Counting Sort

- We will consider three algorithms that are faster and work by not making comparisons.
- Counting sort assumes that the numbers to be sorted are in the range 1 to k where k is small.
- The basic idea is to determine the rank of each number in final sorted array.

Counting Sort

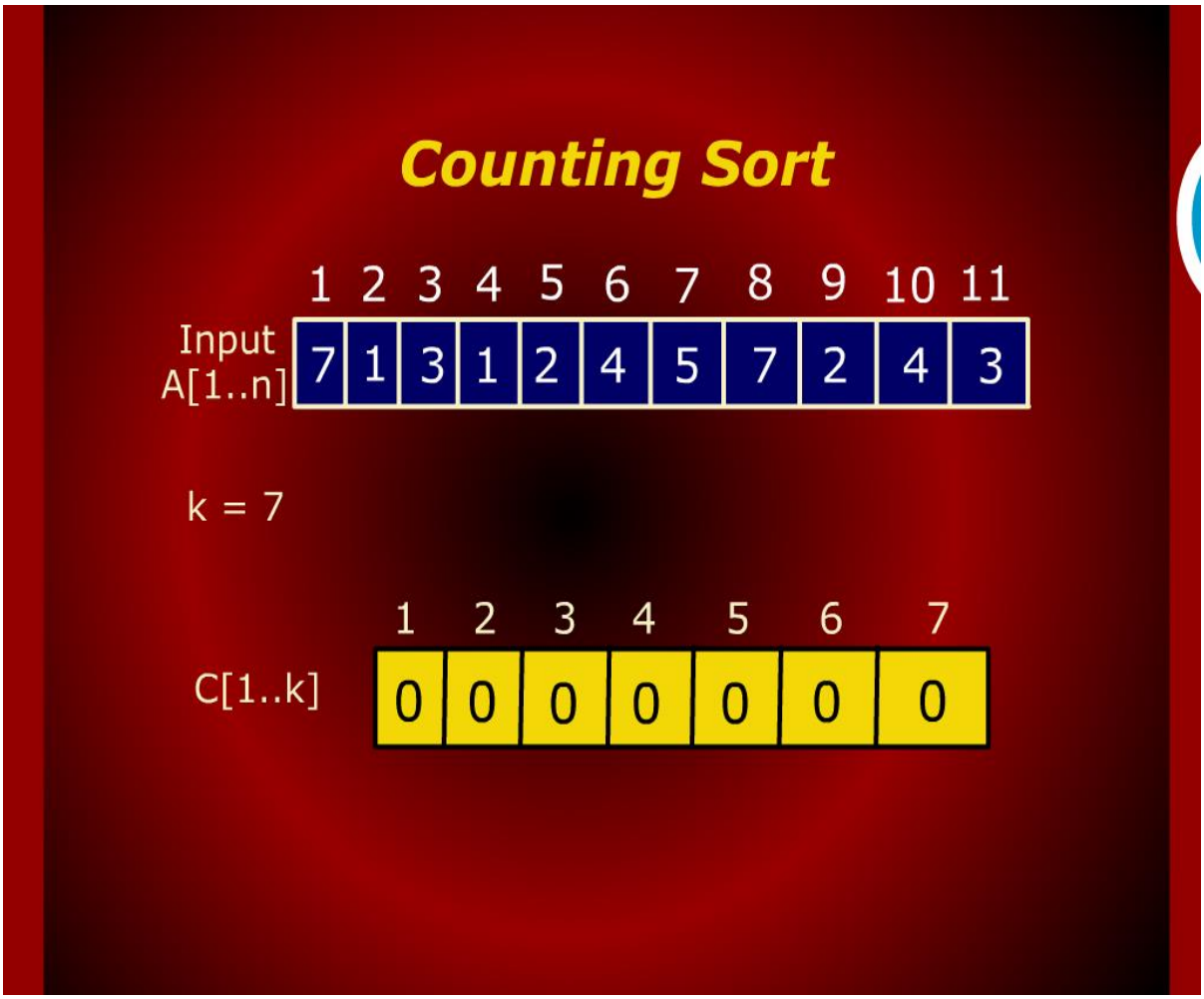
- Recall that the rank of an item is the number of elements that are less than or equal to it.
- Once we know the ranks, we simply copy numbers to their final position in an output array.
- The algorithm sorts in $\Theta(n + k)$.
- If k is $\Theta(n)$ then counting sort is an $\Theta(n)$ time algorithm.

Counting Sort

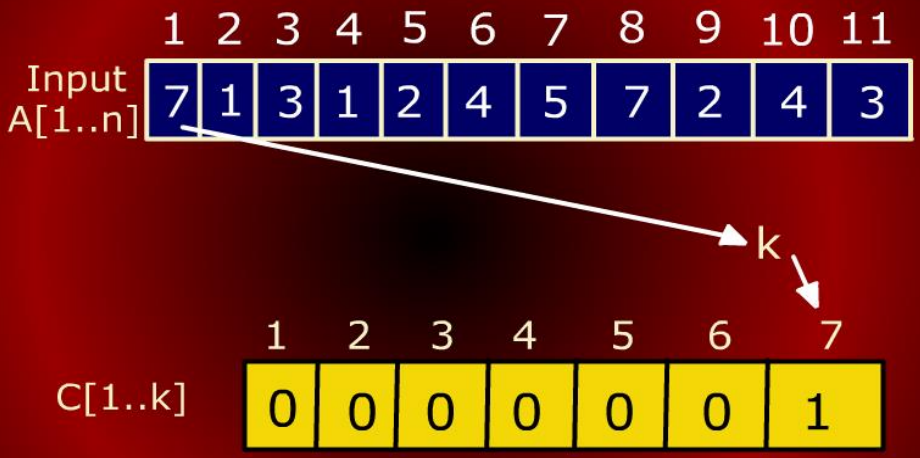
The algorithm uses three arrays.

1. A[1..n]: Holds the initial input.
2. B[1..n]: Array that holds the sorted output.
3. C[1..k]: Array of integers. C[x] is the rank of x in A, where $x \in [1..k]$.

