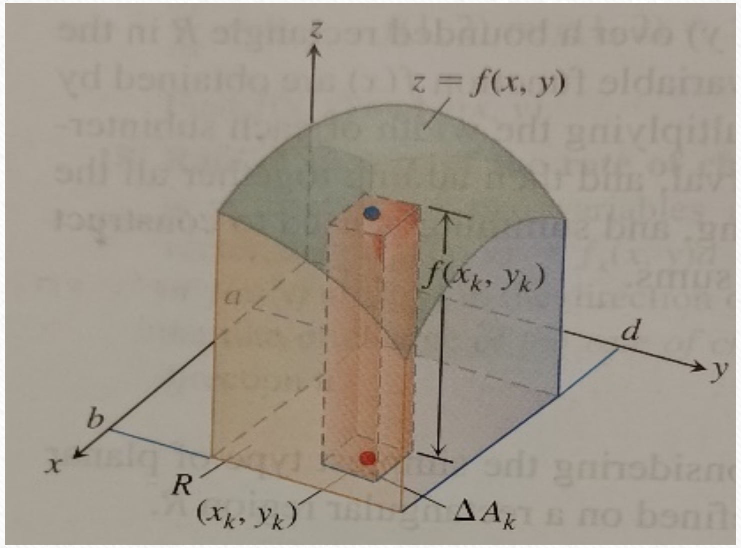
雙重積分、迭代積分

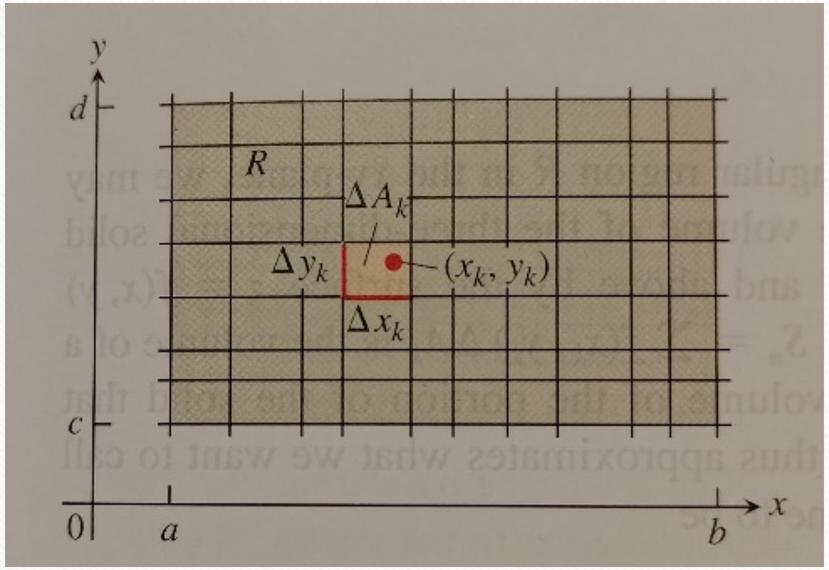
14-1 Double and Iterated Integrals over Rectangles

師大工教一

Double Integrals

Q: Let $R=[a,b]\times[c,d]$, how can we get the volume of the solid below?





1.Partition of R: $P=\{A_1,\cdots,A_n\} \Rightarrow \underbrace{\Delta A_k = \Delta x_k \Delta y_k}_{\text{底面積}}, k=1,\cdots,n.$

