UNIVERSITÉ DE
NEUCHÂTEL

Master of Cognitive Science

Data Science Course

Linear model I

Professor: Moran Steven

Lecturer: Maiolini Marco

Lecture 8: 13/April/2022

Outline

- Part 1: Book report & discussion (15 minutes)
- Part 2: Bases of linear models (45 minutes)

Break (15 minutes)

- Part 3: Practical

Bivariate relationship

When we have two numerical variables, we can distinguish:

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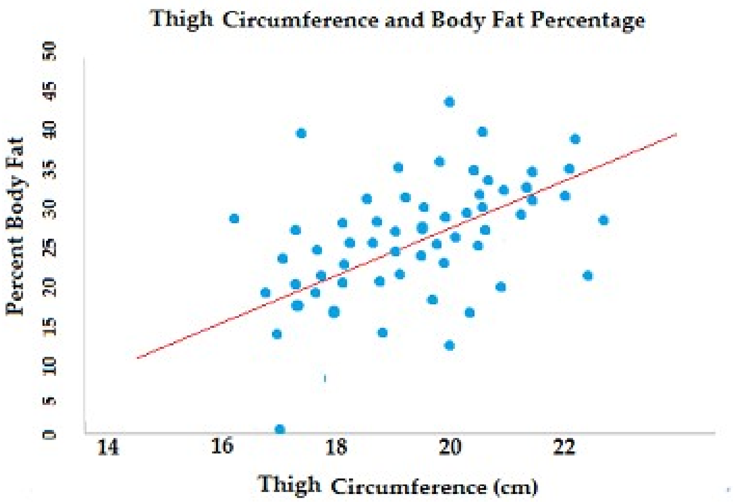

Relationship between X and Y

Techniques based on fitting a straight line to the data:

Relationship between X and Y

Techniques based on fitting a straight line to the data:

Linear regression

Correlation analysis

Linear regression

Example

You want to test Lung Function in children. You measure the forced exhalation volume (FEV), the measure of how much air somebody can forcibly exhale from their lungs, from 6 to 10 year old children. You survey 345 children.

The data here are a part of dataset given in Kahn, Micheal (2005), “An Exhalent Problem for Teaching Statistics”, The Journal of Statistical Education, 13(2).

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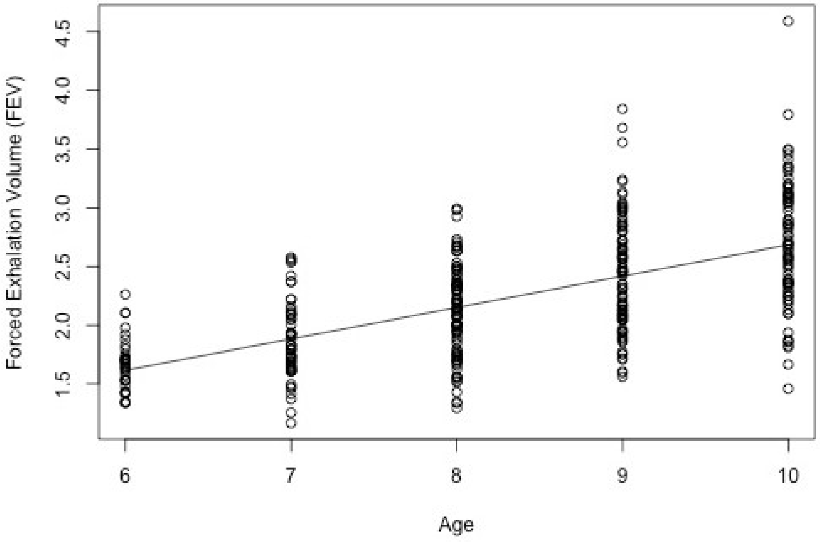