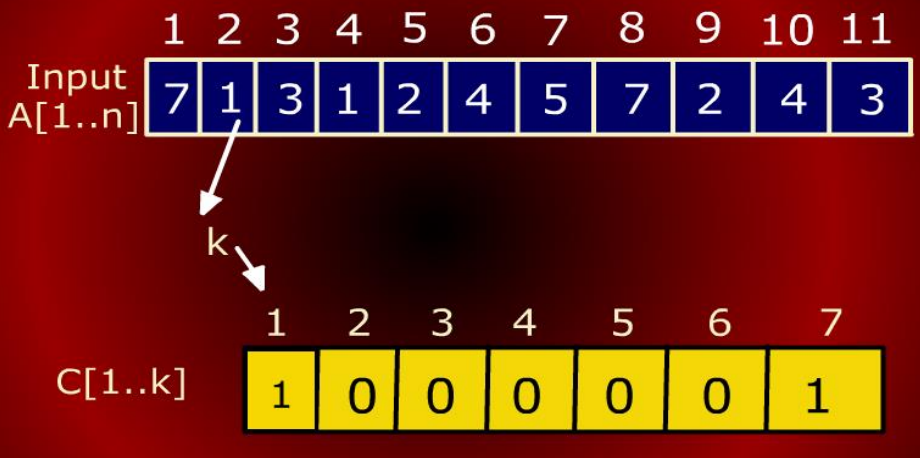
Counting Sort

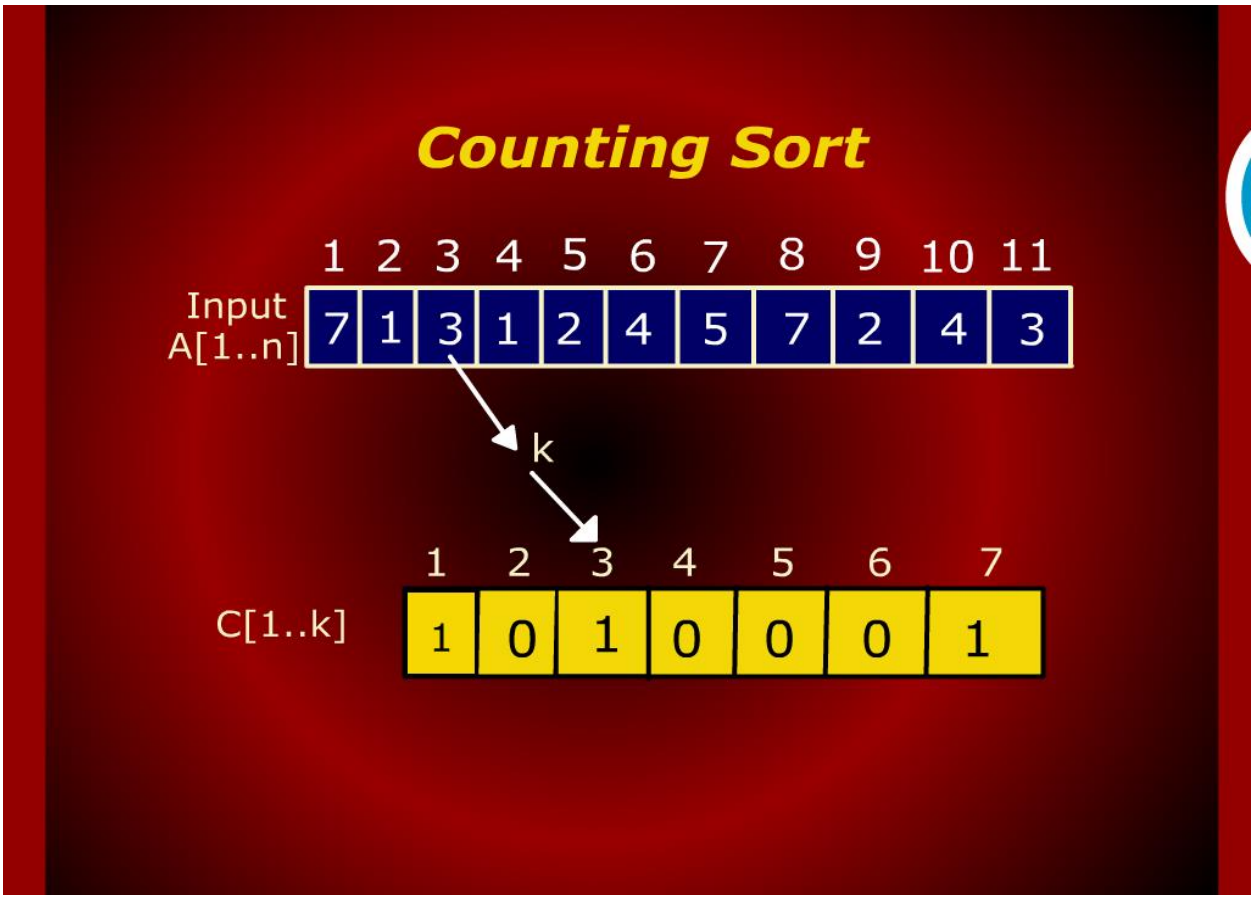


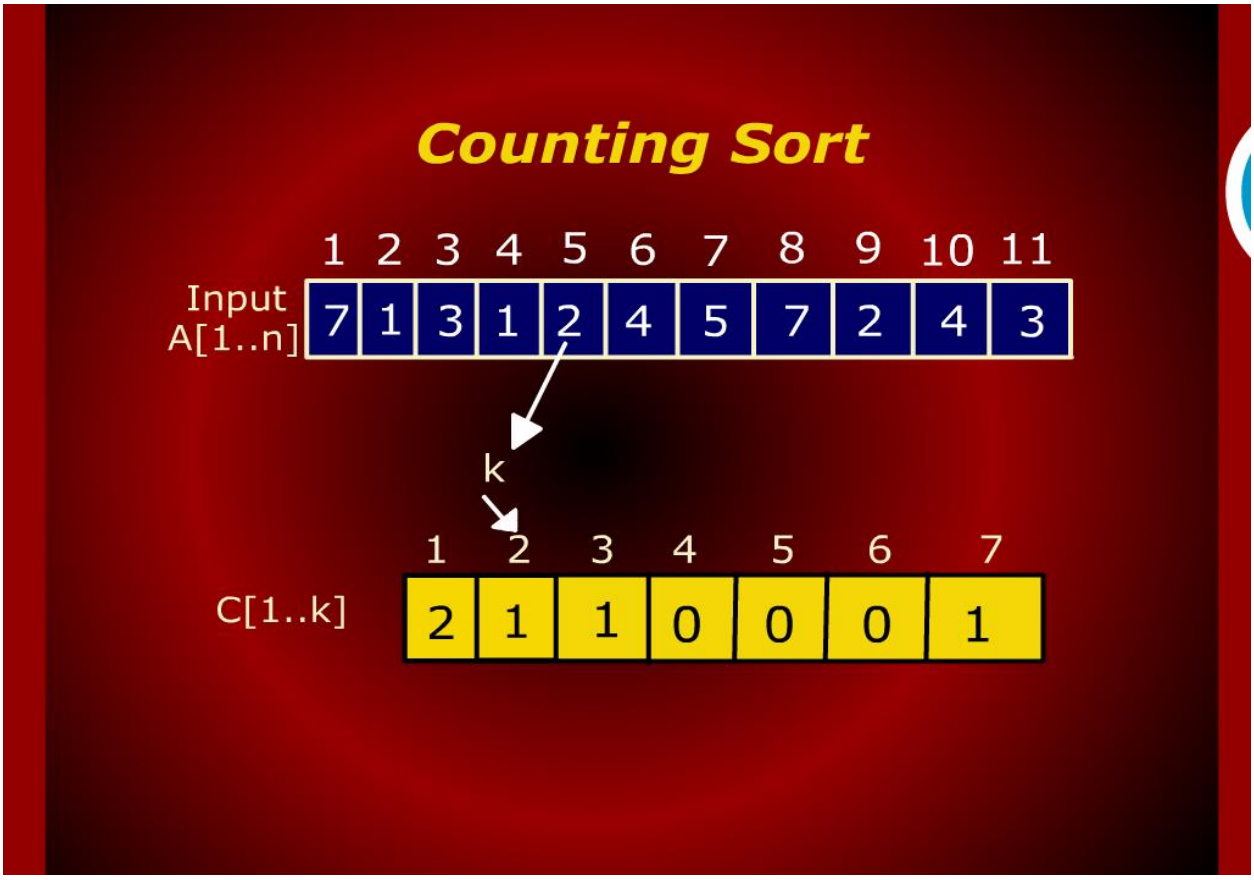
Counting Sort



Counting Sort




Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
k
1 2 3 4 5 6 7
C[1..k]
1 0 1 0 0 0 1


Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
k
1 2 3 4 5 6 7
C[1..k]
2 1 1 0 0 0 1

