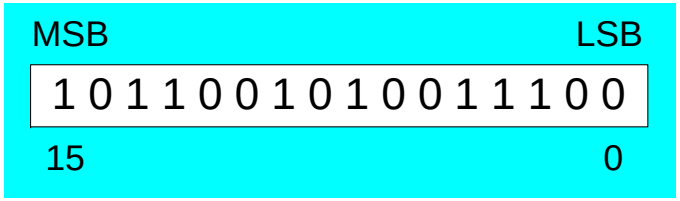
Data Representation

- Binary Numbers
  - Translating between binary and decimal
- Binary Addition
- Integer Storage Sizes
- Hexadecimal Integers
  - Translating between decimal and hexadecimal
  - Hexadecimal subtraction
- Signed Integers
  - Binary subtraction
- Character Storage

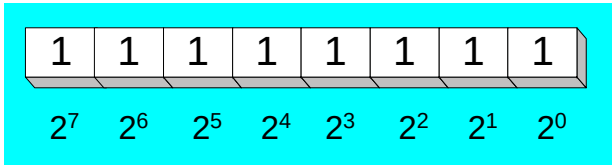
Binary Numbers

- Digits are 1 and 0
  - 1 = true
  - 0 = false
- MSB – most significant bit
- LSB – least significant bit
- Bit numbering:



Binary Numbers

- Each digit (bit) is either 1 or 0
- Each bit represents a power of 2:



Every binary number is a sum of powers of 2

Table 1-3 Binary Bit Position Values.

2^n	Decimal Value	2^n	Decimal Value
2^0	1	2^8	256
2^1	2	2^9	512
2^2	4	2^{10}	1024
2^3	8	2^{11}	2048
2^4	16	2^{12}	4096
2^5	32	2^{13}	8192
2^6	64	2^{14}	16384
2^7	128	2^{15}	32768

Translating Binary to Decimal

Weighted positional notation shows how to calculate the decimal value of each binary bit:

$$dec = (D_{n-1} \times 2^{n-1}) + (D_{n-2} \times 2^{n-2}) + \ldots + (D_1 \times 2^1) + (D_0 \times 2^0)$$

D = binary digit

Convert into Decimal base 10

$$(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$$

Convert binary to Decimal 11010.11

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$$

In general, a number expressed in a base-r system has coefficients multiplied by powers of r:

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1}$$
$$+ a_{-2} \cdot r^{-2} + \cdots + a_{-m} \cdot r^{-m}$$

binary 00001001 = decimal 9:

$$(1 \times 2^3) + (1 \times 2^0) = 9$$

Means Integer

Translating Unsigned Decimal to Binary

- Repeatedly divide the decimal integer by 2. Each remainder is a binary digit in the translated value:

Division	Quotient	Remainder
37 / 2	18	1
18 / 2	9	0
9 / 2	4	1
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1

write it in backward order or reverse order
1 0 0 1 0 1

After decimal point;
write binary in straight order

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

37.6875 = 100101.1011

Irvine, Kip R. Assembly Language for Intel-Based Computers, 2003.