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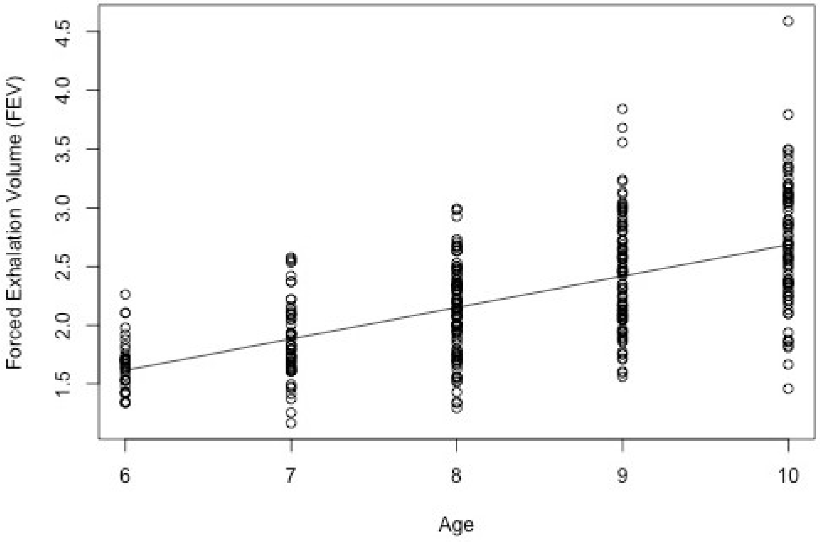

The scatter plot suggest a definite age-relationship, with larger X tending to be associated with bigger values of Y

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Correlation analysis

Example

You investigate whether standardized scores from high school (SAT) are related to academic grades in college (GPA). You predict that there’s a positive correlation: higher SAT scores are associated with higher college GPAs while lower SAT scores are associated with lower college GPAs.

You gather a sample of 5000 college graduates and survey them on their high school SAT scores and college GPAs.

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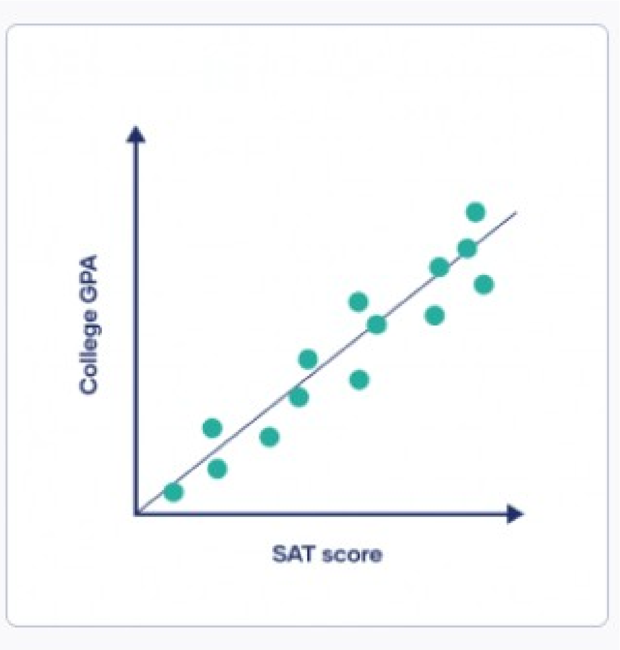

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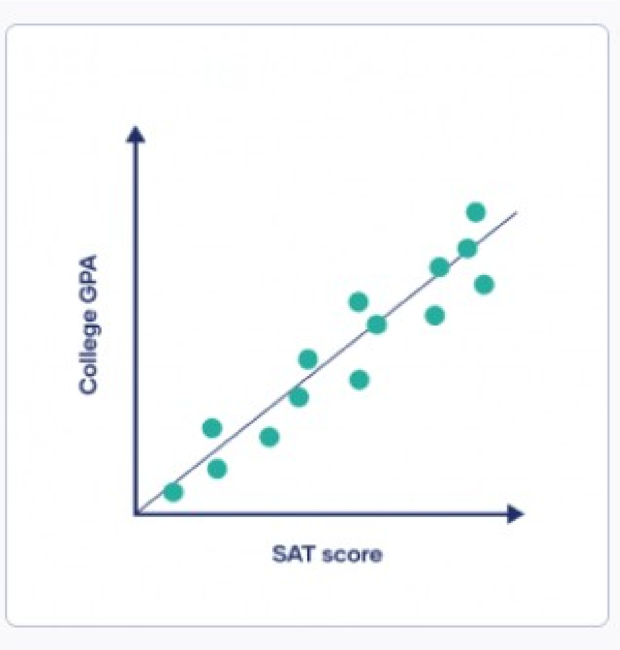

