106 分部 (其它)

2 (1) $\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x$

令 $y=\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x \Rightarrow \ln y = \ln\left(\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x\right) = \lim_{x\to\infty}\left[x\ln\left(1+\frac{1}{x^2}\right)\right] = \lim_{x\to\infty}\left(\frac{\ln(1+1/x^2)}{1/x}\right)\ \left(\frac{0}{0}\right)$

$$\overset{\text{L'Hôpital's}}{\underset{\text{Rule}}{=}} \lim_{x\to\infty}\frac{-2x^{-3}}{(1+1/x^2)\cdot(-x^{-2})} = \lim_{x\to\infty}\frac{2(1/x)}{(1+1/x^2)} = 0$$

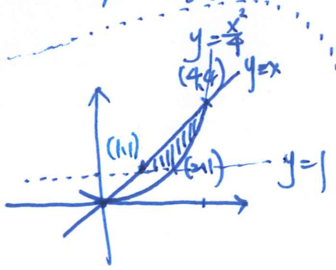
$\Rightarrow \lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x = y = 1$ ※

(2) $\lim_{x\to1^+}\frac{\int_1^x \cos t^2\,dt}{x-1}\ \left(\frac{0}{0}\right) = \lim_{x\to1^+}\frac{\cos x^2}{1} = \cos 1$ ※

3. 找 $y=\ln(\cos x)$ 的弧長 $[0,\frac{\pi}{3}]$. $\Rightarrow y'=\frac{\sin x}{\cos x}=\tan x$

所求 $\int_0^{\frac{\pi}{3}}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^{\frac{\pi}{3}}\sqrt{1+\tan^2 x}\,dx = \int_0^{\frac{\pi}{3}}\sec x\,dx = \ln|\sec x+\tan x|\Big|_0^{\frac{\pi}{3}}$

$=\ln(2+\sqrt{3})-\ln(1+0)=\ln(2+\sqrt{3})$



4. (1) 找面積 $y=\frac{x^2}{4}$. $y=x$. $y=1$.

所求  =  + 

$=\frac{1}{2}(1\cdot1)+\int_2^4\left(x-\frac{x^2}{4}\right)dx = \frac{1}{2}+\left(\frac{1}{2}x^2-\frac{x^3}{12}\right)\Big|_2^4 = \frac{1}{2}+\left(\frac{1}{2}(16-4)-\frac{1}{12}(64-8)\right) = \frac{1}{2}+6-\frac{14}{3}$
$=\frac{11}{6}$

(2) 繞x軸之体積

$V=2\pi\int_1^4(2\sqrt{y}-y)y\,dy = 2\pi\left(\frac{4}{5}y^{\frac{5}{2}}-\frac{1}{3}y^3\right)\Big|_1^4 = 2\pi\left(\frac{4}{5}\cdot31-\frac{1}{3}63\right) = 2\pi\cdot\frac{19}{5} = \frac{38}{5}\pi$

法2 $V=\pi\int_1^2(x^2-1^2)dx+\pi\int_2^4\left(x^2-\frac{x^4}{16}\right)dx = \pi\left(\left(\frac{x^3}{3}-x\right)\Big|_1^2+\left(\frac{x^3}{3}-\frac{1}{80}x^5\right)\Big|_2^4\right) = \pi\left(\frac{7}{3}-1+\frac{56}{3}-\frac{1}{80}(992)\right)$
$=\pi\left(20-\frac{62}{5}\right)=\frac{38}{5}\pi$

5. $F(s)=\int_0^\infty e^{-st}dt = \lim_{a\to\infty}\int_0^a e^{-st}dt = \lim_{a\to\infty}-\frac{1}{s}e^{-st}\Big|_0^a = \lim_{a\to\infty}\left(-\frac{1}{s}e^{-sa}+\frac{1}{s}\right)$

當 $s>0$ 時. $\lim_{a\to\infty}\left(-\frac{1}{s}e^{-sa}\right)=0$. $F(s)=\frac{1}{s}$

