Assignment-1
Solution

The symbol "⊆" means "is a subset of".
The symbol "⊂" means "is a proper subset of".

Q1)

1) Let S={ab, bb} and T={ab, bb, bbbb} Show that $S^* = T^*$ [Hint $S^* \subseteq T^*$ and $T^* \subseteq S^*$]

Ans:

All of the members of set S^* are members of set T^* but all members of set T^* is not members of set S^*

Hence S^* is not equal to T^*

2)

Let S={ab, bb} and T={ab, bb, bbb} Show that $S^* \neq T^*$ But $S^* \subset T^*$

Solution: Since $S \subset T$, so every string belonging to S^* , also belongs to T^* but bbb is a string belongs to T^* but does not belong to S^*.

- 3) Let S={a, bb, bab, abaab} be a set of strings. Are abbabaabab and baabbbabbaabb in S^*? Does any word in S^* have odd number of b's?

  Solution: since abbabaabab can be grouped as (a)(bb)(abaab)ab , which shows that the last member of the group does not belong to S, so abbabaabab is not in S^*, while baabbbabbaabb can not be grouped as members of S, hence baabbbabbaabb is not in S^*. Since each string in S has even number of b's so there is no possiblity of any string with odd number of b's to be in S^*.

Q2)Is there any case when S^+ contains Λ? If yes then justify your answer.

Solution: consider S={Λ,a} then

S^+ ={Λ, a, aa, aaa, …}

Here Λ is in S^+ as member of S. Thus Λ will be in S^+, in this case.

Q2) Prove that for any set of strings S…

i) $(S^+)^+ = S^+$

Solution: since S^+ generates all possible strings that can be obtained by concatenating the strings of S, so $(S^+)^+$ generates all possible strings that can be obtained by concatenating the strings of S^+, will not generate any new string.

Hence $(S^+)^+ = S^+$

Q2) continued…

ii) Is $(S^*)^+=(S^+)^*$

Solution: since Λ belongs to S^* ,so Λ will belong to $(S^*)^+$ as member of S^* .Moreover Λ may not belong to S^+, in general, while Λ will automatically belong to $(S^+)^*$.

Hence $(S^*)^+=(S^+)^*$

iii. $(S^+)^*=(S^*)^*$

Solution: In general Λ is not in S^+, while Λ does belong to S^*. Obviously Λ will now be in $(S^+)^*$, while $(S^*)^*$ and S^* generate the same set of strings. Hence $(S^+)^*=(S^*)^*$.

Q3. Define Language over the given regular expressions:

- Consider the language, defined over

  $\Sigma=\{a, b\}$ of **words beginning with a**, then its regular expression may be

  $$a(a+b)^*$$

- Consider the language, defined over

  $\Sigma=\{a, b\}$ of **words beginning and ending in same letter**, then its regular expression may be

  $$a\,(a + b)^*\,a + b\,(a + b)^*\,b$$

- Consider the language, defined over

  $\Sigma=\{a, b\}$ of **words ending in b**, then its regular expression may be?

  $(a+b)^*b$.

- Consider the language, defined over

  $\Sigma=\{a, b\}$ of **words not ending in a**, then its regular expression may be

  $(a+b)^*b + \Lambda$.

It is to be noted that this language may also be expressed by $((a+b)^*b)^*$.