師大微積分乙 113-102 期末考歷屆

▶ 積分技巧與基本計算

1. (師大微乙 (一)113 暑 #2) Evaluate the following integrals.

   (a) $\displaystyle\int \left(x+\sqrt{x}+1+\frac{1}{\sqrt{x}}+\frac{1}{x}\right)\,\mathrm{d}x$

   (b) $\displaystyle\int \left(x^2\sin x+\cos^2 x\right)\,\mathrm{d}x$

   (c) $\displaystyle\int \left(\tan^2 x+\tan^4 x\right)\,\mathrm{d}x$

   (d) $\displaystyle\int \frac{2x}{\sqrt{1+x^4}}\,\mathrm{d}x$

   (e) $\displaystyle\int \frac{e^x}{e^{2x}-3e^x+2}\,\mathrm{d}x$

   (f) $\displaystyle\int_{-1}^{1}(x^3+3)\sqrt{4-x^2}\,\mathrm{d}x$

2. (師大微乙 (一)113 暑 #3) Let a be a real number and $a>0$.

   (a) Evaluate $\displaystyle\int x\ln x\,\mathrm{d}x$

   (b) If a satisfies the equation $\displaystyle\int_0^a x\ln x\,\mathrm{d}x=0$, find the value of a.

3. (師大微乙 (一)113#2) Evaluate the integrals.

   (a) $\displaystyle\int \frac{\cos^5 x}{\sin^2 x}\,\mathrm{d}x$

   (b) $\displaystyle\int \frac{x^2}{\sqrt{49-x^2}}\,\mathrm{d}x$

   (c) $\displaystyle\int \frac{2x^3+4}{(x-1)^2(x^2+1)}\,\mathrm{d}x$

   (d) $\displaystyle\int \frac{\tan x(\sec^2 x+4\sec x)}{\sec^2 x+5\sec x-6}\,\mathrm{d}x$

   (e) $\displaystyle\int_{-1}^{1}\sqrt{x^{10}}+x^2\sec x\tan x+\frac{1}{1+x^2}\,\mathrm{d}x$

4. (師大微乙 (一)113#6)

   (a) Evaluate $\displaystyle\int_0^1 \ln x\,\mathrm{d}x$. (Note: this integral is an improper integral(瑕積分).)

(b) Let a be a real number and $a > 0$. If a satisfies the equation $\int_0^a \ln x \, \mathrm{d}x = 0$, find a.

5. (師大微乙 (一)112#2) Evaluate the following integrals.

(a) $\int \frac{x^2}{x^4 - 1} \, \mathrm{d}x$

(b) $\int_0^{\pi^2/4} \cos\sqrt{x} \, \mathrm{d}x$

(c) $\int_0^e \ln x \, \mathrm{d}x$

(d) $\int \sec^5 x \, \mathrm{d}x$

(e) $\int \frac{(1-y^2)^{5/2}}{y^8} \, \mathrm{d}y$

6. (師大微乙 (一)111#1) Evaluate the integrals.

(a) $\int \frac{\ln(\ln x)}{x \ln x} \, \mathrm{d}x$

(b) $\int_0^1 x \ln x \, \mathrm{d}x$

(c) $\int \frac{x^3}{x^2 + 2x + 1} \, \mathrm{d}x$

(d) $\int \frac{x}{\sqrt{1 + x^4}} \, \mathrm{d}x$

(e) $\int \sqrt{\sin x} \cos^3 x \, \mathrm{d}x$

(f) $\int \frac{xe^{2x}}{(2x+1)^2} \, \mathrm{d}x$

(g) $\int \frac{\sec^2\theta}{\tan^3\theta - \tan^2\theta} d\theta$

(h) $\int_\infty^1 \frac{9r^2}{\sqrt{1 - r^3}} \, \mathrm{d}r$

7. (師大微乙 (一)110#1) 計算下列積分. (Evaluate the following integrals.)

(a) $\int_1^4 (\frac{2}{x^3} - \sqrt{x}) \, \mathrm{d}x$

(b) $\int x^2\sqrt{x^3 + 1} \, \mathrm{d}x$

(c) $\int_0^3 \frac{x}{\sqrt{x+1}} \, \mathrm{d}x$

(d) $\int \cos\sqrt{x} \, \mathrm{d}x$

(e) $\int e^x \cos 2x \, \mathrm{d}x$

(f) $\int \frac{2}{16x^2 - 1} \, \mathrm{d}x$

(g) $\int \sqrt{9 - x^2} \, \mathrm{d}x$

(h) $\int_4^6 \frac{1}{\sqrt{8x - x^2}} \, \mathrm{d}x$

8. (師大微乙 (一)110#5) 計算以下瑕積分. (Evaluate the following improper integrals.)