Exercises

1.3 Convert the following numbers with the indicated bases to decimal:

(a)* $(4310)_5$ (b)* $(198)_{12}$

(c) $(735)_8$ (d) $(525)_6$

1.3

$$(4310)_5 = 4 * 5^3 + 3 * 5^2 + 1 * 5^1 = 580_{10}$$

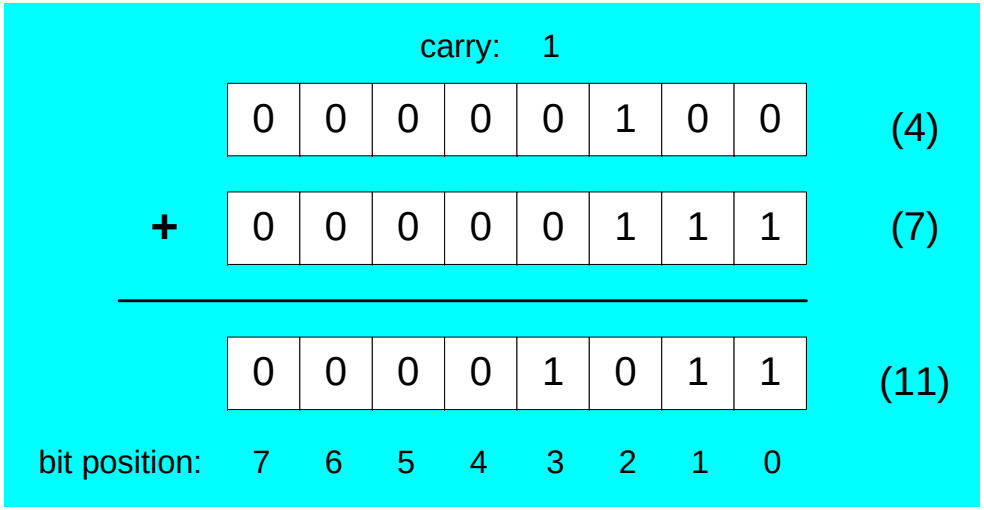
$$(198)_{12} = 1 * 12^2 + 9 * 12^1 + 8 * 12^0 = 260_{10}$$

$$(735)_8 = 7 * 8^2 + 3 * 8^1 + 5 * 8^0 = 477_{10}$$

$$(525)_6 = 5 * 6^2 + 2 * 6^1 + 5 * 6^0 = 197_{10}$$

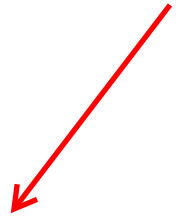
Binary Addition

- Starting with the LSB, add each pair of digits, include the carry if present.



Binary Addition Rules:

1. 0 + 0 = 0 (No carry)
2. 0 + 1 = 1 (No carry)
3. 1 + 0 = 1 (No carry)



4. 1 + 1 = 10 (Sum = 0, Carry = 1)
5. 1 + 1 + 1 = 11 (Sum = 1, Carry = 1)

Integer Storage Sizes

Standard sizes:

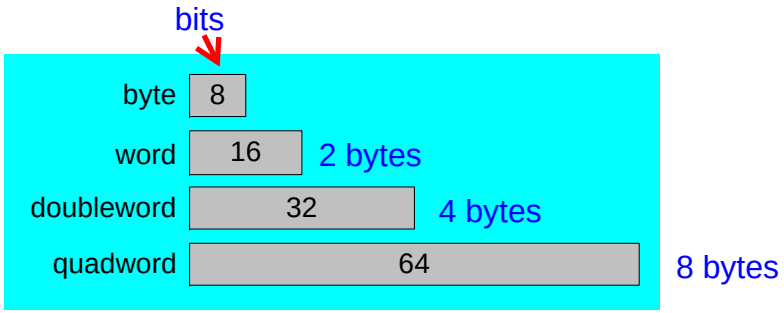


Table 1-4 Ranges of Unsigned Integers.

Storage Type	Range (low–high)	Powers of 2
Unsigned byte	0 to 255	0 to $(2^8 - 1)$
Unsigned word	0 to 65,535	0 to $(2^{16} - 1)$
Unsigned doubleword	0 to 4,294,967,295	0 to $(2^{32} - 1)$
Unsigned quadword	0 to 18,446,744,073,709,551,615	0 to $(2^{64} - 1)$

What is the largest unsigned integer that may be stored in 20 bits?

$(2^{20} - 1) \rightarrow 2^{20} = 1{,}048{,}576 - 1 \rightarrow 1{,}048{,}575$

Hexadecimal Integers

Binary values are represented in hexadecimal.

Table 1-5 Binary, Decimal, and Hexadecimal Equivalents.

Binary	Decimal	Hexadecimal	Binary	Decimal	Hexadecimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	10	A
0011	3	3	1011	11	B
0100	4	4	1100	12	C
0101	5	5	1101	13	D
0110	6	6	1110	14	E
0111	7	7	1111	15	F

Translating Binary to Hexadecimal

- Each hexadecimal digit corresponds to 4 binary bits.
- Example: Translate the binary integer 000101101010011110010100 to hexadecimal:

1	6	A	7	9	4
0001	0110	1010	0111	1001	0100

Converting Hexadecimal to Decimal

- Multiply each digit by its corresponding power of 16:

  $$dec = (D_3 \times 16^3) + (D_2 \times 16^2) + (D_1 \times 16^1) + (D_0 \times 16^0)$$

- Hex 1234 equals $(1 \times 16^3) + (2 \times 16^2) + (3 \times 16^1) + (4 \times 16^0)$, or decimal 4,660.

