106 (分部)

2. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=5$. $\Rightarrow \frac{2}{3}x^{-\frac{1}{3}}+\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\sqrt[3]{\frac{y}{x}} \Rightarrow m=-\frac{1}{2} \Rightarrow y-1=-\frac{1}{2}(x-8) \Rightarrow y=-\frac{1}{2}x+5$

3. (a) $\frac{dy}{dx}=\frac{1}{2}(e^{2x}+e^{-2x})^{-\frac{1}{2}}(2e^{2x}-2e^{-2x})=\frac{e^{2x}-e^{-2x}}{\sqrt{e^{2x}+e^{-2x}}}$

(b) $y=\frac{1}{4}\ln(x+1)-\frac{1}{4}\ln(x-1)+\frac{1}{2}\tan^{-1}x \Rightarrow \frac{dy}{dx}=\frac{1}{4(x+1)}-\frac{1}{4(x-1)}+\frac{1}{2(x^2+1)}$

(c) $\frac{dy}{dx}=2x\log_2(x^2+1)+x^2\frac{1}{\ln 2}\cdot\frac{2x}{x^2+1}$

(d) $x=\sec\frac{1}{y} \Rightarrow \sec^{-1}x=\frac{1}{y} \Rightarrow -y^{-2}\frac{dy}{dx}=\frac{1}{|x|\sqrt{x^2-1}} \Rightarrow \frac{dy}{dx}=\frac{-y^2}{|x|\sqrt{x^2-1}}=\frac{-1}{(\sec^{-1}x)^2|x|\sqrt{x^2-1}}$

$\left(y=\frac{1}{\sec^{-1}x}\right)$

105 (分部)

2. $y=\frac{x+1}{x-1}=1+\frac{2}{x-1}$. $\frac{dy}{dx}=-2(x-1)^{-2}=-\frac{1}{2} \Rightarrow x=3$ 或 $-1$. 切点為 $(3,2)$ 或 $(-1,0)$

所求 $2y+x=7$ 或 $2y+x=-1$

3. (a) $y=e^{(\ln|5x|+(\cos x)^2)^2} \Rightarrow \frac{dy}{dx}=e^{(\ln|5x|+(\cos x)^2)^2}\cdot 2(\ln|5x|+(\cos x)^2)\left(\frac{1}{x}-2\cos x\cdot\sin x\right)$

$\Uparrow$ 或 $-\sin 2x$

(b) $y=\tan^{-1}(\sqrt{1-x^2}) \Rightarrow \frac{dy}{dx}=\frac{1}{(\sqrt{1-x^2})^2+1}\cdot\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x)=\frac{-x}{2-x^2}\cdot\frac{1}{\sqrt{1-x^2}}$

4. $x^2+\sqrt{xy}+y^2=72$. $(x_0,y_0)=(2,8) \Rightarrow 2x+\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}+x^{\frac{1}{2}}\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}+2y\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-(2x+\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}})}{2y+\frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}}$

故過 $(2,8)$ 的切線斜率 $=\frac{-(4+\frac{1}{2}\frac{\sqrt{8}}{\sqrt{2}})}{16+\frac{1}{2}\frac{\sqrt{2}}{\sqrt{8}}}=\frac{-(5)}{16+\frac{1}{4}}=\frac{-4}{13}$

所求: $y-8=\frac{-4}{13}(x-2)$

5. $f(x)=x^3+2x-1$

$f'(x)=3x^2+2$

$(f^{-1}(x))'=\frac{1}{f'(f^{-1}(x))} \Rightarrow (f^{-1}(-4))'=\frac{1}{f'(f^{-1}(-4))}=\frac{1}{f'(-1)}=\frac{1}{5}$ ※

$\left(x^3+2x-1=-4 \Rightarrow x^3+2x+3=0 \Rightarrow (x+1)(x^2-x+3)=0 \Rightarrow x=-1\right)$

104(分部)

3.(a) $y=2^x(1-\frac{4}{x+3}) \Rightarrow \frac{dy}{dx}=\ln 2\cdot 2^x(1-\frac{4}{x+3})+2^x(4(x+3)^{-2})=\ln 2\cdot 2^x(1-\frac{4}{x+3})+\frac{2^{x+2}}{(x+3)^2}$

(b) $y=(\ln\frac{3}{x})^{\frac{x}{3}}=e^{\frac{x}{3}\ln(\ln\frac{3}{x})}$

$\frac{dy}{dx}=(\ln\frac{3}{x})^{\frac{x}{3}}\cdot\left(\frac{\ln(\ln\frac{3}{x})}{3}+\frac{\cancel{x}}{3}\cdot\frac{1}{\ln\frac{3}{x}}\cdot\frac{\cancel{x}}{\cancel{3}}(-\cancel{3}\cancel{x}^{-2})\right)=(\ln\frac{3}{x})^{\frac{x}{3}}\cdot\frac{\ln(\ln\frac{3}{x})-\ln\frac{x}{3}}{3}$

$=\frac{\ln(\ln\frac{3}{x})\cdot\ln(\frac{3}{x})^{\frac{x}{3}}-(\ln\frac{3}{x})^{\frac{x-3}{3}}}{3}$

(c) $y=e^{3x}\sin(\cos^2(3x))=3e^{3x}\sin(\cos^2(3x))+e^{3x}\cos(\cos^2(3x))\cdot 2\cos 3x\cdot -\sin 3x\cdot 3$