(a) $\int_0^\infty xe^{-2x} \, \mathrm{d}x$

(b) $\int_0^1 \ln x \, \mathrm{d}x$

9. (師大微乙 (一)109#1) Evaluate the integrals.

(a) $\int \sqrt{16 - 4x^2} \, \mathrm{d}x$

(b) $\int_1^\infty \frac{1}{x(\ln x)^3} \, \mathrm{d}x$

(c) $\int x^4 \cos(2x) \, \mathrm{d}x$

(d) $\int \sin(7x) \cos(3x) \, \mathrm{d}x$

(e) $\int \frac{\sqrt{x^2 + 3}}{x} \, \mathrm{d}x$

(f) $\int \sec(2x) \tan^2(2x) \, \mathrm{d}x$

(g) $\int_0^{\pi/2} \frac{\cos(\theta)}{(\sin^2(\theta) + 4\cos^2(\theta))^{3/2}} d\theta$

(h) $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} \, \mathrm{d}x$

10. (師大微乙 (一)108#3)

(a) Does the improper integral $\int_1^\infty \frac{\ln t}{2\sqrt{t}} \, \mathrm{d}t$ converge or diverge? Give reasons for your answer.

(b) Use the part (a) to evaluate $\lim_{x \to \infty} \frac{\int_1^x \frac{\ln t}{2\sqrt{t}} \, \mathrm{d}t}{\sqrt{x^3}}$.

11. (師大微乙 (一)108#4) Find each integral.

(a) $\int \frac{8x}{x^3 + x^2 - x - 1} \, \mathrm{d}x$

(b) $\int \tan^4(5x)\,\mathrm{d}x$

(c) $\int e^x\sqrt{1-e^{2x}}\,\mathrm{d}x$

(d) $\int e^x\cos x\,\mathrm{d}x$

(e) $\int_0^{\pi/2}\dfrac{\cos^5 x}{\sin^5 x+\cos^5 x}\,\mathrm{d}x$ (Hint: Use the substitution $u=\frac{\pi}{2}-x$.)

12. (師大微乙 (一)107#1) Evaluate the integrals.

(a) $\int \dfrac{2x+1}{\sqrt{4-x}}\,\mathrm{d}x$

(b) $\int \dfrac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x$

(c) $\int xe^{2x}\,\mathrm{d}x$

(d) $\int \dfrac{-x^3+5x^2-4x+9}{x^4-2x^3+3x^2-4x+2}\,\mathrm{d}x$

(e) $\int \dfrac{x}{x^2-2x+3}\,\mathrm{d}x$

(f) $\int \tan x\cdot\cos(2x)\,\mathrm{d}x$

(g) $\int_0^{\sqrt{3}}\dfrac{x^2}{(4-x^2)^{3/2}}\,\mathrm{d}x$

(h) $\int \sqrt{1+x^{-1}}\,\mathrm{d}x$

13. (師大微乙 (一)107#5) Find the improper integral

$$\int_0^4 \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x.$$

14. (師大微乙 (一)106#1) Evaluate the integrals.

(a) $\int (x+1)5^{(x+1)^2}\,\mathrm{d}x$

(b) $\int \dfrac{2x^3-4x^2-15x+5}{x^2-2x-8}\,\mathrm{d}x$

(c) $\int e^{\sqrt{2x}}\,\mathrm{d}x$

(d) $\int e^x\sqrt{1-e^{2x}}\,\mathrm{d}x$

(e) $\int \sec^4 x\tan^3 x\,\mathrm{d}x$

(f) $\displaystyle\int \frac{\sin x}{\cos x - \cos^2 x}\,\mathrm{d}x$

(g) $\displaystyle\int \frac{\ln\sqrt{x}}{x}\,\mathrm{d}x$

(h) $\displaystyle\int_0^1 x\ln x\,\mathrm{d}x$

15. (師大微乙 (一)106#5) 給定下列之瑕積分 (improper integral)，請做以下兩小題.

$$\int_1^\infty (\frac{c}{x^2} - \frac{1}{3x})\,\mathrm{d}x$$

(a) 找出使得上述瑕積分收斂 (converge) 的 c 值.

(b) 求 (a) 答案 c 值對應的之瑕積分值.

16. (師大微乙 (一)105#1) Evaluate the integrals.

