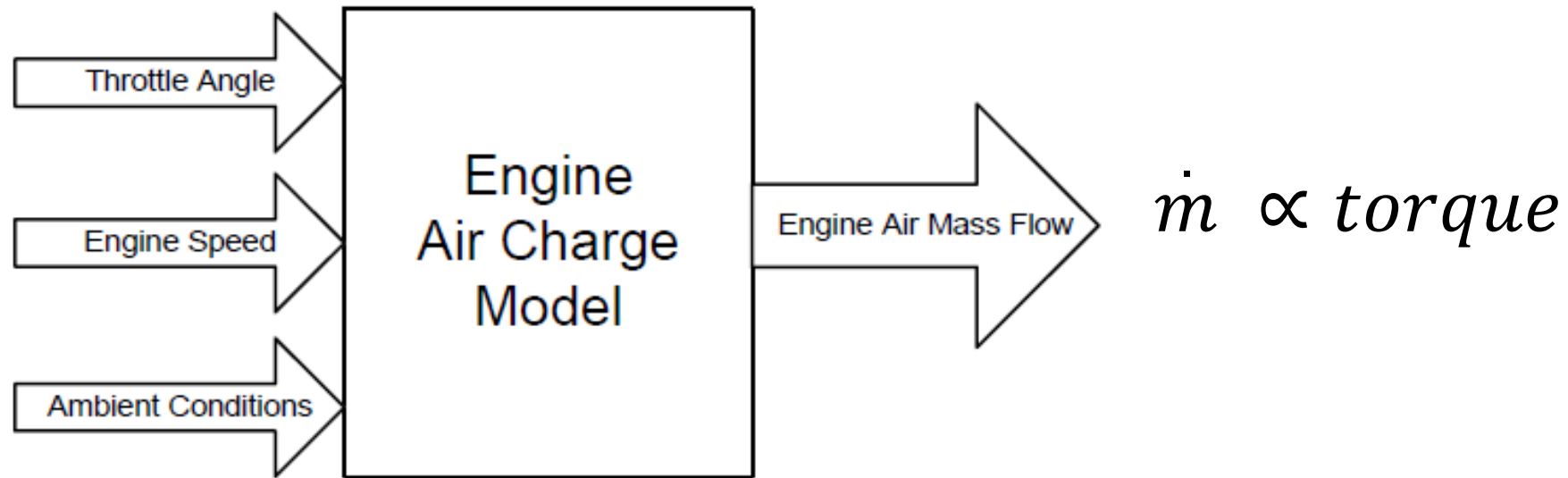


Vehicle Dynamics and Simulation

Dr B Mason

Mean value model creation

- System representation (naturally aspirated/gasoline)



