

Powertrain Calibration Optimisation

Introduction to Design of Experiments



Overview

□ Section 1

- Process Overview

□ Section 2

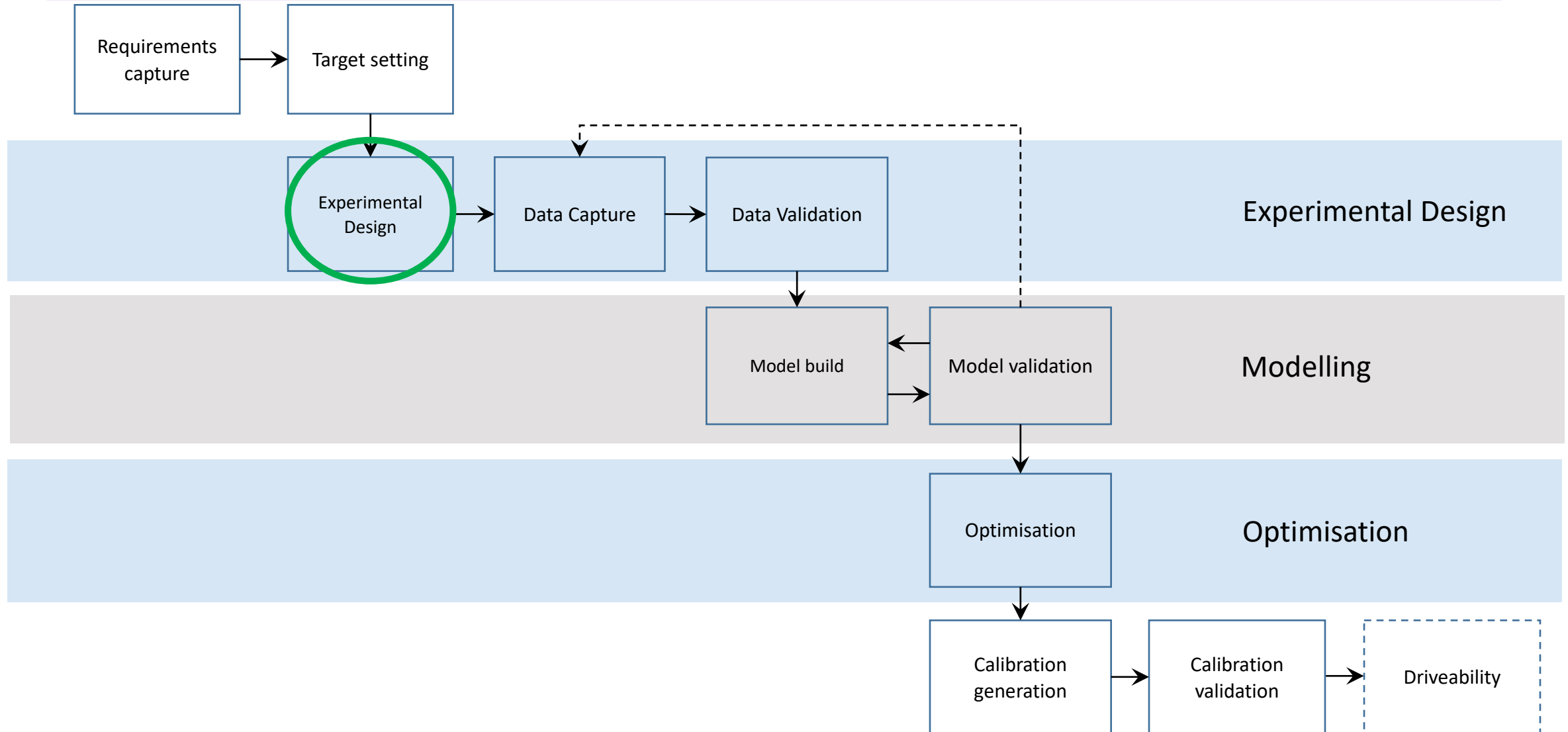
- Objectives for Design of Experiments (DOE)
- Outline of the DOE process
 - Classical design
 - Space Filling design
 - Optimal design
- Criteria for the DOE process



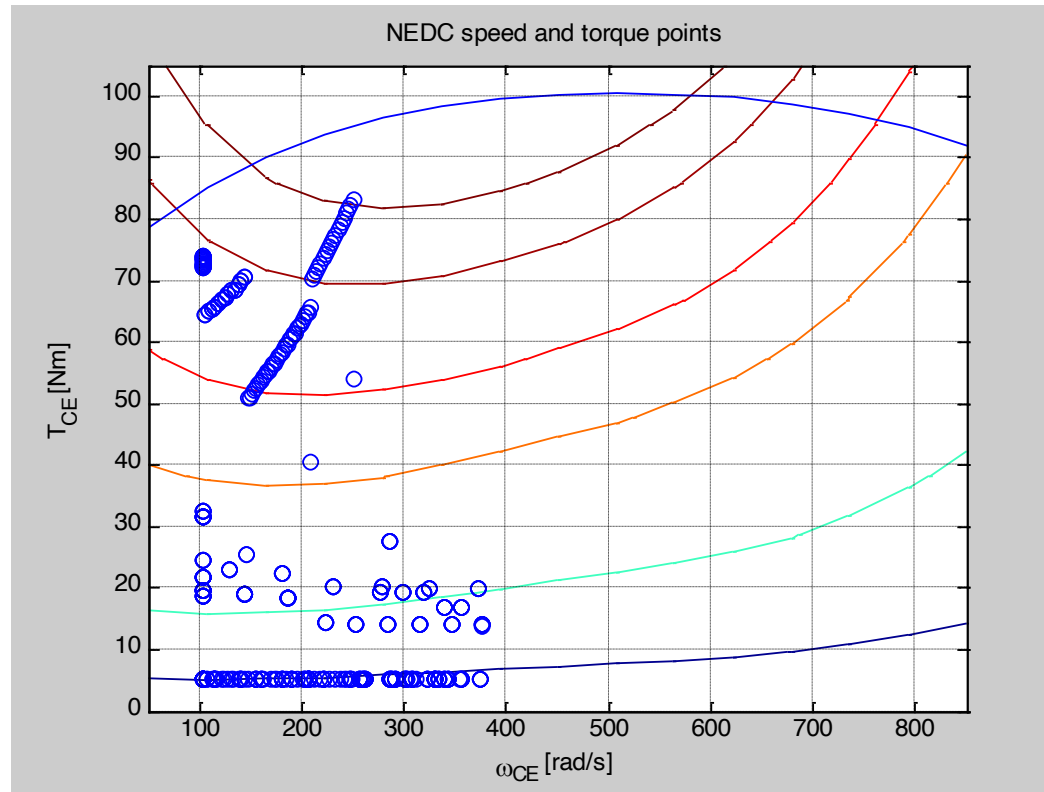
Four major steps in calibration

- Plan the experiments
 - With limited test bed time what is the best way to gather data? Identify modal points – plan experiments.
- Acquire the data
 - There is always a significant volume of data; automated methods are essential
- Fit models
 - Models will be quick to fit and accurate and represent engine behaviour
- Conduct optimisation
 - Using models, identify the combinations of controls that give *best* engine behaviour





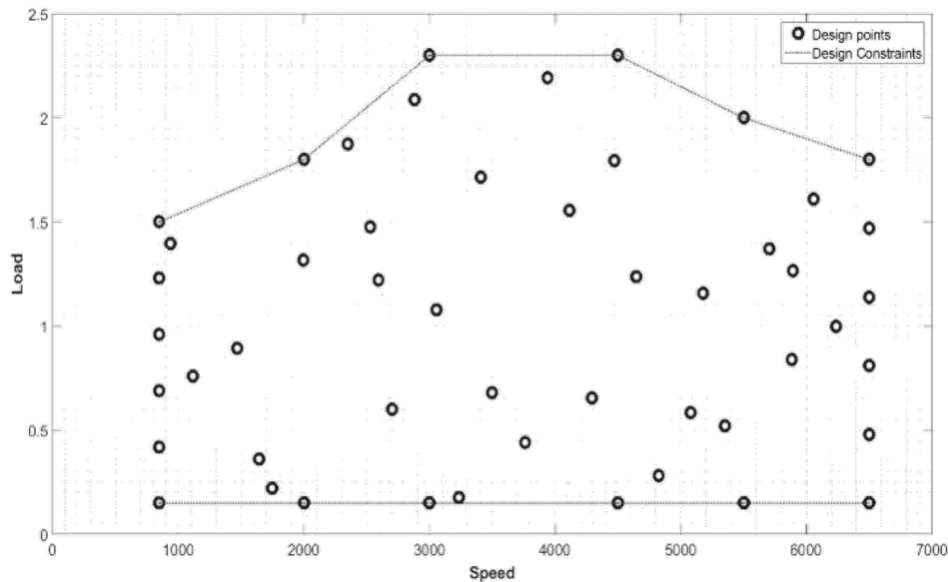
NEDC speed and torque points



Design of Experiments is used to plan engine testing

- Design of Experiments (DOE) provides efficient experimentation
- DOE is widely used in the process and medical industries

Torque experiment



3^k factorial experiments
(i.e. 3 levels, k factors)

Why 3 levels?



k	Test Points
2	9
3	27
4	81
5	243
6	729
7	2187



Design of Experiments (DOE) - What do you do?