當 $s<0$ 時. $\lim_{a\to\infty}\left(-\frac{1}{s}e^{-sa}\right)=\infty$ 發散

當 $s=0$ 時. $F(s)=\int_0^\infty 1\,dt=\infty$ 發散

故當 $s>0$ 時. $F(s)$ 收斂
且 $F(s)=\frac{1}{s}$ ※

6. $F(x)=\int_0^{x^2}x\,f(t)\,dt = x\cdot\int_0^{x^2}f(t)\,dt$

$F'(x)=\int_0^{x^2}f(t)\,dt + x\cdot f(x^2)\cdot 2x = \int_0^{x^2}f(t)\,dt+2x^2 f(x^2)$ ※

105分部(其它)

2. 找 p s.t. $\int_0^1 \frac{1}{x^p}dx$ 收斂

① 當 p=1 時. $\int_0^1 \frac{1}{x^p}dx = \int_0^1 \frac{1}{x}dx = \lim_{a\to0^+} \ln x\Big|_a^1 = 0-\lim_{a\to0^+}\ln a = \infty$. 發散

② 當 p>1 時 $\int_0^1 \frac{1}{x^p}dx > \int_0^1\frac{1}{x} dx = \infty$. 發散

③ 當 p<1 時 $\int_0^1 \frac{1}{x^p}dx = \int_0^1 x^{-p}dx = \frac{1}{1-p}x^{1-p}\Big|_0^1 = \frac{1}{1-p}$ 收斂　故所求 $p<1$

3. $\lim_{x\to\infty}(1+\frac{2}{x})^{3x}$

令 $y=\lim_{x\to\infty}(1+\frac{2}{x})^{3x} \Rightarrow \ln y = \lim_{x\to\infty} 3x\ln(1+\frac{2}{x}) = \lim_{x\to\infty} \frac{\ln(1+\frac{2}{x})}{1/3x}\ \left(\frac{0}{0}\right) \xrightarrow[\text{Rule}]{\text{L'Hopital's}} \lim_{x\to\infty} \frac{-2x^{-2}\cdot 3}{(1+\frac{2}{x})(-x^{-2})} = 6$

$\Rightarrow \lim_{x\to\infty} (1+\frac{2}{x})^{3x} = y = e^6$

法2 $\lim_{x\to\infty}(1+\frac{2}{x})^{3x} = \lim_{x\to\infty}\left[(1+\frac{2}{x})^{\frac{x}{2}}\right]^6 = e^6$

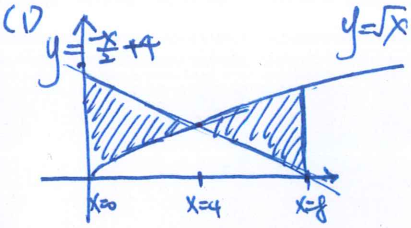
4. $y=\sqrt{9-x^2}=(9-x^2)^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2}(9-x^2)^{-\frac{1}{2}}\cdot(-2x) = \frac{-x}{\sqrt{9-x^2}}$

(1) 弧長 over $[-1,1]$ ⇒ $\int_{-1}^1\sqrt{1+(y')^2}\,dx = \int_{-1}^1\sqrt{1+\frac{x^2}{9-x^2}}\,dx = 3\int_{-1}^1\frac{1}{\sqrt{9-x^2}}dx \xrightarrow[dx=3du]{令x=3u} 3\int_{-\frac{1}{3}}^{\frac{1}{3}}\frac{3du}{3\sqrt{1-u^2}}$
$= 3\sin^{-1}u\Big|_{-\frac{1}{3}}^{\frac{1}{3}} = 6\sin^{-1}\frac{1}{3}$ #

(2) 繞 x 軸轉之表面積 ⇒ $S = 2\pi\int_{-1}^1\sqrt{9-x^2}\cdot\sqrt{1+\frac{x^2}{9-x^2}}\,dx = 2\pi\int_{-1}^1 3\,dx = 12\pi$ ※

5. $y=\sqrt{x}$. $y=\frac{-x}{2}+4$. $x=0$. $x=8$

(1)



