106 台部

1 (1) $\int_1^e x\ln x\,dx = \int_1^e \ln x\, d(\frac{1}{2}x^2) = \frac{1}{2}x^2\ln x\Big|_1^e - \int_1^e \frac{1}{2}x^2\frac{1}{x}dx = \frac{1}{2}x^2\ln x\Big|_1^e - \frac{1}{4}x^2\Big|_1^e = \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4}$

$= \frac{1}{4}(e^2+1)$

(2) $\int e^{\tan 2x}\sec^2 2x\,dx = \frac{1}{2}e^{\tan 2x} + C$ $\quad$ (令 $\tan 2x = u.\ \sec^2 2x \cdot 2\,dx = du$)

$\left(\int e^u \frac{1}{2}du = \frac{1}{2}e^u + C\right)$

(3) $\int x^5 e^{x^3}dx \overset{\text{令 } x^3=u,\ 3x^2dx=du}{=\!=\!=\!=\!=} \int \frac{1}{3}ue^u du = \frac{1}{3}\int u\,de^u = \frac{1}{3}\left(ue^u - \int e^u du\right) + C$

令 $x^3 = u$

$3x^2dx = du$

$= \frac{1}{3}ue^u - \frac{1}{3}e^u + C$

$= \frac{1}{3}x^3e^{x^3} - \frac{1}{3}e^{x^3} + C$

(4) $\int \tan^4 x\,dx = \int \tan^2 x(\sec^2 x - 1)dx$

$= \int \tan^2 x\sec^2 x\,dx - \int \tan^2 x\,dx = \int \tan^2 x\sec^2 x\,dx - \int \sec^2 x\,dx + \int 1\,dx$

$= \int \tan^2 x\,d\tan x - \int 1\,d\tan x + \int 1\,dx$

$= \frac{1}{3}\tan^3 x - \tan x + x$

check: $\left(\frac{1}{3}\tan^3 x - \tan x + x\right)' = \tan^2 x\sec^2 x - \sec^2 x + 1$

$= \tan^2 x(\tan^2 x + 1) - (\tan^2 x + 1) + 1$

$= \tan^4 x$. ok!

(5) $\int \frac{dx}{x\sqrt{25 - \ln^2 x}} = \int \frac{du}{\sqrt{25 - u^2}} = \int \frac{5\,dv}{5\sqrt{1 - v^2}} = \sin^{-1} v + C = \sin^{-1}\left(\frac{u}{5}\right) + C$

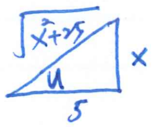
令 $\ln x = u$ $\quad$ 令 $u = 5v$ $\quad$ $= \sin^{-1}\left(\frac{\ln x}{5}\right) + C$

$\frac{1}{x}dx = du$ $\quad$ $du = 5dv$

(6) $\int \frac{dx}{x^2\sqrt{x^2+25}} = \int \frac{5\sec^2 u\,du}{25\tan^2 u \cdot 5\sec u} = \frac{1}{25}\int \frac{\cos^2 u\sec u\,du}{\sin^2 u} = \frac{1}{25}\int \frac{d\sin u}{\sin^2 u}$

令 $x = 5\tan u$

$dx = 5\cdot\sec^2 u\cdot du$



$= \frac{-1}{25}\frac{1}{\sin u} + C = \frac{-1}{25}\frac{\sqrt{x^2+25}}{x} + C$

$= \frac{-\sqrt{x^2+25}}{25x} + C$

(7) $\int \frac{1}{x(x-1)^2}dx = \int\left(\frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}\right)dx = \ln|x| - \ln|x-1| - \frac{1}{x-1} + C$

解: $\frac{1}{x(x-1)^2} = \frac{a}{x} + \frac{b}{x-1} + \frac{c}{(x-1)^2} = \frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}$

$= \frac{(a+b)x^2 + (-2a-b+c)x + a}{x(x-1)^2}$

$$\begin{array}{ccc} a & -2a & a \\ b & -b & \\ & c & \\ \hline & & 1 \end{array}$$

$a = 1$

$b = -1$

$c = 1$

106 分部

1(8) $\displaystyle\int \frac{\sin x\,dx}{\cos x-\cos^2 x} = \int \frac{-1}{t-t^2}dt = \int \frac{1}{t(t-1)}dt = \int\left(\frac{1}{t-1}-\frac{1}{t}\right)dt$

令 $\cos x = t$

$-\sin x\,dx = dt$

$= \ln|t-1| - \ln|t| + C$

$= \ln\left|1-\frac{1}{t}\right| + C = \ln|1-\sec x| + C$

Check: $(\ln|1-\sec x|)' = \dfrac{-\sec x\tan x}{1-\sec x} = \dfrac{-\sin x}{\cos^2 x-\cos x}$. ok!