- Find the variables which influence the output (speed, load, ignition timing ..)
- Estimate the levels that are of interest (high, low ..)
- Two levels and k variables gives a 2^k design
 - 2^k is likely to be too many
 - select a fraction
- There are many ways to select a fraction
- Estimate *main effects* first - then *first order* interactions - and so on.

Quadratic surface model

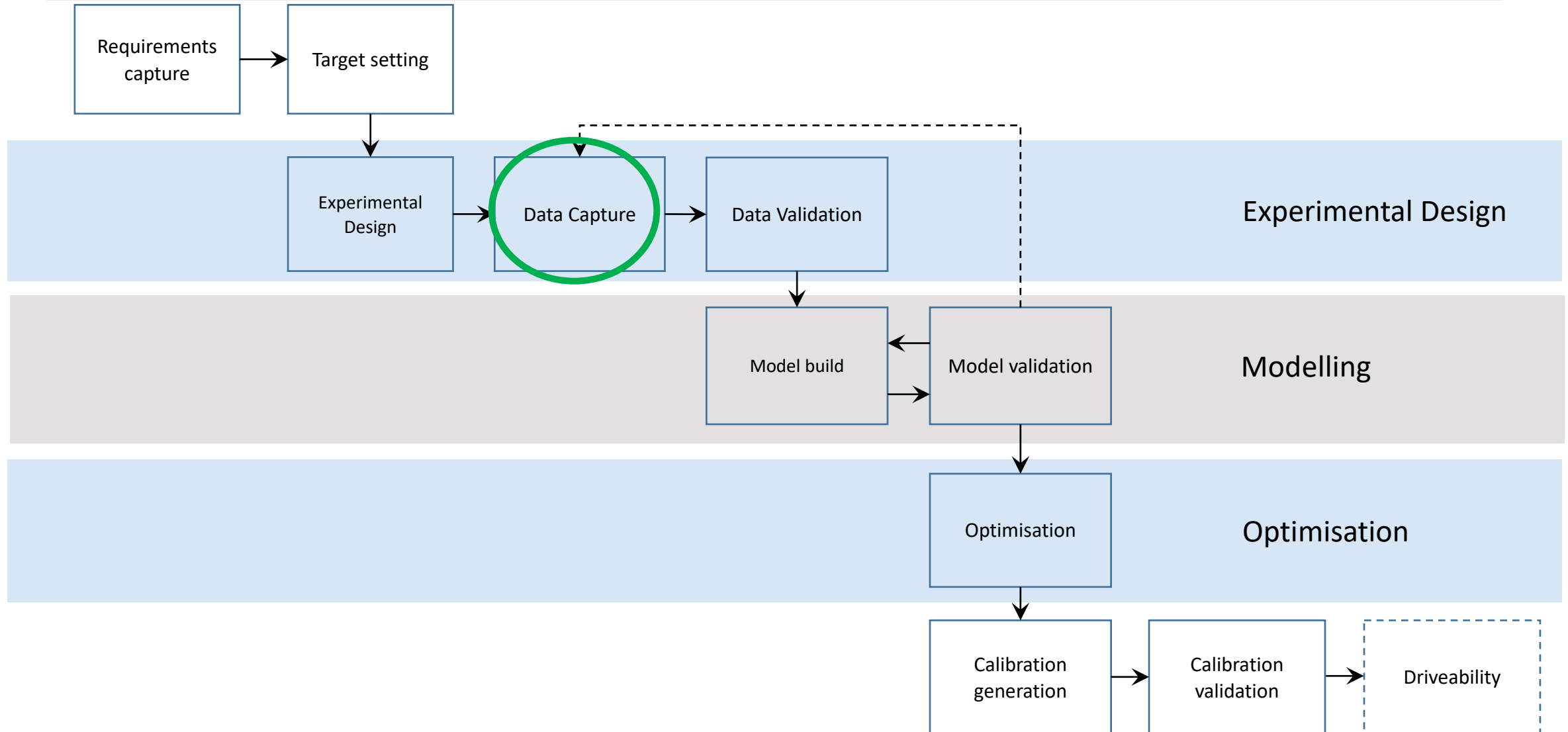
$$\widehat{y}_q = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2$$

first order

second order

higher order





Different testing environments are used throughout the Calibration Development Process:

- ☐ Engine Testbed (Dyno)
- ☐ Chassis-rolls Dyno/ Powertrain Testbed (Vehicle)
- ☐ Public roads, Test Trips (Vehicle)
- ☐ Hardware-in-the-Loop, HiL (Simulation Environment)



❑ Engine Testbed

- ❑ Engine connected to a dynamometer
Control of engine speed and load
- ❑ Control of coolant and oil temperatures
- ❑ Instrumentation of Engine and Exhaust
Temperatures, pressures, ...
- ❑ Emissions Measurement Systems
- ❑ Test Automation
- ❑ Controlled testing environment for repeatable
and steady conditions calibration tasks



❑ Chassis-rolls Dyno

- ❑ Testing of vehicle with complete powertrain on rolls with simulation of various driving resistances
- ❑ Simulation of various environmental conditions: cold/hot climate, altitude,...
- ❑ Vehicle and engine with additional instrumentation (temperature, pressure sensors,...)
- ❑ Emissions Measurement Systems
- ❑ Tests Automation
- ❑ Controlled testing environment for transient/dynamic conditions



❑ Road Testing/Test Trip

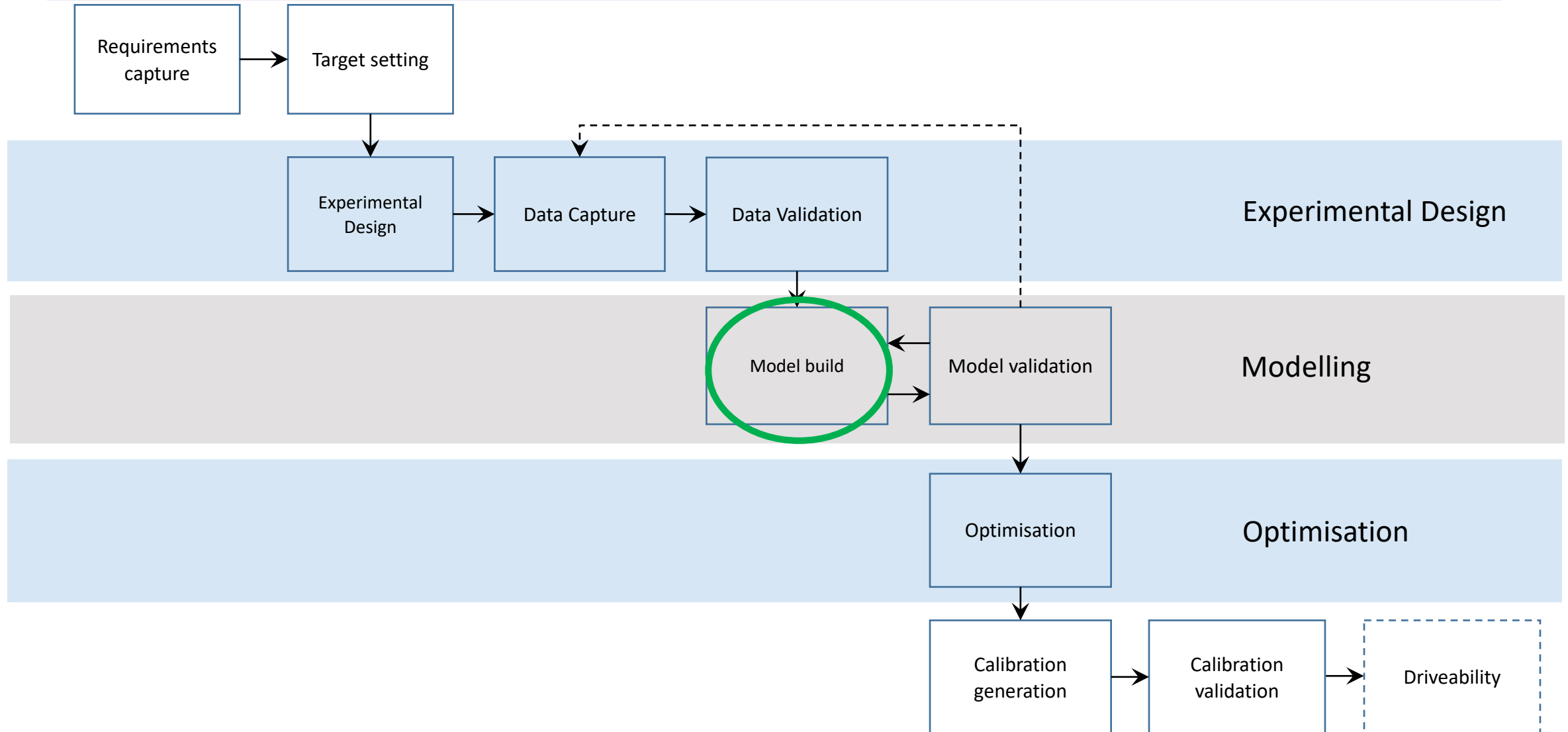
- ❑ Cold Climate
- ❑ Hot Climate
- ❑ Altitude
- ❑ Tests tracks for specific manoeuvres (high speed testing,...)
- ❑ Testing environment in real conditions