分割

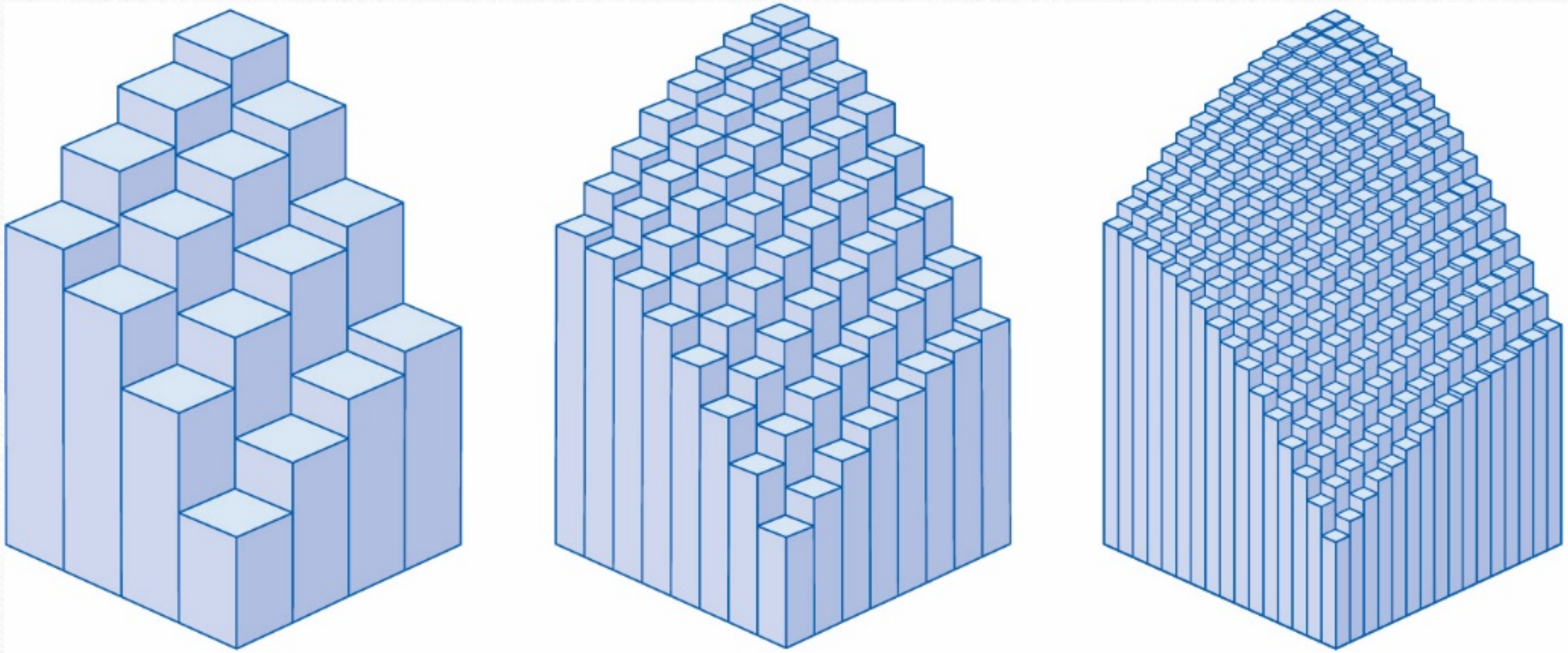
2.Choose a point $\left(x_k^*, y_k^*\right) \in A_k$ → $f(x_k^*, y_k^*)$ 高

3.Riemann Sum: $S_n = \sum_{k=1}^{n} f\left(x_k^*, y_k^*\right)\Delta A_k$

4.Let $\|P\|$ be the largest width or height of any rectangle in the partition.

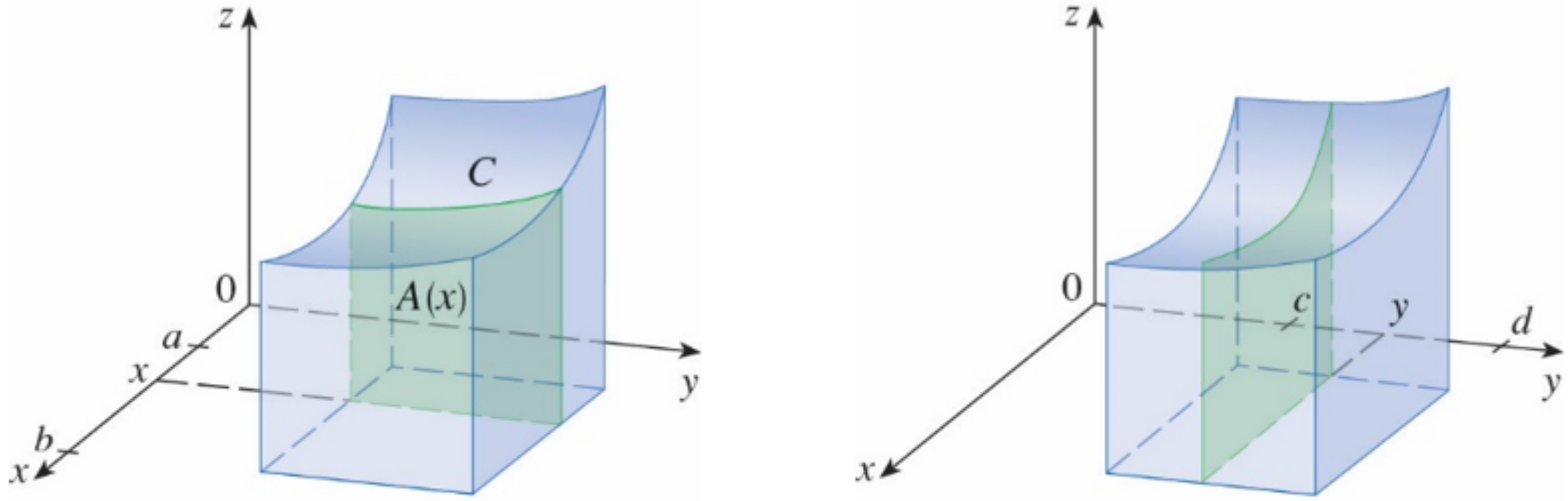
If the limit $\boxed{\lim_{\|P\|\to 0} \sum_{k=1}^{n} f\left(x_k^*, y_k^*\right)\Delta A_k}$ exists, the function f is said to be integrable and the limit is called the **double integral** of f over R, which is written as