(2) $D = \int_0^4 \left(\frac{-x}{2}+4-\sqrt{x}\right)dx + \int_4^8\left[\sqrt{x} - \left(\frac{-x}{2}+4\right)\right]dx$

$= \left(-\frac{1}{4}x^2+4x-\frac{2}{3}x^{\frac{3}{2}}\right)\Big|_0^4 + \left(\frac{2}{3}x^{\frac{3}{2}}+\frac{1}{4}x^2-4x\right)\Big|_4^8 = \left(-4+16-\frac{16}{3}\right) + \left(\frac{32\sqrt{2}-16}{3}+12-16\right)$

$= 8+\frac{32\sqrt{2}-32}{3}$

$= \frac{32\sqrt{2}-8}{3}$ ※

(3) 繞 x 軸體積 $= \pi\int_0^4\left[\left(\frac{-x}{2}+4\right)^2-(\sqrt{x})^2\right]dx + \pi\int_4^8\left[(\sqrt{x})^2-\left(\frac{-x}{2}+4\right)^2\right]dx$

$=\pi\int_0^4\left(\frac{x^2}{4}-5x+16\right)dx + \pi\int_4^8\left(-\frac{x^2}{4}+5x-16\right)dx$

$=\pi\left[\left(\frac{1}{12}x^3-\frac{5}{2}x^2+16x\right)\Big|_0^4 + \left(-\frac{1}{12}x^3+\frac{5}{2}x^2-16x\right)\Big|_4^8\right] = \pi\left[\left(\frac{16}{3}-40+64\right)+\left(-\frac{112}{3}+120-64\right)\right]$

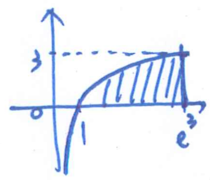
$=\pi(80-32) = 48\pi$

6. $\int\sqrt{\frac{x}{1-x^3}}\,dx$ $(0<x<1)$ $= \int\frac{\sqrt{x}}{\sqrt{1-x^3}}dx \xrightarrow[\frac{3}{2}x^{\frac{1}{2}}dx=du]{令x^{\frac{3}{2}}=u} \int\frac{\frac{2}{3}du}{\sqrt{1-u^2}} = \frac{2}{3}\arcsin(x^{\frac{3}{2}})+C$

104(其它)

2. $y=\ln x$, x軸 (a) 繞x軸轉之體積 $=\pi\int_1^{e^3}\left((\ln x)^2-0^2\right)dx \overset{\substack{令\ln x=u\\ \frac{1}{x}dx=du\\ dx=e^u du}}{=\!=\!=} \pi\int_0^3 u^2e^u\,du = \pi\left(u^2-2u+2\right)e^u\Big|_0^3 = \pi(5e^3-2)$

又 $\int x^2e^x dx=\int x^2 d(e^x)=x^2e^x-2\int xe^x dx$
$=x^2e^x-2(xe^x-e^x)+C$
$=(x^2-2x+2)e^x+C$

方法 

體積 $=2\pi\int_0^3 y(e^3-e^y)dy$
$=2\pi e^3\int_0^3 y\,dy-2\pi\int_0^3 ye^y dy = 9\pi e^3-2\pi(ye^y-e^y)\Big|_0^3$
$=9\pi e^3-2\pi(2e^3+1)$
$=\pi(5e^3-2)$

(b) 繞y軸轉之體積 $=2\pi\int_1^{e^3} x\ln x\,dx = 2\pi\left(\frac{x^2}{2}\ln x-\frac{x^2}{4}\right)\Big|_1^{e^3} = 2\pi\left[\frac{3e^6}{2}-\frac{e^6}{4}+\frac{1}{4}\right] = \frac{5}{2}\pi e^6+\frac{1}{2}\pi$

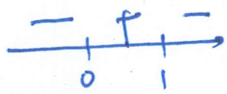
方法 $3\pi(e^3)^2-\pi\int_0^3(e^y)^2dy = 3\pi e^6-\pi\frac{1}{2}e^{2y}\Big|_0^3 = 3\pi e^6-\frac{1}{2}\pi e^6+\frac{1}{2}\pi = \frac{5}{2}\pi e^6+\frac{1}{2}\pi$ ※