Note: Boosted engines will also have wastegate position / duty cycle

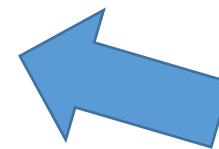
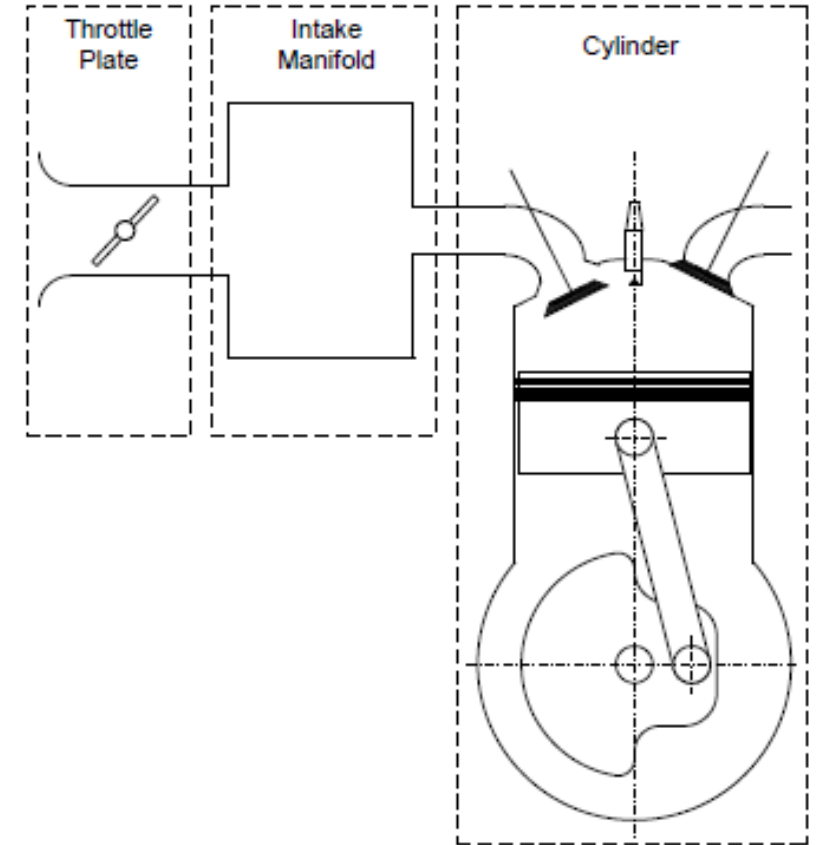
Mean value engine model

- Origin of air flow is induction into cylinder.
- Airflow is throttled.
- Volumetric flow is given;

$$\dot{V} = V_{disp} \frac{N_{eng}}{120}$$

- Mass flow (speed density equation);

$$\dot{m} = \frac{P_{man}}{RT_{man}} \frac{V_{disp}}{120} N_{eng}$$



Easy so far? Things get worse from here!

Mean value engine model – volumetric efficiency

- Volumetric efficiency, η

$$\dot{m} = \eta \frac{P_{man}}{RT_{man}} \frac{V_{disp}}{120} N_{eng}$$

$$0.5 < \eta < 1.2$$

Max is ≈ 1 for Naturally aspirated

- Modifies the speed density equation
- Depends on;
 - Intake and exhaust geometry
 - Intake and exhaust manifold pressure
 - Engine speed
 - Valve timing
 - Acoustic and inertial air effects
 - etc
- Perhaps the most important parameter in all of the mean value models!

Throttle

- Can be modelled as Laval nozzle of variable throat area (projected cross sectional area).
- For $\frac{P_{man}}{P_{atm}} > 0.528$ mass flow depends on P_{atm} and P_{man} ;

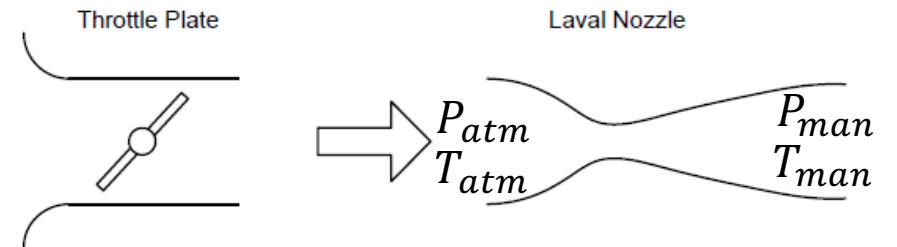
$$\dot{m} = \frac{C_d A_{th} P_{atm}}{\sqrt{RT_{atm}}} \left(\frac{P_{man}}{P_{atm}} \right)^{\frac{1}{\gamma}} \left\{ \frac{2\gamma}{\gamma - 1} \left[1 - \left(\frac{P_{man}}{P_{atm}} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$

- For $\frac{P_{man}}{P_{atm}} \leq 0.528$ flow depends on P_{in} alone (sonic/choked flow);

$$\dot{m} = \frac{C_d A_{th} P_{atm}}{\sqrt{RT_{atm}}} \sqrt{\gamma} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