Counting Sort

	1	2	3	4	5	6	7	8	9	10	11
Input A[1..n]	7	1	3	1	2	4	5	7	2	4	3

Finally

	1	2	3	4	5	6	7
C[1..k]	2	2	2	2	1	0	2

Counting Sort

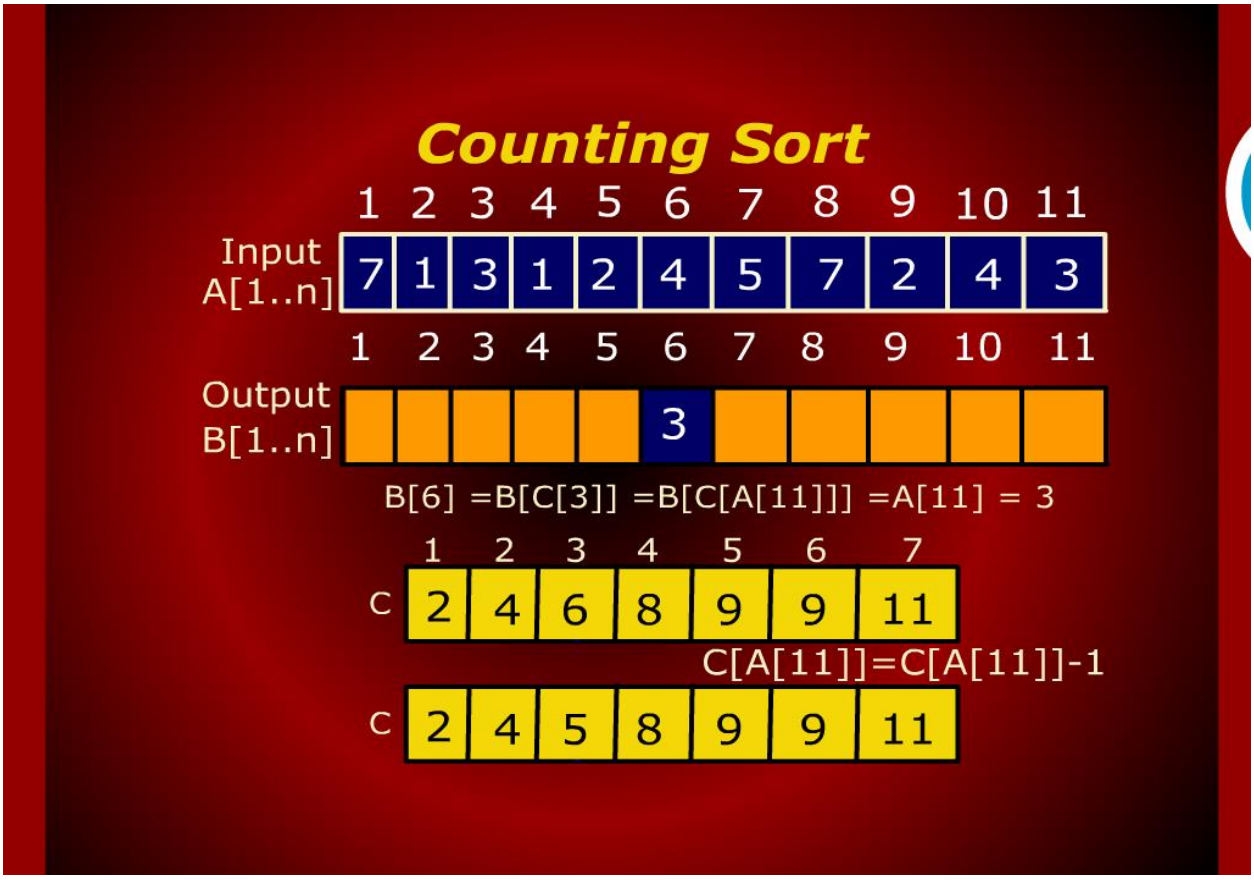
	1	2	3	4	5	6	7	8	9	10	11
Input A[1..n]	7	1	3	1	2	4	5	7	2	4	3