105 分部

1 (1) $\displaystyle\int \sin 2x\cos 5x\,dx = \int \frac{1}{2}(\sin 7x+\sin -3x)dx = \frac{1}{2}\int(\sin 7x - \sin 3x)dx$

$= \frac{1}{2}\left(-\frac{1}{7}\cos 7x + \frac{1}{3}\cos 3x\right) + C$

☆ 積化和差, 和差化積

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\Rightarrow \sin X\cos Y = \frac{1}{2}\left(\sin(X+Y) + \sin(X-Y)\right)$

(2) $\displaystyle\int x^3\cos 2x\,dx = \int x^3\,d\frac{\sin 2x}{2} = \frac{1}{2}x^3\sin 2x - \int \frac{1}{2}\sin 2x\cdot 3x^2\,dx$

$\displaystyle\int x^2\sin 2x\,dx = \int x^2\,d\frac{-\cos 2x}{2} = -\frac{1}{2}x^2\cos 2x + \int\frac{\cos 2x}{2}\cdot 2x\,dx = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\ (+C)$

$\displaystyle\int x\cos 2x\,dx = \int x\,d\frac{\sin 2x}{2} = \frac{1}{2}x\sin 2x - \int\frac{1}{2}\sin 2x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\ (+C)$

$\therefore \displaystyle\int x^3\cos 2x\,dx = \frac{1}{2}x^3\sin 2x - \frac{3}{2}\left(-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right) + C$

$= \frac{1}{2}x^3\sin 2x + \frac{3}{4}x^2\cos 2x - \frac{3}{4}x\sin 2x - \frac{3}{8}\cos 2x + C$

(3) $\displaystyle\int \sec^5 x\tan^3 x\,dx = \int \sec^4 x\tan^2 x\,d\sec x = \int \sec^4 x(\sec^2 x-1)\,d\sec x = \frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

hint: $(\sec x)' = \sec x\tan x$

(4) $\displaystyle\int \cos^4(2x)\,dx = \int\left(\frac{1+\cos 4x}{2}\right)^2 dx = \frac{1}{4}\int(\cos^2 4x + 2\cos 4x + 1)dx$

hint: $\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$= \frac{1}{4}\int\left(\frac{1+\cos 8x}{2}\right)dx + \frac{1}{2}\int\cos 4x\,dx + \frac{1}{4}\int 1\,dx$

$= \frac{1}{8}x + \frac{1}{64}\sin 8x + \frac{1}{8}\sin 4x + \frac{1}{4}x + C$

☆ Answer: $\frac{1}{2}\left(\cos^3 2x\sin 2x + \frac{3}{8}\left(2x - \frac{1}{4}\sin 8x\right)\right) + C$

$= \frac{3}{8}x + \frac{1}{2}\cos^3 2x\sin 2x - \frac{3}{32}\sin 8x + C$

105 分部

1 (5) $\int \frac{\sqrt{x^2-3}}{x} dx = \int \frac{\sqrt{3}\tan u}{\sqrt{3}\sec u} \sqrt{3}\sec u \tan u \, du = \sqrt{3}\int (\sec^2 u - 1)\, du = \sqrt{3}\tan u - \sqrt{3} u$

$= -\sqrt{3}\sec^{-1}\frac{x}{\sqrt{3}} + \sqrt{x^2-3}$

令 $x = \sqrt{3}\sec u$

$dx = \sqrt{3}\sec u \tan u \, du$

$\sec u = \frac{x}{\sqrt{3}}$

$\sec^2 u - 1 = \frac{x^2}{3} - 1$

$\tan u = \sqrt{\frac{1}{3}(x^2-3)}$

(6) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^3(3x)\cos^3(3x) + \sin(3x)\cos 3x}{\sin^2(3x)+3} dx = 0$

令 $f(x) = \frac{\sin^3 3x \cos^3 3x + \sin 3x \cos 3x}{\sin^2 3x + 3}$ 則 $f(-x) = -f(x)$

