

1-6 Inverse Functions and Logarithms

反函數
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師大工教一

※Inverse Functions

反函數

Definition: A function $f(x)$ is **one-to-one** if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

Definition: Suppose that $f(x)$ is a one-to-one function with domain A and range B . The **inverse function** f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x \Leftrightarrow f(x) = y$.

Note: $f^{-1}(x)$ does not mean $\frac{1}{f(x)}$.
↳ 讀作: f inverse

Cancellation Equations: $f^{-1}(f(x)) = x, x \in A, f(f^{-1}(y)) = y, y \in B$

消去律

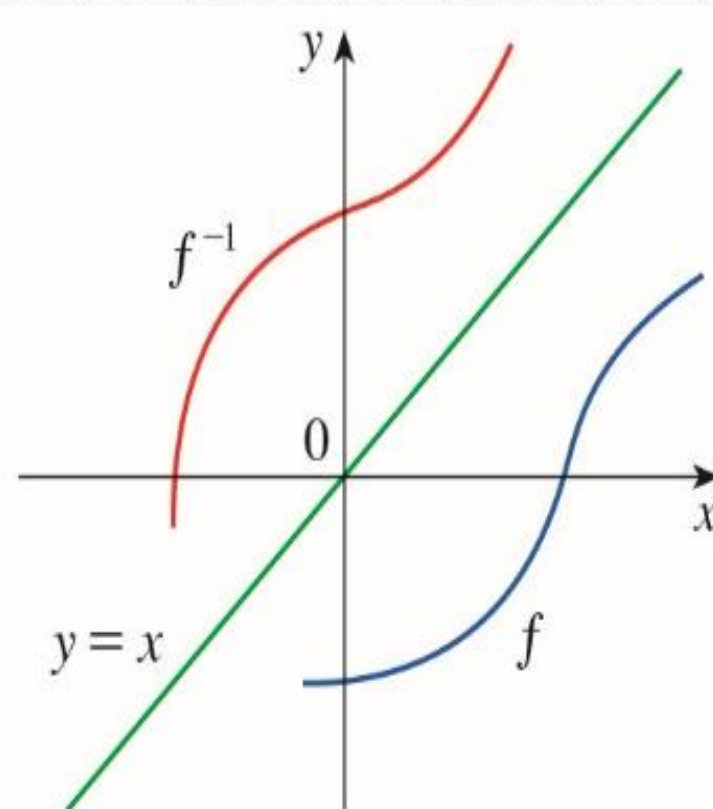
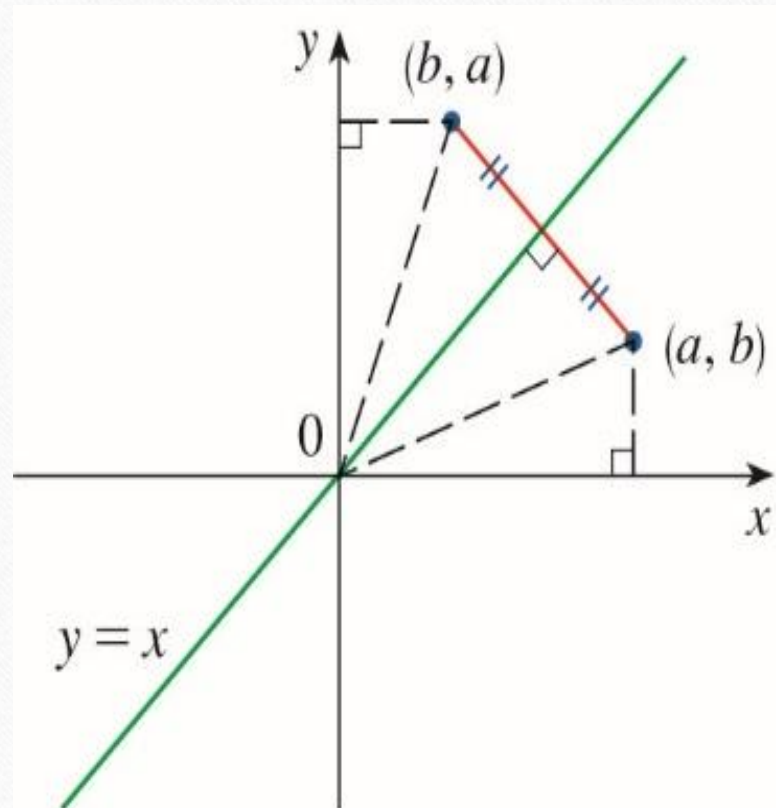
合成 function $\mathbb{R} \times \mathbb{R}$:

