4-1 Extreme Values of Functions on Closed Intervals

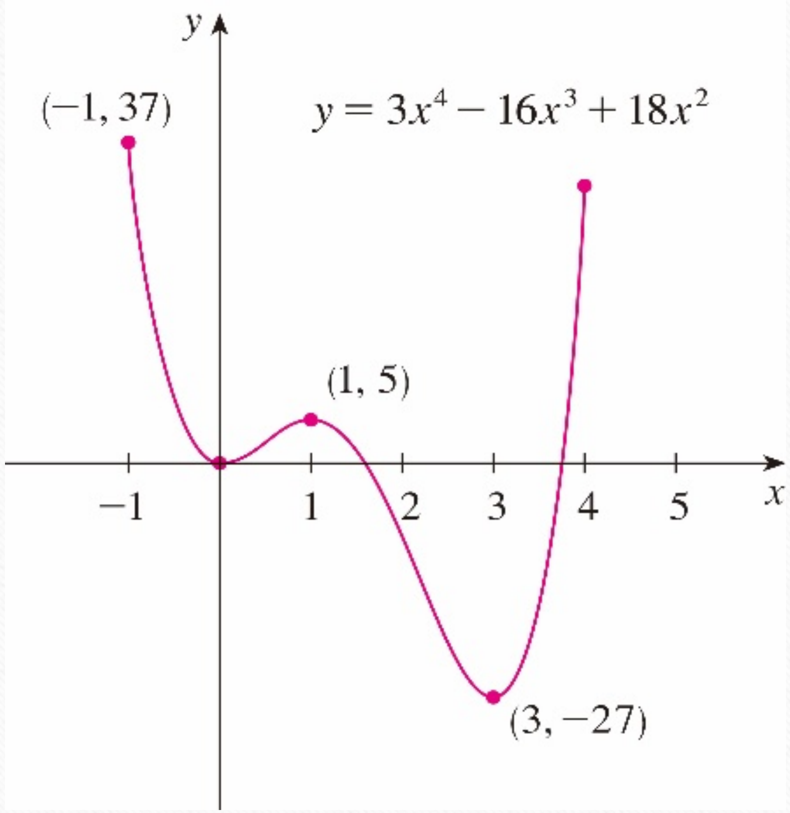
師大工教一

Definition: Let f be a function with domain D. Then f has an **absolute maximum** value on D at a point c if $f(x) \le f(c)$ for all x in D and an **absolute minimum** value on D at a point c if $f(x) \ge f(c)$ for all x in

global 整体的

D.

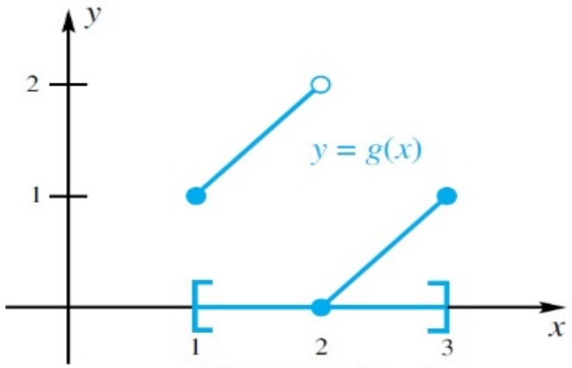
Note: Maximum and minimum values are called **extreme values** of the function f. Absolute maxima or minima are also referred to as **global** maxima or minima.


y
(−1, 37)
y = 3x^4 − 16x^3 + 18x^2
(1, 5)
x
−1
1
2
3
4
5
(3, −27)

Theorem 1—The Extreme Value Theorem

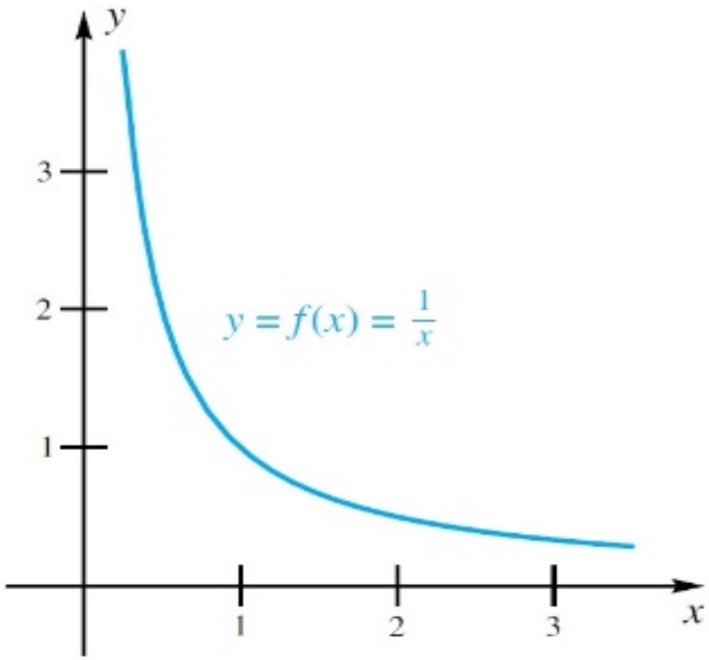
极值定理

If f is continuous on a closed interval $[a,b]$, then f attains both an absolute maximum value M and an absolute minimum value m in $[a,b]$. That is, there are numbers x_1 and x_2 in $[a,b]$ with $f(x_1)=M, f(x_2)=m$, and $m \le f(x) \le M$ for every other x in $[a,b]$.