❑ Hardware-in-the-Loop HiL

- ❑ Engine simulation model connected to a physical ECU
- ❑ Vehicle simulation model can be integrated
- ❑ The HiL simulation controller supply the sensors inputs to the ECU and reads the actuator outputs to simulate the engine
- ❑ Depending on model accuracy a various range of calibration tasks can be realized on the HiL environment
- ❑ Extreme environmental boundary conditions can be simulated without risk of damaging engine or vehicle prototype

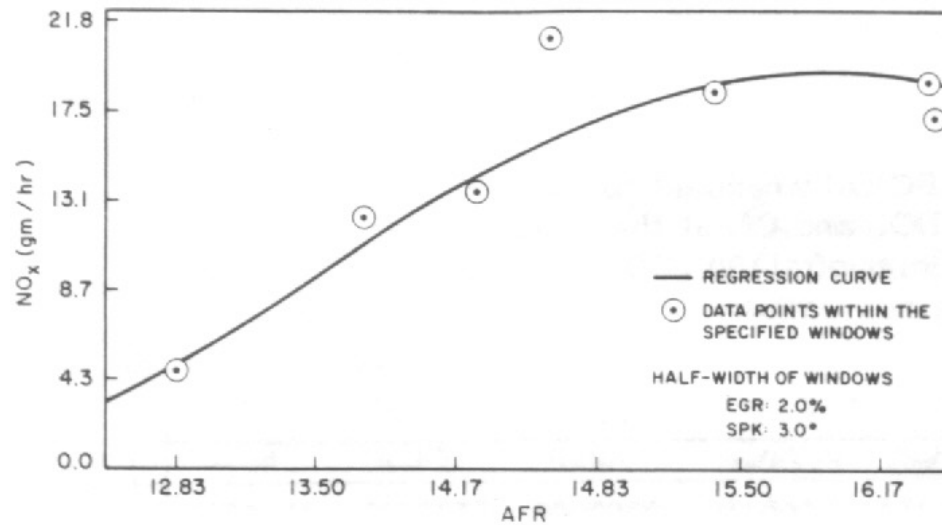




Creating models of the data

The data generated during the engine mapping process is reduced to a form that is easy to work with

- A model is fitted to the data
- Optimisation is conducted on the model



Radial Basis Functions

- Polynomial functions remain the most popular model type
- Radial basis functions are gaining in popularity
 - They offer a broader range of representation
 - A radial basis function is based on the sum of functions located at a number of centres

$$f_i = K \left(\frac{\|x_i - c_i\|}{\sigma} \right)$$

$$\hat{y}(\vec{x}) = \sum_{i=1}^n \beta_i f_i(\vec{x})$$



Types of models - Radial basis function

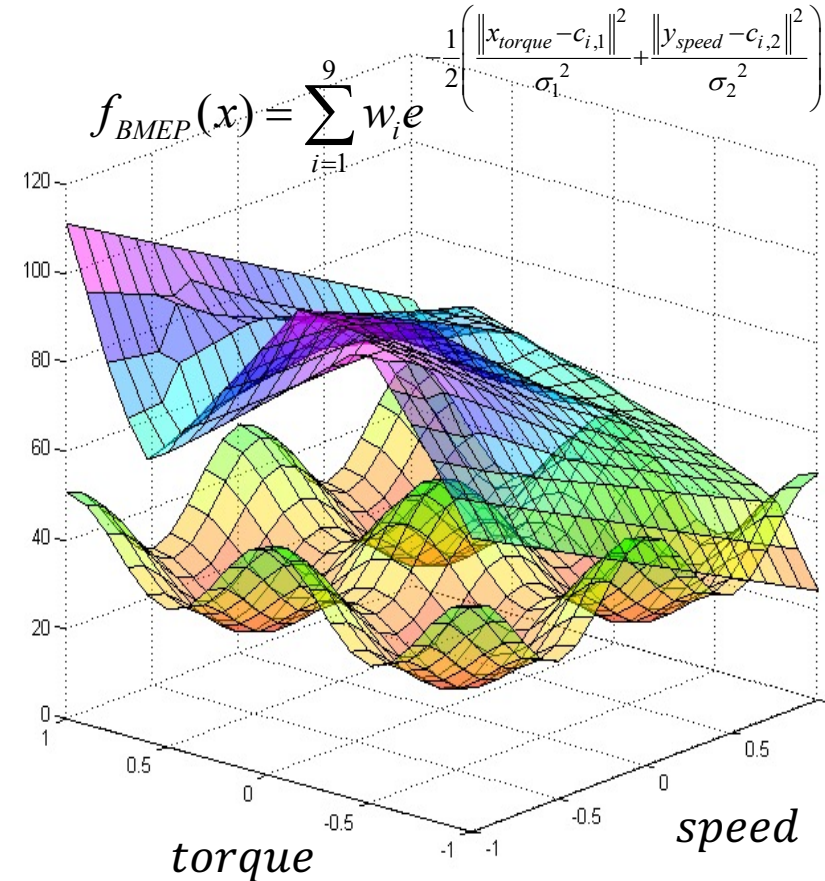
A BMEP response surface model using RBF with two inputs (torque and speed):

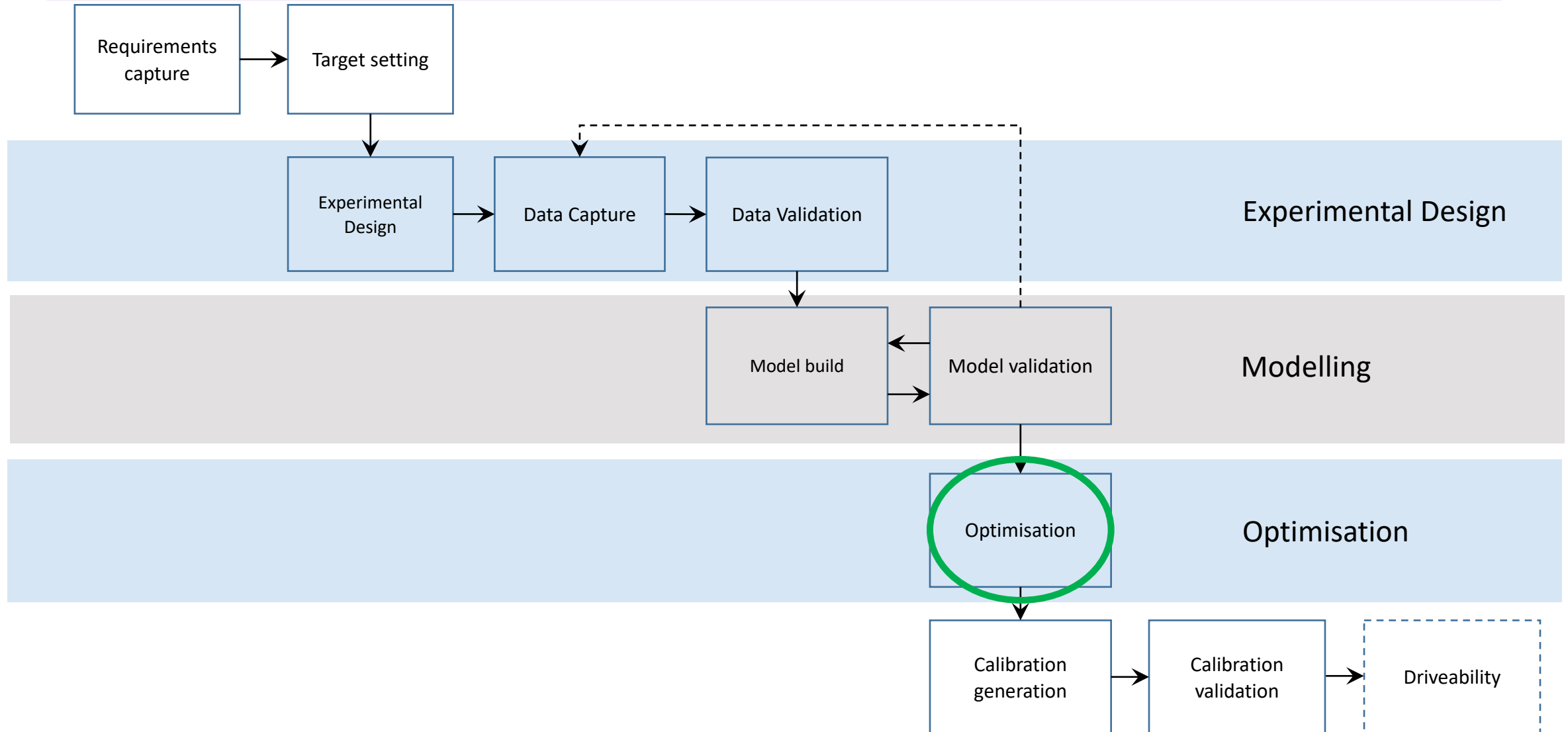
Parameters requiring training:

1. Weights w_i
2. Centers $c_{i,1}$ $c_{i,2}$
3. Widths σ_1^2 σ_2^2

In MBC, the training is done automatically. It only needs training data and maximum number of centers to use.