(a) $\displaystyle\int_1^4 (3 - |x-3|)\,\mathrm{d}x$

(b) $\displaystyle\int \frac{3}{x^2+3x-10}\,\mathrm{d}x$

(c) $\displaystyle\int \sin^2\theta\tan\theta\sqrt{\cos\theta}d\theta$

(d) $\displaystyle\int \frac{\sec x\tan x}{2\sec x - 1}\,\mathrm{d}x$

(e) $\displaystyle\int \frac{\ln x}{x^3}\,\mathrm{d}x$

(f) $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\,\mathrm{d}x$

(g) $\displaystyle\int \frac{x^3e^{x^2}}{(x^2+1)^2}\,\mathrm{d}x$

(h) $\displaystyle\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,\mathrm{d}x$

17. (師大微乙 (一)104#1) 計算下列積分：

(a) $\displaystyle\int_0^1 (x^3+2x)\,\mathrm{d}x$

(b) $\displaystyle\int \frac{2x+1}{x^2-7x+12}\,\mathrm{d}x$

(c) $\displaystyle\int_0^{\pi/2} \cos^3(x)\,\mathrm{d}x$

(d) $\displaystyle\int_1^e \frac{\mathrm{d}y}{y(1+(\ln y)^2)}$

(e) $\displaystyle\int \sec^3(z)\,\mathrm{d}z$

(f) $\displaystyle\int_1^9 \frac{\log_3 x}{x}\,\mathrm{d}x$

(g) $\displaystyle\int \sin\sqrt{2x}\,\mathrm{d}x$

(h) $\displaystyle\int \frac{\mathrm{d}s}{1+e^s}$

18. (師大微乙 (一)104#5) 判斷下列瑕積分收斂或發散 (請標明使用何種檢定法).

$$\int_1^\infty \frac{1}{\sqrt{x^6+1}}\,\mathrm{d}x$$

19. (師大微乙 (一)103#1) 底下共有 8 題積分計算題

(a) $\displaystyle\int_1^4 x^2+3x+1\,\mathrm{d}x$

(b) $\displaystyle\int \frac{1}{x^2-5x+6}\,\mathrm{d}x$

(c) $\displaystyle\int \sin^3 x\cos^4 x\,\mathrm{d}x$

(d) $\displaystyle\int \tan^2 x\sec^3 x\,\mathrm{d}x$

(e) $\displaystyle\int \frac{\sqrt{1-x^2}}{x}\,\mathrm{d}x$

(f) $\displaystyle\int \ln x\,\mathrm{d}x$

(g) $\displaystyle\int \frac{\ln x}{x}\,\mathrm{d}x$

(h) $\displaystyle\int \frac{2z}{\sqrt[3]{z^2+1}}\,\mathrm{d}z$

20. (師大微乙 (一)103#5)

(a) 驗算 $\displaystyle\int_0^2 x^2\,\mathrm{d}x+\int_0^4 \sqrt{x}\,\mathrm{d}x=8$

(b) 求 $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\,\mathrm{d}x+\int_0^1 \sin^{-1} x\,\mathrm{d}x$

(d) 判斷下列瑕積分收斂或發散，若收斂請計算其值.

$$\int_0^\infty \frac{e^{\tan^{-1}x}}{1+x^2}\,\mathrm{d}x$$

21. (師大微乙 (一)102#1) 計算下列積分:

(a) $\displaystyle\int_0^1 (x^2+3x+2)\,\mathrm{d}x$

(b) $\displaystyle\int \frac{1}{x^2+3x+2}\,\mathrm{d}x$

(c) $\displaystyle\int_0^1 x\sqrt{1-x}\,\mathrm{d}x$

(d) $\displaystyle\int_0^{2/3} \sqrt{4-9x^2}\,\mathrm{d}x$

(e) $\displaystyle\int \tan^3 x \sec^4 x\,\mathrm{d}x$

(f) $\displaystyle\int \frac{1+\sin x}{\cos^2 x}\,\mathrm{d}x$

(g) $\displaystyle\int x^2 e^{-x}\,\mathrm{d}x$

(h) $\displaystyle\int \sin\sqrt{x}\,\mathrm{d}x$

22. (師大微乙 (一)102#5) 判斷下列瑕積分收斂或發散，若收斂，計算其值.

(a) $\displaystyle\int_0^\infty \frac{1}{x^2+1}dx$

(b) $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}+1}dx.$

▶ **洛必達法則與微積分基本定理**

23. (師大微乙 (一)113 暑 #1) Apply the L'Hôpital rule to evaluate the following limits.

(a) $\displaystyle\lim_{x\to 0^+} x^2 \ln x$

(b) $\displaystyle\lim_{k\to\infty} \left(1+\frac{\pi}{k}\right)^k.$

24. (師大微乙 (一)113 暑 #5) Let f be continuous on its domain. Suppose that

$$\int_0^{x^3} f(3t^2-6t)\,\mathrm{d}t = \ln x$$

Find the value of $f(2025)$. ($\star$ Hint: $2025 = 27\times 75$)

25. (師大微乙 (一)113#1) Find the limit if it exists.