$$f(x) = x+2, g(x) = x^2$$

$$(f \circ g)(x) = f(g(x)) = f(x^2)$$

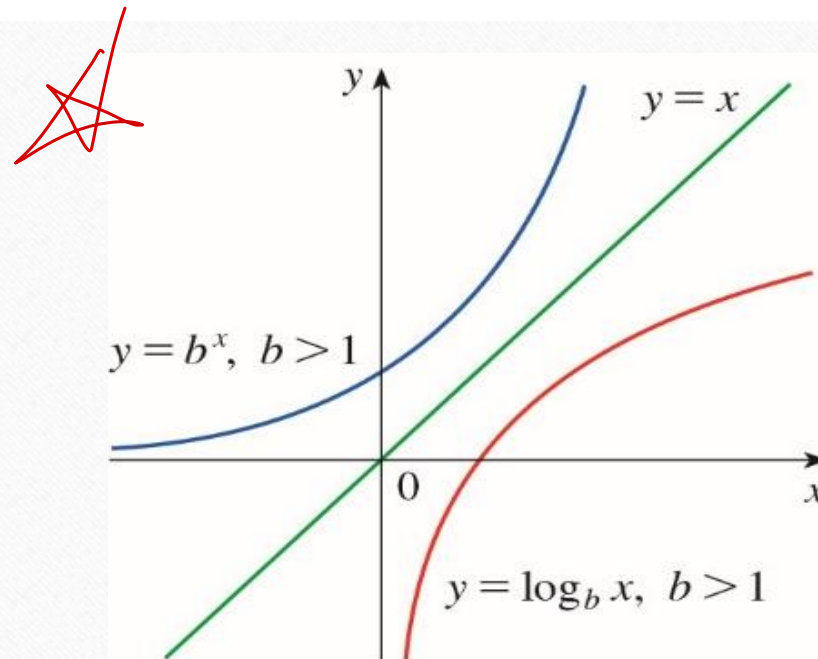
circle $= x^2 + 2$

$$(g \circ f)(x) = g(f(x)) = g(x+2) = (x+2)^2 = x^2 + 4x + 4$$



※Logarithmic Functions

Definition: The logarithmic function with base b , written $y = \log_b x$, is the inverse of the base b exponential function $y = b^x, (b > 0, b \neq 1)$.



$$y = \log_b x \Leftrightarrow y = b^x$$

The natural logarithmic function $y = \log_e x = \ln x$

The common logarithmic function $y = \log_{10} x = \log x$

※ Properties of Logarithms

$$1. \ln(bx) = \ln b + \ln x \quad 2. \ln\left(\frac{b}{x}\right) = \ln b - \ln x \quad 3. \ln x^r = r \ln x$$