BMEP





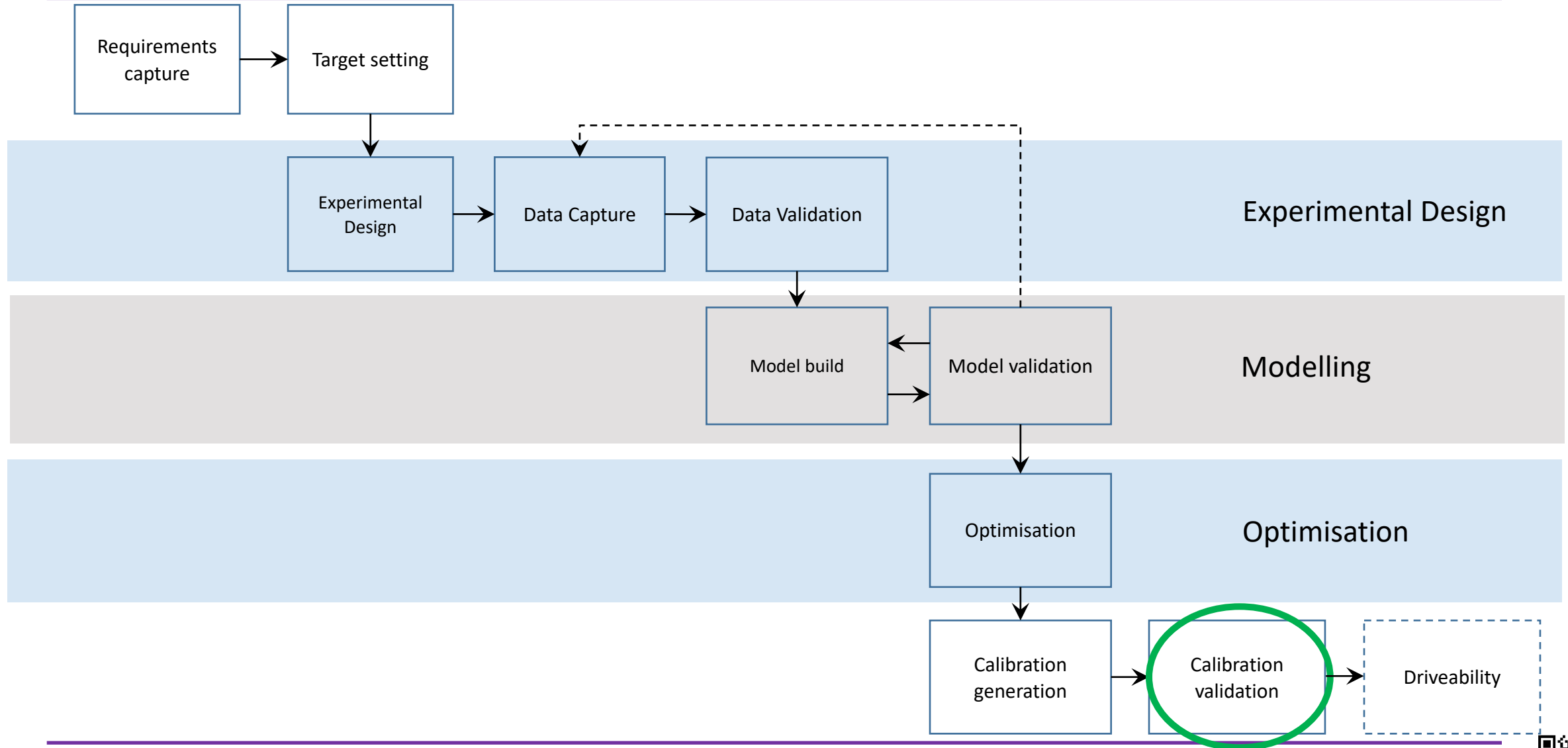
Optimisation

- Optimisation is the process of finding the best combination of controls to meet a specified task
- In an optimisation process a cost function is formulated and minimised
- The cost function contains quantities to be minimised
- This is a simple example of a cost function to be minimised that would result in low fuel consumption and torque delivery

$$J = \sum f + \left(\frac{\partial T}{\partial t} \Big|_{n,T,\dots} \right)^{-1}$$

- f is a measure of fuel consumption
- $\frac{\partial T}{\partial t} \Big|_{n,T}$ is a measure of torque delivery at a given engine state



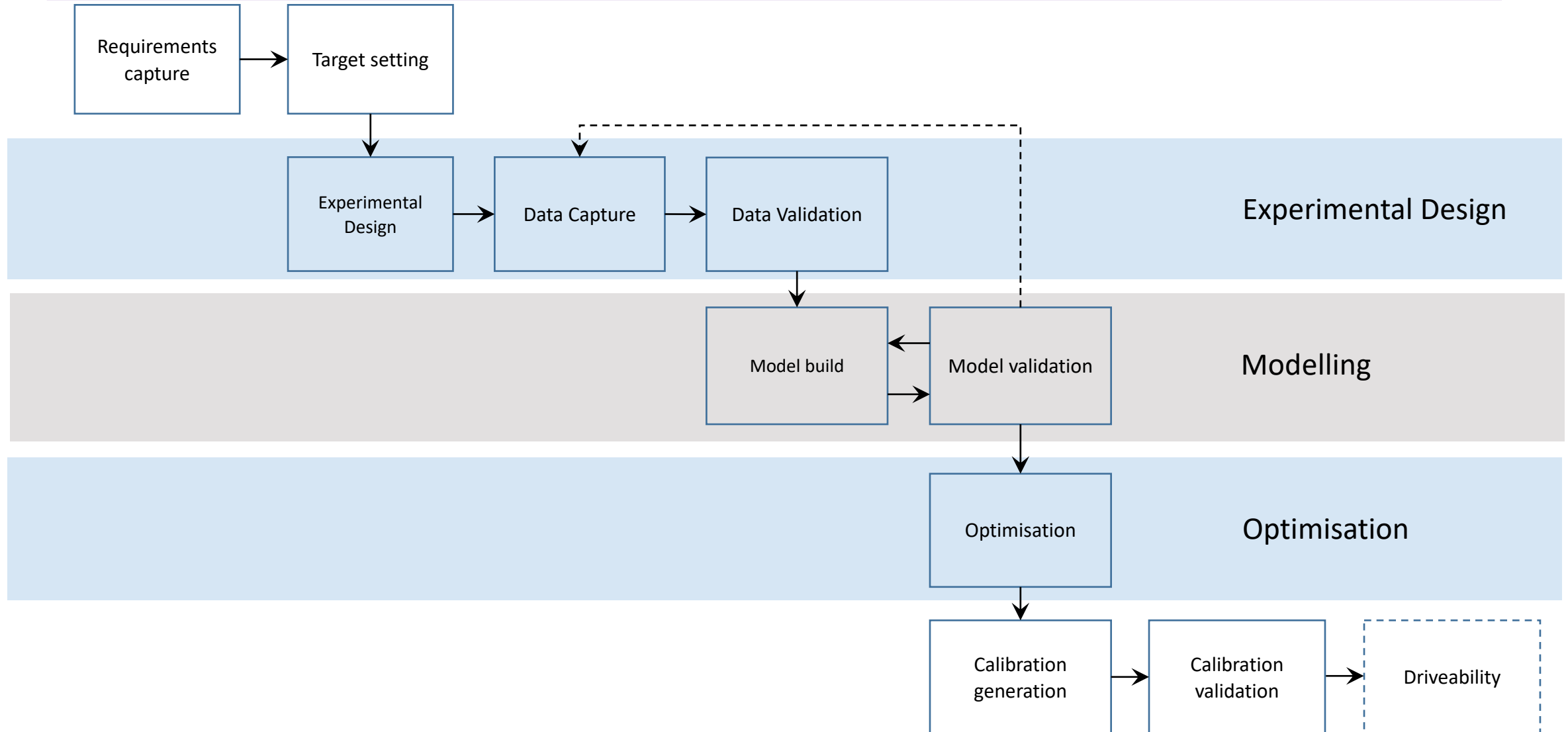


Driveability - delivering the customer appeal

- The job is not complete until the driveability is considered satisfactory
- The focus in this phase of work is transient effects
 - Avoidance of engine knock
 - “Good” delivery of torque
- Driveability is likely to lead to compromise
- Other vehicle level attributes that may require further iteration



