- Data is usually obtained experimentally and incorporated within a regression model.
- Friction torque is deducted (imep – bmep) to establish output torque.
- Fmep is calculated;

$$fmep = 0.97 + 0.15\left(\frac{N}{1000}\right) + 0.05\left(\frac{N}{1000}\right)^2$$

- And;

$$T_f = \frac{fmepV_{sw}}{4\pi} = \frac{0.97 + 0.15\left(\frac{N}{1000}\right) + 0.05\left(\frac{N}{1000}\right)^2 V_{sw}}{4\pi}$$

Parameterisation effort

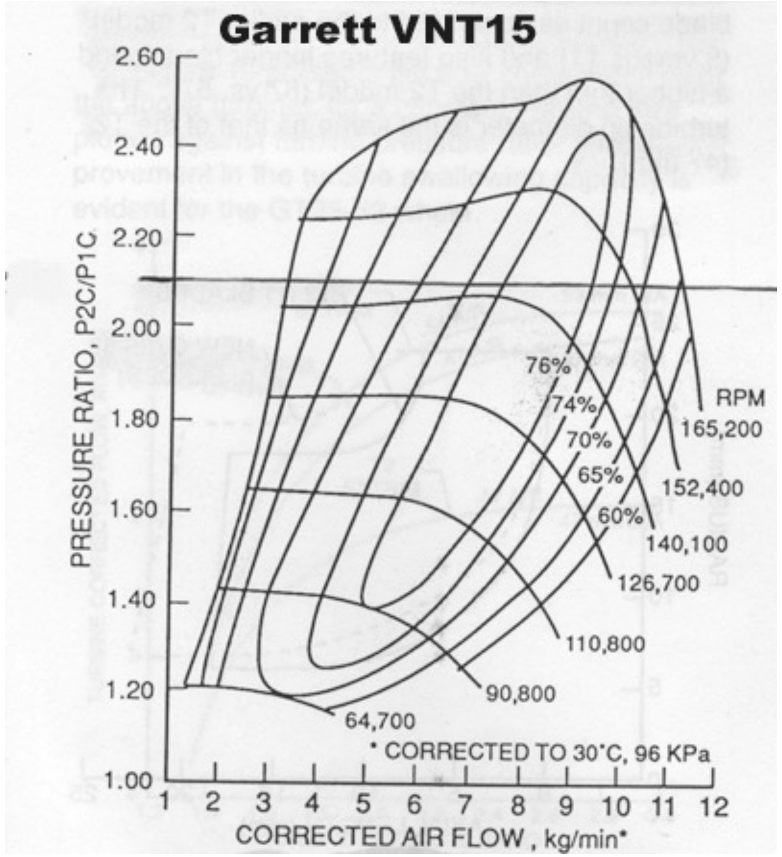
- Model has 5 unknown parameters, C_d, η_{vol}, h, V and V_{disp}.
- With $\eta_{vol} = f(P, T, N, IVO, EVC)$
- η_{vol} is obtained by experiment at some P, T, N, IVO, EVC. Recall;

