平面曲線的參數化

10-1 Parametrizations of Plane Curves

不會考

師大工教一

know 觀念

Definition If x and y are given as functions $x = f(t), y = g(t)$ over an interval I of $t-$value, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a **parametric curve**. The equations are **parametric equations**.

The variable t is a **parameter** for the curve. If I is a closed interval, $a \le t \le b$, the point $(f(a), g(a))$ is the **initial point** of the curve and $(f(b), g(b))$ is the **terminal point**.

$(x(t), y(t))$, $t \in I$

$(f(a), g(a))$ intial point 起点

$(f(b), g(b))$ terminal point 终点

parametric curves 參數化曲線

$x = f(t)$, $y = g(t)$, $t \in I$

parametric equations 參數化方程式

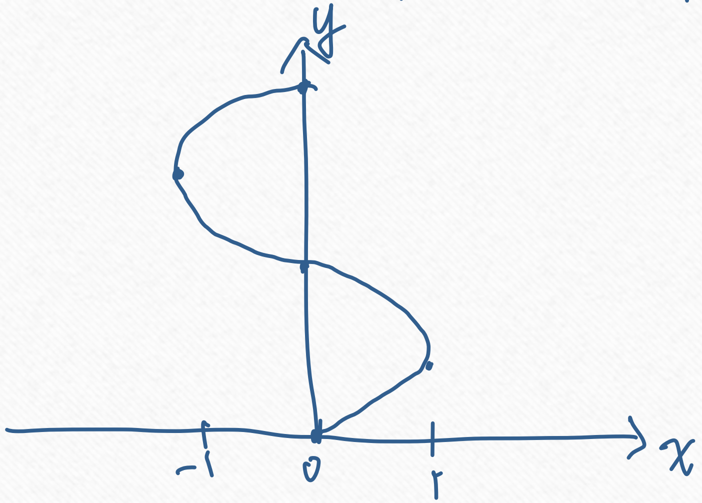
t : parameter

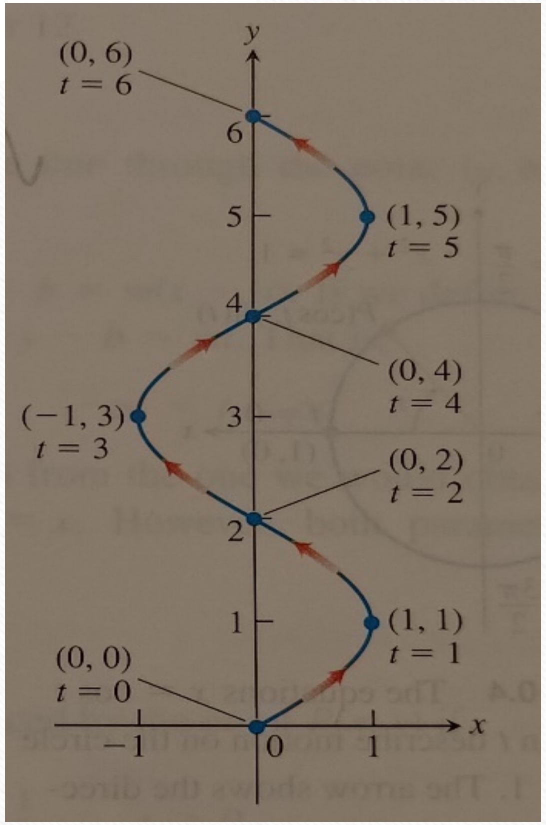
$I = [a, b]$, $t \in I \Leftrightarrow a \le t \le b$

Ex1(p598) Sketch the curve defined by the parametric equations

$$x = \sin\frac{\pi t}{2}, y = t, 0 \le t \le 6.$$

t	0	1	2	3	4	5	6
x	0	1	0	-1	0	1	0
y	0	1	2	3	4	5	6





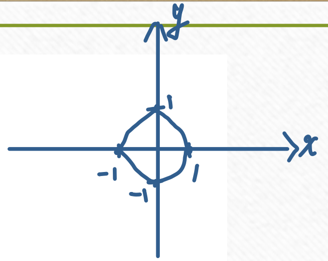
Ex3(p600) Graph the parametric curves.

(a) $x = \cos t, \quad y = \sin t, \quad 0 \le t \le 2\pi$

(b) $x = a\cos t, \quad y = a\sin t, \quad 0 \le t \le 2\pi$

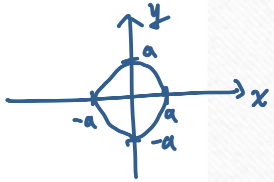
(a)

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	1	0	-1	0	1
y	0	1	0	-1	0



(b)

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	a	0	$-a$	0	a
y	0	a	0	$-a$	0



Ex7(p601) Sketch and identify the path traced by the point $P(x, y)$ if

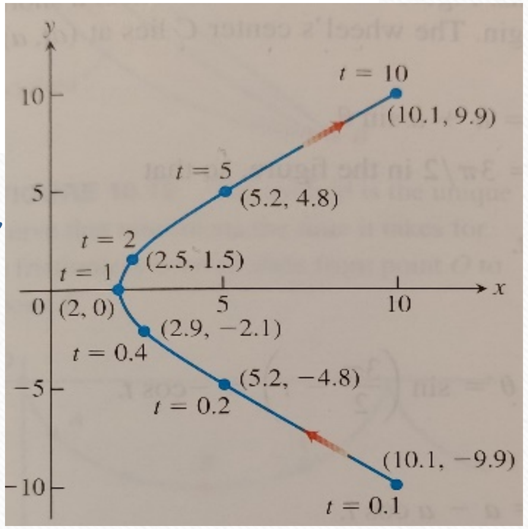
$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t > 0$$

t			$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	
x			$\frac{17}{4}$	$\frac{10}{3}$	$\frac{5}{2}$	2	$\frac{5}{2}$	$\frac{10}{3}$	$\frac{17}{4}$	
y			$-\frac{15}{4}$	$-\frac{8}{3}$	$-\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{8}{3}$	$\frac{15}{4}$	

$$\left.\begin{array}{l} x + y = 2t \\ x - y = \frac{2}{t} \end{array}\right\} (x+y)(x-y) = 4$$

$$x^2 - y^2 = 4$$

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$



HW10-1

- **HW: 5,7,10,20,31**