3. $y=2\sqrt{x}$, $1\le x\le 2$ 對x軸轉之表面積
$y'=2\left(\frac{1}{2}x^{-\frac{1}{2}}\right)=\frac{1}{\sqrt{x}}$. 所求 $=2\pi\int_1^2 2\sqrt{x}\cdot\sqrt{1+\frac{1}{x}}\,dx = 4\pi\int_1^2\sqrt{x+1}\,dx = 4\pi\,\frac{2}{3}(x+1)^{\frac{3}{2}}\Big|_1^2$
$=\frac{8}{3}\pi(3\sqrt{3}-2\sqrt{2})$

4. $\int_{x^2}^2 f(t)dt=\frac{1}{2}\sin\left(\frac{\pi}{2}x\right) \Rightarrow -f(x^2)\cdot 2x=\frac{1}{2}\cos\left(\frac{\pi}{2}x\right)\cdot\frac{\pi}{2} \Rightarrow x=1$時: $-2f(1)=0 \Rightarrow f(1)=0$

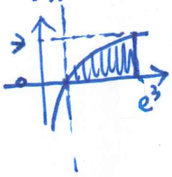
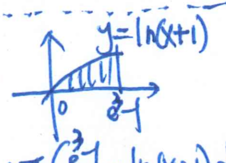
5. $0<\frac{1}{\sqrt{x^6+1}}<\frac{1}{x^3}$ (當 $1\le x<\infty$ 時) $\Rightarrow \int_1^\infty 0\,dx\le\int_1^\infty\frac{1}{\sqrt{x^6+1}}dx\le\int_1^\infty\frac{1}{x^3}dx \Rightarrow$ 收斂

$\int_1^\infty 0\,dx = 0$, $\int_1^\infty\frac{1}{x^3}dx = \lim_{a\to\infty}-\frac{1}{2}x^{-2}\Big|_1^a=\frac{1}{2}$

6. $x-x^2=x(1-x)$ 

$\int_a^b\sqrt{x-x^2}\,dx$ 代表 $\sqrt{x-x^2}$ 從 $x=a$ 到 $x=b$ 之間的面積. 又 $a,b\in[0,1]$. 且 $\sqrt{x-x^2}$ 在 0~1 時恆 ≥ 0

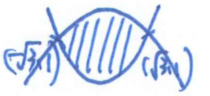
故最大值為 $\int_0^1\sqrt{x-x^2}\,dx=\int_0^1\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2}\,dx \overset{\substack{令x-\frac{1}{2}=\frac{1}{2}\sin\theta\\ dx=\frac{1}{2}\cos\theta d\theta}}{=\!=\!=} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{2}\cos\theta\cdot\frac{1}{2}\cos\theta\,d\theta \overset{\cos^2\theta=\frac{1+\cos2\theta}{2}}{=\!=\!=} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos2\theta+1}{8}d\theta$
$=\left(\frac{\sin2\theta}{16}+\frac{\theta}{8}\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$
$=\frac{\pi}{8}$

7.  向左平移1 $\Rightarrow$ 

所求 $=2\pi\int_0^{e^3-1}x\ln(x+1)\,dx \overset{令u=x+1}{=} 2\pi\int_1^{e^3}(u-1)\ln u\,du$
$=2\pi\left(\frac{u^2}{2}\ln u-\frac{u^2}{4}-u\ln u+u\right)\Big|_1^{e^3} = 3e^6\pi-\frac{\pi}{2}e^6+\frac{\pi}{2}-6e^3\pi+2e^3\pi-2\pi$
$=\frac{5}{2}e^6\pi-\frac{3}{2}\pi-4e^3\pi$ ※

10.3 (其它)

2. $y=7-2x^2$, $y=x^2-2$ ⇒ 交点 $(\sqrt{3},1)$, $(-\sqrt{3},1)$

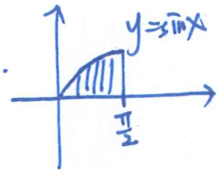


所求 $=\int_{-\sqrt{3}}^{\sqrt{3}}\left[(7-2x^2)-(x^2-2)\right]dx$

$= (9x-x^3)\Big|_{-\sqrt{3}}^{\sqrt{3}} = 18\sqrt{3}-6\sqrt{3}$

$=12\sqrt{3}$ ※

3.