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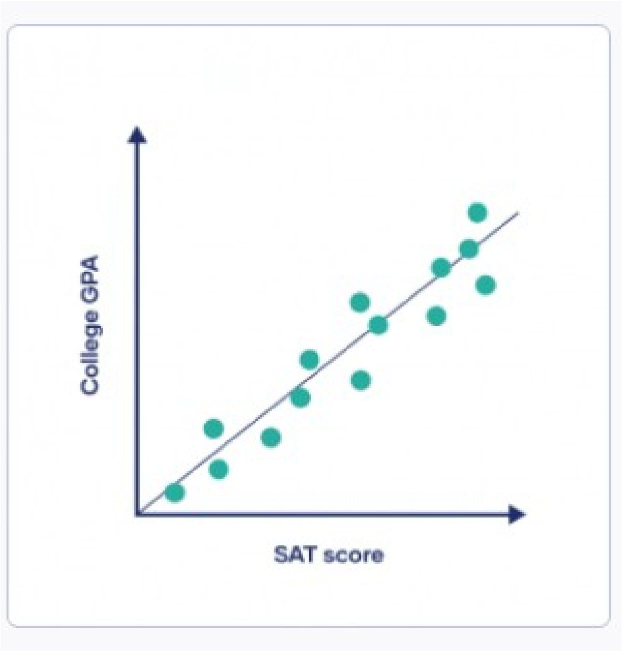

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The scatter plot seems to confirm our prediction, with higher SAT scores associated with higher GPA values.

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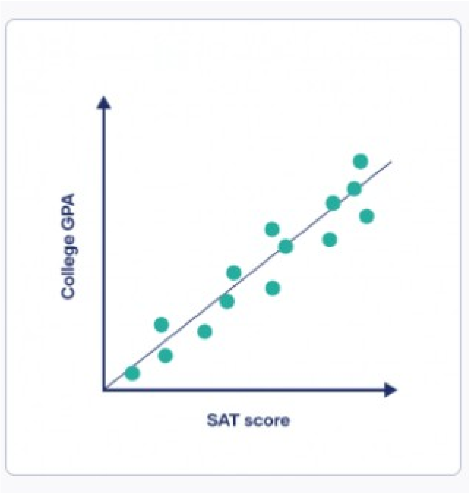

The Pearson's r correlation test:

- Variables are quantitative
- Variables normally distributed

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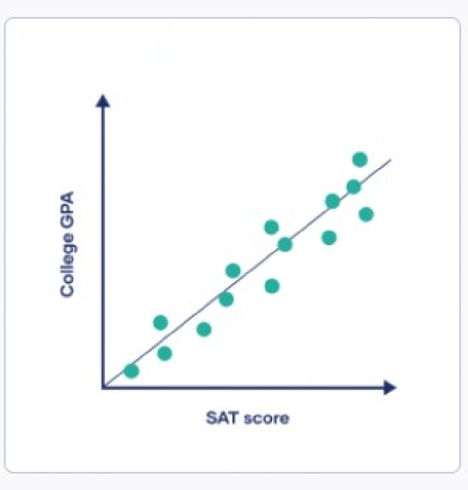

The Pearson's r correlation test:

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In R you see if two variable are correlated:

cor(x, y)

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A strong correlation between two variables does not indicate any causal connection between them. It is important to remember this concept when interpreting correlation.

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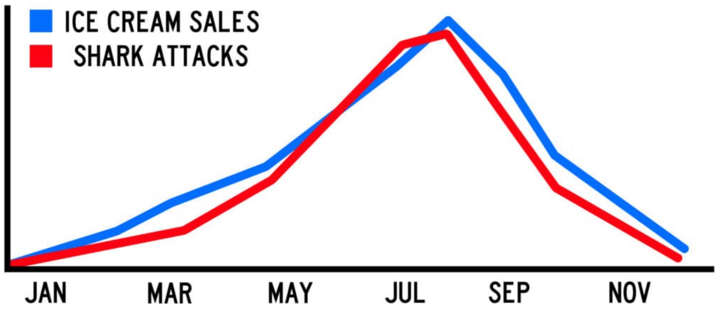

Both ice cream sales and shark attacks increase when the weather is hot and sunny, but they are not caused by each other (they are caused by good weather, with lots of people at the beach, both eating ice cream and having a swim in the sea)