- Hex 3BA4 equals $(3 \times 16^3) + (11 * 16^2) + (10 \times 16^1) + (4 \times 16^0)$, or decimal 15,268.

Powers of 16

Used when calculating hexadecimal values up to 8 digits long:

16^n	Decimal Value	16^n	Decimal Value
16^0	1	16^4	65,536
16^1	16	16^5	1,048,576
16^2	256	16^6	16,777,216
16^3	4096	16^7	268,435,456

Converting Decimal to Hexadecimal

Division	Quotient	Remainder
422 / 16	26	6
26 / 16	1	A
1 / 16	0	1

reverse order

decimal 422 = 1A6 hexadecimal

Convert decimal 153 to octal. The required base r is 8. First, 153 is divided by 8 to give an integer quotient of 19 and a remainder of 1. Then 19 is divided by 8 to give an integer quotient of 2 and a remainder of 3. Finally, 2 is divided by 8 to give a quotient of 0 and a remainder of 2. This process can be conveniently manipulated as follows:

153	
19	1
2	3
0	$2 = (231)_8$

Convert Decimal Fraction to Octal Fraction

$$0.513 \times 8 = 4.104$$

$$0.104 \times 8 = 0.832$$

$$0.832 \times 8 = 6.656$$

$$0.656 \times 8 = 5.248$$

$$0.248 \times 8 = 1.984$$

$$0.984 \times 8 = 7.872$$

ant figures, is obtained from the int

$$(0.513)_{10} = (0.406517\ldots)_8$$

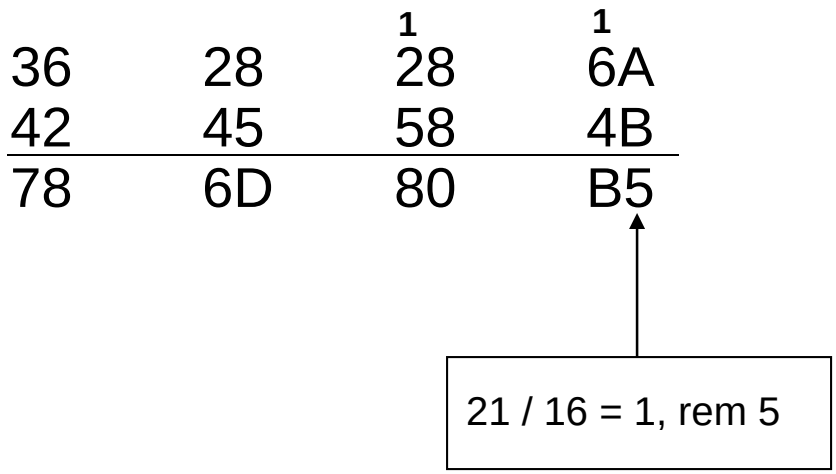
straight order
only digit before
decimal

Table 1.2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Hexadecimal Addition

- Divide the sum of two digits by the number base (16). The quotient becomes the carry value, and the remainder is the sum digit.



A+B = 10 +11=21

21/16 = 1, rem 5

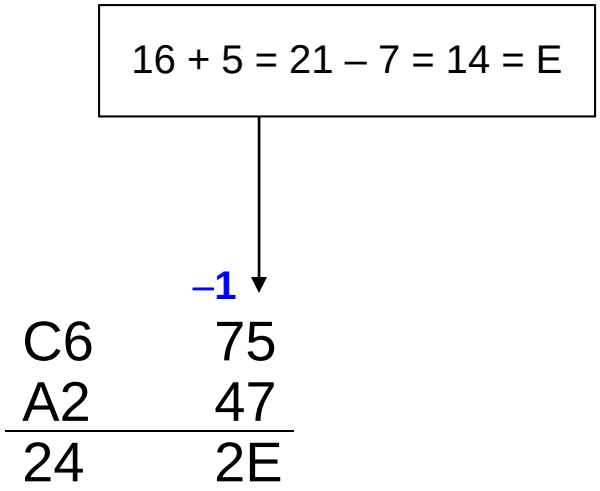
Decimal

convert to Hexdecimal

Important skill: Programmers frequently add and subtract the addresses of variables and instructions.