$$\eta = \frac{120\dot{m}_{actual}}{\rho V_{disp} N_{eng}}$$

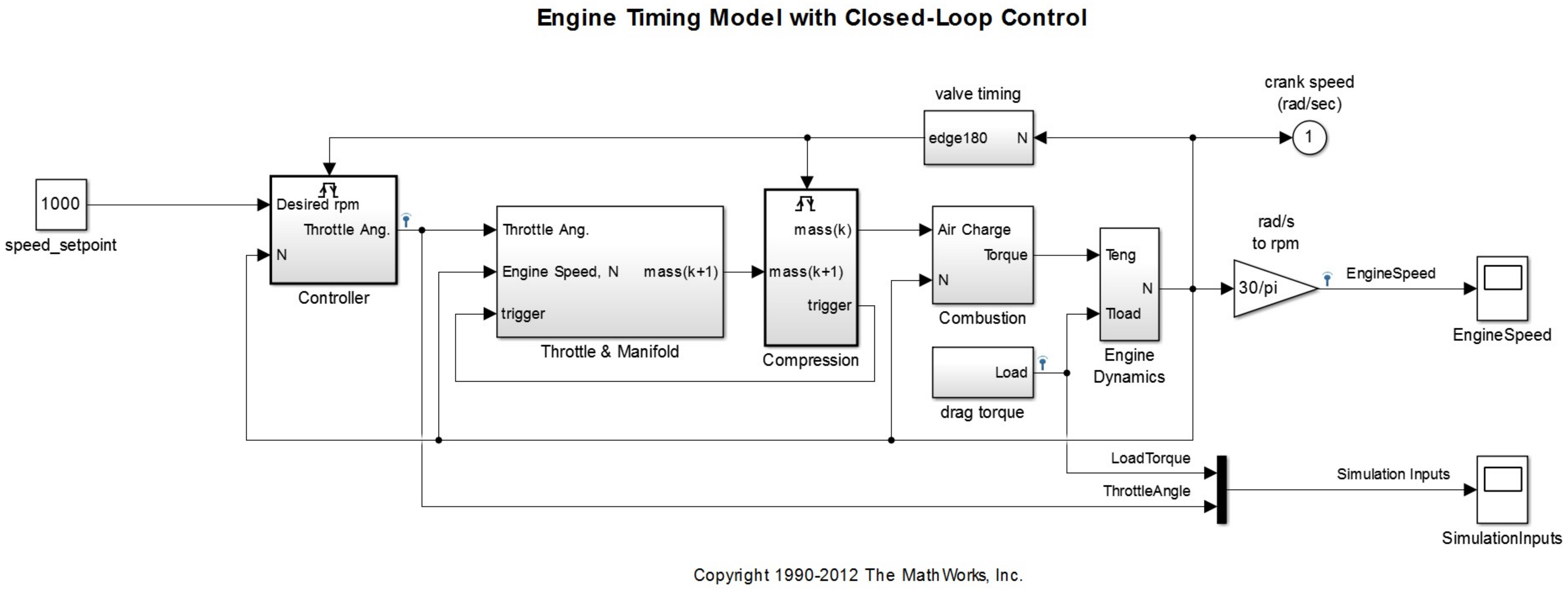
- C_d is also experimentally obtained (usually on flow rigs)
- Obtaining h in reality is very difficult and this is normally one of the tuned parameters.
- V and V_{disp} are obtained relatively easily but can also be used to tune the model response to match reality.

Other considerations

- Adding a turbocharger complicates matters significantly and introduces a causality loop.
  - The loop is normally broken by a delay.
- Heat transfer from the exhaust manifold has a significant effect on the turbo performance.
- Errors in the ‘turbo loop’ are accumulated within the loop.
- Each additional volume adds two model states (T and P) increasing significantly the computational burden.
- Volumes of very different sizes result in stiff models i.e. slow and fast dynamics.