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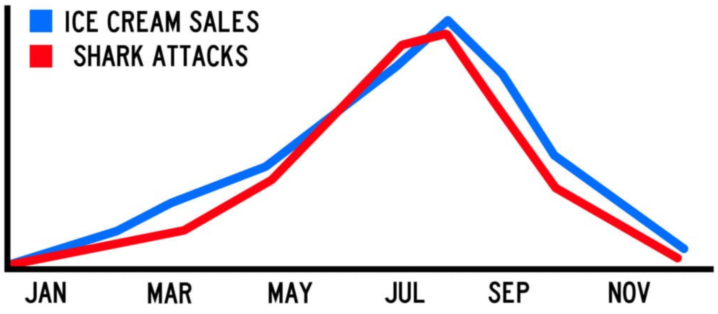

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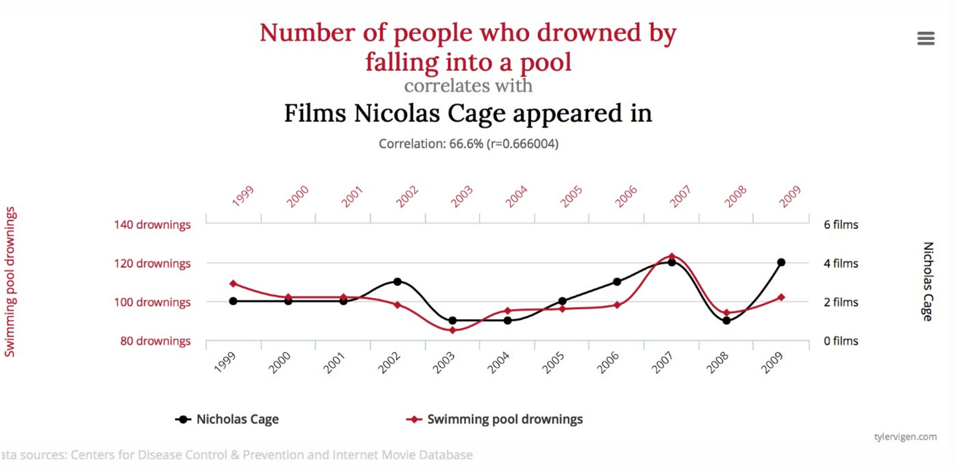

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The cat didn’t crush the awning

The Regression Line

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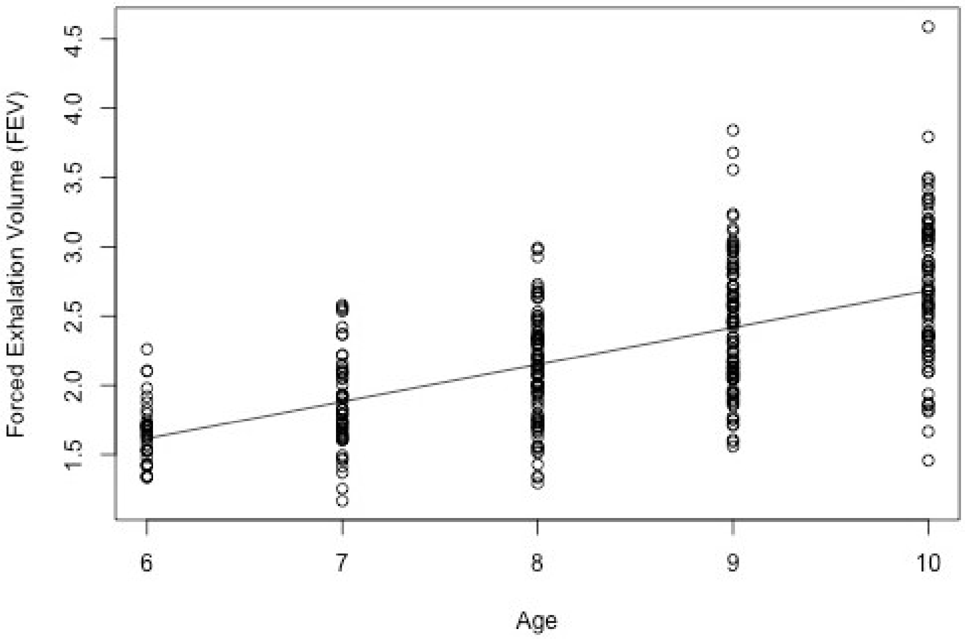
Intercept = 0.01165

