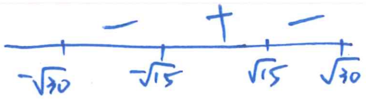
106 (分部)

4. $f(x)=x\sqrt{30-x^2} \Rightarrow f'(x)=\sqrt{30-x^2}+x\cdot\frac{\frac{1}{2}(-2x)}{\sqrt{30-x^2}}=\sqrt{30-x^2}-\frac{x^2}{\sqrt{30-x^2}}=\frac{2(15-x^2)}{\sqrt{30-x^2}}$

(a) critical points : $(\sqrt{15},\sqrt{30}).(-\sqrt{15},-\sqrt{30}).(\sqrt{30},0).(-\sqrt{30},0)$



(b) f is increasing on $(-\sqrt{15},\sqrt{15})$
decreasing on $(-\sqrt{30},-\sqrt{15})\cup(\sqrt{15},\sqrt{30})$

5. $f(x)=x\sin\frac{1}{x}$

$\lim\limits_{x\to\infty} x\sin\frac{1}{x}=\lim\limits_{y\to0^+}\frac{\sin y}{y}=1$, $\lim\limits_{x\to-\infty} x\sin\frac{1}{x}=\lim\limits_{y\to0^-}\frac{\sin y}{y}=1$

horizontal asymptote : $y=1$

6. $f(x)=\frac{x+3}{\sqrt{x-3}}=\sqrt{x-3}+\frac{6}{\sqrt{x-3}}$

(a) vertical asymptote : $x=3$

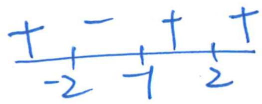
$f'(x)=\frac{1}{2}(x-3)^{-\frac{1}{2}}+6\cdot(-\frac{1}{2})(x-3)^{-\frac{3}{2}}=\frac{1}{2\sqrt{x-3}}-\frac{6}{2(x-3)\sqrt{x-3}}=\frac{(x-9)}{2(x-3)\sqrt{x-3}}$

$f''(x)=\frac{1}{2}(-\frac{1}{2})(x-3)^{-\frac{3}{2}}+6(-\frac{1}{2})(-\frac{3}{2})(x-3)^{-\frac{5}{2}}=-\frac{1}{4}\left(\frac{1}{(x-3)^{\frac{3}{2}}}-18\frac{1}{(x-3)^{\frac{5}{2}}}\right)=-\frac{1}{4}\frac{(x-21)}{(x-3)^{\frac{5}{2}}}$

(b) point of inflection : $(21, 4\sqrt{2})$
f is concave upward on $(3,21)$ and concave downward on $(21,+\infty)$

(c) $f'(9)=0$. $f''(9)>0 \Rightarrow (9.2\sqrt{6})$ is a relative minimum.

105 (分部)



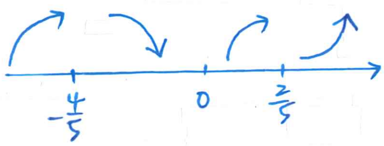
6. (a) $f(x)=\ln\frac{3(x^2-x-2)}{x^2-4} \Rightarrow \frac{3(x^2-x-2)}{x^2-4}>0 \Rightarrow (x+1)(x+2)(x-2)^2>0$

Domain of $f(x)$:
$(-\infty,-2)\cup(-1,2)\cup(2,+\infty)$

(b) $f(x)=\ln\frac{3(x+1)}{(x+2)} \Rightarrow$ vertical asymptote : $x=-2$
horizontal asymptote : $y=\ln 3$ $(\because \lim\limits_{x\to\pm\infty}f(x)=\ln 3)$

7. $f(x)=x^{\frac{2}{3}}(x+2)$

$f'(x)=\frac{2}{3}x^{-\frac{1}{3}}(x+2)+x^{\frac{2}{3}}=\frac{5}{3}x^{\frac{2}{3}}+\frac{4}{3}x^{-\frac{1}{3}}=\frac{5x+4}{3\sqrt[3]{x}}$



$f''(x)=\frac{10}{9}x^{-\frac{1}{3}}-\frac{4}{9}x^{-\frac{4}{3}}=\frac{10x-4}{9\cdot x^{\frac{4}{3}}}$

(a) f is increasing on $(-\infty,-\frac{4}{5})\cup(0,+\infty)$ and is decreasing on $(-\frac{4}{5},0)$