(a) $\displaystyle\lim_{x\to 0} \frac{x^2}{\ln\cos 2x}$

(b) $\displaystyle\lim_{x\to 0^+} \left(e^{2x}+2x\right)^{\frac{2}{x}}$

(c) $\displaystyle\lim_{x\to 0^+} \frac{\int_{\sqrt{x}}^0 \sin t^2\,\mathrm{d}t}{\sqrt{x^3}}$

26. (師大微乙 (一)112#4) Let $F(x) = \int_{x^2}^{0} \arctan(t)\,\mathrm{d}t$ for all $x \in \mathbb{R}$.
    (a) Find the open intervals on which F is increasing or decreasing.
    (b) Evaluate $\lim_{x\to-\infty} \frac{F(x)}{x^2}$.

27. (師大微乙 (一)110#2) 使用羅畢達法則計算以下的極限. (Apply L'Hopital's rule to evaluate the following limits.)
    (a) $\lim_{x\to0^+} x\ln x$
    (b) $\lim_{x\to1} \frac{\cos(\pi x - \frac{\pi}{2})}{x-1}$

28. (師大微乙 (一)109#2) Evaluate the limit.
    (a) $\lim_{x\to0^+} \frac{e^x - 1 - x}{x^2}$
    (b) $\lim_{x\to0^+} (\sin(2x))^{3x}$

29. (師大微乙 (一)107#2) Evaluate the limit
$$\lim_{x\to1^+} (\ln x)^{x-1}.$$

30. (師大微乙 (一)106#4) 請用羅必達法則 (L' Hôpital Rule) 求極限 $\lim_{x\to5} \frac{\sqrt{25-x^2}}{x-5}$.

31. (師大微乙 (一)105#2) 已知 $\int_0^{x^5} f(2t^2 - t + 1)\,\mathrm{d}t = \ln x$，求 $f(2017)$ 的值.

32. (師大微乙 (一)105#5) 求極限 $\lim_{x\to1} \frac{\frac{1}{4} + \int_1^x t\ln t\,\mathrm{d}t}{x\ln x - x + 1}$ 的值.

33. (師大微乙 (一)104#4) 已知 $\int_{x^2}^{2} f(t^3)\,\mathrm{d}t = \frac{1}{2}\sin(\frac{\pi}{2}x)$, 求 $f(1)$.

34. (師大微乙 (一)103#5) 已知 $\int_0^{x^2} f(t)\,\mathrm{d}t = \cos\pi x$, 求 $f(2)$ =?

35. (師大微乙 (一)102#6) 令函數 $f(x) = \int_0^{x^2} \cos t^3 + 1\,\mathrm{d}t$, 求 $f'(x)$.

▶ 黎曼和與積分式分析

36. (師大微乙 (一)113 暑 #6) Let the function $f(x) = \frac{1}{1+x}$ be defined on the interval $[0, 2]$. Now, divide (分割) the interval $[0, 2]$ into n equal sub-intervals (n 等份的子區間). For each(每個) $k = 1, 2, \ldots, n$, let the k-th sub-interval be
$$\left[\frac{2(k-1)}{n}, \frac{2k}{n}\right]$$

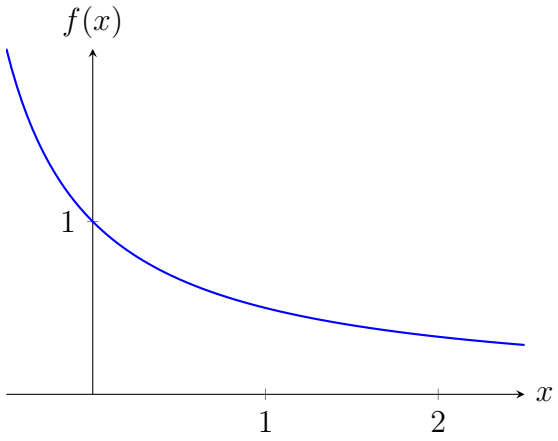
and choose an arbitrary point c_k within the k-th sub-interval (在第 k 個子區間中任意選擇一個點 c_k). Also, define the sequence S_n (數列 S_n) as

$$S_n = \frac{2}{n}[f(c_1) + f(c_2) + \cdots + f(c_n)].$$

(a) (3 pts) Is $S_1 > S_2$ always true? Give your reason!