- At max throat flow velocity;

$$\frac{P_{man}}{P_{atm}} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}}$$



Throttle

- Throttle effective area

$$A_{th} = \frac{\pi D^2}{4} \left\{ \left(1 - \frac{\cos \psi}{\cos \psi_0} \right) + \frac{2}{\pi} \left[\frac{a}{\cos \psi} (\cos^2 \psi - a^2 \cos^2 \psi_0)^{\frac{1}{2}} - \frac{\cos \psi}{\cos \psi_0} \sin^{-1} \left(\frac{a \cos \psi_0}{\cos \psi} \right) - a(1 - a^2)^{\frac{1}{2}} + \sin^{-1} a \right] \right\}$$

- Where

$$a = \frac{d}{D}$$

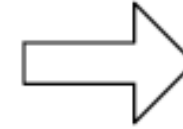
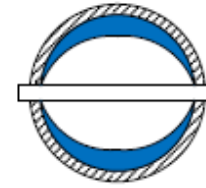
- Throttle max occurs when

$$\psi = \cos^{-1}(a \cos \psi_0)$$

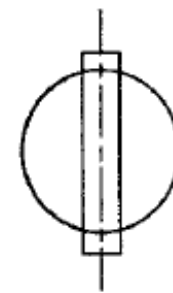
- So that at ψ_{max}

$$A_{th} \approx \frac{\pi D^2}{4} - dD$$

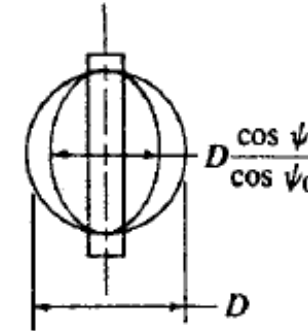
Projected Open Area



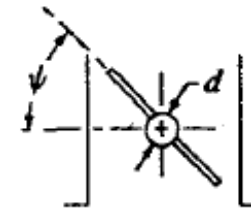
Equivalent Nozzle Throat Area



Closed



Open to angle ψ



Throttle

- Model is valid for frictionless, adiabatic flow through smoothly convergent-divergent nozzle only!
- Discharge coefficient, C_d is used to 'correct' for reality i.e.
- C_d is not constant it depends on;
 - Throttle position, α
 - Throttle pressure ratio, $\frac{P}{P_{amb}}$
- In reality this tends to be mapped for a specific throttle using a flow bench

Intake manifold

- Can be represented as open system of constant volume.
- System stores mass and energy, represented by state variables P and T .

- Mass balance;

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (1)$$

- Energy balance;

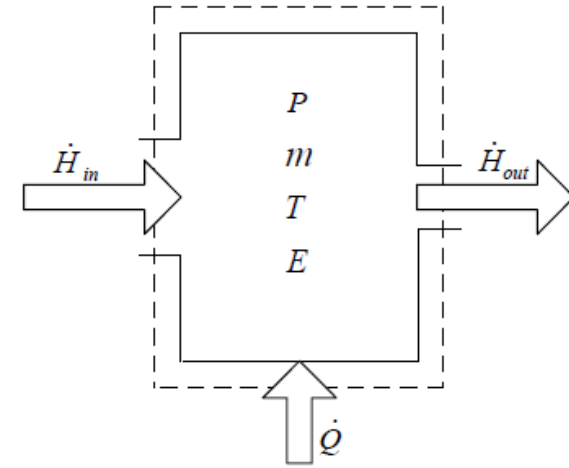
$$\frac{dE}{dt} = \dot{m}_{in}h_{0_{in}} - \dot{m}_{out}h_{0_{out}} + \dot{Q} \quad (2)$$

- Where;

$$h_0 = C_p T + \frac{u^2}{2}$$

And the energy within the volume is;

$$E = mc_v T + \frac{mu^2}{2} + mgz$$



Intake manifold

- Making some assumptions
 - GPE change is 0
 - KE change is 0

- So that;
$$E = mc_vT + \frac{mu^2}{2} + mgz = mc_vT$$
$$h_0 = C_pT + \frac{u^2}{2} = C_pT \quad (3)$$