$$\text{※} 4. a^{\log_a x} = x, \log_a a^x = x, e^{\ln x} = x, \ln e^x = x$$

$$\text{※} 5. a^x = (e^{\ln a})^x = e^{\ln a \cdot x}$$

$$6. \log_a x = \frac{\ln x}{\ln a}$$

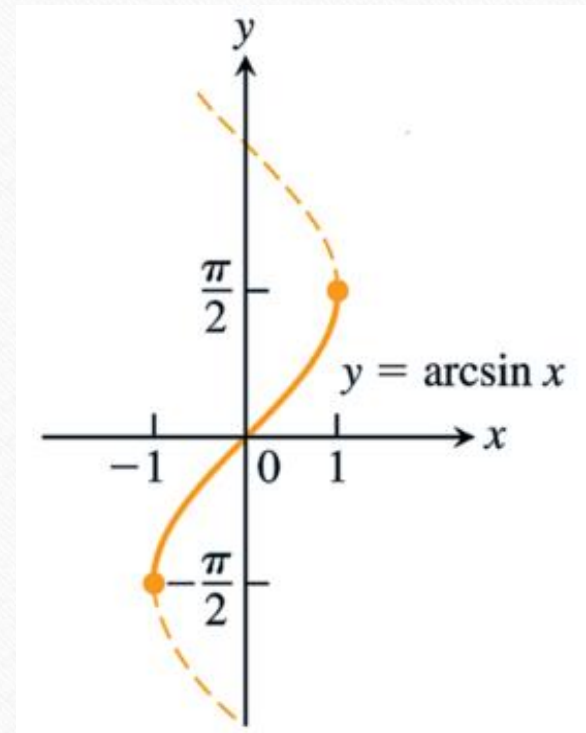
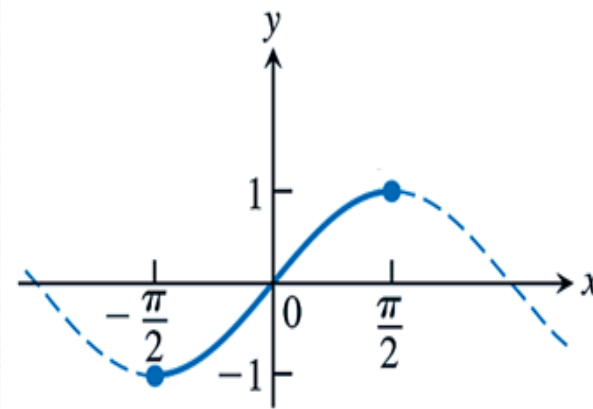
$$\log_a x = \frac{\log_b x}{\log_b a}$$

※Inverse Trigonometric Functions

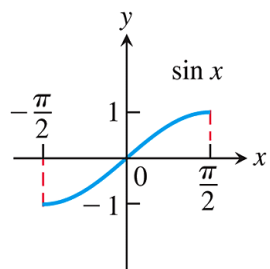
$$\sin \frac{\pi}{6} = \frac{1}{2} \Leftrightarrow \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{6} = \arcsin \left(\frac{1}{2} \right) = \frac{\pi}{6}$$

Definition: $\sin \theta = a \Leftrightarrow \sin^{-1} a = \theta$, etc.

Properties: 1. $\sin(\sin^{-1} a) = a$, 2. $\sin^{-1}(\sin \theta) = \theta$



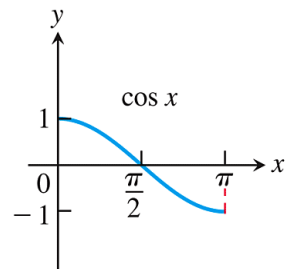
Domain restrictions that make the trigonometric functions one-to-one