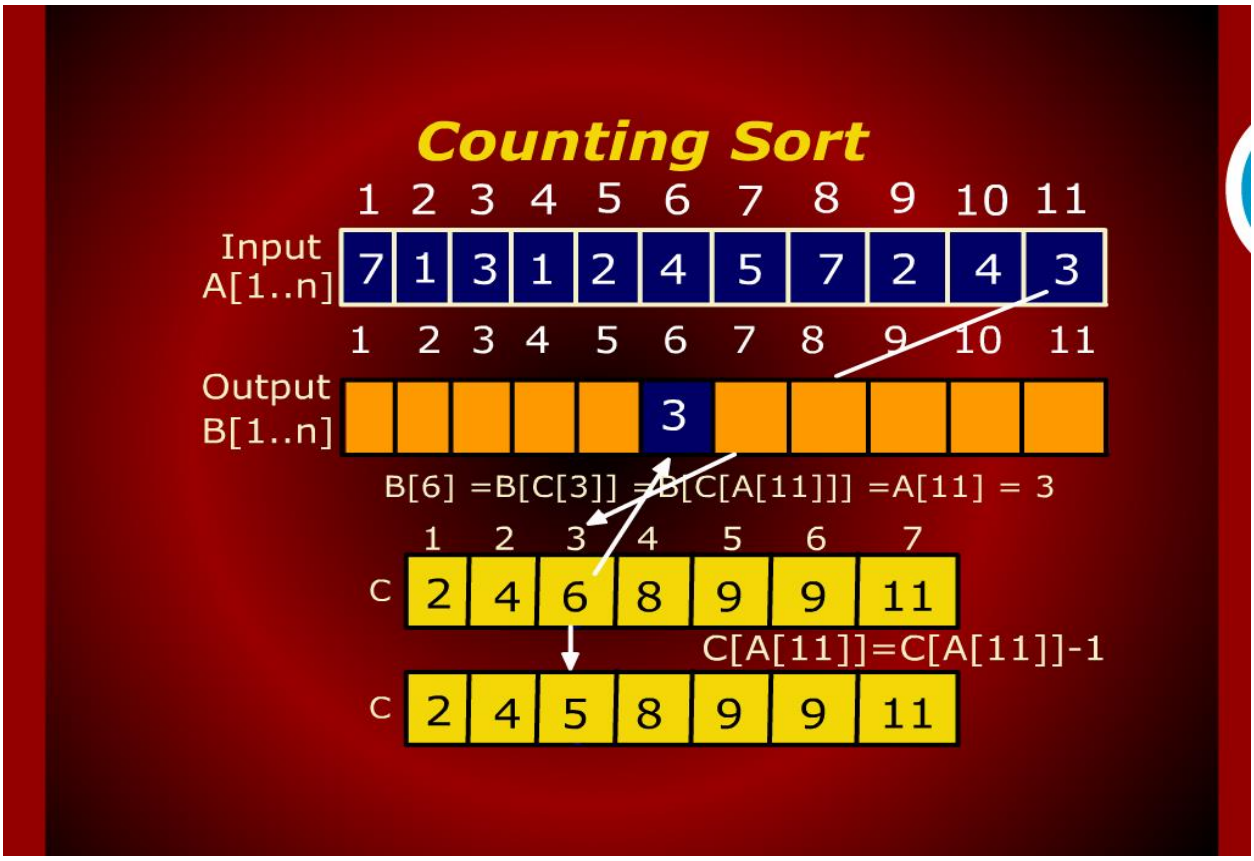
	1	2	3	4	5	6	7
C[1..k]	2	2	2	2	1	0	2

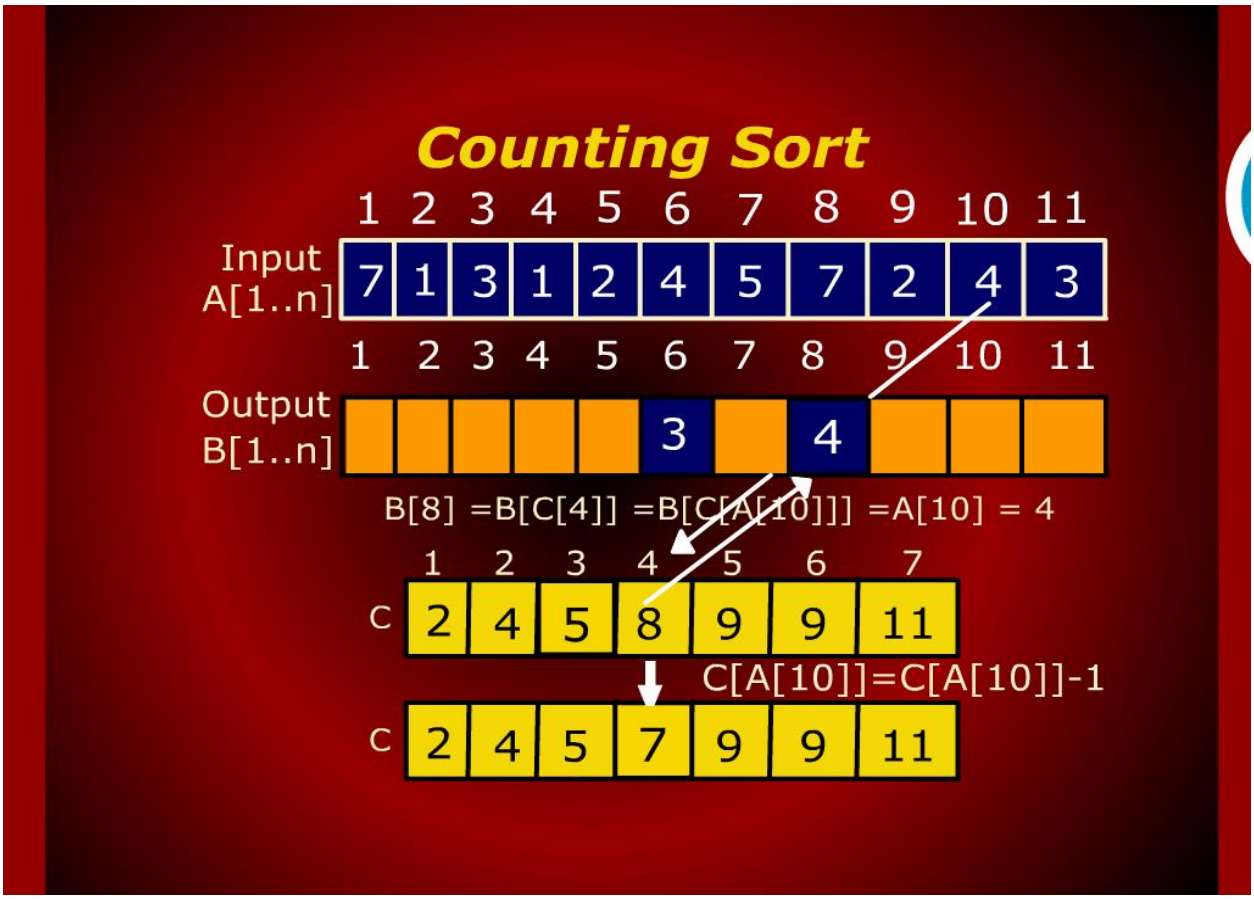
for i= 2 to 7
doC[i] =C[i] +C[i- 1]