Powertrain Calibration Optimisation



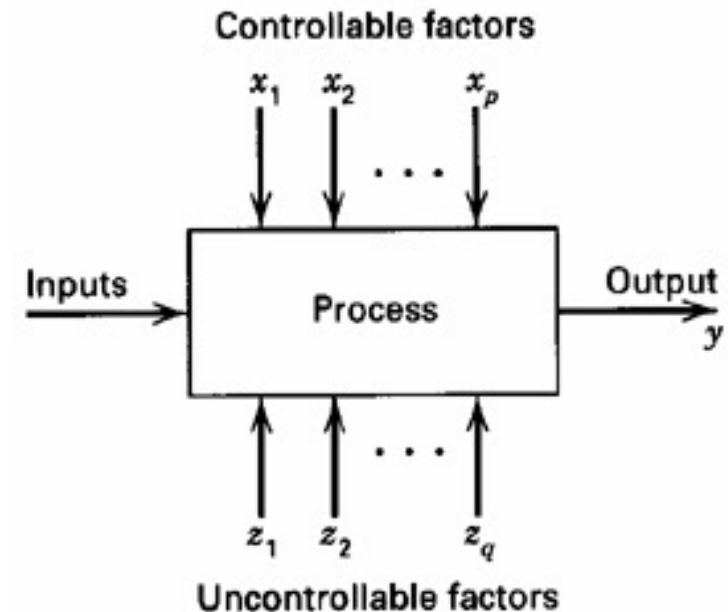
Section 2

- Objectives for this lecture (and the lab)
 - How to choose inputs
 - Advantages and disadvantages of different methods
 - Review of designs
 - Classical: Full Factorial, Fractional Factorial, Box-Behnken, Central-Composite
 - Space-filling: Latin Hypercube, Lattice, stratified Latin Hypercube
 - Optimal: “alphabet soup” A, D and V optimal



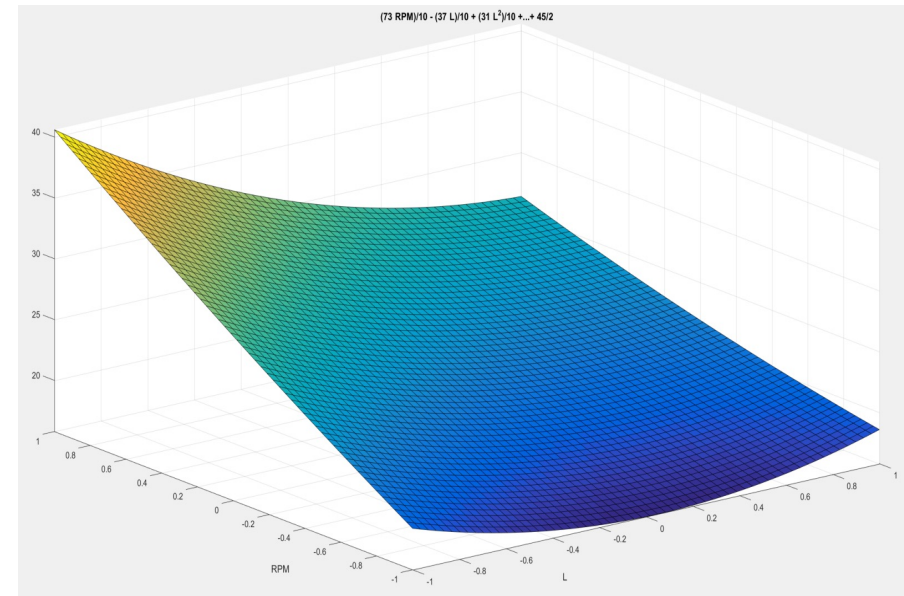
Why DOE?

- ❑ Much time required for full factorial experiments
- ❑ Characterisation of engine for optimisation
- ❑ The use of DOE improves the yield of information compared with ad-hoc experimental methods
- ❑ The result is better use of resources
- ❑ A DOE process allows the inclusion of explicit constraints: speed load, EGR limits



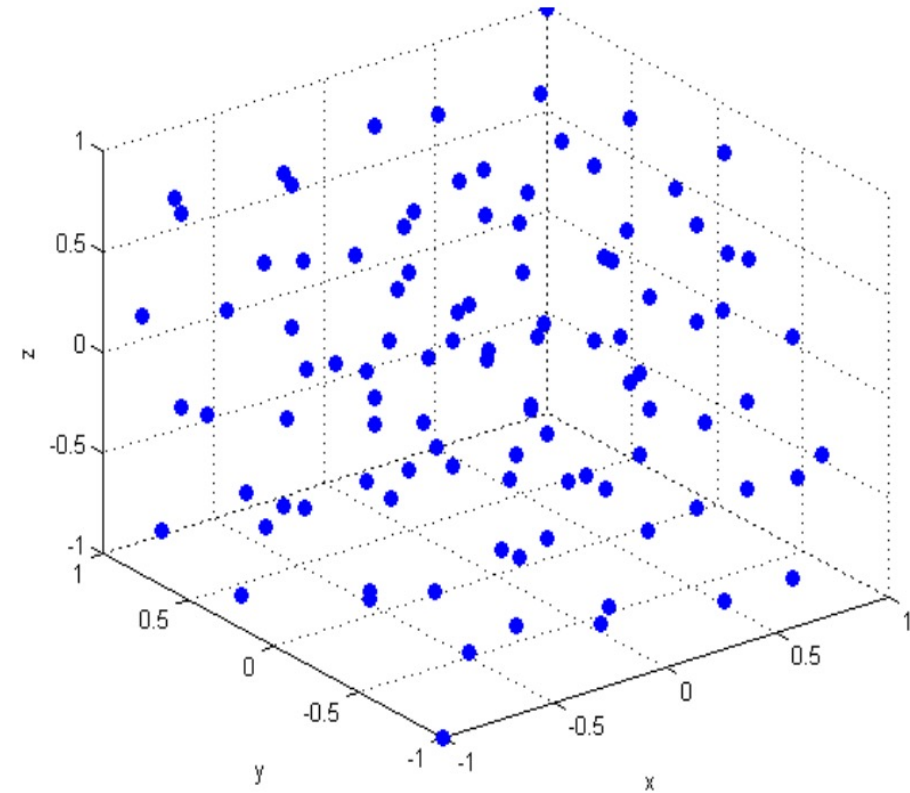
Mathematical Model

$$y = f(\boldsymbol{\beta}, \mathbf{X}) = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_1x_2\beta_1 + x_1x_2\beta_2 + x_1^2\beta_{11} + x_2^2\beta_{22}$$



Categories of DOE

- **Classical:** Full Factorial, Fractional Factorial, Box-Behnken, Central-Composite
- **Space-filling:** Latin Hypercube, Lattice, stratified Latin Hypercube
- **Optimal:** “alphabet soup” A, D and V optimal



Space filling design



How to choose different design styles

- Decide on the aim of the experiment.
 - A/B testing
 - Factor screening
 - Response surface modelling
- Evaluate how much you already know
 - Classic designs:
 - Simple regions (linear models, quadratic models)
 - Space-filling:
 - Low system knowledge
 - Optimal designs:
 - High system knowledge

**The DOE process
is typically
iterative**



Terminology (1)

- Randomisation
 - Randomising the order of experiments so as to avoid systematic errors
- Blocking
 - Explicitly accounting for key factors in the planning of experiments – test bed, operator
- Confounding
 - Independent variable a and b are said to be confounded when they both influence dependent variable c



Terminology (2)

- Response variable
 - Measured output value
- Factors
 - Input variables that can be changed
 - E.g. torque, speed, voltage, frequency, current
- Interaction
 - Effect of one input factor depends on level of another input factor

Quadratic surface model

$$\widehat{y}_q = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2$$

first order

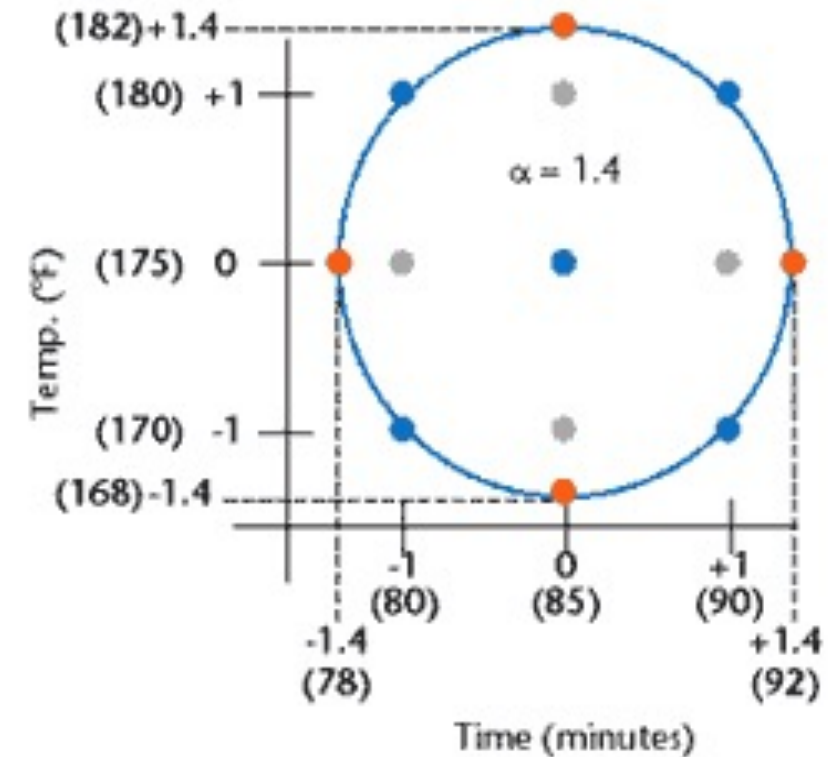
second order

higher order



Terminology (3)

