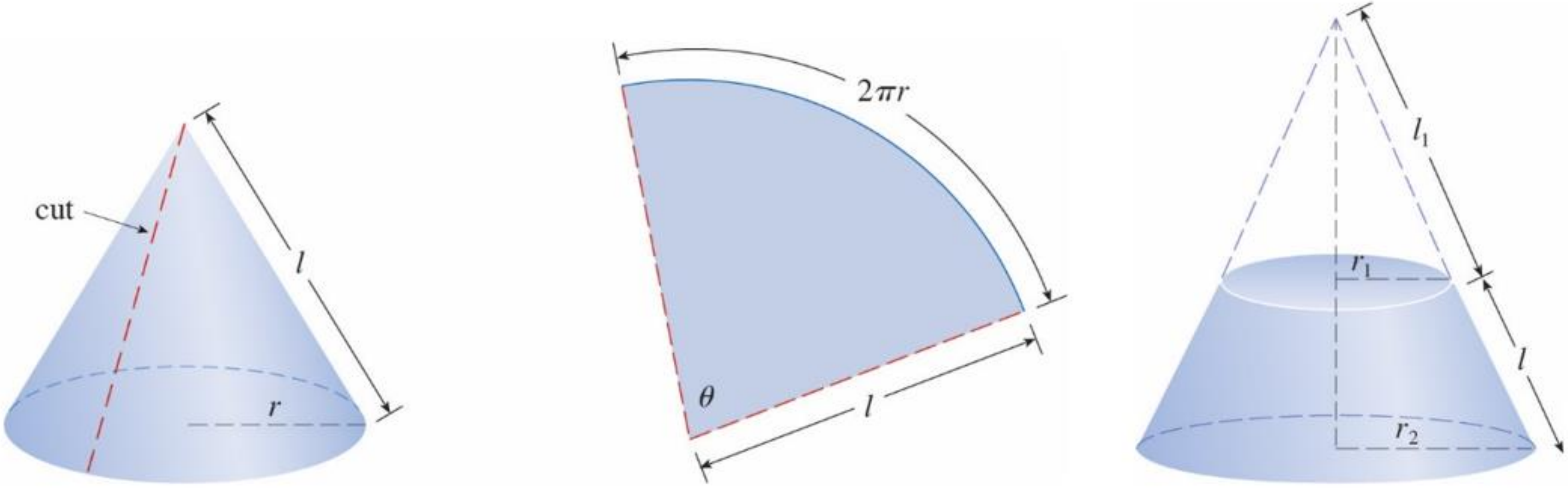
旋転表面積

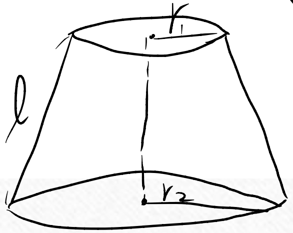
6-4 Areas of Surfaces of Revolution

師大工教一

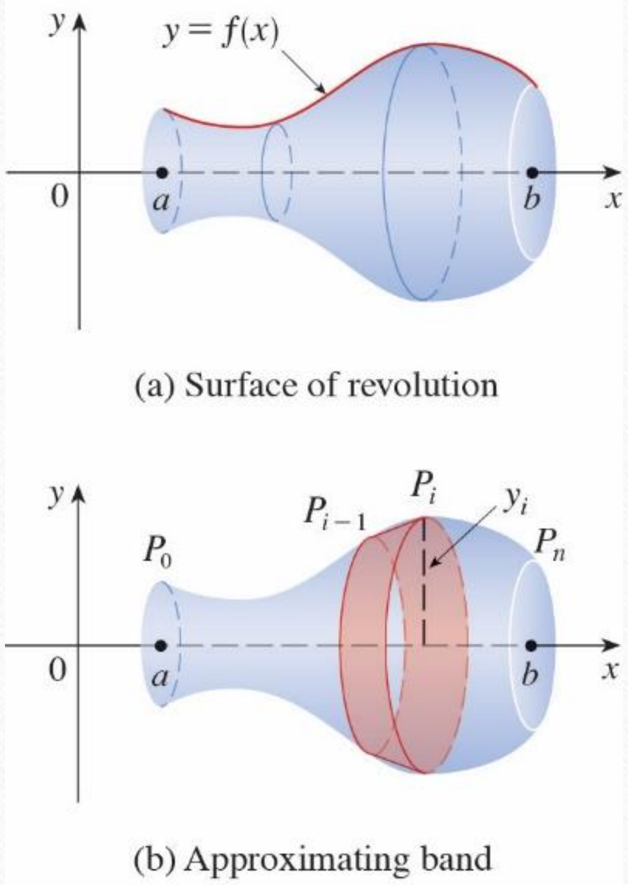


The area of surface of the frustum of a cone is $2\pi \frac{r_1 + r_2}{2} l$.

圆锥台



$2\pi\left(\frac{r_1+r_2}{2}\right)l$



(1 piece)Frustum surface area

$$= 2\pi \cdot \frac{f\left(x_{k-1}\right)+f\left(x_k\right)}{2} \sqrt{\left(\Delta x_k\right)^2+\left(\Delta y_k\right)^2}$$

$$
\text{(n pieces) surface area}\begin{cases} = \sum_{k=1}^{n} 2\pi \cdot \frac{f(x_{k-1}) + f(x_k)}{2} \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2} \\ = \sum_{k=1}^{n} 2\pi \cdot \frac{f(x_{k-1}) + f(x_k)}{2} \sqrt{(\Delta x_k)^2 + \left(f'(c_k)\Delta x_k\right)^2} \\ = \sum_{k=1}^{n} 2\pi \cdot \frac{f(x_{k-1}) + f(x_k)}{2} \sqrt{1 + \left(f'(c_k)\right)^2}\,\Delta x_k \end{cases}
