112 學年度第 1 學期 **微積分乙 (一) 期末考試題** 2023.12.20

注意事項：**做計算過程或說明不給分，總分 100 分。**

1. (8 pts) Find the area of the surface generated by revolving the following curve

$$x = 2\sqrt{4-y},\quad 0 \le y \le \frac{15}{4}$$

about the y-axis.

2. (50 pts) Evaluate the following integrals.

(a) $\int \frac{x^2}{x^2-1}\,dx$ (b) $\int_0^{\pi/4} \cos\sqrt{x}\,dx$ (c) $\int_0^{\pi} \ln x\,dx$

(d) $\int \sec^2 x\,dx$ (e) $\int \frac{(1-y^{1/2})^2}{y}\,dy$

3. (8 pts) Let $\mathcal{R}$ be the region bounded by the curves $x^2 - x + y = 0$, $y = 0$, $x = 0$ and $1 - x = 0$. Use the Shell Method to find the volume of the solid generated by revolving $\mathcal{R}$ about the y-axis.

4. Let $F(x) = \int_{x^2}^{0} \arctan(t)\,dt$ for all $x \in \mathbb{R}$.

(a) (4 pts) Find the open intervals on which F is increasing or decreasing.

(b) (6 pts) Evaluate $\lim_{x\to\infty} \frac{F(x)}{x^2}$.

5. (8 pts) Find the average value of $g(x) = \frac{1}{e^x + e^{-x}}$ on the interval $[-\ln 3, 0]$.

6. Let Ω be the region bounded by the curves $y = x^2$ and $y = x + 2$.

(a) (8 pts) Find the area of the plane region Ω.

(b) (8 pts) Use the Washer Method to find the volume of the solid generated by revolving Ω about the x-axis.

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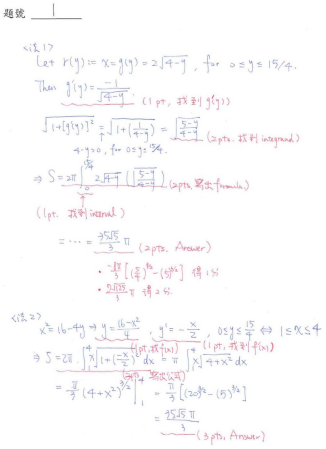
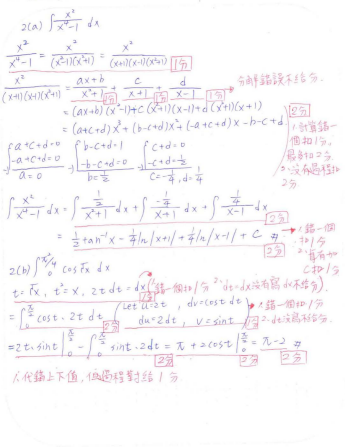
題號 1



11201 微積分乙期末考評分標準

題號 2(a)(b)



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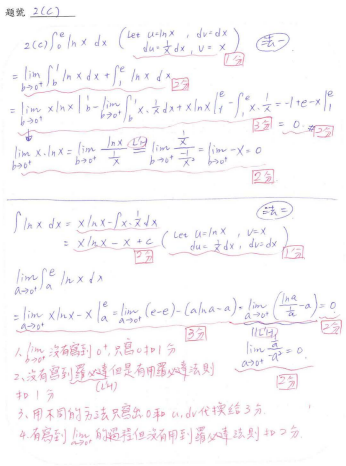

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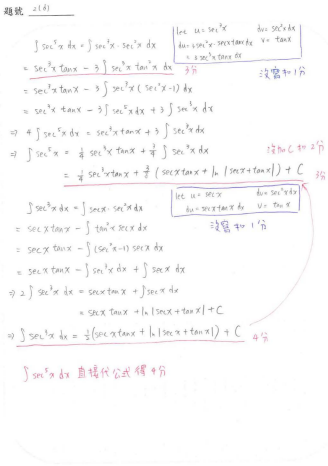

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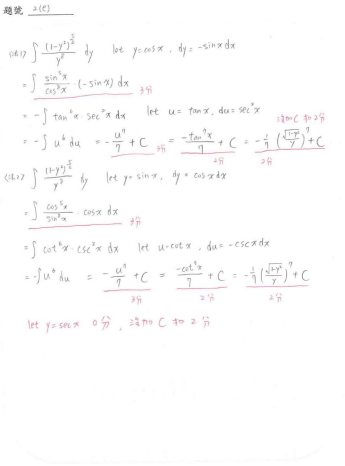

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題號 3.

$$V = 2\pi \int_0^1 x\left(x - x^3\right)dx$$

(2pt) (2pt) (2pt)

$$= 2\pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \frac{4}{15}\pi \quad (2pt)$$

若寫成 2 倍 $\left(\frac{2}{15}\pi\right)$ 給 6 分

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題號 4(a).