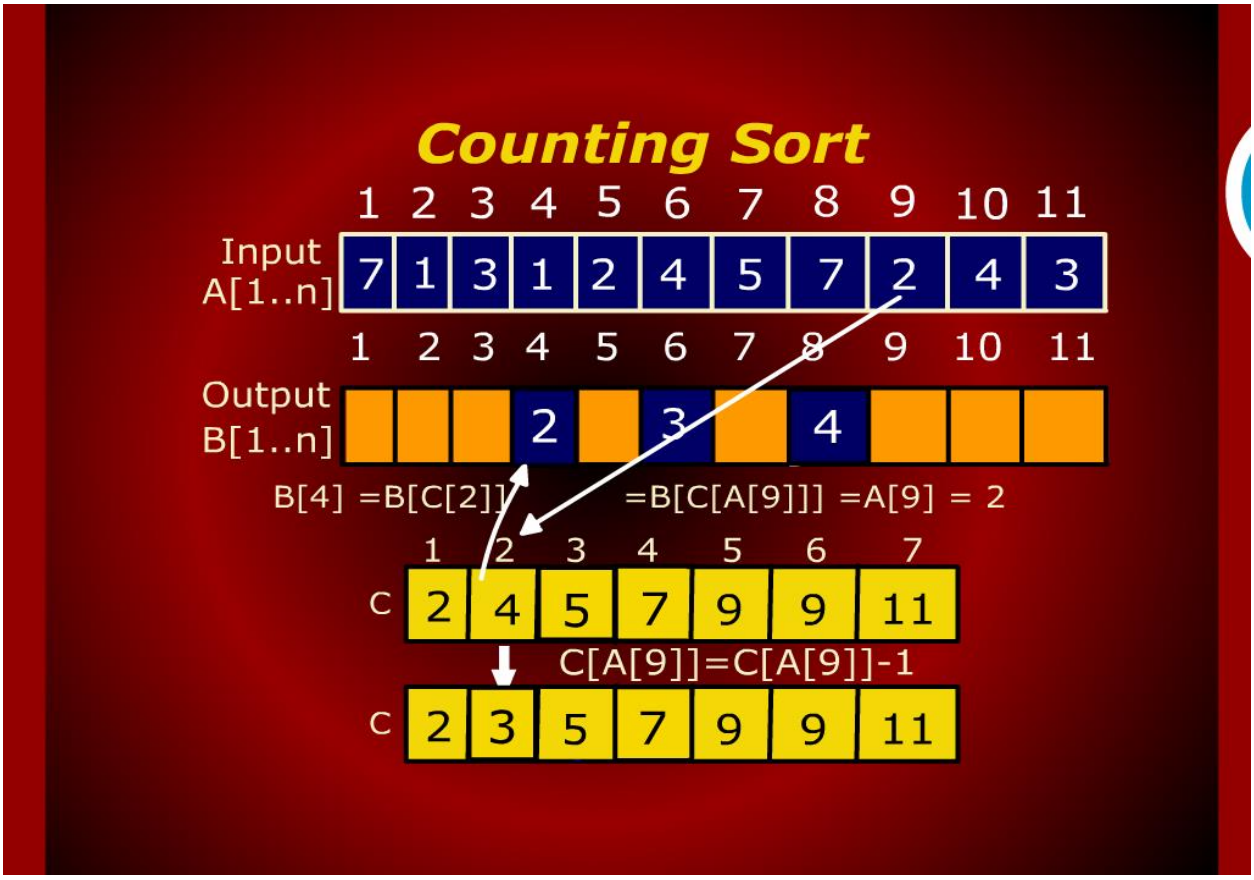
	1	2	3	4	5	6	7
C	2	4	6	8	9	9	11

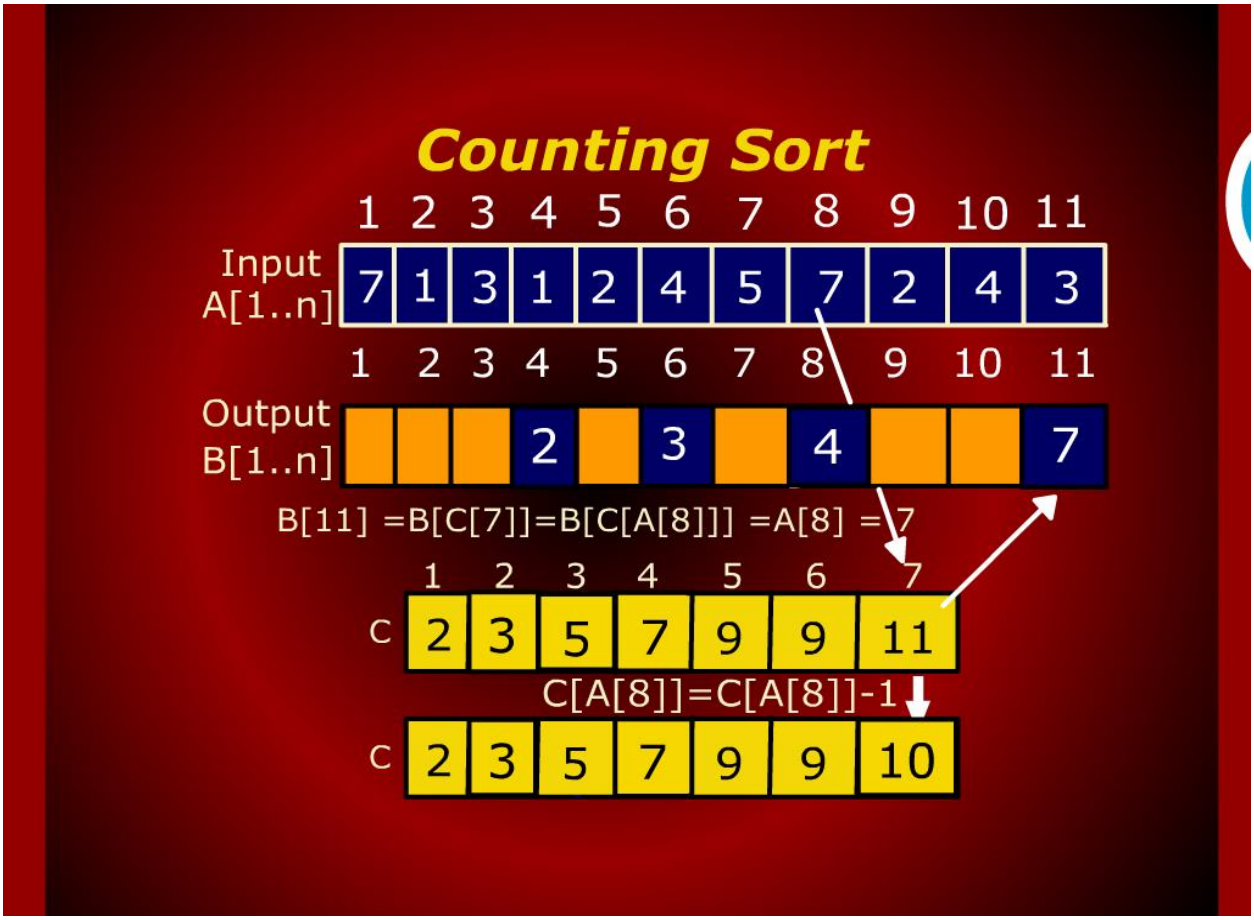
6 elements <= 3

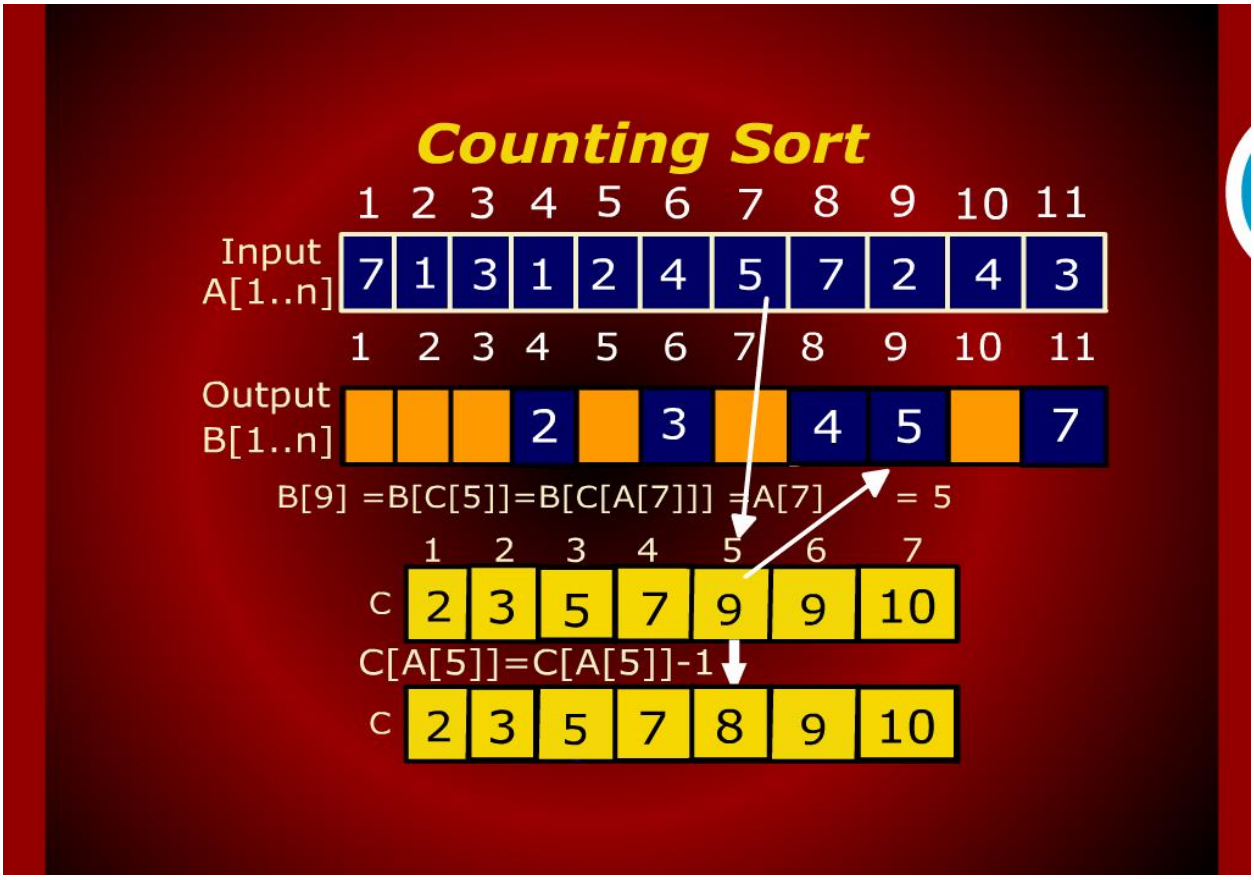

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output B[1..n]
3
B[6] =B[C[3]] =B[C[A[11]]] =A[11] = 3
1 2 3 4 5 6 7
C
2 4 6 8 9 9 11
C[A[11]]=C[A[11]]-1
C
2 4 5 8 9 9 11