$$

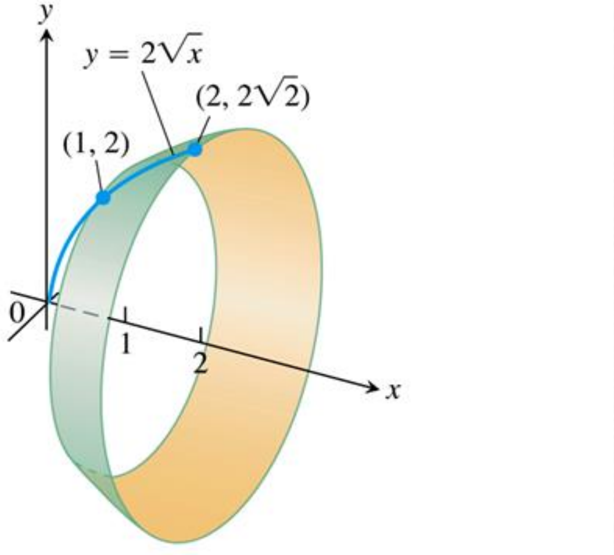
$$
\text{Surface area}\begin{cases} = \lim_{n\to\infty} \sum_{k=1}^{n} 2\pi \cdot \frac{f(x_{k-1}) + f(x_k)}{2} \sqrt{1 + \left(f'(c_k)\right)^2}\,\Delta x_k \\ = \int_a^b 2\pi f(x) \sqrt{1 + \left(f'(x)\right)^2}\, dx \end{cases}
$$

Definition If the function $f(x) \geq 0$ is continuously differentiable on $[a,b]$, the area of surface generated by revolving the graph of $y = f(x)$ about the $x-$ axis is $S = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \boxed{\int_a^b 2\pi f(x)\sqrt{1+\left(f'(x)\right)^2}\, dx}$

Ex1(p401) Find the area of surface generated by revolving the curve

$y = 2\sqrt{x}$, $1 \le x \le 2$, about the $x-$axis.