(7) $\int \frac{2x^4-3x^2-16}{x^4-2x^2-8} dx = \int \left(2 + \frac{x^2}{x^4-2x^2-8}\right) dx$

$x^4-2x^2-8 = (x-2)(x+2)(x^2+2)$

$\frac{x^2}{x^4-2x^2-8} = \frac{a}{x-2} + \frac{b}{x+2} + \frac{cx+d}{x^2+2} = \frac{1}{6}\left(\frac{1}{x-2} + \frac{-1}{x+2} + \frac{2}{x^2+2}\right)$

$\frac{1}{6}$ $\quad -\frac{1}{6}$ $\quad \frac{1}{3}$

∴ $\int \frac{2x^4-3x^2-16}{x^4-2x^2-8} dx = \int \left[2 + \frac{1}{6}\left(\frac{1}{x-2} - \frac{1}{x+2} + \frac{2}{x^2+2}\right)\right] dx = 2x + \frac{1}{6}\ln|x-2| - \frac{1}{6}\ln|x+2| + \frac{1}{6}\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}$

$+ C$

☆ $\int \frac{2}{x^2+2} dx = \int \frac{2}{2\sec^2\theta}\sqrt{2}\sec^2\theta \, d\theta = \sqrt{2}\,\theta = \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

令 $x = \sqrt{2}\tan\theta$

$dx = \sqrt{2}\sec^2\theta \, d\theta$

104

1(1) $\int_0^1 (x^3+2x)\,dx = \frac{1}{4}x^4+x^2\Big|_0^1 = \frac{5}{4}$

(2) $\int \frac{2x+1}{x^2-7x+12}\,dx = \int\left(\frac{-7}{x-3}+\frac{9}{x-4}\right)dx = -7\ln|x-3|+9\ln|x-4|+C$

$\triangle\ \frac{2x+1}{x^2-7x+12} = \frac{a}{x-3}+\frac{b}{x-4} = \frac{-7}{x-3}+\frac{9}{x-4}$

$$\begin{array}{cc} a & -4a \\ b & -3b \\ \hline 2 & 1 \end{array} \qquad \begin{array}{l} a+b=2 \\ 4a+3b=-1 \\ a=-7,\ b=9 \end{array}$$

(3) $\int_0^{\frac{\pi}{2}} \cos^3 x\,dx = \left(\sin x-\frac{1}{3}\sin x^3\right)\Big|_0^{\frac{\pi}{2}} = \frac{2}{3}$

$\int \cos^3 x\,dx = \int \cos^2 x\,d\sin x = \int (1-\sin^2 x)\,d\sin x = \sin x-\frac{1}{3}\sin x^3$

(4) $\int_1^e \frac{dy}{y(1+(\ln y)^2)} = \tan^{-1}\ln|y|\Big]_1^e = \tan^{-1}1-\tan^{-1}0 = \frac{\pi}{4}-0=\frac{\pi}{4}$.

令 $\ln y = x$ $\quad \int\frac{dy}{y(1+(\ln y)^2)} = \int\frac{1}{1+x^2}\,dx = \tan^{-1}x\,(+C) = \tan^{-1}\ln|y|+C$

$\frac{1}{y}dy=dx$

(5) $\int \sec^3 z\,dz = \int \sec z\,d\tan z = \sec z\tan z-\int \tan z\cdot\sec z\tan z\,dz$

$= \sec z\tan z-\int\sec^3 z\,dz+\int\sec z\,dz$

$\Rightarrow \int\sec^3 z\,dz = \frac{1}{2}\sec z\tan z+\frac{1}{2}\int\sec z\,dz$

$= \frac{1}{2}\sec z\tan z+\frac{1}{2}\ln|\sec z+\tan z|+C$

(6) $\int_1^9 \frac{\log_3 x}{x}\,dx = \frac{1}{2\ln 3}(\ln x)^2\Big|_1^9 = \frac{1}{2\ln 3}(2\ln 3)^2 = 2\ln 3$

$\int\frac{\log_3 x}{x}\,dx = \frac{1}{\ln 3}\int\frac{\ln x}{x}\,dx = \frac{1}{\ln 3}\cdot\frac{1}{2}(\ln x)^2+C$