No max, min = 0

Figure 3



On $(0, \infty)$, no max or min

On $[1, 3]$, max = 1, min = $\frac{1}{3}$

On $(1, 3]$, no max, min = $\frac{1}{3}$

Figure 2

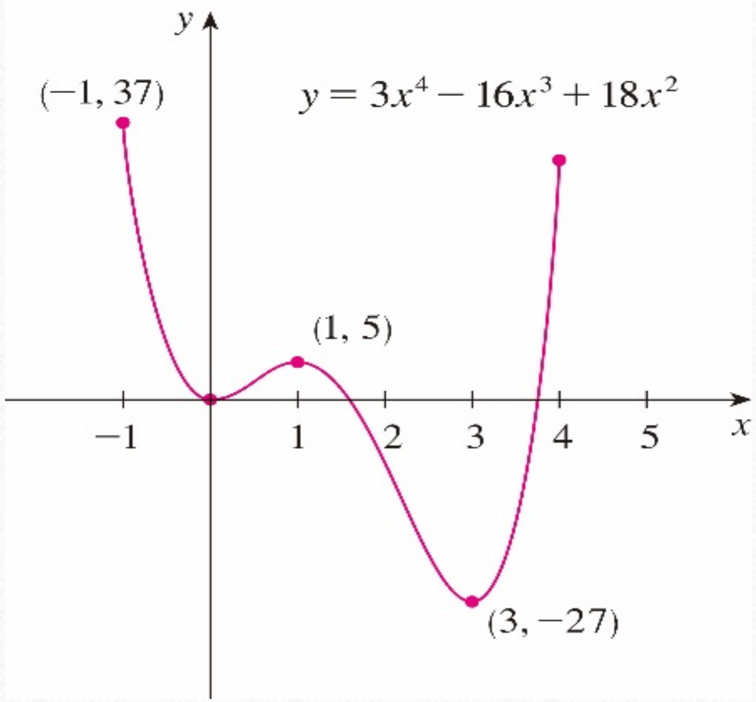


Figure 4.3(p232) shows that extreme values could happen at endpoints or interior points of the function.

局部極值

Local (Relative) Extreme Values

Definition: A function f has a **local maximum** value at a point c within its domain D if $f(x) \le f(c)$ for all $x \in D$ lying in some open interval containing c.

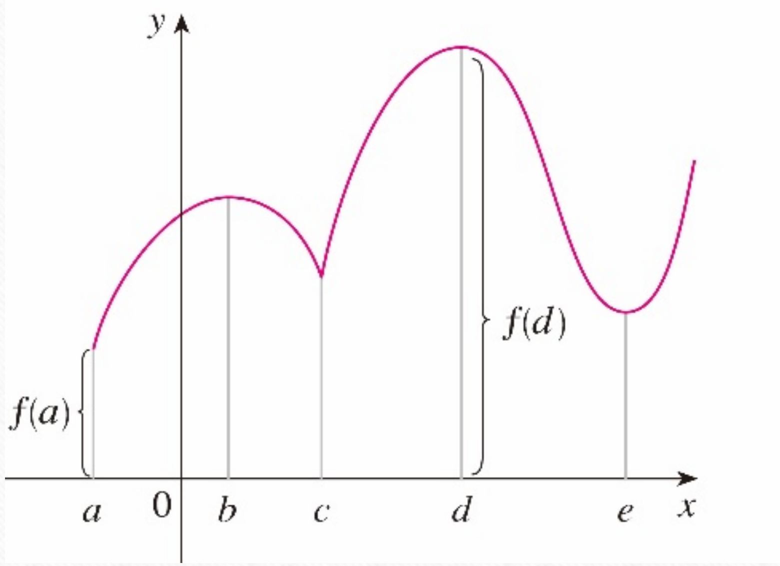
A function f has a **local minimum** value at a point c within its domain D if $f(x) \ge f(c)$ for all $x \in D$ lying in some open interval containing c.

Local extrema are also called **relative extrema**.

Finding Extrema

Theorem 2—The First Derivative Theorem for Local Extreme Values

If f has a local maximum or minimum value at an interior point c of its domain, and if f' is defined at c, then $f'(c)=0$.



Definition: An interior point of the domain of a function f where f' is zero or undefined is a **critical point** of f.

臨界点

Finding the Absolute Extrema of a Continuous Function f on a Finite Closed Interval

1. Find all critical points of f on the interval.
2. Evaluate f at all critical points and endpoints.
3. Take the largest and smallest of these values.

Ex3(p234): Find the absolute maximum and minimum values of

$f(x) = 10x(2 - \ln x)$ on the interval $\left[1, e^2\right]$.

$$f'(x) = 10(2 - \ln x) + 10x\left(-\frac{1}{x}\right) = 20 - 10\ln x - 10$$
$$= 10 - 10\ln x \overset{\text{let}}{=} 0$$
$$\ln x = 1$$
$$x = e \leftarrow \text{(critical point)}$$

$f(e) = 10e$ is the absolute maximum Value.

$f(1) = 20$

$f(e^2) = 0$ is the absolute minimum Value

Ex4(p235): Find the absolute maximum and minimum values of $f(x) = x^{\frac{2}{3}}$ on the interval $[-2,3]$.

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} \overset{\text{Let}}{=} 0$ $\quad x=0$ is a critical point

$x^{-\frac{1}{3}} = 0$

$x = 0$

$f(0) = 0$ is absolute minimum value

$f(-2) = \sqrt[3]{4}$

$f(3) = \sqrt[3]{9}$ is absolute maximum value

HW4-1

- **HW: 24,29,38,47,50,69,72**