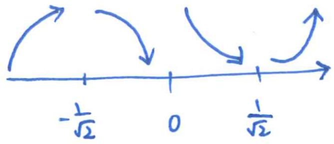
(b) point of inflection : $(\frac{2}{5}.\frac{12}{25}\sqrt[3]{20})$

104

4. $f(x)=2x+\frac{1}{x}$

$f'(x)=2-x^{-2}=2-\frac{1}{x^2}$

$f''(x)=2\cdot\frac{1}{x^3}$



(a) critical points: $(-\frac{1}{\sqrt{2}},-2\sqrt{2})$, $(\frac{1}{\sqrt{2}},2\sqrt{2})$

(b) f is increasing on $(-\infty,\frac{-1}{\sqrt{2}})\cup(\frac{1}{\sqrt{2}},+\infty)$

decreasing on $(\frac{-1}{\sqrt{2}},0)\cup(0,\frac{1}{\sqrt{2}})$

(c) $(-\frac{1}{\sqrt{2}},-2\sqrt{2})$ is a relative maximum

$(\frac{1}{\sqrt{2}},2\sqrt{2})$ is a relative minimum

5. $f(x)=x^3-9x^2$

$f'(x)=3x^2-18x=3x(x-6)$

$f''(x)=6x-18$

(a) point of inflection: $(3,-54)$

(b) f is concave upward on $(3,+\infty)$ and concave downward on $(-\infty,3)$

(c) $(0,0)$ is a relative maximum

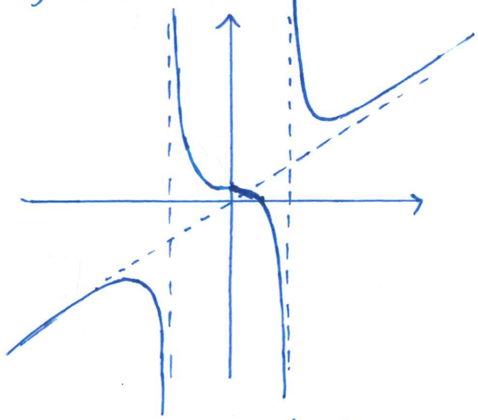
$(6,-108)$ is a relative minimum

2. $f(x)=\frac{x^3-1}{2x^2-4}=\frac{1}{2}x+\frac{2x-1}{2x^2-4}$.

horizontal asymptote: X

vertical asymptote: $x=2$, $x=-2$

slant asymptote: $y=\frac{1}{2}x$

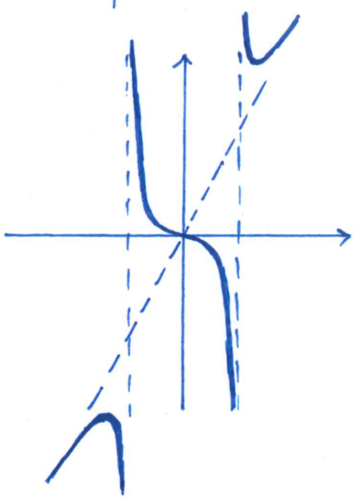


103.

4. (a) $f(x)=\frac{2x^3}{x^2-1}=2x+\frac{2x}{x^2-1}$

vertical asymptote: $x=1$, $x=-1$

slant asymptote: $y=2x$



(b) $f'(x)=2+\frac{2(x^2-1)-2x(2x)}{(x^2-1)^2}=2-\frac{2x^2+2}{(x^2-1)^2}$

$f'(x)=0 \Rightarrow (x^2-1)^2=x^2+1$

$\Rightarrow x^4=3x^2 \Rightarrow x=\pm\sqrt{3}$

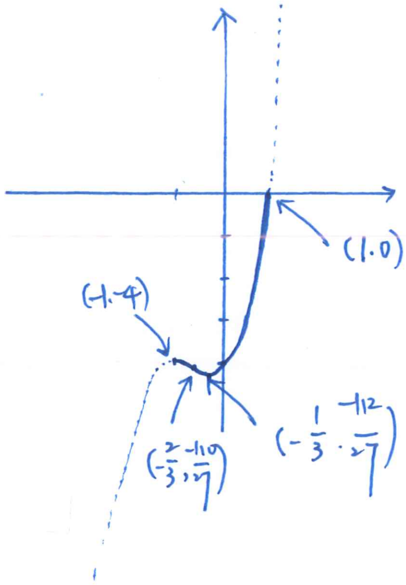
critical points: $(-\sqrt{3},-3\sqrt{3})$, $(0,0)$, $(\sqrt{3},3\sqrt{3})$

103

5. $g(x) = x^3 + 2x^2 + x - 4.$

$g'(x) = 3x^2 + 4x + 1 = (3x+1)(x+1)$

$g''(x) = 6x + 4.$



(a) $(-1, -4)$ is a relative (local) maximum.

