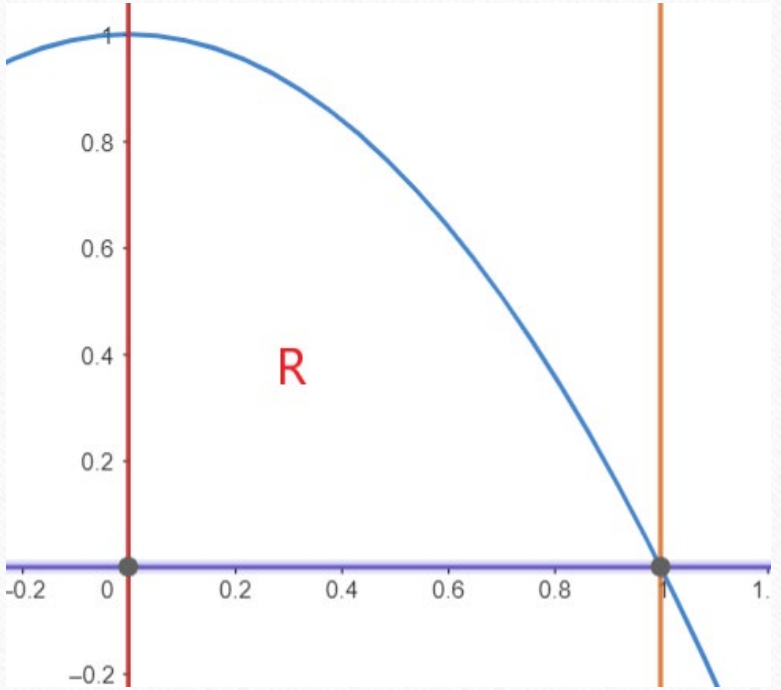
5-1 Area and Estimating with Finite Sums

師大工教一

Q: How can we find the area A of the region R enclosed by $y = 1 - x^2$, $x-$ axis, $x = 0$, $x = 1$?



Method 1: **Upper Sums** (see Figure 5.2)

$$A \approx 1 \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2} = \frac{7}{8} = 0.875$$

$$A \approx 1 \cdot \frac{1}{4} + \frac{15}{16} \cdot \frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \frac{7}{16} \cdot \frac{1}{4} = \frac{25}{32} = 0.78125$$

Method 2: **Lower Sums**(see Figure 5.3(a))

$$A \approx \frac{15}{16}\cdot\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}+\frac{7}{16}\cdot\frac{1}{4}+0\cdot\frac{1}{4}=\frac{17}{32}=0.53125$$

$$0.53125 < A < 0.78125$$

Method 3: **Midpoint rule**(see Figure 5.3(b))

$$A \approx \frac{63}{64}\cdot\frac{1}{4}+\frac{55}{64}\cdot\frac{1}{4}+\frac{39}{64}\cdot\frac{1}{4}+\frac{15}{64}\cdot\frac{1}{4}=\frac{172}{64}\cdot\frac{1}{4}=0.671875$$

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- **HW: 2,3**