- Taking the derivative of $E = mc_vT$ wrt to t ;

$$\frac{dE}{dt} = c_vT \frac{dm}{dt} + c_vm \frac{dT}{dt} \quad (4)$$

- And by substituting 1, 3, 4 into 2;

$$c_vT(\dot{m}_{in} - \dot{m}_{out}) + c_vm \frac{dT}{dt} = \dot{m}_{in}C_pT_{in} - \dot{m}_{out}C_pT_{out} + \dot{Q} \quad (5)$$

Intake manifold

- Coupling the energy and mass balances using the ideal gas law;

$$m = \frac{PV}{RT} \quad (6)$$

- Taking the derivative;

$$\frac{dm}{dt} = \frac{V}{RT} \frac{dP}{dt} - \frac{PV}{RT^2} \frac{dT}{dt} \quad (7)$$

- Substituting (1, 6 and 7 into 5) and assuming $T_{out} = T$;

$$\frac{dT}{dt} = \left[c_p \dot{m}_{in} T_{in} - c_p \dot{m}_{out} T - c_v T \dot{m}_{in} + \frac{dQ}{dt} \right] \frac{RT}{c_v PV} \quad (8)$$

$$\frac{dP}{dt} = \left[c_p \dot{m}_{in} T_{in} - c_p \dot{m}_{out} T + \frac{dQ}{dt} \right] \frac{R}{c_v V} \quad (9)$$

And;

$$\frac{dQ}{dt} = hA_{wall}(T_{wall} - T) \quad (7)$$

Torque model

- Torque produced is a function of;
 - Spark advance
 - Inducted air mass flow
 - AFR
- Data is usually obtained experimentally and incorporated within a regression model.
- Friction torque is deducted (imep – bmep) to establish output torque.
- Fmep [bar] is calculated;

$$fmep = 0.97 + 0.15 \left(\frac{N}{1000} \right) + 0.05 \left(\frac{N}{1000} \right)^2$$

- And;

$$T_f = \frac{fmep V_{sw}}{4\pi} = \frac{0.97 + 0.15 \left(\frac{N}{1000} \right) + 0.05 \left(\frac{N}{1000} \right)^2 V_{sw}}{4\pi}$$

Parameterisation effort

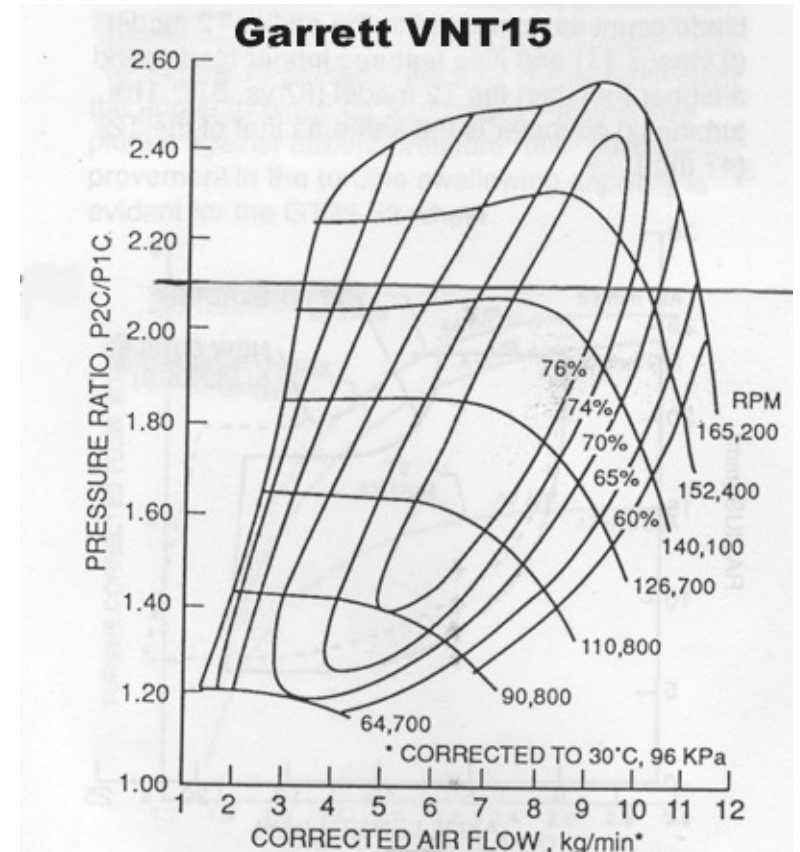
- Model has 5 unknown parameters, C_d , η_{vol} , h , V and V_{disp} .
- With $\eta_{vol} = f(P, T, N, IVO, EVC)$
- η_{vol} is obtained by experiment at some P, T, N, IVO, EVC . Recall;