$(-\frac{1}{3}, \frac{-112}{27})$ is a relative (local) minimum.

(b) f is concave upward on $(-\frac{2}{3}, 1)$

is concave downward on $(-1, -\frac{2}{3})$

(c) absolute maximum $= 0$ at (1.0)

absolute minimum $= \frac{-112}{27}$ at $(-\frac{1}{3}. \frac{-112}{27})$

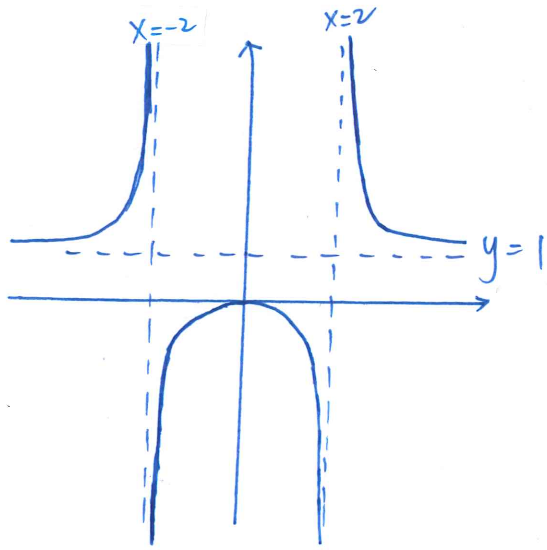
102.

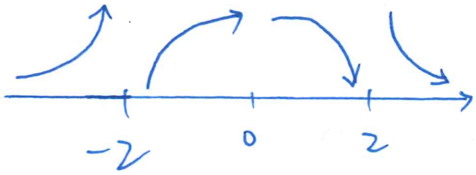
$f(x) = \frac{x^2}{x^2-4} = 1 + \frac{4}{x^2-4}$

$f'(x) = \frac{-8x}{(x^2-4)^2}$

$f''(x) = \frac{-8(x^2-4)^2 - (-8x)2(x^2-4)\cdot 2x}{(x^2-4)^4}$

$= \frac{-8x^2+32+32x^2}{(x^2-4)^3} = \frac{24x^2+32}{(x^2-4)^3}$





(a) f is increasing on $(-\infty, -2) \cup (-2, 0)$

decreasing on $(0, 2) \cup (2, +\infty)$

(b) f is concave upward on $(-\infty, -2) \cup (2, +\infty)$

concave downward on $(-2, 2)$

(c) horizontal asymptote : $y = 1$

vertical asymptote : $x = -2, x = 2$

Bonus

105 (合卷)

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0} x\sin\frac{1}{x}=0 \quad \left(\because -x\le x\sin\frac{1}{x}\le x \;\&\; \lim_{x\to 0}-x=0=\lim_{x\to 0}x\right)$$

104.

(a) $\forall\, \varepsilon>0$. choose $\delta=\varepsilon$. when $0<|x-3|<\delta=\varepsilon$

then $|(8-x)-5|=|x-3|<\varepsilon$, therefore. $\lim_{x\to 3}(8-x)=5$

(b) By definition of e ※

102.

(a) $-x\le x\sin\frac{1}{x}\le x$. $\lim_{x\to 0}-x=0=\lim_{x\to 0}x \Rightarrow \lim_{x\to 0}x\sin\frac{1}{x}=0$

(b) IVT:

If f is continuous on $[a,b]$, $f(a)\ne f(b)$, and k is any number between $f(a)$ and $f(b)$

then there is at least one number c in $[a,b]$ s.t. $f(c)=k$

(c) $\forall\, \varepsilon>0$. when $0<|x-2|<\delta$ then $|f(x)-5|<\varepsilon$