- Levels
 - Specific values of factors (inputs)
- Replication
 - Completely re-run experiment with same input levels
 - Used to determine impact of “noise” (measurement error, random effects)
- Rotatability
 - A design is rotatable if the variance of the predicted response at any point \mathbf{x} depends only on the distance of \mathbf{x} from the design centre



Classical design

Key steps in designing an experiment

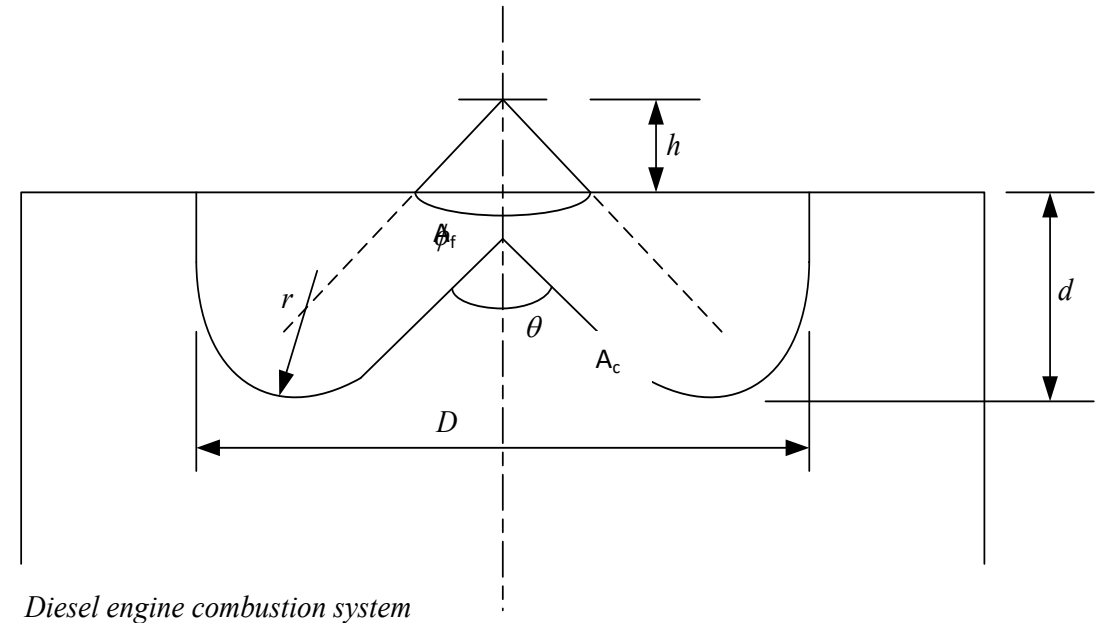
1. Identify factors of interest and a response variable
2. Determine appropriate level for each explanatory variable
3. Determine a design structure
4. Randomise the order in which each set of conditions is run and collect the data
5. Organise the results in order to draw appropriate conclusions
6. Replicate to give “noise” information



Classical Design Example: Optimising a diesel engine combustion system

- Output BSFC
- Overall diameter D , radius r
- Depth d , Angle of central cone A_c
- Angle of fuel jets A_f , Height of injector h , Injection pressure p

- The number of experiments at 2 levels = 2^k (k =number of variables)
 - $k = 3$: 8 experiments
 - $k = 4$: 16 experiments
 - $k = 7$: 128 experiments



Classical design: Determine factors, range and variable level

- Angle of fuel jets A_f , height of injector h , injector pressure p
- Two level values, denoted by + and -:
- $A_{f-} = 110$, $A_{f+} = 130$
- $H_- = 2\text{mm}$, $H_+ = 8\text{mm}$
- $p_- = 800\text{bar}$, $p_+ = 1200\text{bar}$



Determine the design structure


- Full factorial design: To list each factor combination exactly once
- Structure the list
 - 1st Column – alternate every 4 rows
 - 2nd Column – alternate every 2 rows
 - 3rd Column – alternate every other row

A	Af	h	p
1	-	-	-
2	-	-	+
3	-	+	-
4	-	+	+
5	+	-	-
6	+	-	+
7	+	+	-
8	+	+	+

Experiment design



Classical design: Organise the results to draw conclusions

- Run the experiments according to the design
- To determine what effect changing the level of p, A_f and h has on BSFC
 - For p
 - $\frac{1}{2}(\text{average}(-) - \text{average}(+)) = -4.5$  main effect
 - For A_f
 - $\frac{1}{2}((\text{average}-) - (\text{average}+)) = -1.75$
 - For h
 - $\frac{1}{2}((\text{average}-) - (\text{average}+)) = -0.25$

A	Af	h	p	bsfc
1	-	-	-	218
2	-	-	+	207
3	-	+	-	220
4	-	+	+	210
5	+	-	-	216
6	+	-	+	208
7	+	+	-	212
8	+	+	+	205

Experiment results



Classical design: Replications

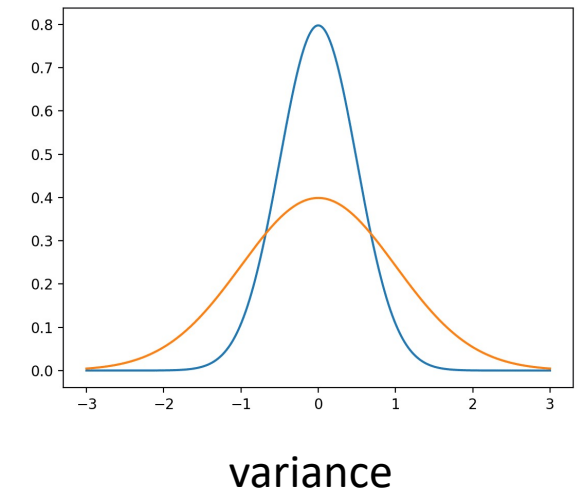
		Factor A					
		Factor A					
		Factor A					
		1	2	...	j	...	a
Factor B	1
	2

	i	y_{ijk}

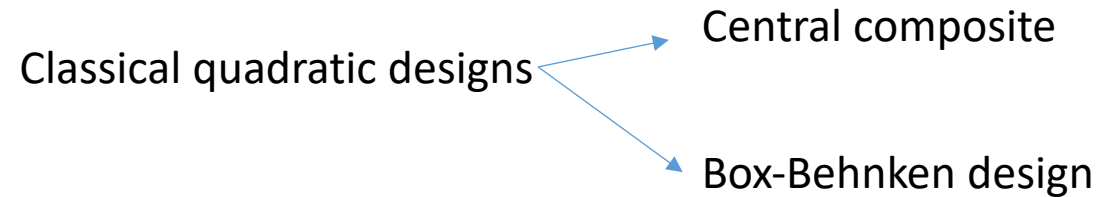
	b

Two Factors
 n Replications

n replications



Quadratic designs



e.g. MBT model, naturally aspirated

$$\begin{aligned} MBT \\ = 22.5 + 7.3RPM - 3.7L + 0.6RPM^2 + 3.1L^2 \\ - 0.6RPM \cdot L - 2.8RPM \cdot L \end{aligned}$$

Quadratic surface

$$y_q = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 + \beta_{23}x_2x_3 + \beta_{11}x_1^2 + \beta_{22}x_2^2 + \beta_{33}x_3^2$$

Cubic surface

$$\begin{aligned} y_c = y_q + \beta_{123}x_1x_2x_3 + \beta_{112}x_1^2x_2 + \beta_{113}x_1^2x_3 + \beta_{122}x_1x_2^2 + \beta_{133}x_1x_3^2 + \beta_{223}x_2^2x_3 + \\ \beta_{233}x_2x_3^2 + \beta_{111}x_1^3 + \beta_{222}x_2^3 + \beta_{333}x_3^3 \end{aligned}$$



Classical design: Box-Behnken designs

- The design is intended to fit a quadratic model, containing squared terms and products of two factors
- Suitable for small number of factors (three or less) and at least three levels (to get quadratic curvature)
- The ratio of the number of experimental points to the number of coefficients in the range of 1.5 to 2.6
 - More efficient i.e. few tests than full factorial

$$y = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_1x_2\beta_1 + x_1x_2\beta_2 + x_1^2\beta_{11} + x_2^2\beta_{22}$$

products of 2 factors

squared terms

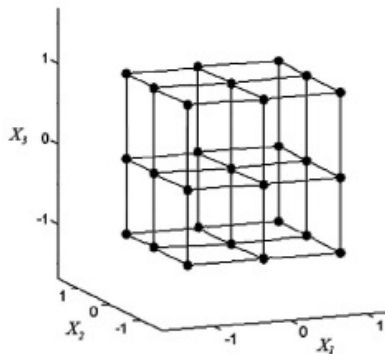


Classical design: Box-Behnken designs

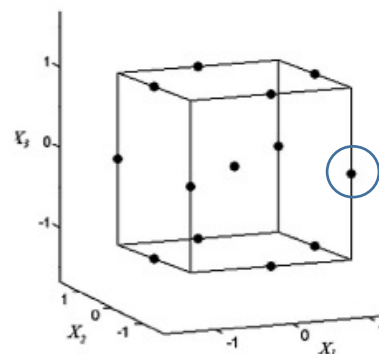
- Midpoints of edges of the input space at the centre (avoiding corner/extreme points)
- Fewer points than fractional factorial
- A combination of a two-level factorial with an *incomplete block design*.
- All combinations for the factorial design, while the other factors are kept at the central values.