Slope = 0.26721



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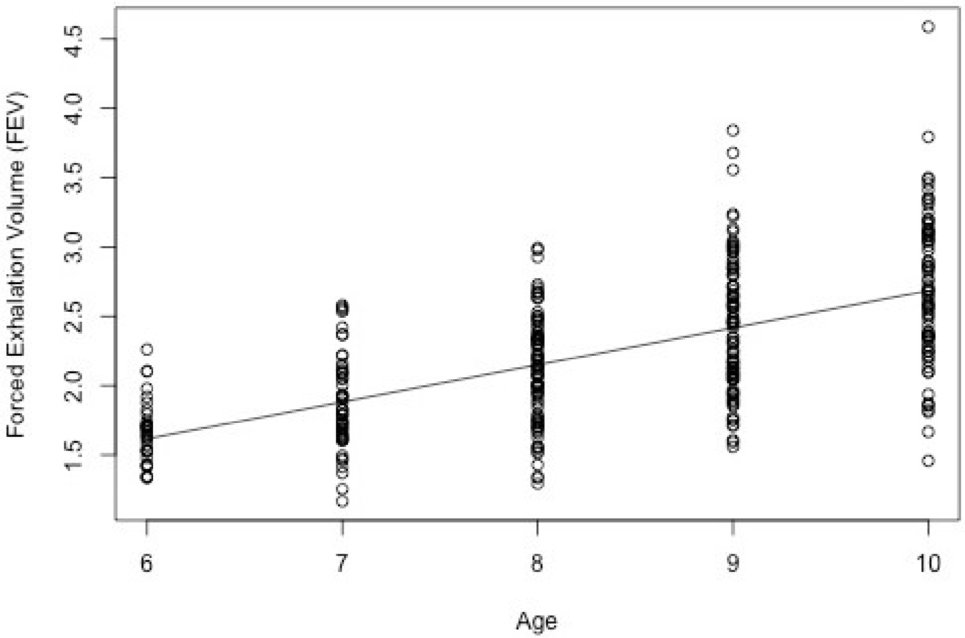

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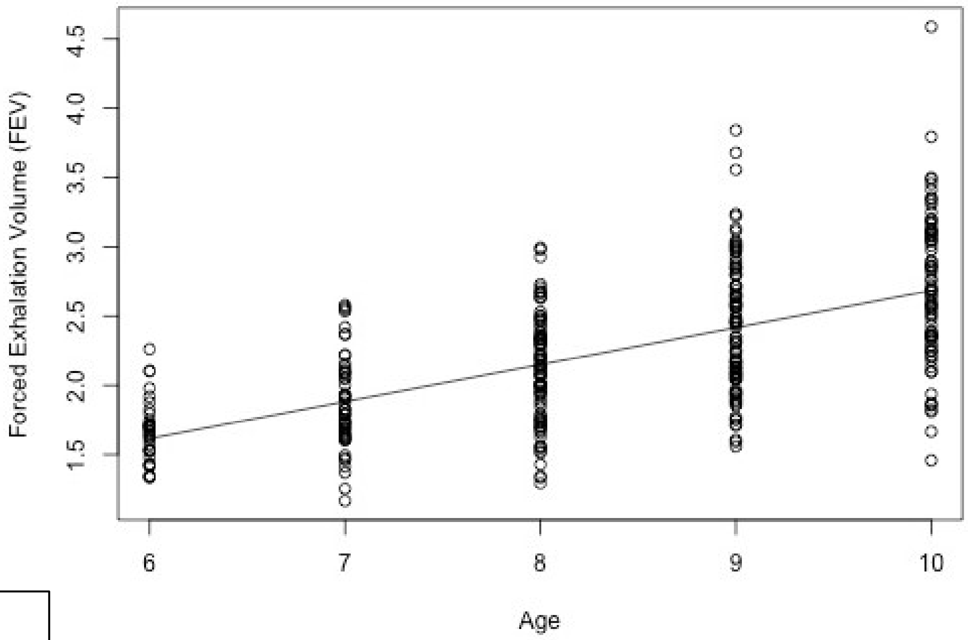

In R you can estimate slope & intercept:

lm(formula = Response ~ Explanatory, data = dataset)

(General) Linear Models

The General Linear Models are used to predict one Response variable from one or more Explanatory variables

Image taken from “Mixed Effects Models and Extensions in Ecology with R” Alain F. Zuur, Elena N. Ieno, Neil Walker, Anatoly A. Saveliev, Graham M. Smith - (2009, Springer-Verlag New York)