Hexadecimal Subtraction

- When a borrow is required from the digit to the left, add 16 to the current digit's value:



Practice: The address of **var1** is 00400020. The address of the next variable after var1 is 0040006A. How many bytes are used by var1?

Addition and Multiplication Examples

Add and multiply the following numbers without converting them to decimal.
(a) Binary numbers 1011 and 101.
(b) Hexadecimal numbers 2E and 34.

(a) 10000 and 110111

$$\begin{array}{r} 1011 \\ +101 \\ \hline 10000 \end{array} = 16_{10}$$

$$\begin{array}{r} 1011 \\ \times 101 \\ \hline 1011 \\ 1011 \\ \hline 110111 \end{array} = 55_{10}$$

(b) 62_h and 958_h

$$\begin{array}{rr} 2E_h & 0010\_1110 \\ +34_h & 0011\_0100 \\ \hline 62_h & 0110\_0010 \end{array} = 98_{10}$$

$$\begin{array}{r} 2E_h \\ \times 34_h \\ \hline B^{3}8 \\ 8^{2}A \\ \hline 9\ 5\ 8_h \end{array} = 2392_{10}$$

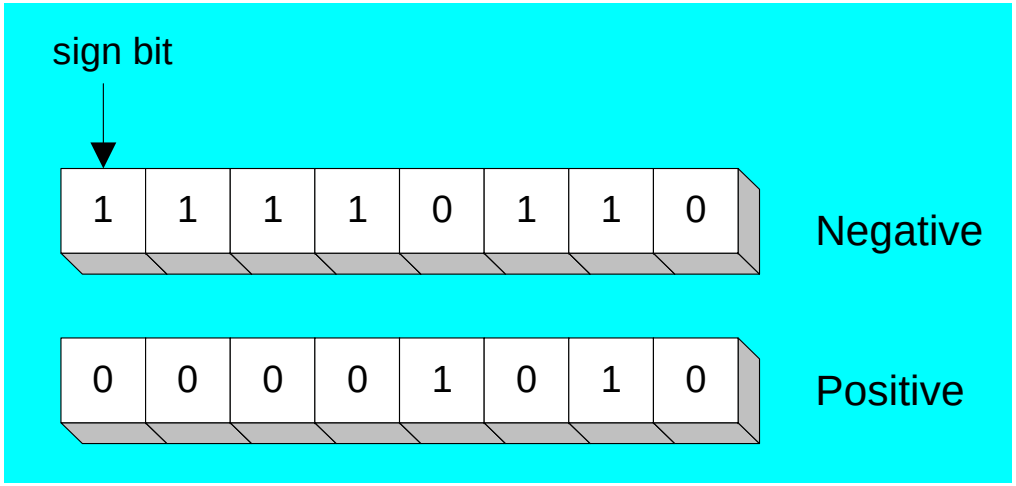
Hexadecimal Complement

(a) Find the 16's complement of B2FA.
(b) Convert B2FA to binary.
(c) Find the 2's complement of the result in (b).
(d) Convert the answer in (c) to hexadecimal and compare with the answer in (a).

	B2FA
15s comp:	4D05
16s comp:	4D06

Signed Integers

The highest bit indicates the sign. 1 = negative, 0 = positive



If the highest digit of a hexadecimal integer is > 7, the value is negative. Examples: 8A, C5, A2, 9D

signed-magnitude representation:	10001001
signed-1's-complement representation:	11110110
signed-2's-complement representation:	11110111

Ranges of Signed Integers

The highest bit is reserved for the sign. This limits the range:

Storage Type	Range (low–high)	Powers of 2
Signed byte	−128 to +127	-2^7 to $(2^7 - 1)$
Signed word	−32,768 to +32,767	-2^{15} to $(2^{15} - 1)$
Signed doubleword	−2,147,483,648 to 2,147,483,647	-2^{31} to $(2^{31} - 1)$
Signed quadword	−9,223,372,036,854,775,808 to +9,223,372,036,854,775,807	-2^{63} to $(2^{63} - 1)$

Practice: What is the largest positive value that may be stored in 20 bits?