$=3e^{3x}\sin(\cos^2(3x))-3e^{3x}\cos(\cos^2(3x))\sin 6x$

(d) $y=\sqrt{2+\sqrt{2+\sqrt{x}}}=\frac{1}{2}\cdot\frac{1}{\sqrt{2+\sqrt{2+\sqrt{x}}}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{2+\sqrt{x}}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}=\frac{1}{8\sqrt{2+\sqrt{2+\sqrt{x}}}\cdot\sqrt{2+\sqrt{x}}\cdot\sqrt{x}}$

(e) $\ln(xy)-e^{x+y}+x^2y=1 \Rightarrow \frac{1}{x}+\frac{1}{y}\frac{dy}{dx}-e^xe^y-e^xe^y\frac{dy}{dx}+2xy+x^2\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-\frac{1}{x}+e^{x+y}-2xy}{x^2-e^{x+y}+\frac{1}{y}}=\frac{-y+xye^{x+y}-2x^2y^2}{x^3y+x-xye^{x+y}}$

103(分部)

2.(a) $y=\left(\frac{\cos x}{1+\sin x}\right)^2 \Rightarrow \frac{dy}{dx}=2\cdot\frac{\cos x}{1+\sin x}\cdot\frac{(-\sin x)(1+\sin x)-\cos x\cdot\cos x}{(1+\sin x)^2}=2\cdot\frac{\cos x}{1+\sin x}\cdot\frac{-\sin x-1}{(1+\sin x)^2}$

$=\frac{-2\cos x}{(1+\sin x)^2}$

(b) $x\cdot\sin 5y=y\cdot\cos 5x+\frac{1}{2} \Rightarrow \sin 5y+x\cos 5y\cdot 5\cdot\frac{dy}{dx}=\frac{dy}{dx}\cos 5x-5y\sin 5x$

$\Rightarrow \frac{dy}{dx}=\frac{-5y\sin 5x-\sin 5y}{5x\cdot\cos 5y-\cos 5x}$

(c) $y=\ln(|\tan^{-1}x^3|) \Rightarrow \frac{dy}{dx}=\frac{1}{\tan^{-1}x^3}\cdot\frac{3x^2}{1+x^6}$

(d) $y=\sqrt[3]{\frac{x^5(x+1)^{16}}{(x+2)}} \Rightarrow \ln y=\frac{5}{3}\ln x+\frac{16}{3}\ln(x+1)-\frac{1}{3}\ln(x+2)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{5}{3}\cdot\frac{1}{x}+\frac{16}{3}\cdot\frac{1}{x+1}-\frac{1}{3}\cdot\frac{1}{x+2}$

$\Rightarrow \frac{dy}{dx}=\sqrt[3]{\frac{x^5(x+1)^{16}}{x+2}}\left(\frac{5}{3}\cdot\frac{1}{x}+\frac{16}{3}\cdot\frac{1}{x+1}-\frac{1}{3}\frac{1}{x+2}\right)$

102(分部)

2. (a) $y = \dfrac{\sin x - x}{x^3} \Rightarrow \dfrac{dy}{dx} = \dfrac{(\cos x - 1)x^3 - (\sin x - x)3x^2}{x^6} = \dfrac{2x + x\cos x - 3\sin x}{x^4}$

(b) $y = e^{2x}\cdot x^5 \cdot \sin x \Rightarrow \dfrac{dy}{dx} = 2e^{2x}\cdot x^5 \sin x + 5x^4\cdot e^{2x}\sin x + e^{2x}x^5\cos x$

(c) $y = \cos x^{\sqrt{3}} \Rightarrow \dfrac{dy}{dx} = -\sin x^{\sqrt{3}} \cdot \sqrt{3}\cdot x^{\sqrt{3}-1}$

(d) $y = (\cos x)^{\sqrt{3}} \Rightarrow y = e^{\sqrt{3}\ln\cos x} \Rightarrow \dfrac{dy}{dx} = (\cos x)^{\sqrt{3}} \cdot \sqrt{3} \cdot \dfrac{-\sin x}{\cos x}$

$= -\sqrt{3}\sin x \cdot (\cos x)^{\sqrt{3}-1}$

或 $-\sqrt{3}(\cos x)^{\sqrt{3}} \cdot \tan x$

(e) $y = \sqrt{3}^{\sin x} + \sqrt{3}\tan x + \sec(\sqrt{3}x) \Rightarrow \dfrac{dy}{dx} = \sqrt{3}^{\sin x}\cdot(\ln\sqrt{3}\cdot\cos x) + \sqrt{3}\sec^2 x + \sec(\sqrt{3}x)\tan(\sqrt{3}x)\cdot\sqrt{3}$

$= e^{\sin x \ln\sqrt{3}} + \sqrt{3}\tan x + \sec(\sqrt{3}x)$

(f) $y = \dfrac{(x+2)^5(e^x-1)^2}{x^3(2x-1)^3} \Rightarrow \ln y = 5\ln(x+2) + 2\ln(e^x-1) - 3\ln x - 3\ln(2x-1)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{5}{x+2} + \dfrac{2\cdot e^x}{e^x-1} - \dfrac{3}{x} - \dfrac{6}{2x-1}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{(x+2)^5(e^x-1)^2}{x^3(2x-1)^3}\left(\dfrac{5}{x+2} + \dfrac{2e^x}{e^x-1} - \dfrac{3}{x} - \dfrac{6}{2x-1}\right)$

3. $y = \sin\left(\dfrac{\pi y}{x}\right) \Rightarrow \dfrac{dy}{dx} = \cos\left(\dfrac{\pi y}{x}\right)\cdot\pi\left(\dfrac{dy}{dx}\cdot x^{-1} + y\cdot(-x^{-2})\right)$

$\Rightarrow \dfrac{dy}{dx} = 0$ 所求即 $y = 1$