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- Multiple Regression → $Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 ...$

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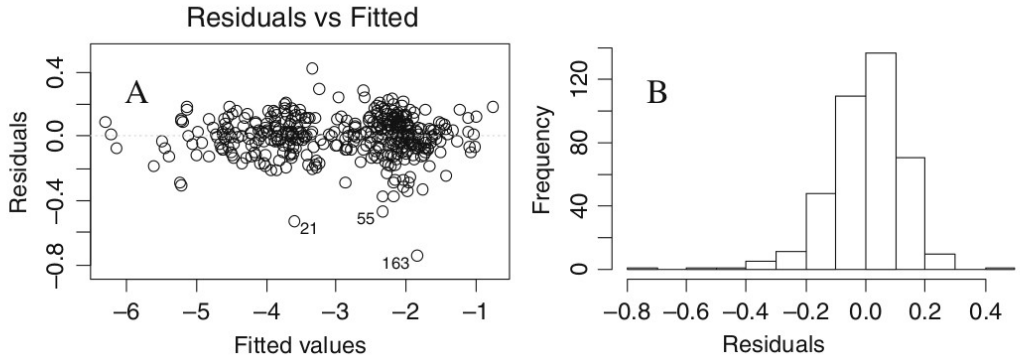
<u>Assumption</u>

- Linearity
- Normality of residuals
- Homoscedasticity
(Homogeneity of variance)



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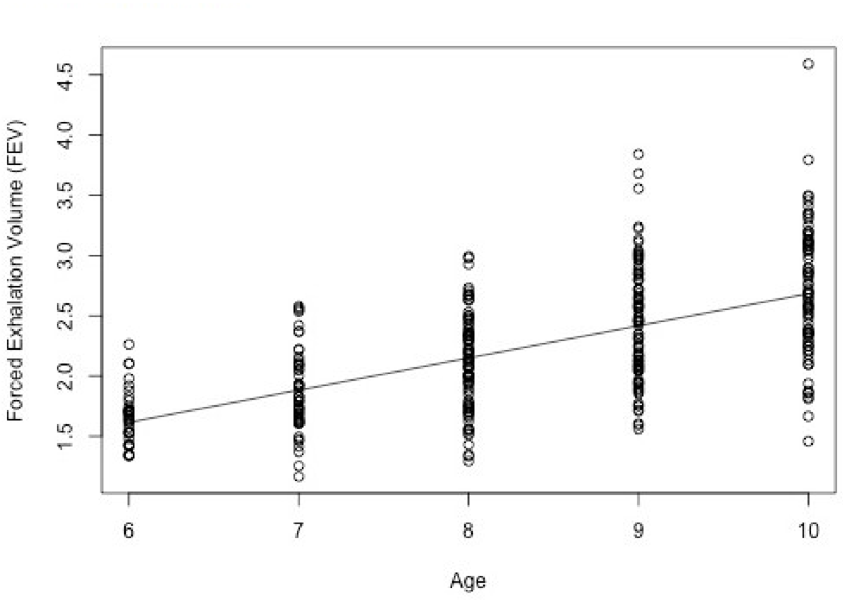

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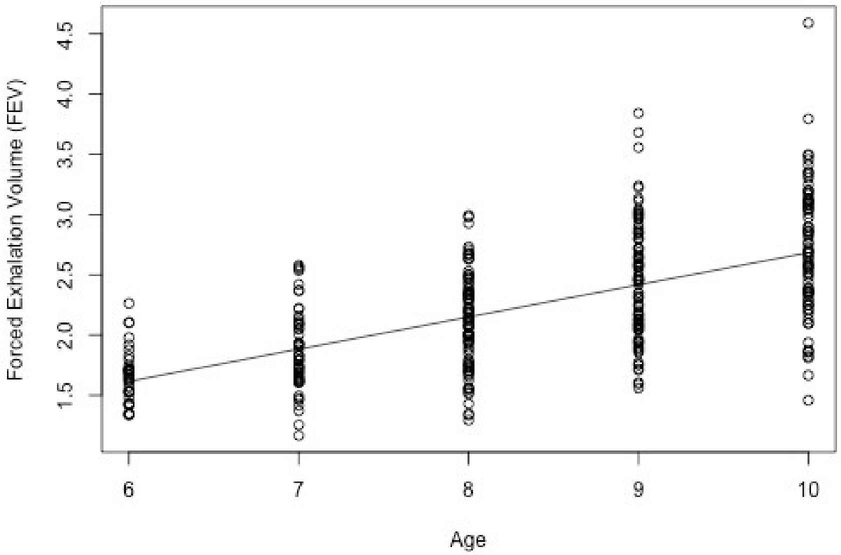
Linear Regression