Incomplete block

Factor 1	Factor 2	Factor 3
+	-	0
+	+	0
-	0	-
-	0	+
+	0	-
+	0	+
0	-	-
0	-	+
0	+	-
0	+	+
-	-	0
-	+	0
0	0	0



Full factorial (27)



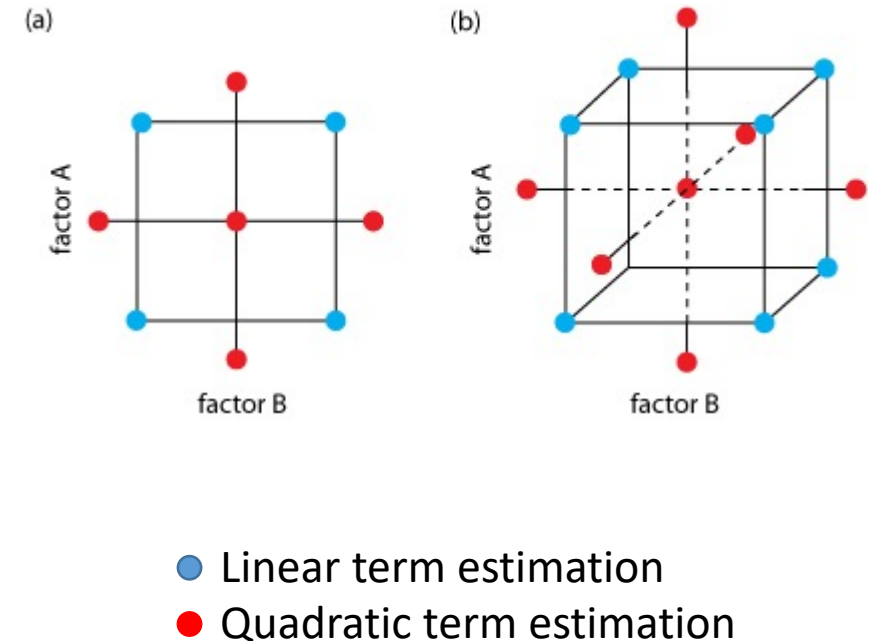
Box-Behnken (13)

$$\begin{aligned} x_1 &= 1 \\ x_2 &= -1 \\ x_3 &= 0 \end{aligned}$$



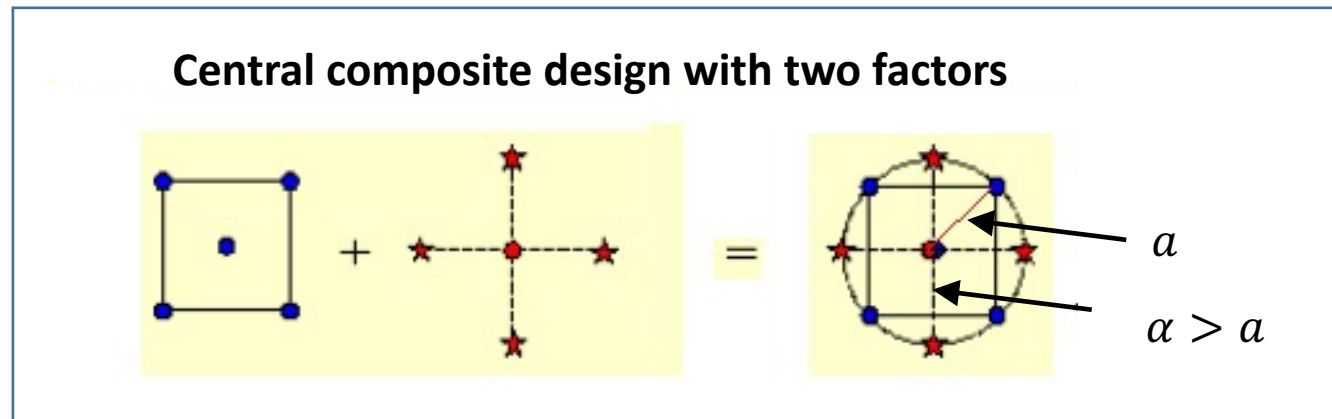
Classical design: Central Composite Design (CCD)

- Used when second order model is suspected in 2^k design
- Similar to Box-Behnken with corner and extreme points
- A set of centre points: the medians of the values used in the factorial portion – usually repeated
- A set of axial points, experimental runs identical to the centre points except for one factor, which will take on values both below and above the median of the two factorial levels



Classical design: Central Composite Design (CCD)

- Start with factorial design (with centre points)
- Add 'star' points to get an estimate of curvature



- Star points may not be achievable



Space filling designs

- Good when little is known about the system under study
- Distributes design points (in hyperspace) as far from each other as possible
- Various strategies for achieving this
- Fill-out the n-dimensional space that are regularly spaced

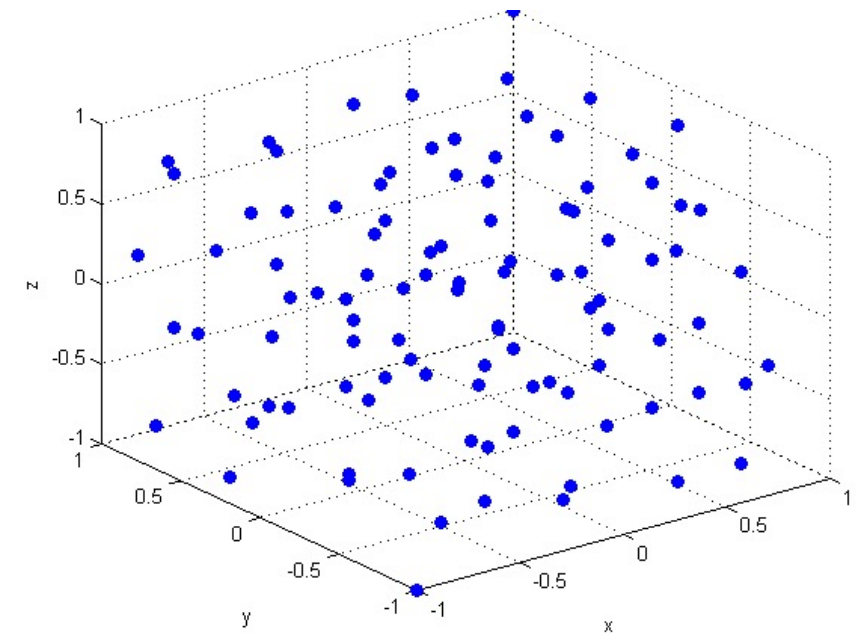
	1	2	3	4	5
1					
2					
3					
4					
5					



Space filling: Motivation

- Predictors for response are often based on interpolations
- Prediction error at any point is relative to its distance from closest design point
- Uneven designs can yield predictors that are very inaccurate in sparsely observed parts of experimental region
- Disadvantage: **superfluous points may be placed in regions of the design space**

So what??



Space filling design in 3 factors



Space filling: Latin Hypercube

- ❑ Scheme for generating design points
- ❑ Efficient algorithm – same number of points for increased dimensions (factors)
- ❑ Generate sets of design points that, for an N point design, project onto M different levels in each factor.
- ❑ Try several such sets of randomly generated points and choose the one that best satisfies user-specified criteria (augment??)



Latin Hypercube design: construct an LHD

- Partition experimental region into a square with M^2 cells (M along each dimension)
- Label the cells with integers from $\{1, \dots, M\}$ such that a Latin square is obtained, each integer occurs exactly once in each row and column
- Select one of the integers, say i , at random
- Sample one point from each cell labelled with i

LHD generated may not be space filling. Design requirements need to be assessed.

1				●		
2		●				
3			●			
4					●	
5	●					
6						●
	1	2	3	4	5	6

1	●					
2		●				
3			●			
4				●		
5					●	
6						●
	1	2	3	4	5	6

Not space filling!