(7) $\int \sin\sqrt{2x}\,dx = \int u\sin u\,du = -u\cos u+\sin u+C$

$= -\sqrt{2x}\cos\sqrt{2x}+\sin\sqrt{2x}+C$

令 $\sqrt{2x}=u$ 即 $2x=u^2$

$\Rightarrow dx = u\,du$

(8) $\int\frac{1}{1+e^s}\,ds = \int\left(1-\frac{e^s}{1+e^s}\right)ds = s-\ln|1+e^s|+C$

103

(a) $\int_1^4 (x^2+3x+1)dx = \left(\frac{1}{3}x^3+\frac{3}{2}x^2+x\right)\Big|_1^4 = \frac{1}{3}\cdot 63+\frac{3}{2}\cdot 15+3 = 21+\frac{45}{2}+3 = \frac{93}{2}$

(b) $\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-2)(x-3)}dx = \int\left(\frac{1}{x-3}-\frac{1}{x-2}\right)dx = \ln|x-3|-\ln|x-2|+C$

(c) $\int \sin^3 x\cos^4 x\,dx = \int (1-\cos^2 x)\cos^4 x\cdot d(-\cos x) = \int(\cos^6 x-\cos^4 x)\,d\cos x = \frac{1}{7}\cos^7 x-\frac{1}{5}\cos^5 x+C$

(d) $\int \tan^2 x\sec^3 x\,dx = \int \sec^5 x\,dx - \int\sec^3 x\,dx$

$= \int \sec^3 x\,d\tan x - \int \sec^3 x\,dx$

$= \sec^3 x\tan x - \int \tan x\cdot 3\sec^2 x\cdot\sec x\cdot\tan x\,dx - \int\sec^3 x\,dx$

$= \sec^3 x\tan x - 3\int\tan^2 x\sec^3 x\,dx - \int\sec^3 x\,dx$

$\Rightarrow \int\tan^2 x\sec^3 x\,dx = \frac{1}{4}\sec^3 x\tan x - \frac{1}{4}\int\sec^3 x\,dx$

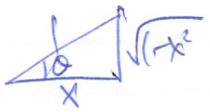
又 $\int\sec^3 x\,dx = \int\sec x\,d\tan x = \sec x\tan x - \int\tan x\cdot\sec x\tan x\,dx$

$= \sec x\tan x - \int\sec^3 x\,dx + \int\sec x\,dx$

$\Rightarrow \int\sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x+\tan x|$

$\therefore \int\tan^2 x\sec^3 x\,dx = \frac{1}{4}\sec^3 x\tan x - \frac{1}{8}\sec x\tan x - \frac{1}{8}\ln|\sec x+\tan x| + C$

(e) $\int\frac{\sqrt{1-x^2}}{x}dx \xlongequal[\substack{令 x=\cos\theta \\ dx=-\sin\theta d\theta}]{} \int\frac{\sin\theta}{\cos\theta}(-\sin\theta)d\theta = \int\frac{\cos^2\theta-1}{\cos\theta}d\theta = \int(\cos\theta-\sec\theta)d\theta$



$= \sin\theta - \ln|\sec\theta+\tan\theta| + C$

$= \sqrt{1-x^2} - \ln\left|\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right| + C$

$= \sqrt{1-x^2} + \ln\left|\frac{x}{1+\sqrt{1-x^2}}\right| + C$

(f) $\int\ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}dx$

$= x\ln x - x + C$

(g) $\int\frac{\ln x}{x}dx = \int\ln x\,d(\ln x) = (\ln x)^2 - \int\frac{\ln x}{x}dx \Rightarrow \int\frac{\ln x}{x}dx = \frac{1}{2}(\ln x)^2 + C$

(h) $\int\frac{2z}{\sqrt[3]{z^2+1}}dz \xlongequal[\substack{令 z^2+1=t \\ 2z\,dz=dt}]{} \int t^{-\frac{1}{3}}dt = \frac{3}{2}t^{\frac{2}{3}}+C = \frac{3}{2}(z^2+1)^{\frac{2}{3}}+C$

102

1 (1) $\int_0^1 (x^2+3x+2)dx = \left(\frac{1}{3}x^3+\frac{3}{2}x^2+2x\right)\Big|_0^1 = \frac{1}{3}+\frac{3}{2}+2 = \frac{23}{6}$

(2) $\int \frac{1}{x^2+3x+2}dx = \int \frac{1}{(x+1)(x+2)}dx = \int\left(\frac{1}{x+1}-\frac{1}{x+2}\right)dx = \ln|x+1| - \ln|x+2| + C$