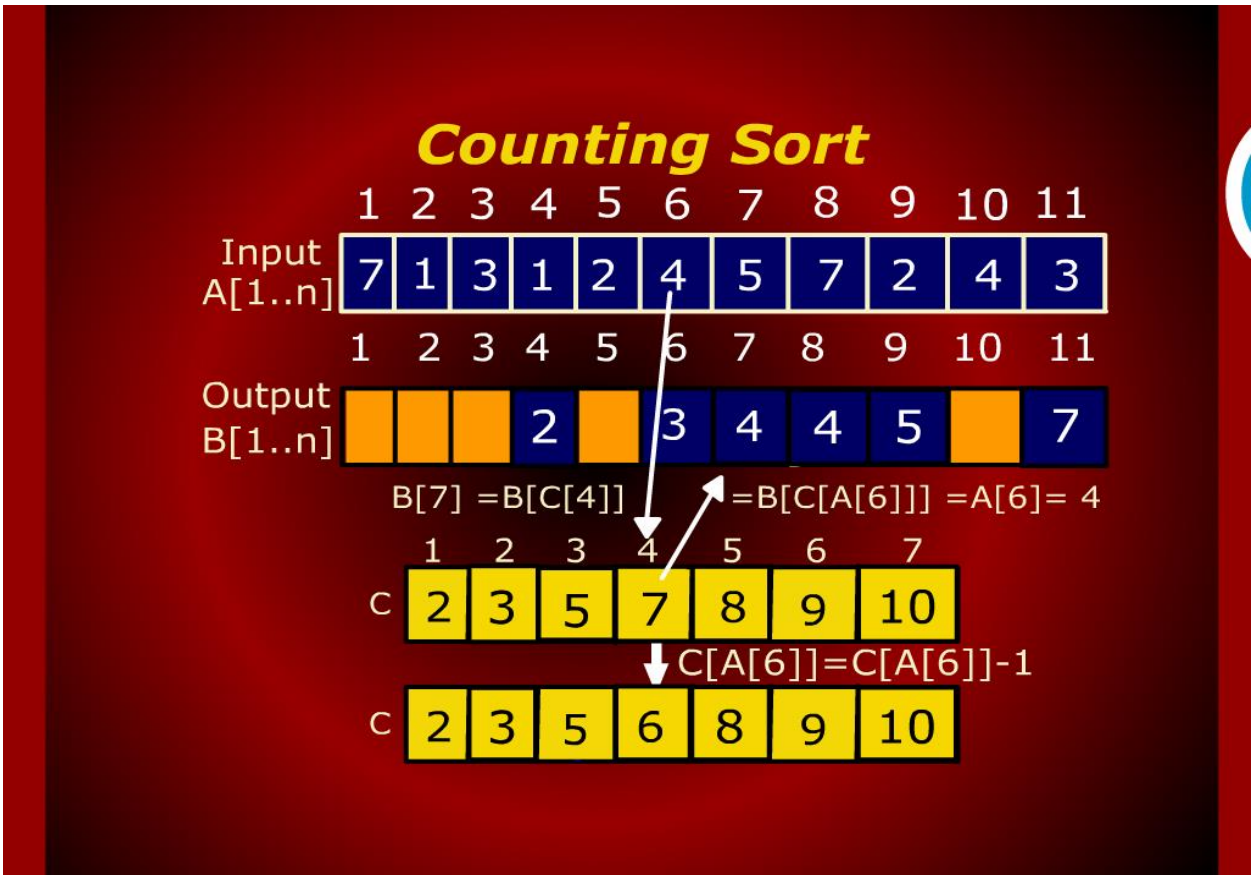

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output
B[1..n]
3
B[6] =B[C[3]] =B[C[A[11]]] =A[11] = 3
1 2 3 4 5 6 7
C
2 4 6 8 9 9 11
C[A[11]]=C[A[11]]-1
C
2 4 5 8 9 9 11

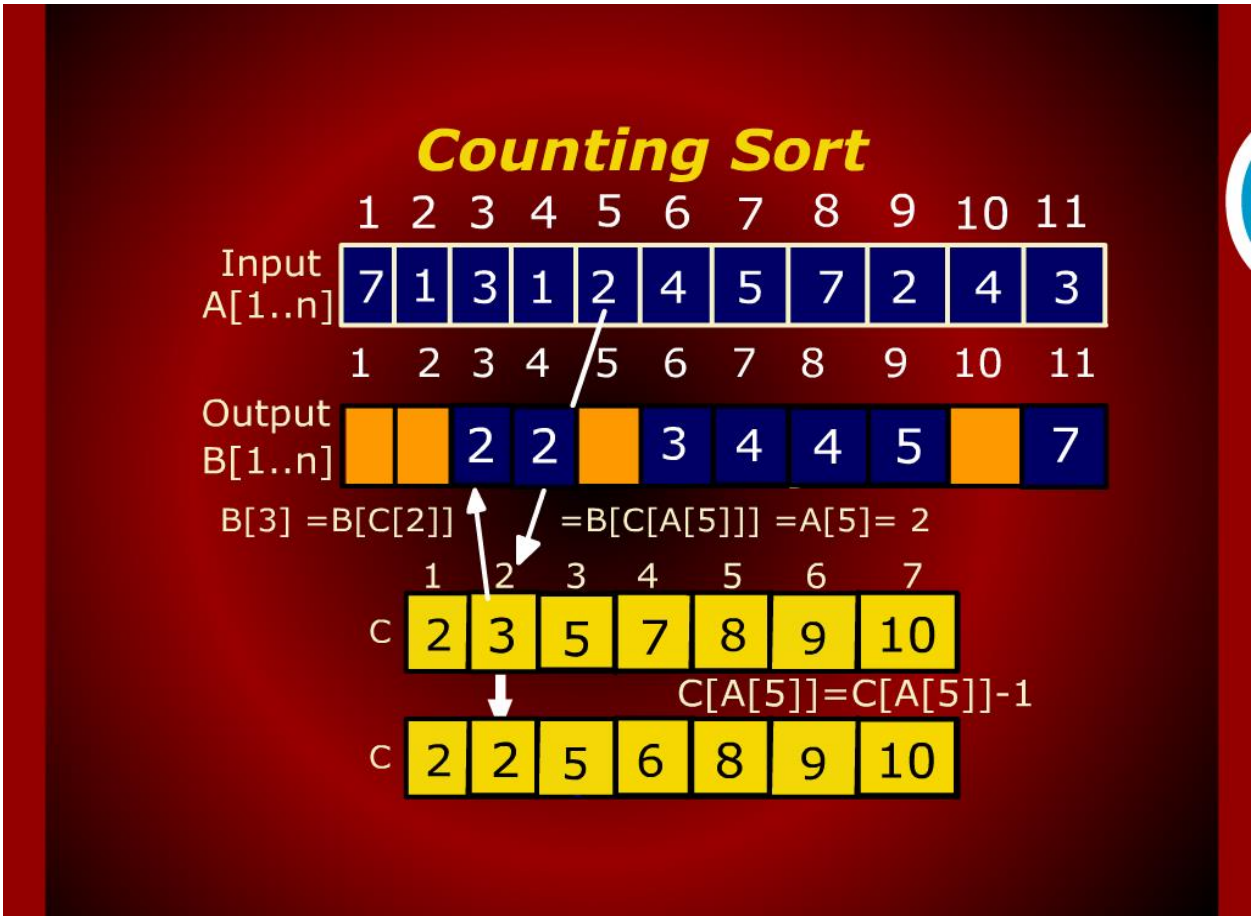

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output
B[1..n]
3 4
B[8] =B[C[4]] =B[C[A[10]]] =A[10] = 4
1 2 3 4 5 6 7
C
2 4 5 8 9 9 11
C[A[10]]=C[A[10]]-1
C
2 4 5 7 9 9 11