X

Y

X explain Y

X ~ Y



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Additive independent effects

$$\begin{tikzcd}
\mathbf{X} \arrow[dr] & & \mathbf{Y} \arrow[dl] \\
& \mathbf{Z} &
\end{tikzcd}$$

X and Y explain the variation in Z independently

Z ~ X + Y

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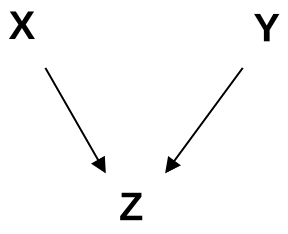

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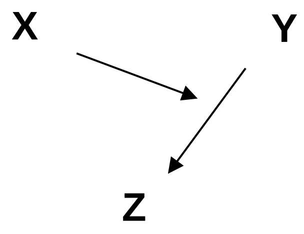
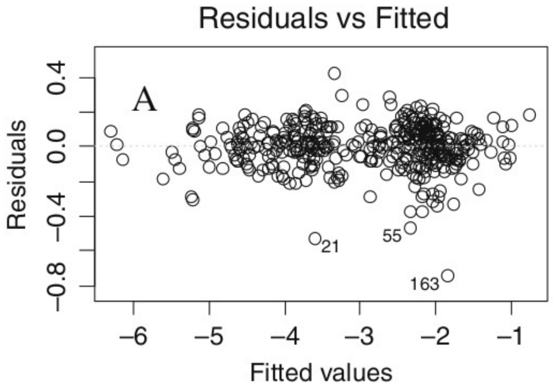
Interaction among variable



X modifies how Y affects Z

Z ~ X + Y + X*Y

Residuals



- *Residuals:* Difference between observation and fitted values
- *Fitted values: Estimation of an observation using all previous ones*

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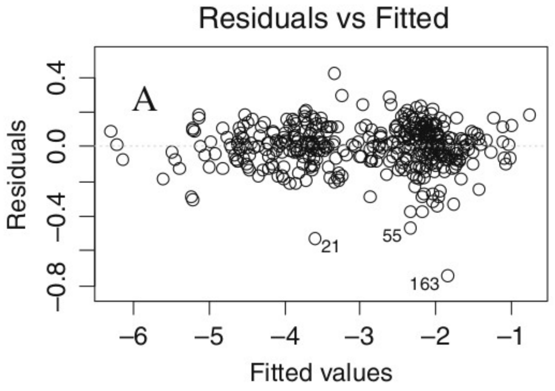

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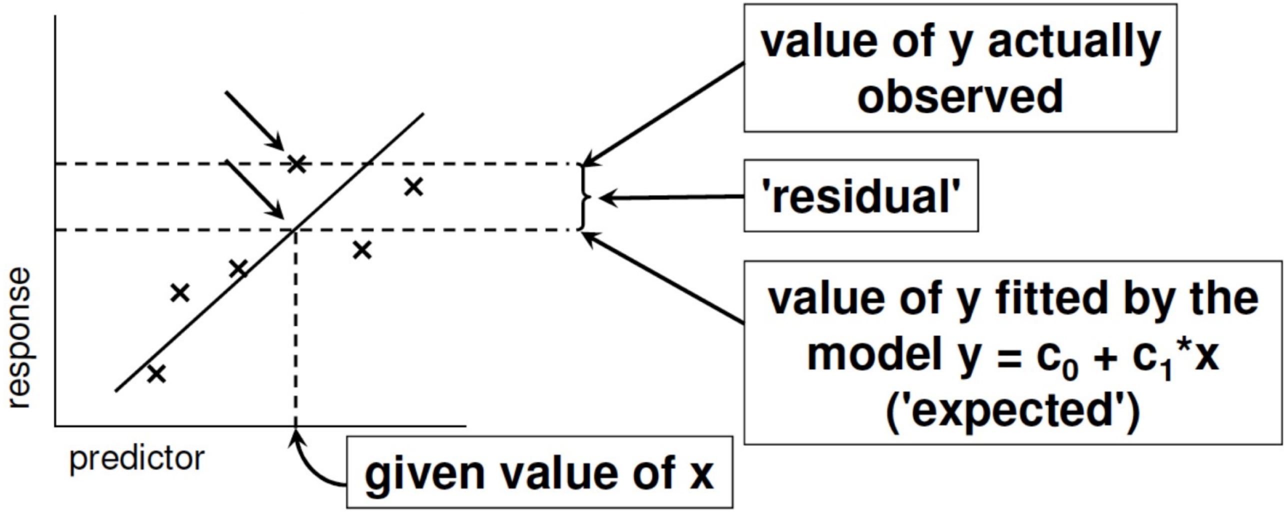

