応用数学演習問題（Practice Mathematics Q）

☆第1章　ベクトルと行列の演算 I

1.1

$\vec{a} = \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$ のとき

問1.1.1

$$\vec{a} + \vec{b} = \begin{pmatrix} 1+5 \\ 6+2 \\ 3+4 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 7 \end{pmatrix}$$ ┘

問1.1.2

$$\vec{a} - \vec{b} = \begin{pmatrix} 1-5 \\ 6-2 \\ 3-4 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ -1 \end{pmatrix}$$ ┘

問1.1.3

$$7\vec{a} = 7\begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 42 \\ 21 \end{pmatrix}$$ ┘

問1.1.4　$8(\vec{a} + \vec{b}) = 8 \cdot \begin{pmatrix} 6 \\ 8 \\ 7 \end{pmatrix} = \begin{pmatrix} 48 \\ 64 \\ 56 \end{pmatrix}$ ┘

1.2

$A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ 1 & 5 \end{pmatrix}$ のとき

問1.2.1

$$A+B = \begin{pmatrix} 2+1 & 1+4 \\ 5+1 & 3+5 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 6 & 8 \end{pmatrix}$$ ┘

問1.2.2

$$A-3B=\begin{pmatrix}2-3\cdot1 & 1-3\cdot4\\ 5-3\cdot1 & 3-3\cdot5\end{pmatrix}=\begin{pmatrix}-1 & -11\\ 2 & -12\end{pmatrix}$$

☆第2章　ベクトルと行列の演算II

2.1

$$\vec{v}=\begin{pmatrix}1\\0\\3\end{pmatrix},\quad A=\begin{pmatrix}1&3&4\\5&9&0\\3&1&2\end{pmatrix},\quad B=\begin{pmatrix}1&0&3\\0&2&5\end{pmatrix}$$

のとき．

問2.1.1

$$A\vec{v}=\begin{pmatrix}1&3&4\\5&9&0\\3&1&2\end{pmatrix}\begin{pmatrix}1\\0\\3\end{pmatrix}=\begin{pmatrix}13\\5\\9\end{pmatrix}$$

問2.1.2

$$B\vec{v}=\begin{pmatrix}1&0&3\\0&2&5\end{pmatrix}\begin{pmatrix}1\\0\\3\end{pmatrix}=\begin{pmatrix}10\\15\end{pmatrix}$$

問2.1.3

$$BA=\begin{pmatrix}1&0&3\\0&2&5\end{pmatrix}\begin{pmatrix}1&3&4\\5&9&0\\3&1&2\end{pmatrix}=\begin{pmatrix}10&6&10\\25&23&10\end{pmatrix}$$

問 2 1.4

$$B^T = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 5 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 5 \end{pmatrix}$$

2.2

$$A = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \text{ のとき}$$

問2 2.1

$$AB = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 7 & 13 \end{pmatrix}$$

問2.2.2

$$\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{cc|cc} 2 & 0 & -1 & 1 \\ 0 & -1 & -2 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -0.5 & 0.5 \\ 0 & 1 & 2 & -1 \end{array}\right)$$

$$\therefore A^{-1} = \begin{pmatrix} -0.5 & 0.5 \\ 2 & -1 \end{pmatrix}$$

例 2 2.3

$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -8 & -3 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{cc|cc} 1 & 0 & -\frac{1}{8} & \frac{3}{8} \\ 0 & -8 & -3 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -\frac{1}{8} & \frac{3}{8} \\ 0 & 1 & \frac{3}{8} & -\frac{1}{8} \end{array}\right)$$

$$\therefore B^{-1} = \begin{pmatrix} -\frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & -\frac{1}{8} \end{pmatrix}$$

例 2 2.4

$$BAB^{-1}$$

$$= \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} -\frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & -\frac{1}{8} \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 4 \\ 10 & 4 \end{pmatrix}\begin{pmatrix} -\frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & -\frac{1}{8} \end{pmatrix} = \begin{pmatrix} -\frac{2}{8} & \frac{38}{8} \\ \frac{2}{8} & \frac{26}{8} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{4} & \frac{19}{4} \\ \frac{1}{4} & \frac{13}{4} \end{pmatrix}$$