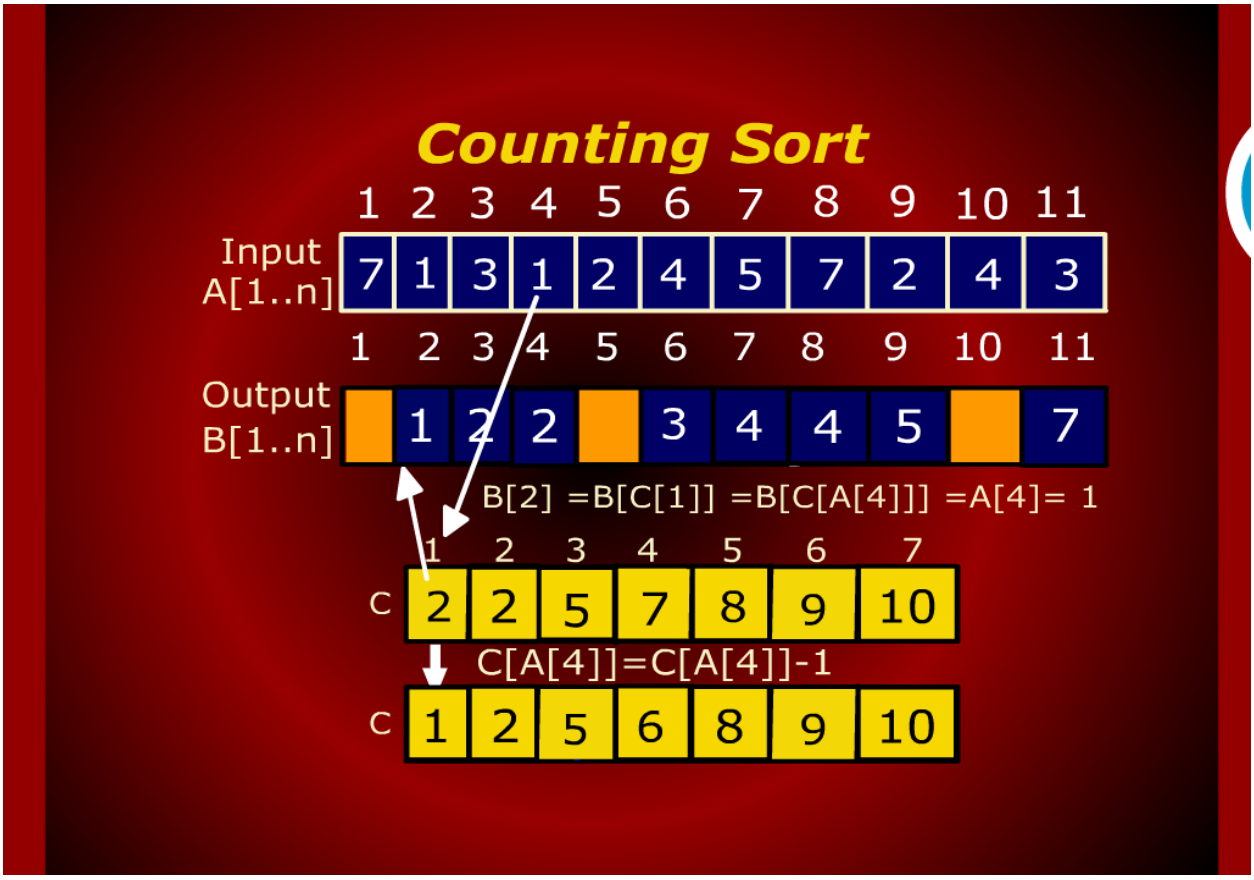

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output
B[1..n]
2 3 4
B[4] =B[C[2]] =B[C[A[9]]] =A[9] = 2
1 2 3 4 5 6 7
C
2 4 5 7 9 9 11
C[A[9]]=C[A[9]]-1
C
2 3 5 7 9 9 11

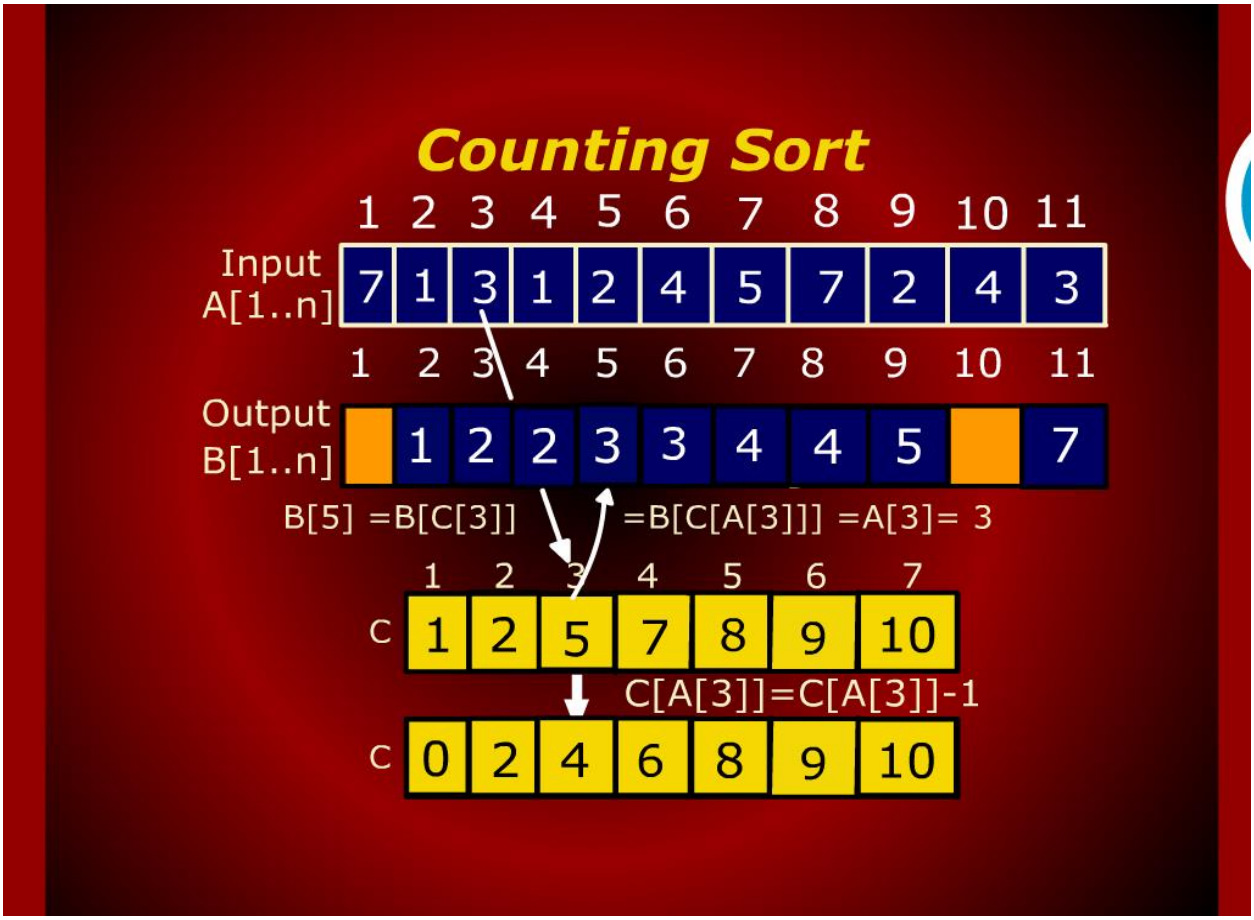

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output
B[1..n]
2 3 4 7
B[11] =B[C[7]]=B[C[A[8]]] =A[8] =7
1 2 3 4 5 6 7
C
2 3 5 7 9 9 11
C[A[8]]=C[A[8]]-1
C
2 3 5 7 9 9 10