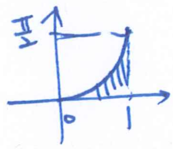
所求 $=\pi\int_0^{\frac{\pi}{2}}\sin^2 x\,dx = \pi\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}dx$

$=\pi\left(\frac{x}{2}-\frac{\sin 2x}{4}\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi^2}{4}$ ※

4. $y=\int_0^x \tan t\,dt$, $y'=\tan x$. $0\le x\le\frac{\pi}{4}$ 之長度 $=\int_0^{\frac{\pi}{4}}\sqrt{1+\tan^2 x}\,dx = \ln|\sec x+\tan x|\Big|_0^{\frac{\pi}{4}} = \ln(\sqrt{2}+1)$ ※

5. (a) $\int_0^2 x^2\,dx = \frac{1}{3}x^3\Big|_0^2 = \frac{8}{3}$, $\int_0^4\sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^4 = \frac{16}{3}$. 故 $\int_0^2 x^2dx+\int_0^4\sqrt{x}\,dx = 8$ ※

(b) $\int_0^{\frac{\pi}{2}}\sin x\,dx = -\cos x\Big|_0^{\frac{\pi}{2}} = 1$, $\int_0^1\sin^{-1}x\,dx = \frac{\pi}{2}-\int_0^{\frac{\pi}{2}}\sin y\,dy = \frac{\pi}{2}-1$ 所求 $=1+\left(\frac{\pi}{2}-1\right)=\frac{\pi}{2}$



(c) $\int_0^{x^2} f(t)\,dt = \cos\pi x$

⇒ $f(x^2)\cdot 2x = -\sin\pi x\cdot\pi$ ⇒ $x=\sqrt{2}$ 代入: $2\sqrt{2}f(2) = -\sin\sqrt{2}\pi\cdot\pi$ ⇒ $f(2)=\frac{-\sin(\sqrt{2}\pi)}{2\sqrt{2}}\cdot\pi$

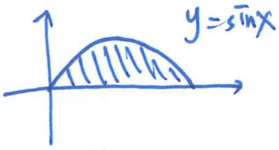
(d) $\int_0^\infty\frac{e^{\tan^{-1}x}}{1+x^2}dx = \lim_{a\to\infty}e^{\tan^{-1}x}\Big|_0^a = e^{\frac{\pi}{2}}-e^0 = e^{\frac{\pi}{2}}-1$ (收斂)

102(其它)

2. 所求 $= \int_1^3 \ln x\,dx = (x\ln x - x)\Big|_1^3 = 3\ln 3 - 2$ ※

3. $y=\int_0^x \sqrt{\cos 4t}\,dt$. $y'=\sqrt{\cos 4x}$. $0\le x\le \frac{\pi}{4}$ 之長 $= \int_0^{\frac{\pi}{4}}\sqrt{1+\cos 4x}\,dx = \int_0^{\frac{\pi}{4}}\sqrt{2}\cdot\cos 2x\,dx$

$= \sqrt{2}\cdot\frac{1}{2}\sin 2x\Big|_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2}$

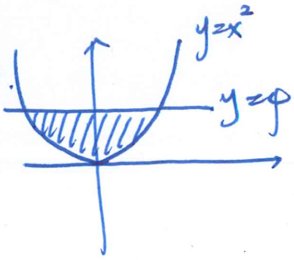
4. 