$$\eta = \frac{120\dot{m}_{actual}}{\rho V_{disp} N_{eng}}$$

- C_d is also experimentally obtained (usually on flow rigs)
- Obtaining h in reality is very difficult and this is normally one of the tuned parameters.
- V and V_{disp} are obtained relatively easily but can also be used to tune the model response to match reality.

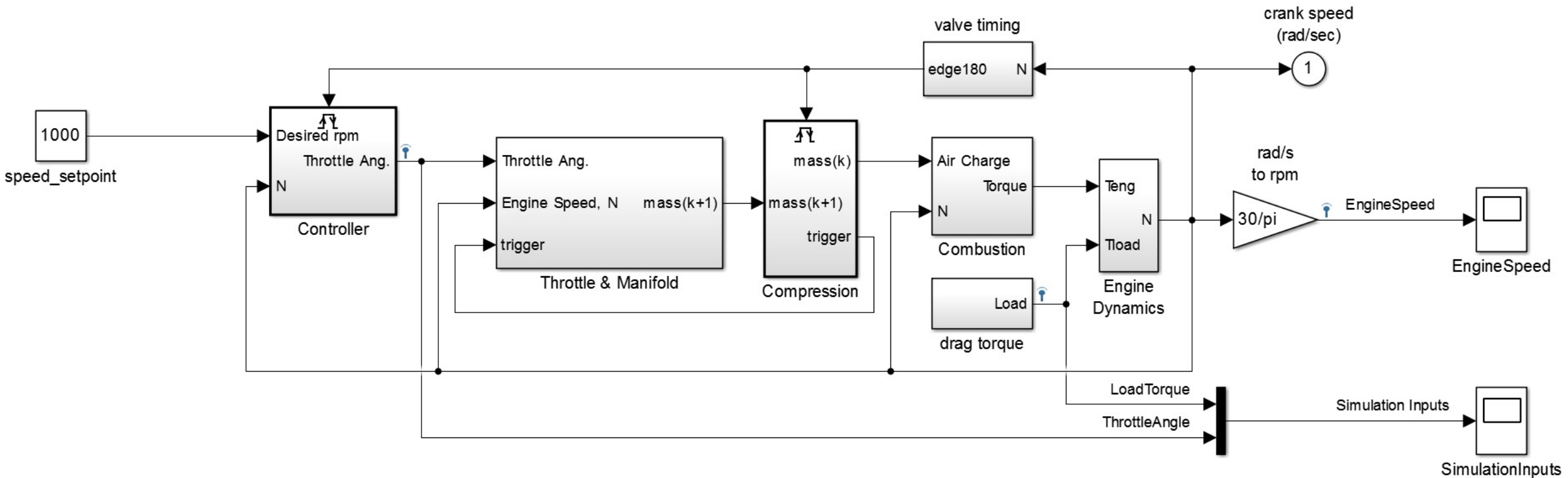
Other considerations

- Adding a turbocharger complicates matters significantly and introduces a causality loop.
 - The loop is normally broken by a delay.
- Heat transfer from the exhaust manifold has a significant effect on the turbo performance.
- Errors in the 'turbo loop' are accumulated within the loop.
- Each additional volume adds two model states (T and P) increasing significantly the computational burden.
- Volumes of very different sizes result in stiff models i.e. slow and fast dynamics.