Crossley and Cook Model



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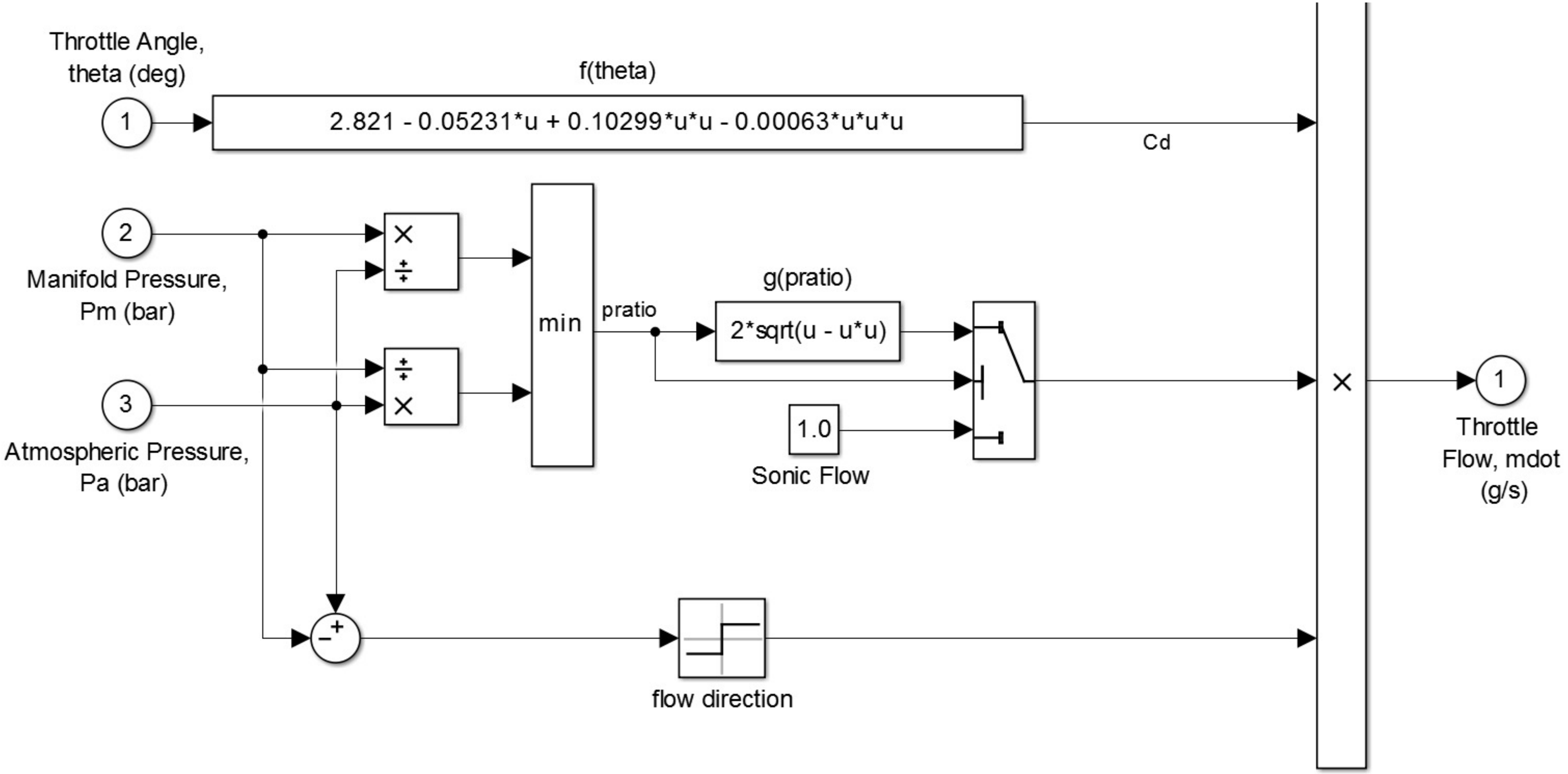
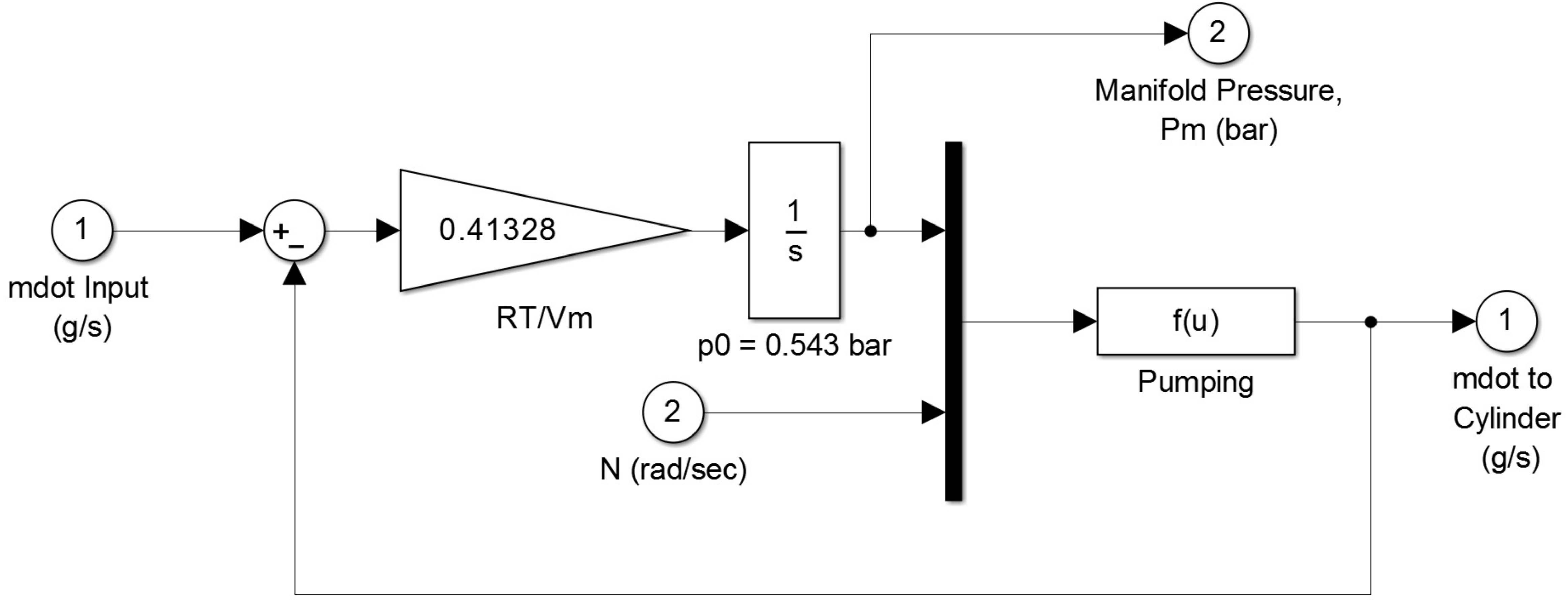
Crossley and Cook Model

Throttle



Crossley and Cook Model

Intake Manifold



Crossley and Cook Model

Torque Generation