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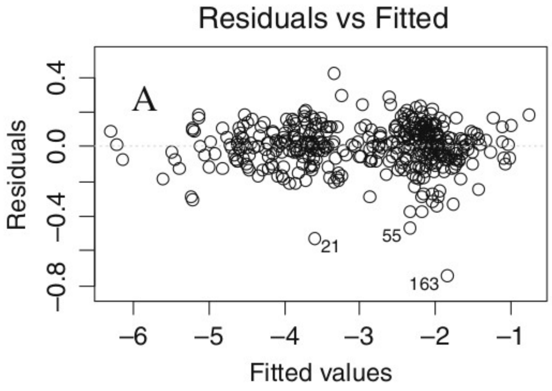

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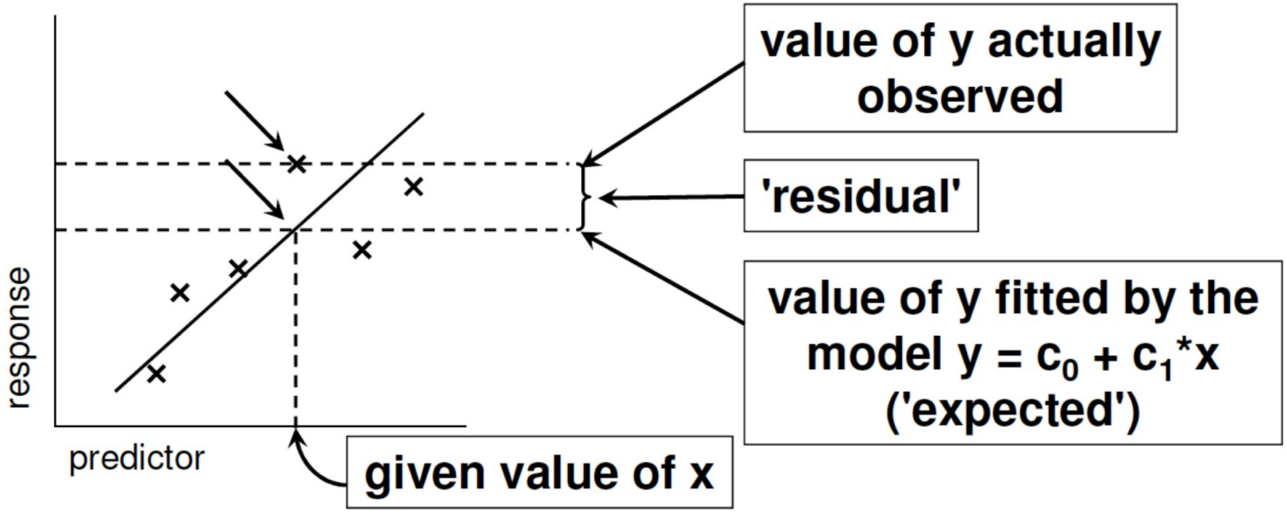
Error (so called Residual)

$$Y = b_1X + b_0 + e$$



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In R you can fit your data in a General Linear Model:

*lm(formula = Response ~ Explanatory + Z + Z*Y, data = dataset)*

In R you can fit your data in a GLM:

*glm(formula = Response ~ Explanatory + Z + Z*Y, family = binomial, data = dataset)*

Summary

Model	Variables	Distribution	R code
Linear Regression	$Y = b_0 + b_1x$	Normal	*lm(formula, data)*
General Linear Models	$Y = b_0 + b_1x_1 + b_2x_2 + ...$	Normal	*lm(formula, data)*
Generalized Linear Models (GLM)	$Y = b_0 + b_1x_1 + b_2x_2 + ...$	Any	*glm(formula, family, data)*

datacamp

R