Crossley and Cook Model

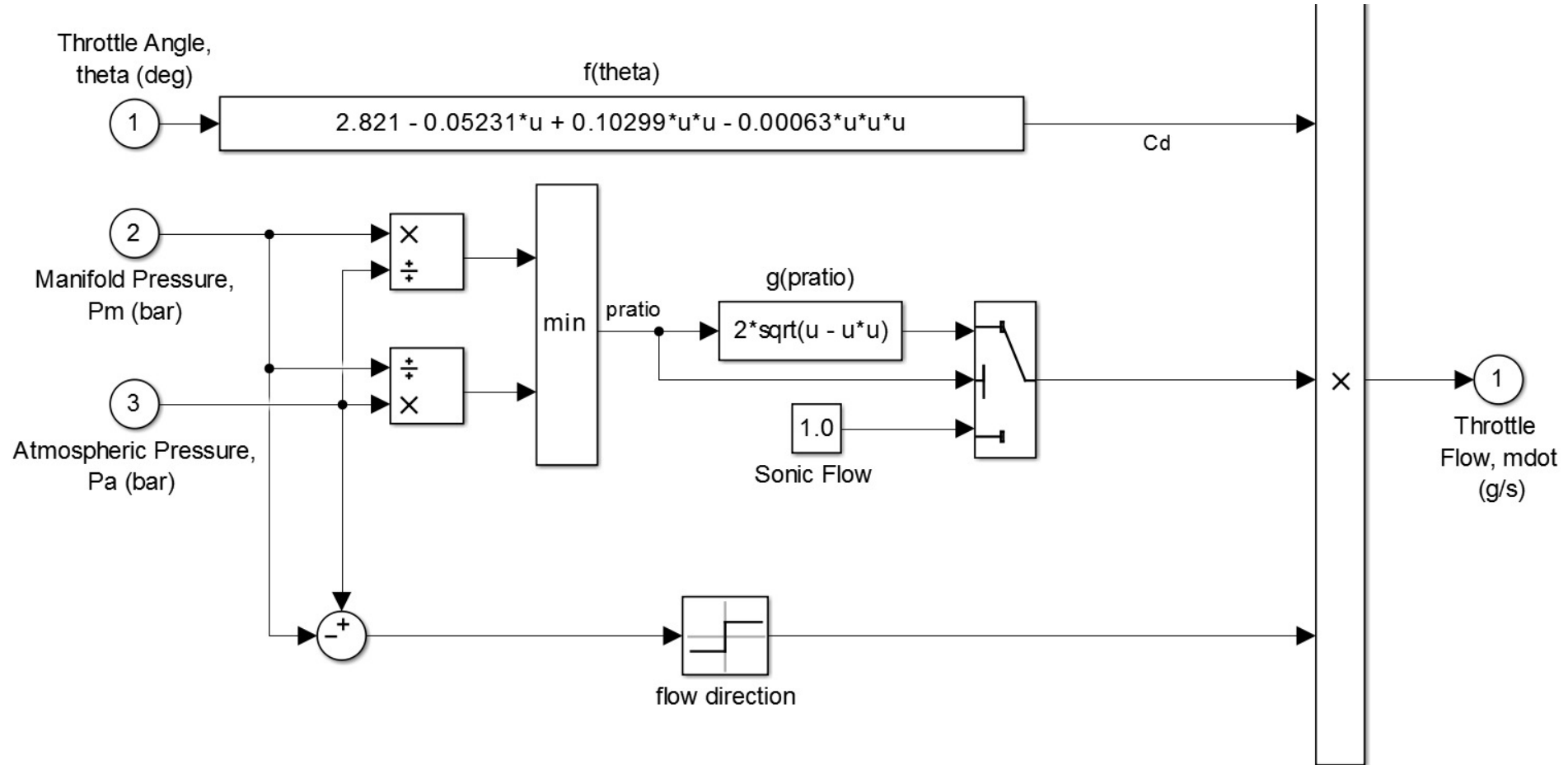
Engine Timing Model with Closed-Loop Control