$\iint_R f(x,y)\,dA$ or $\iint_R f(x,y)\,dxdy$.



(a) $m = n = 4$, $V \approx 41.5$ (b) $m = n = 8$, $V \approx 44.875$ (c) $m = n = 16$, $V \approx 46.46875$

Fubini's Theorem for Calculating Double Integrals



$$\iint_R f(x,y)\,dA = \int_a^b A(x)\,dx \qquad \int_c^d A(y)\,dy \text{ 亦可}$$

$$= \int_a^b \left(\int_c^d f(x,y)\,dy\right)dx \qquad = \int_c^d \left[\int_a^b f(x,y)\,dx\right]dy$$

Iterated integrals
迭代積分

Volume= $\int_{x=a}^{x=b} A(x)\,dx = \int_{x=a}^{x=b}\left(\int_{y=c}^{y=d} f(x,y)\,dy\right)dx$,

Or Volume= $\int_{y=c}^{y=d} A(y)\,dy = \int_{y=c}^{y=d}\left(\int_{x=a}^{x=b} f(x,y)\,dx\right)dy$,

We may simplify the expression by $\int_a^b \int_c^d f(x,y)\,dydx$ or $\int_c^d \int_a^b f(x,y)\,dxdy$,

and call them **iterated integrals**.

Theorem 1—Fubini's Theorem (First Form)

If $f(x,y)$ is continuous throughout the rectangular region

$R: a \le x \le b, c \le y \le d$, then $\iint\limits_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$.

Ex1(p800) Calculate $\iint\limits_R f(x,y)\,dA$ for $f(x,y)=100-6x^2y$ and $R: 0\le x\le 2, -1\le y\le 1$.

$$\int_{-1}^{1}\left[\int_0^2 (100-6x^2y)\,dx\right]dy \qquad (100x-2x^3y)_x = 100-6x^2y$$

偏導

$$=\int_{-1}^{1}\left(100x-2x^3y\Big|_0^2\right)dy=\int_{-1}^{1}[200-16y-0]\,dy=\int_{-1}^{1}(200-16y)\,dy$$

代入x

$$=200y-8y^2\Big|_{-1}^{1}=200\cancel{-8}-(-200\cancel{-8})$$

$$=400 \#$$

Ex2(p801) Find the volume of the region bounded above by the elliptical paraboloid $z=10+x^2+3y^2$ and below by $R: 0\le x\le 1, 0\le y\le 2$.

$=f(x,y)$

$$\int_0^2\left[\int_0^1 (10+x^2+3y^2)\,dx\right]dy=\int_0^2\left(10x+\frac{x^3}{3}+3xy^2\Big|_0^1\right)dy=\int_0^2\left(10+\frac{1}{3}+3y^2\right)dy=\int_0^2\left(\frac{31}{3}+3y^2\right)dy$$

$$=\frac{31}{3}y+y^3\Big|_0^2=\frac{62}{3}+8=\frac{86}{3} \#$$

HW14-1

- **HW: 3,7,11,13,15,23,24**