(3) $\int_0^1 x\sqrt{1-x}\,dx \xlongequal[dx=-dt]{令1-x=t} \int_1^0 (1-t)t^{\frac{1}{2}}(-dt) = \int_0^1 (t^{\frac{1}{2}}-t^{\frac{3}{2}})dt = \frac{2}{3}t^{\frac{3}{2}} - \frac{2}{5}t^{\frac{5}{2}}\Big|_0^1$

$= \frac{2}{3}-\frac{2}{5} = \frac{4}{15}$

(4) $\int_0^{2/3} \sqrt{4-9x^2}\,dx \xlongequal[则\ dx=\frac{2}{3}dt]{令\ x=\frac{2}{3}t} \int_0^1 2\sqrt{1-t^2}\cdot\frac{2}{3}dt = \frac{4}{3}\int_0^1\sqrt{1-t^2}\,dt = \frac{4}{3}\cdot\frac{\pi}{4}$

$= \frac{\pi}{3}$

(5) $\int \tan^3 x\sec^4 x\,dx = \int \tan^3 x(\tan^2 x+1)\,d\tan x = \frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + C$

(6) $\int \frac{1+\sin x}{\cos^2 x}dx = \int(\sec^2 x + \tan x\sec x)dx = \tan x + \sec x + C$

(7) $\int x^2 e^{-x}dx = \int x^2 d(-e^{-x}) = -x^2e^{-x} + \int e^{-x}\cdot 2x\,dx$

$= -x^2e^{-x} + 2\int x\,d(-e^{-x}) = -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}dx$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

(8) $\int \sin\sqrt{x}\,dx \xlongequal[\substack{\Rightarrow x=t^2 \\ dx=2t\,dt}]{令\sqrt{x}=t} \int \sin t\cdot 2t\,dt = 2\int t\,d(-\cos t)$

$= -2t\cos t + 2\int \cos t$

$= -2t\cos t + 2\sin t + C$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

※

106 林郭

1 (1) $\int (x+1)\cdot 5^{(x+1)^2}\,dx = \frac{1}{2\ln 5}5^{(x+1)^2}+C$

(2) $\int \frac{2x^3-4x^2-15x+5}{x^2-2x-8}\,dx = \int\left(2x+\frac{x+5}{(x-4)(x+2)}\right)dx = \int\left(2x+\frac{\frac{3}{2}}{x-4}+\frac{-\frac{1}{2}}{x+2}\right)dx$

$= x^2+\frac{3}{2}\ln|x-4|-\frac{1}{2}\ln|x+2|+C$

(3) $\int e^{\sqrt{2x}}\,dx \underset{\substack{令\sqrt{2x}=t\\ 2x=t^2\\ dx=t\,dt}}{=} \int t\cdot e^t\,dt = te^t-e^t+C = \sqrt{2x}e^{\sqrt{2x}}-e^{\sqrt{2x}}+C$

(4) $\int e^x\sqrt{1-e^{2x}}\,dx \xlongequal[\substack{令 e^x=t\\ e^x dx=dt}]{} \int\sqrt{1-t^2}\,dt \xlongequal[\substack{令 t=\sin\theta\\ dt=\cos\theta d\theta}]{} \int\cos^2\theta\,d\theta$

$= \int\left(\frac{\cos 2\theta+1}{2}\right)d\theta = \frac{1}{4}\sin 2\theta+\frac{1}{2}\theta = \frac{1}{2}t\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t = \frac{1}{2}e^x\sqrt{1-e^{2x}}+\frac{1}{2}\sin^{-1}e^x+C$

(5) $\int \sec^4 x\tan^3 x\,dx$ (同 107 年 1(5)) $= \int \tan^3 x(\tan^2 x+1)\,d\tan x = \frac{1}{6}\tan^6 x+\frac{1}{4}\tan^4 x+C$

(6) $\int\frac{\sin x}{\cos x-\cos^2 x}\,dx = \int\frac{d\cos x}{\cos x(\cos x-1)} = \int\frac{1}{\cos x-1}\,d\cos x-\int\frac{1}{\cos x}\,d\cos x.$

$= \ln|\cos x-1|-\ln|\cos x|+C = \ln|1-\sec x|+C$

(7) $\int\frac{\ln\sqrt{x}}{x}\,dx \xlongequal[\substack{令 x=t^2\\ dx=2t\,dt}]{} \int\frac{\ln t}{t^2}2t\,dt = 2\int\frac{\ln t}{t}\,dt = 2\int\ln t\,d\ln t = 2(\ln t)^2-2\int\ln t\,d\ln t$