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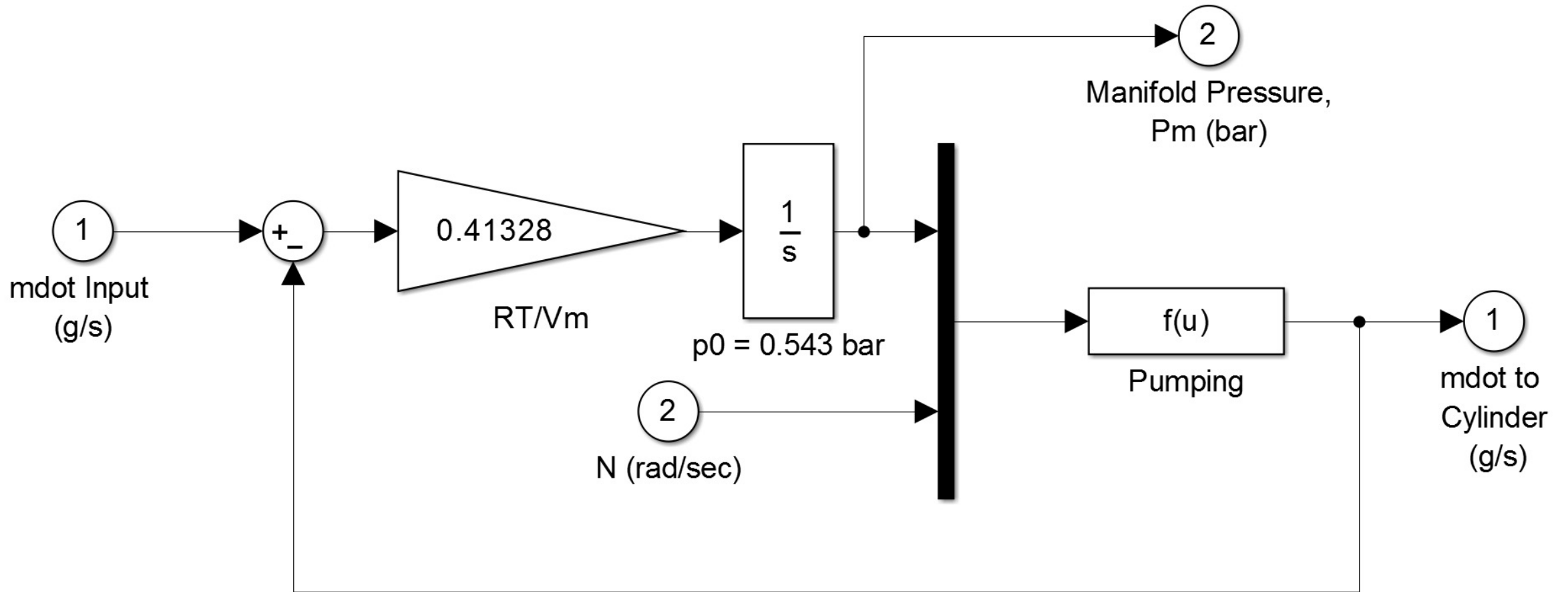
Crossley and Cook Model

Throttle



Crossley and Cook Model

Intake Manifold



Crossley and Cook Model

Torque Generation