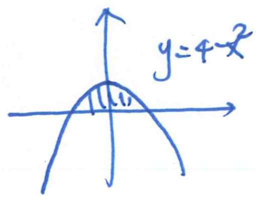
繞y軸轉之體積 $= \int_0^{\pi} 2\pi x\sin x = 2\pi(-x\cos x+\sin x)\Big|_0^{\pi} = 2\pi^2$

5. (a) $\int_0^{\infty}\frac{1}{x^2+1}\,dx = \lim\limits_{a\to\infty}\tan^{-1}x\Big|_0^a = \frac{\pi}{2}-0 = \frac{\pi}{2}$ (收斂)

(b) $\int_0^{\infty}\frac{1}{\sqrt{x+1}}\,dx > \int_1^{\infty}\frac{1}{\sqrt{x+1}}\,dx > \int_1^{\infty}\frac{1}{x}\,dx = \lim\limits_{a\to\infty}\ln x\Big|_1^a = \infty$ (發散)

6. $f'(x) = \cos(x^6+1)\cdot 2x$

7. 

所求即 

繞x軸轉之體積

$= \int_{-2}^{2}\pi(4-x^2)^2\,dx$

$= \pi\int_{-2}^{2}(x^4-8x^2+16)\,dx$

$= \pi\left(\frac{1}{5}x^5-\frac{8}{3}x^3+16x\right)\Big|_{-2}^{2} = 2\pi\left(\frac{32}{5}-\frac{64}{3}+32\right)$

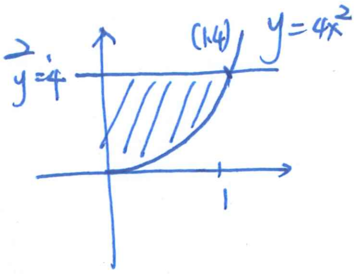
$= \frac{512}{15}\pi$

8. 所求 $= \int_0^1 \sqrt{1+x}\,dx$

$= \frac{2}{3}(1+x)^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}(2\sqrt{2}-1)$ ※

106 本部(其它)

2.



所求 $= \pi\int_0^1\left(4^2-(4x^2)^2\right)dx = 16\pi - \pi\frac{16}{5}x^5\Big|_0^1 = \frac{64}{5}\pi.$

3. $y=\int_{\frac{\pi}{4}}^{x}\cos t\,dt \Rightarrow y' = \cos x \Rightarrow$ 所求長度 $=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\sqrt{1+\cos^2 x}\,dx = -\cos x\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \sqrt{2}$

4. $\because \lim_{x\to5^-}\sqrt{25-x^2} = 0 = \lim_{x\to5^-}(x-5)$ $\therefore \lim_{x\to5^-}\frac{\sqrt{25-x^2}}{x-5}\left(\frac{0}{0}\right) = \lim_{x\to5^-}\frac{\frac12(25-x^2)^{-\frac12}(-2x)}{1} = \lim_{x\to5^-}\frac{-x}{\sqrt{25-x^2}}$

$= -\infty$

5. $\int_1^\infty\left(\frac{cx}{x^2+2}-\frac{1}{3x}\right)dx = \lim_{a\to\infty}\left(\frac{c}{2}\ln(x^2+2)-\frac13\ln x\right)\Big|_1^a = \lim_{a\to\infty}\left(\ln\frac{(a^2+2)^{\frac{c}{2}}}{\sqrt[3]{a}} - \frac{c}{2}\ln 3\right)$

$\Rightarrow$

(1) 唯有當 $c=\frac13$ 時收斂

若 $c>\frac13$. 則所求積分 $=\infty$.

若 $c<\frac13$ 則所求積分 $=-\infty$.

(2) $\int_1^\infty\left(\frac{\frac13 x}{x^2+2}-\frac{1}{3x}\right)dx = \frac13\int_1^\infty\left(\frac{x}{x^2+2}-\frac{1}{x}\right)dx = \lim_{a\to\infty}\frac13\ln\frac{\sqrt{x^2+2}}{x}\Big|_1^a = -\frac13\ln\sqrt{3}$

6. $\int_0^1 x^a(1-x)^b\,dx \underset{\substack{\text{令 } 1-x=u\\ \text{則 } x=1-u\\ \text{且 } dx=-du}}{=\!=\!=\!=} \int_1^0(1-u)^a u^b(-du) = \int_0^1(1-u)^a u^b\,du$