Both LHD but not both space filling i.e. distributed evenly over the space



Space filling: Latin Hypercube design

Use measure of spread to assess quality of design

Examples:

- Maxmin distance design: design D that maximises smallest distance between any 2 points in D
- Minmax distance design: design D that minimises the largest distance between any point in the experimental region and the design
- Optimal average distance design: design D that minimises average distance between pairs of points in D



Optimal Designs

Good where;

- Factorial or fractional factorial require too many runs
- The design space is constrained

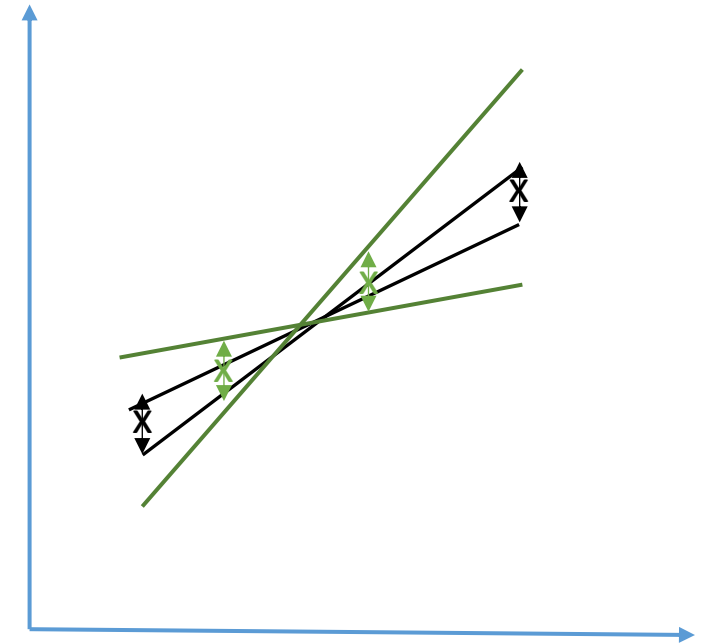
Require good knowledge about the model type -> hence system

- Formulate purpose of experiment in terms of optimising an objective
- Select design such that the design (i.e., set of points from experimental region) optimises some objective

Example:

- Fit straight line to given data $x=[x_1, x_2, \dots, x_n]$, response variable y
- Goal: select design to give most precise (**minimum variance**) estimate of slope

↖ D-optimal



Optimal design: D-Optimality Criterion

- Linear system

$$y = \beta_1 X_1 + \beta_2 X_2$$

- The fitted model will be

$$\hat{y} = \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2$$

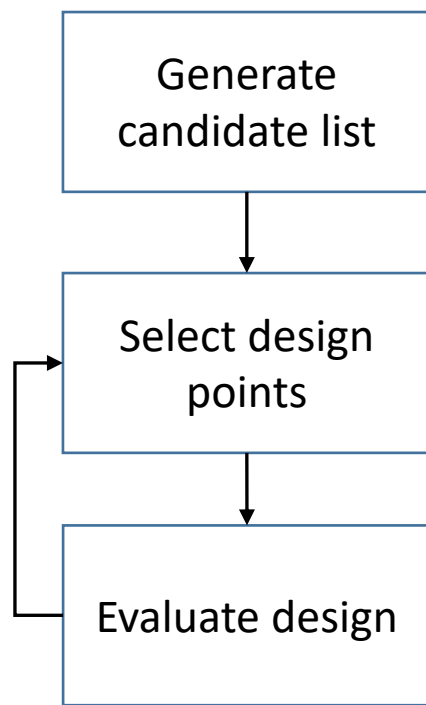
D-optimal design
minimises the
covariance of the
parameter
estimates for a
specified model

Where $\hat{\beta}_1$ and $\hat{\beta}_2$ are sample based estimates of β_1 and β_2 (*true value*)



Optimal design: D-Optimality Criterion

Designs A, B and C.
All two-factor, six-point designs

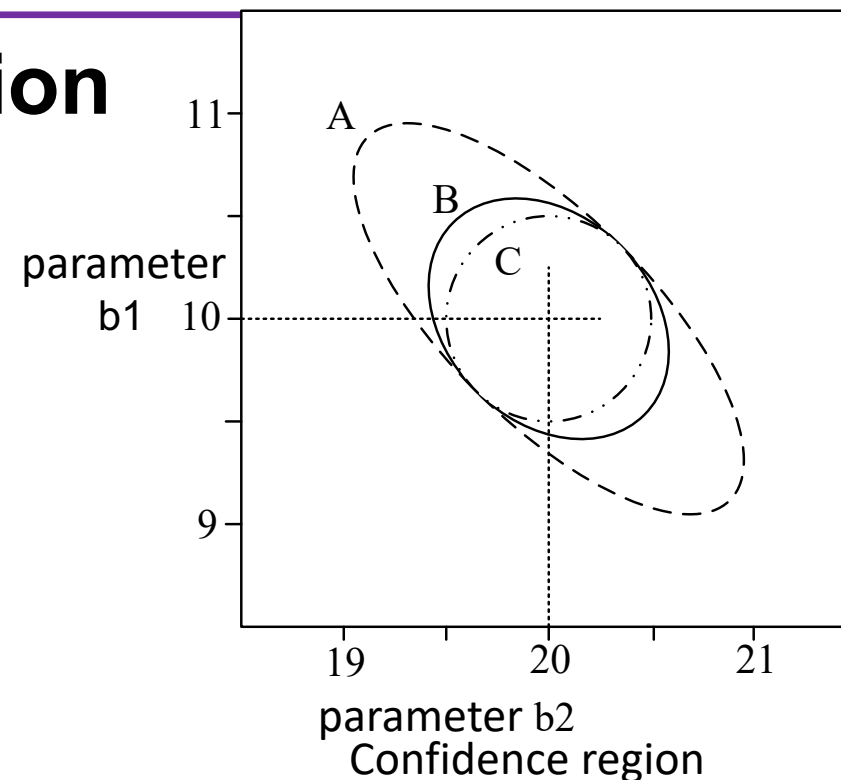


Design					
A		B		C	
0.75	0.25	1.0	0.0	1.0	0.0
0.75	0.25	1.0	0.0	1.0	0.0
0.50	0.50	0.5	0.5	1.0	0.0
0.50	0.50	0.5	0.5	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0

X_A

X_B

X_C



D-optimal design minimises the covariance of the parameter estimates for a specified model i.e. $\text{main}(D = |X^T X|^{-1})$



Optimal design: D-Optimality Criterion

- Table displays three possible six-point designs
- Figure displays joint confidence region for parameters b_1 and b_2 on the assumption that $b_1=10$, $b_2=20$, and $s=0.25$
- The largest ellipse is a 95% joint confidence region for b_1 and b_2 based on design A.
- The middle-sized ellipse is the corresponding region based on design B, while the smallest ellipse is for design C.
- The joint confidence region gets smaller and smaller, and estimates of b_1 and b_2 have become more and more precise



Confidence intervals for parameters

linear regression model

$$y_i = b_0 + b_1 x_{i,1} + b_2 x_{i,2} + \dots + b_K x_{i,K} + e_i$$

Confidence interval for estimated coefficients

$$\hat{b}_1 \pm t_{\alpha/2} \sqrt{\text{var}(\hat{b}_1)}$$

$t_{\alpha/2}$ is obtained from a t distribution with $n - 2$ degrees of freedom

$$\text{var}(\hat{\beta}) = s_e^2 (\mathbf{X}^T \mathbf{X})^{-1}$$

Need to maximise this

variances of the parameters along the diagonal,
and the covariances as the off-diagonal elements

degrees of freedom of the t distribution is

$$df = n - K$$

where K is the number of predictors in the model,
and n is the sample size.

df	0.95	0.99
2	4.303	9.925
3	3.182	5.841
4	2.776	4.604
5	2.571	4.032
8	2.306	3.355
10	2.228	3.169
20	2.086	2.845
50	2.009	2.678
100	1.984	2.626

t table

	b1		b2	
Design	Low limit	High limit	Low limit	High limit
A	9.25	10.75	19.25	20.75
B	9.55	10.45	19.55	20.45
C	9.60	10.4	19.60	20.4



Optimal design: D-Optimality Criterion

Design A is put in the form of a design matrix and covariance matrix

Design					
A		B		C	
0.75	0.25	1.0	0.0	1.0	0.0
0.75	0.25	1.0	0.0	1.0	0.0
0.50	0.50	0.5	0.5	1.0	0.0
0.50	0.50	0.5	0.5	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0

Determinant of the information matrix

$$|X'X| = \begin{vmatrix} 1.75 & 1.25 \\ 1.25 & 1.75 \end{vmatrix} = (1.75)^2 - (1.25)^2 = 1.5$$

Information matrix

$$X'X =$$

$$\begin{bmatrix} 0.75 & 0.75 & 0.50 & 0.50 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.50 & 0.50 & 0.75 & 0.75 \end{bmatrix}$$

$$\begin{bmatrix} 0.75 & 0.25 \\ 0.75 & 0.25 \\ 0.50 & 0.50 \\ 0.50 & 0.50 \\ 0.25 & 0.75 \\ 0.25 & 0.75 \end{bmatrix}$$

$$= \begin{bmatrix} 1.75 & 1.25 \\ 1.25 & 1.75 \end{bmatrix}$$

Thus for design A



Optimal design: D-Optimality Criterion

The relative areas of ellipses A, B, and C in Figure, are:

Designs A, B, C
 $X'X$ and $|X'X|$
determinants:

$$\frac{1}{\sqrt{1.5}} : \frac{1}{\sqrt{6}} : \frac{1}{\sqrt{9}} \equiv 1.0 : 0.50 : 0.41$$

Design	$X'X$	$ X'X $
A	$\begin{bmatrix} 1.75 & 1.25 \\ 1.25 & 1.75 \end{bmatrix}$	1.5
B	$\begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 2.5 \end{bmatrix}$	6.0
C	$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	9.0

Best design – not necessarily optimal



Other Optimal designs

- D-optimal designs minimise the covariance estimates of the model parameters
- A-optimal designs minimises the average variance of the estimates of the model parameters
- V-optimal designs: minimise average prediction variance over specific points

$$\frac{(y_1 - m) + (y_2 - m) + \dots (y_i - m) + \dots + (y_n - m)}{n}$$



Summary

- Classical design:
 - Linear model: full factorial, fractional factorial
 - Quadratic model : central composite, Box-Behnken design
- Space filling: low knowledge about the system
- Optimal design: know system well and know the model



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