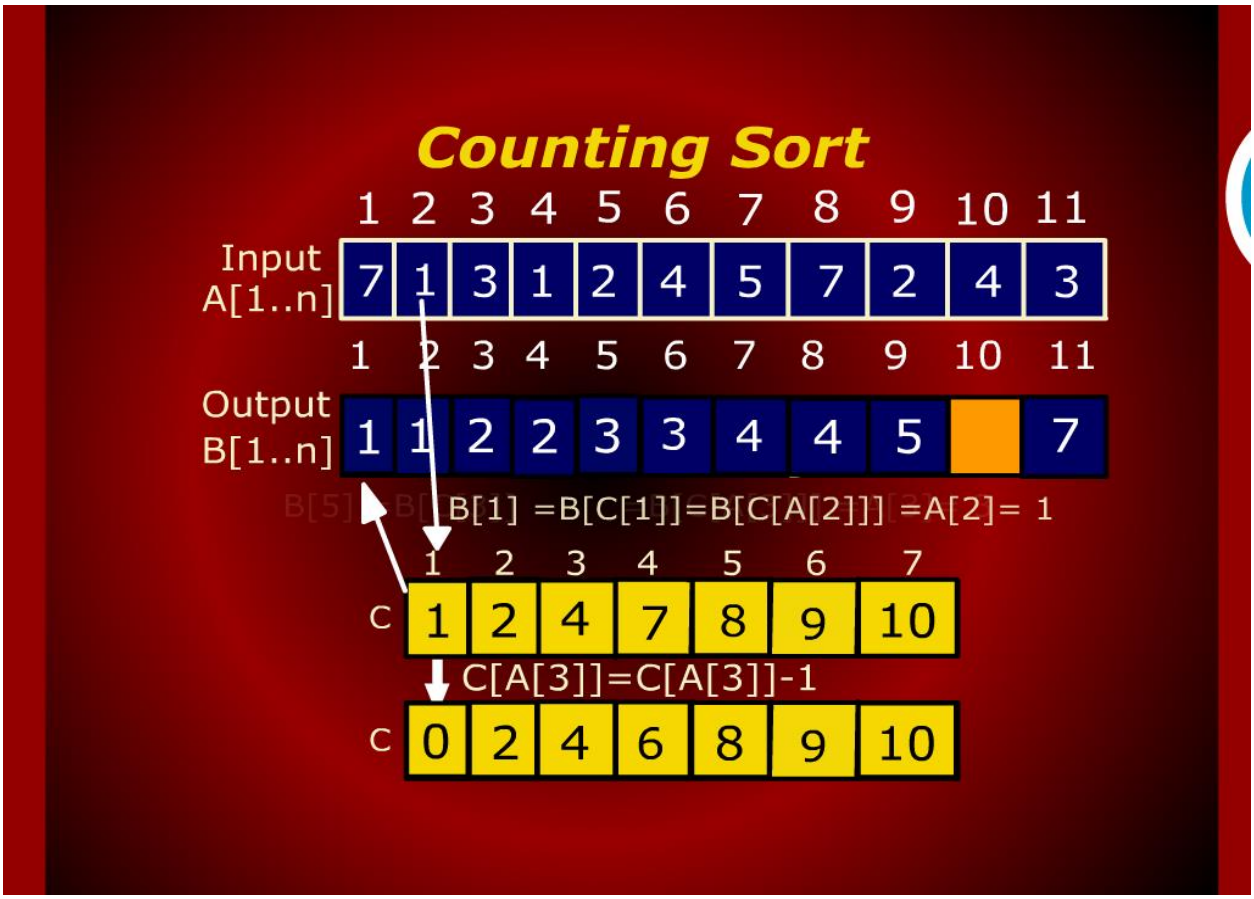

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output B[1..n]
2 3 4 5 7
B[9] =B[C[5]]=B[C[A[7]]] =A[7] = 5
1 2 3 4 5 6 7
C 2 3 5 7 9 9 10
C[A[5]]=C[A[5]]-1
C 2 3 5 7 8 9 10


Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output B[1..n]
2 3 4 4 5 7
B[7] =B[C[4]] =B[C[A[6]]] =A[6]= 4
1 2 3 4 5 6 7
C 2 3 5 7 8 9 10
C[A[6]]=C[A[6]]-1
C 2 3 5 6 8 9 10

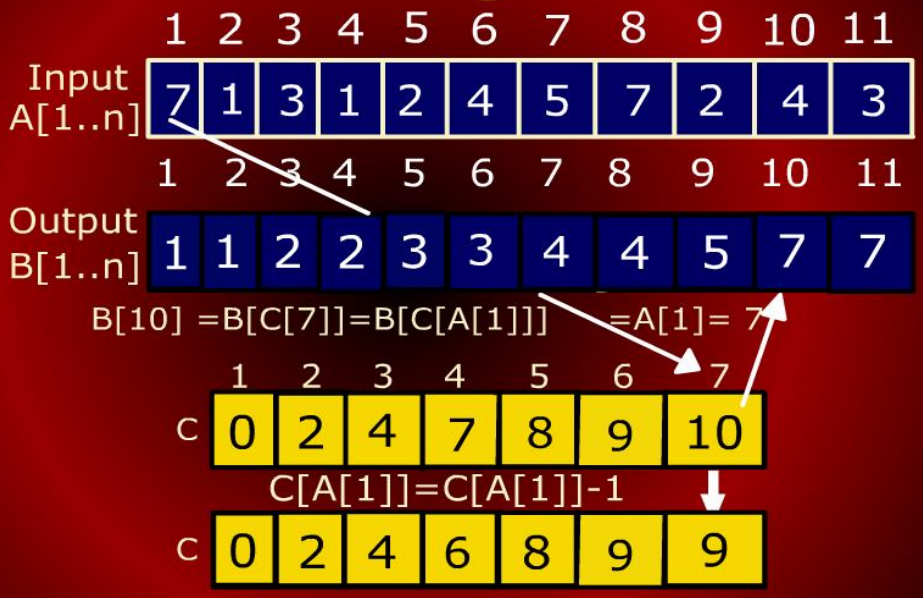

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output
B[1..n]
2 2 3 4 4 5 7
B[3] =B[C[2]]
=B[C[A[5]]] =A[5]= 2
1 2 3 4 5 6 7
C
2 3 5 7 8 9 10
C[A[5]]=C[A[5]]-1
C
2 2 5 6 8 9 10


Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output B[1..n]
1 2 2 3 4 4 5 7
B[2] =B[C[1]] =B[C[A[4]]] =A[4]= 1
1 2 3 4 5 6 7
C
2 2 5 7 8 9 10
C[A[4]]=C[A[4]]-1
C
1 2 5 6 8 9 10


Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input
A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output
B[1..n]
1 2 2 3 3 4 4 5 7
B[5] =B[C[3]] =B[C[A[3]]] =A[3]= 3
1 2 3 4 5 6 7
C
1 2 5 7 8 9 10
C[A[3]]=C[A[3]]-1
C
0 2 4 6 8 9 10

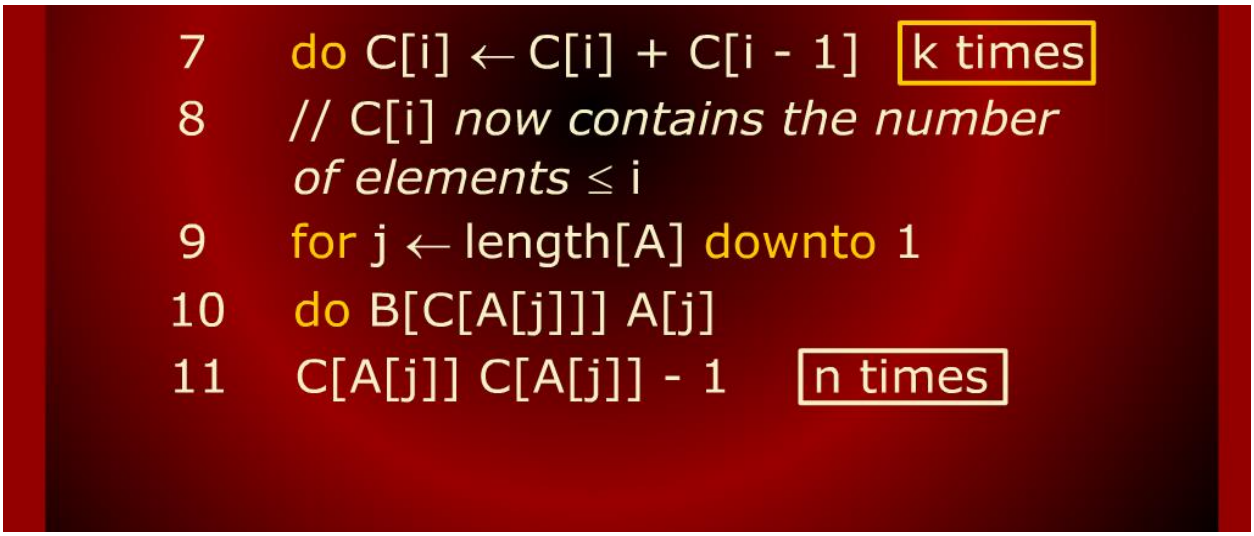

Counting Sort
1 2 3 4 5 6 7 8 9 10 11
Input A[1..n]
7 1 3 1 2 4 5 7 2 4 3
1 2 3 4 5 6 7 8 9 10 11
Output B[1..n]
1 1 2 2 3 3 4 4 5 7
B[1] =B[C[1]]=B[C[A[2]]] =A[2]= 1
1 2 3 4 5 6 7
C 1 2 4 7 8 9 10
C[A[3]]=C[A[3]]-1
C 0 2 4 6 8 9 10

Counting Sort



Counting Sort

```
COUNTING-SORT
( array A, array B,int k)
1     for i ← 1 to k
2     do C[i] ← 0          k times
3     for j ← 1 to length[A]
4     do C[A[j]] ← C[A[j]] + 1   n times
5     // C[i] now contains the number
      of elements = i
6     for i ← 2 to k
```


7 do C[i] ← C[i] + C[i - 1] k times
8 // C[i] now contains the number of elements ≤ i
9 for j ← length[A] downto 1
10 do B[C[A[j]]] A[j]
11 C[A[j]] C[A[j]] - 1 n times