$F(x) = \int_{x^2}^{0} \arctan(t)\,dt$

2分 1. $\frac{d}{dx}F(x) = -\arctan(x^4)\cdot\frac{d(x^2)}{dx} = -2x\arctan(x^4)$

有畫出微分 1分

(但微的 $F(x)$ 不對不給分)

微錯或沒正確 1分.

2分 2. $\arctan(x^2) \ge 0 \quad \forall x \in \mathbb{R} \Rightarrow$ ① $F'(x) = -2x\arctan(x^4) > 0 \quad \forall x < 0$

② $F'(x) = -2x\arctan(x^4) < 0 \quad \forall x > 0$

$\Rightarrow$ ① when $x > 0$: $x \in (0, \infty)$: we have $F(x)$ is decreasing.

② when $x < 0$: $x \in (-\infty, 0)$: we have $F(x)$ is increasing.

2. 全對給 2分.

有錯或沒過程 (但結果正確)給 1分.

過程錯太多且結果錯誤：0分.

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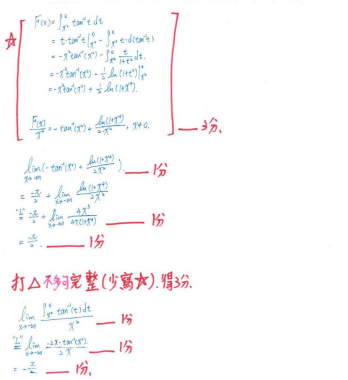
題號 4(b).



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題號 5

$$I = \int_{-\ln 3}^{0} g(x)\,dx = \int_{-\ln 3}^{0} \frac{1}{(e^x + e^{-x})}\,dx = \int_{-\ln 3}^{0} \frac{e^x}{(e^x)^2+1}\,dx$$

$$= \int_{-\ln 3}^{0} \frac{1}{(e^x)^2+1}\,d(e^x) = \int_{\frac{1}{3}}^{1} \frac{1}{u^2+1}\,du$$

↑ let $u = e^x$

$$= \tan^{-1}(u)\Big|_{\frac{1}{3}}^{1} = \tan^{-1}(1) - \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3}\right)$$ 積分正確 +3分

2° $av(g) = \frac{1}{0-(-\ln 3)}\int_{-\ln 3}^{0} g(x)\,dx$ 寫出 average value 的定義 +3 分 (2)

$$= \frac{1}{\ln 3}\left[\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3}\right)\right]$$ 正確答案 +2分

註：若未化簡答案，(如 $\tan^{-1}(1)$ 未化簡為 $\frac{\pi}{4}$)，則扣 1 分。

(2) 若看得出知道 average value 的定義，積分之部份因計算錯誤，則定義部份可給 1 分。

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題號 6(a)

$x^2 = y = x+2 \Rightarrow x^2 - x - 2 = (x+1)(x-2) = 0$

$\Rightarrow x = -1, 2$ (有寫出即得 2分)

$$A = \int_{-1}^{2} (x+2-x^2)\,dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right) = \frac{9}{2}$$

列式正確得 4分

(計算錯誤扣 2分)

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題號 6. (b)

$$\int_{-1}^{2} \pi\left((x+2)^2 - \pi(x^2)^2\right)dx$$ → 列式對 +4

$$= \pi\int_{-1}^{2} x^2+4x+4-x^4\,dx$$

$$= \pi\left[\frac{1}{3}x^3 + 2x^2 + 4x - \frac{1}{5}x^5\right]_{-1}^{2}$$ → 積到這 +2

$$= \frac{72}{5}\pi$$ ← 算對答案 +2：① 少寫 π -1 ② 多寫 +C -1

① 若積分式寫對，上下限寫錯則錯一個扣 1 分，最多扣 2 分

② 寫 $\int \pi(x)^2 - \pi(x+2)^2\,dx$ 但未說明理由 +3

• 若使用其他方法，需完全正確 (過程、答案皆正確) 才給 4 分。