

Task3:

The Invariant: $x * y = k * n + \text{res}$

Step 1: checked if the formula works before the loop starts.

At the beginning, $k = x$, $n = y$, and $\text{res} = 0$.

If we plug these in:

$$x * y = x * y + 0$$

This is obviously true.

Step 2: We assume the formula is true at the start of the loop. We need to show it stays true after we change the variables.

- **When k is even:**

We just cut k in half and double n .

Mathematically, $(k/2) * (2n)$ is the same as $k * n$.

Since res doesn't change here, the total stays the same.

- **When k is odd:**

Since k is odd, when we divide by integer 2, we lose a value. So we handle that first by adding n to res .

After that, we divide k by 2 and multiply n by 2.

Because we moved that extra value into res , the equation $(k*n) + \text{res}$ still equals $x*y$.

Step 3:

The loop only stops when k becomes 0.

Using the invariant formula:

$$x * y = 0 * n + \text{res}$$

This simplifies to $x * y = \text{res}$.

This means **res** holds the correct answer, so the code is correct.

Task4:

Precondition: The array A exists.

Post-condition: The function returns the largest index i where $A[i] == x$, or -1 if x is not in the array.

The Loop Invariant:

For every index k such that $i < k < n$, $A[k] \neq x$.

//scanned the right side of i and verified x isn't there.

A. Initialization

At the start, $i = n - 1$.

The range of "indices greater than i " is empty (there are no indices greater than $n-1$).

Therefore, the statement "all values to the right of i are not x " is true.

B. Maintenance (Inside the loop)

In each step, we check $A[i]$.

- **If $A[i] == x$:**
We return i . Since the invariant told us that no index *greater* than i holds x , finding x at i guarantees that i is the **last** occurrence. This satisfies the post-condition immediately.
- **If $A[i] \neq x$:**
The invariant previously said x is not at indices $i+1, i+2, \dots$.
Now we checked index i and it is also not x .
So, x is not at indices $i, i+1, i+2, \dots$.
We decrement i to $i-1$. The statement " x is not to the right of the new i " holds true.

C. Termination

The loop stops when $i = -1$.

Using our invariant one last time: "For all $k > -1$, $A[k] \neq x$."

Since indices $0, 1, \dots, n-1$ are all > -1 , this means x is nowhere in the array.

The function returns -1 , which is the correct answer.