$\frac{dy}{dx} = \frac{1}{\sqrt{x}}$



$S = \int_1^2 2\pi (2\sqrt{x}) \sqrt{1 + \frac{1}{x}} \, dx$

$= 4\pi \int_1^2 \sqrt{x+1} \, dx$

Let $u = x+1$

$du = dx$

$= 4\pi \int_2^3 \sqrt{u} \, du$

$= 4\pi \left[\frac{2}{3} u^{\frac{3}{2}} \Big|_2^3 \right]$

$= \frac{8}{3}\pi \left(3^{\frac{3}{2}} - 2^{\frac{3}{2}} \right)$

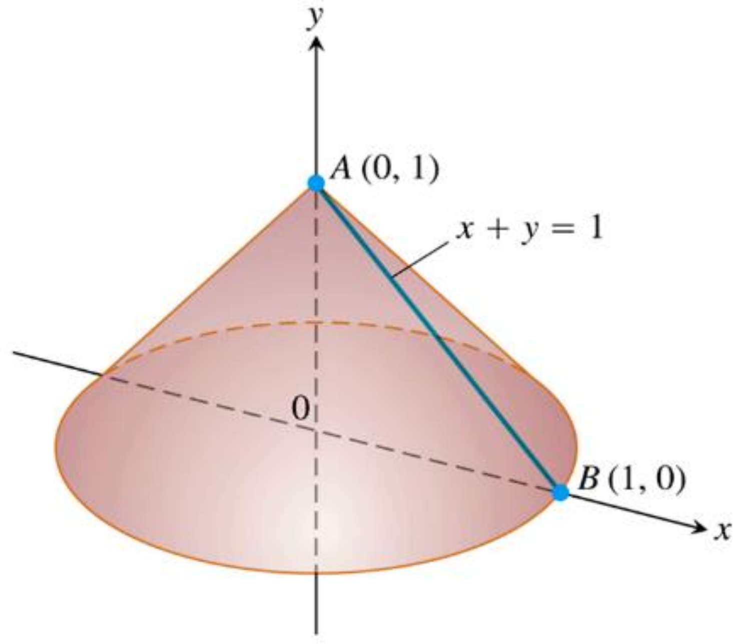
Surface Area for Revolution About the $y-$Axis

If the function $x=g(y)\geq 0$ is continuously differentiable on $[c,d]$, the area of surface generated by revolving the graph of $x=g(y)$ about the $y-$axis is

$$S=\int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy=\int_a^b 2\pi g(y)\sqrt{1+\left(g'(y)\right)^2}\,dy$$

Ex2(p402) The line segment $x = 1 - y$, $0 \le y \le 1$, is revolved about the $y-$ axis to generate a cone. Find its lateral surface area(which excludes the base area).

$\frac{dx}{dy} = -1$



$S = \int_0^1 2\pi(1-y)\sqrt{1+1}\, dy$

$= 2\sqrt{2}\pi \int_0^1 (1-y)\, dy$

$= 2\sqrt{2}\pi \left[y - \frac{y^2}{2} \Big|_0^1 \right]$

$= 2\sqrt{2}\pi \left(1 - \frac{1}{2}\right) = \pi\sqrt{2}$ #

HW6-4

- **HW: 4,7,9,11,19,29**

(105分部) 4 (共 14 分) Let A be the curve $y=\sqrt{9-x^2},\ -1 \le x \le 1$.

(1). (8 分) Find the arc length of the curve A.

(2). (6 分) Find the area of the surface generated by revolving the curve A about the x-axis.

$\frac{dy}{dx} = \frac{1}{\cancel{2}}(9-x^2)^{-\frac{1}{2}} \cdot (-\cancel{2}x) = \frac{-x}{\sqrt{9-x^2}}$

(1) $L = \int_{-1}^{1} \sqrt{1+\frac{x^2}{9-x^2}}\, dx$

(2) $S = \int_{-1}^{1} 2\pi \sqrt{9-x^2}\sqrt{1+\frac{x^2}{9-x^2}}\, dx$

$= 2\pi \int_{-1}^{1} \sqrt{9\cancel{-x^2+x^2}}\, dx = 2\pi \int_{-1}^{1} 3\, dx$

$= 6\pi \int_{-1}^{1} dx = 6\pi \left(x\Big|_{-1}^{1}\right) = 12\pi$