$= \int_0^1 x^b(1-x)^a\,dx$

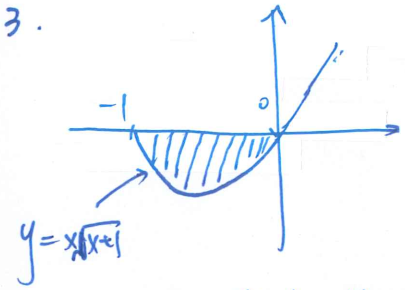
105 插班(其它)

2. $\int_0^{x^5} f(2t^2-t+1)\,dt = \ln x$ ← 知 $x>0$

$\Rightarrow f(2x^{10}-x^5+1)\cdot 5x^4 = \frac{1}{x} \Rightarrow f(2x^{10}-x^5+1) = \frac{1}{5x^5} \Rightarrow f(2t^2-t+1) = \frac{1}{5t}$

令 $2t^2-t+1=2017 \Rightarrow t=32$ 或 $\frac{-63}{2}$ 故 $f(2017) = \frac{1}{160}$ (因 $x>0$)

3.



(1) 所求 $= \int_{-1}^{0} x\sqrt{x+1}\,dx = \int_{-1}^{0}\left[(x+1)^{\frac{3}{2}}-(x+1)^{\frac{1}{2}}\right]dx$

$= \left[\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}\right]\Big|_{-1}^{0} = \frac{-4}{15}$ ※

(2) 繞x軸轉之體積 $= \pi\int_{-1}^{0} x^2(x+1)\,dx = \pi\left(\frac{1}{4}x^4+\frac{1}{3}x^3\right)\Big|_{-1}^{0} = \frac{1}{12}\pi$

(3) 繞y軸轉之體積 $= 2\pi\int_{-1}^{0} x(x\sqrt{x+1})\,dx \xlongequal[\text{令 } x+1=u]{} 2\pi\int_0^1 (u-1)^2\sqrt{u}\,du$

$= 2\pi\int_0^1\left[u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right]du$

$= 2\pi\left(\frac{2}{7}u^{\frac{7}{2}}-\frac{4}{5}u^{\frac{5}{2}}+\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_0^1$

$= 2\pi\left(\frac{2}{7}+\frac{2}{3}-\frac{4}{5}\right) = 2\pi\cdot\frac{30+70-84}{105}$

$= \frac{32}{105}\pi$

4. $y = 1-\frac{x^2}{4}$, $y' = -\frac{x}{2}$ 在 $0\le x\le 2$ 對y軸轉之表面積

所求 $= \int_0^2 2\pi\cdot x\cdot\left(\sqrt{1+\frac{x^2}{4}}\right)dx \xlongequal[\text{令 } x=2\tan\theta]{} \int_0^{\frac{\pi}{4}} 4\pi\tan\theta\sec\theta\; 2\sec^2\theta\,d\theta$

$= 8\pi\int_0^{\frac{\pi}{4}} \sec^2\theta\, d\sec\theta = \frac{8\pi}{3}\sec^3\theta\Big|_0^{\frac{\pi}{4}}$

$= \frac{8\pi}{3}(2\sqrt{2}-1) = \frac{16\sqrt{2}\pi-8\pi}{3}$

※

5. $\int_0^1 t\ln t\,dt = \left(\frac{t^2}{2}\ln t - \frac{t^2}{4}\right)\Big|_0^1 = -\frac{1}{4}$

故 $\lim\limits_{x\to 1}\left(\frac{1}{4}+\int_0^x t\ln t\,dt\right) = 0 = \lim\limits_{x\to 1}(x\ln x - x+1)$

$\Rightarrow \lim\limits_{x\to 1}\frac{\frac{1}{4}+\int_0^x t\ln t\,dt}{x\ln x - x+1}\left(\frac{0}{0}\right) \xlongequal[\text{Rule}]{\text{L'Hôpital's}} \lim\limits_{x\to 1}\frac{x\ln x}{\ln x} = \lim\limits_{x\to 1} x = 1$ ※