$$y = \sin x$$

Domain: $[-\pi/2, \pi/2]$

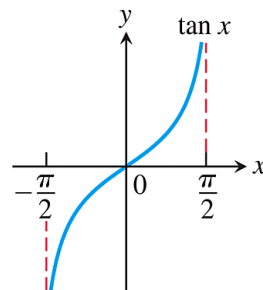
Range: $[-1, 1]$



$$y = \cos x$$

Domain: $[0, \pi]$

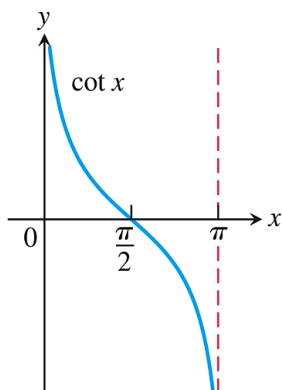
Range: $[-1, 1]$



$$y = \tan x$$

Domain: $(-\pi/2, \pi/2)$

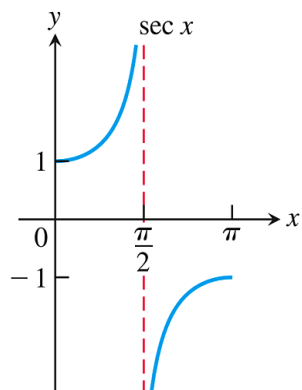
Range: $(-\infty, \infty)$



$$y = \cot x$$

Domain: $(0, \pi)$

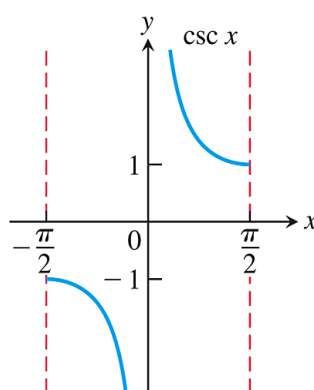
Range: $(-\infty, \infty)$



$$y = \sec x$$

Domain: $[0, \pi/2) \cup (\pi/2, \pi]$

Range: $(-\infty, -1] \cup [1, \infty)$

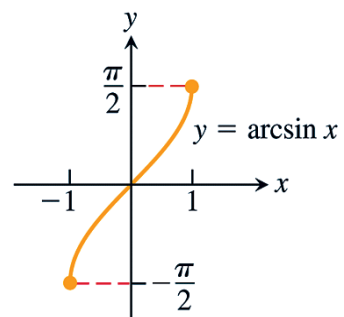


$$y = \csc x$$

Domain: $(-\pi/2, 0) \cup (0, \pi/2]$

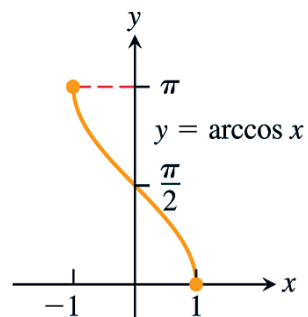
Range: $(-\infty, -1] \cup [1, \infty)$

Domain: $-1 \leq x \leq 1$
 Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



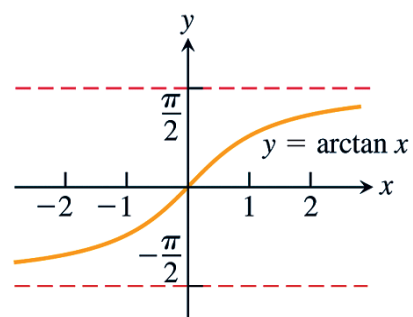
(a)

Domain: $-1 \leq x \leq 1$
 Range: $0 \leq y \leq \pi$



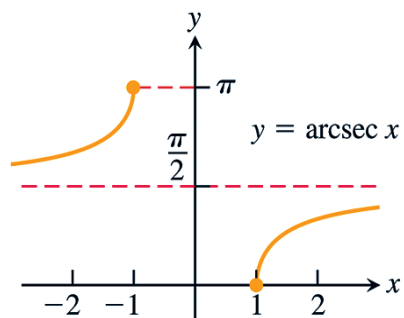
(b)

Domain: $-\infty < x < \infty$
 Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



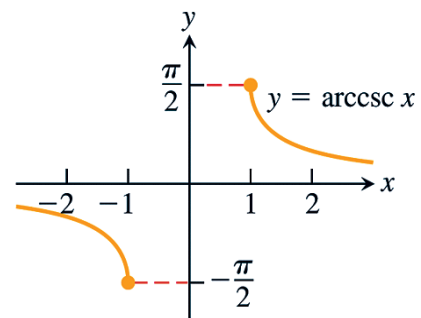
(c)

Domain: $x \leq -1$ or $x \geq 1$
 Range: $0 \leq y \leq \pi, y \neq \frac{\pi}{2}$



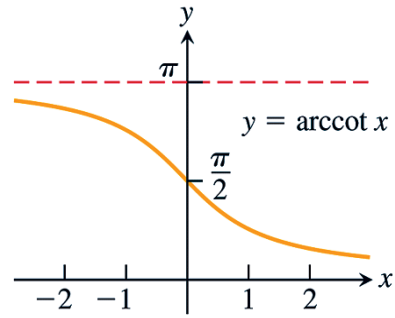
(d)

Domain: $x \leq -1$ or $x \geq 1$
 Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$



(e)

Domain: $-\infty < x < \infty$
 Range: $0 < y < \pi$



(f)

FIGURE 1.64 Graphs of the six basic inverse trigonometric functions.

HW1-6

- HW: 14,60,62,71

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1. (3 pts) Evaluate $\sin(2 \sin^{-1}(-\frac{3}{5}))$.(note : $\sin^{-1} x = \arcsin x$)

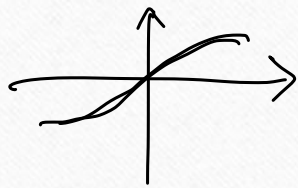
求

$$\text{Let } \sin^{-1}(-\frac{3}{5}) = \theta$$

$$\Rightarrow \sin(\sin^{-1}(-\frac{3}{5})) = \sin \theta$$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$



$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$\begin{aligned} &\downarrow \\ &\sin(2\theta) \\ &= 2 \sin \theta \cos \theta \\ &= 2 \times -\frac{3}{5} \times \frac{4}{5} \\ &= \frac{-24}{25} \end{aligned}$$