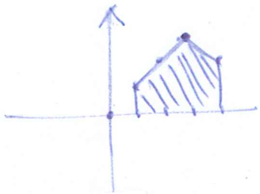
$\parallel\ (\ln t)^2+C = (\ln\sqrt{x})^2+C$

(8) $\int_0^1 x\ln x\,dx = \left(\frac{x^2}{2}\ln x-\frac{1}{4}x^2\right)\Big|_0^1 = -\frac{1}{4}$

105 中部

1 (1) $\int_1^4 (3-|x-3|)\,dx = \int_1^3 x\,dx + \int_3^4 (6-x)\,dx = \frac{1}{2}x^2\Big|_1^3 + \left(6x-\frac{1}{2}x^2\right)\Big|_3^4 = 4+6-\frac{1}{2}\cdot 7$

$= \frac{13}{2}$ ※



(2) $\int \frac{3}{x^2+3x-10}\,dx = \int \frac{3}{(x+5)(x-2)}\,dx = \frac{3}{7}\int\left(\frac{1}{x-2}-\frac{1}{x+5}\right)dx = \frac{3}{7}\ln|x-2| - \frac{3}{7}|x+5| + C$

(3) $\int \sin^2\theta \tan\theta \sqrt{\cos\theta}\,d\theta = \int \frac{\sin^3\theta}{\sqrt{\cos\theta}}\,d\theta = \int \frac{1-\cos^2\theta}{\sqrt{\cos\theta}}\,d(-\cos\theta) = \int\left(\cos^{\frac{3}{2}}\theta - \cos^{-\frac{1}{2}}\theta\right)d\cos\theta$

$= \frac{2}{5}\cos^{\frac{5}{2}}\theta - 2\cos^{\frac{1}{2}}\theta + C$

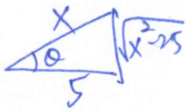
(4) $\int \frac{\sec x \tan x}{2\sec x - 1}\,dx = \frac{1}{2}\ln|2\sec x - 1| + C$

(5) $\int \frac{\ln x}{x^3}\,dx = \int \ln x\, d\left(-\frac{1}{2}x^{-2}\right) = \frac{-\ln x}{2x^2} + \int \frac{1}{2}x^{-2}\cdot\frac{1}{x}\,dx = \frac{-\ln x}{2x^2} - \frac{1}{4}x^{-2} + C$

$= \frac{-1-2\ln x}{4x^2} + C$

(6) $\int \frac{\sqrt{x^2-25}}{x}\,dx \underset{\substack{令 x=5\sec\theta \\ dx=5\sec\theta\tan\theta\,d\theta}}{=\!=\!=\!=} \int \frac{5\tan\theta}{5\sec\theta}\,5\sec\theta\tan\theta\,d\theta = 5\int \tan^2\theta\,d\theta$

$= 5\int(\sec^2\theta - 1)\,d\theta$



$= 5(\tan\theta - \theta) = 5\cdot\frac{\sqrt{x^2-25}}{5} - 5\sec^{-1}\left(\frac{x}{5}\right) + C$

$= -5\sec^{-1}\left(\frac{x}{5}\right) + \sqrt{x^2-25} + C$

(7) $\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx \underset{\substack{令 x^2=y \\ 2x\,dx=dy}}{=\!=\!=} \int \frac{y\,e^y\,\frac{1}{2}dy}{(y+1)^2} = \frac{1}{2}\int \frac{(y+1)-1}{(y+1)^2}e^y\,dy = \frac{1}{2}\left(\int\frac{e^y}{y+1}\,dy - \int\frac{e^y}{(y+1)^2}\,dy\right)$

$= \frac{1}{2}\left(\frac{e^y}{y+1} - \int e^y(-1)(y+1)^{-2}\,dy - \int\frac{e^y}{(y+1)^2}\,dy\right)$

$= \frac{1}{2}\frac{e^y}{y+1} + C = \frac{e^{x^2}}{2(x^2+1)} + C$

(8) $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx \underset{\substack{令 \sqrt[3]{x-1}=y \\ x-1=y^3 \\ dx=3y^2dy}}{=\!=\!=\!=} \int_{-1}^{1}\frac{3y^2\,dy}{y} = \frac{3}{2}y^2\Big|_{-1}^{1} = 0$

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