(b) (3 pts) What is the maximum value of S_4?

(c) (4 pts) Find the value of $\lim_{n\to\infty} S_n$.



(The above figure shows the graph of the function $f(x) = \frac{1}{1+x}$)

37. (師大微乙 (一)113 暑 #7) Let f be a function such that f is differentiable on $[a, b]$ and f' is continuous on $[a, b]$.

(a) Suppose f is one-to-one on $[a, b]$, prove that

$$\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(x)\,dx = bf(b) - af(a)$$

(⋆ Hint: Let $y = f^{-1}(x)$, then $y = f^{-1}(x) \iff x = f(y)$)

(b) Evaluate the integral $\int_{11}^{30} \sqrt[3]{\sqrt{x} - 3} - 2\,dx$.

(⋆ Hint: Note that $f(x) = \sqrt[3]{\sqrt{x} - 3} - 2$ is one-to-one on $[11, 30]$)

38. (師大微乙 (一)113#7) Evaluate $\lim_{n\to\infty} \left(\sum_{k=1}^{n} \left(\frac{7 \cdot 2^k \cdot k^6}{n^7} - \frac{4 \cdot 2^k \cdot k^3}{n^4} + \frac{2}{n} \right) \right)$.

(Hint: The question is about a limit of some Riemann sum (黎曼和). Find the definite integral that the limit represents and evaluate it.)

39. (師大微乙 (一)111#3) Solve the initial value problem for y as a function of x

$$\sqrt{x^2 - 9}\frac{dy}{dx} = 1, x > 3, \quad y(5) = \ln 3.$$

40. (師大微乙 (一)111#4) Consider the well-known standard logistic function(標準型邏輯斯諦函數)

$$f(x) = \frac{1}{1+e^{-x}}.$$

A typical(典型的) application is a model(模型) of population growth(人口成長), and the logistic functions are widely(廣泛地) used in probability(機率), statistics(統計), machine learning(機器學習), chemistry(化學), social science(社會科學) and so on.

(a) Find all antiderivatives of $f(x)$.

(b) Show that $f(x)$ is a solution of the first-order non-linear ordinary differential equation(一階非線性常微分方程)

$$\frac{dy}{dx} = y(1-y)$$

with the initial value condition $y(0) = \frac{1}{2}$.

41. (師大微乙 (一)111#5) Consider the error function(誤差函數)

$$F(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

in areas(領域) of probability(機率) and statistics(統計).

(a) Is $F(x)$ continuous?

(b) Does the graph of $F(x)$ have horizontal tangent lines?

(c) Does the graph of $F(x)$ have inflection points(反曲點)?

(d) (Bonus) Does the graph of $F(x)$ have horizontal asymptotes(水平漸近線)? (Hint: The direct comparison test below will be useful.)

42. (師大微乙 (一)110#6) (Bonus) 證明下列積分值相等 (Prove the following identity.)

$$\int_0^{\pi/2} \sin^n x \, \mathrm{d}x = \int_0^{\pi/2} \cos^n x \, \mathrm{d}x$$

其中 n 是正整數. (where n is a positive integer.)

43. (師大微乙 (一)108#2) Evaluate the following limit

$$\lim_{n\to\infty} \frac{\sqrt{n^2+1} + \sqrt{n^2+4} + \sqrt{n^2+9} + \cdots + \sqrt{n^2+(n-1)^2}}{n^2}$$

by using an appropriate Riemann sum.

44. (師大微乙 (一)106#6) 證明下列積分值是相等的.

$$\int_0^1 x^a(1-x)^b \,\mathrm{d}x = \int_0^1 x^b(1-x)^a \,\mathrm{d}x$$

其中 $a, b > 0$.

45. (師大微乙 (一)104#6) 給一積分式，如下：

$$\int_a^b \sqrt{x-x^2} \,\mathrm{d}x$$

其中 $a, b \in [0,1]$. 式之最大值為何？請說明原因.

46. (師大微乙 (一)102#8) 求 $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \sqrt{\left(1+\frac{k}{n}\right)}$ (將里曼和 (Riemann sum) 之極限表成定積分，再求出定積分的值.)

► **計算面積或體積問題**

47. (師大微乙 (一)113#3) Find the volume of solid generated by revolving the region bounded by the semicircle $y = \sqrt{4-x^2}$ and the x-axis about the line $y = -1$.

48. (師大微乙 (一)113#4) Let D be the plane region enclosed by $y = -\sqrt{x}$, $x = y$, $y = -2$.

    (a) Sketch the region D.

    (b) Find the volume of the solid generated by revolving the region D about the x-axis.

49. (師大微乙 (一)113#5) Find the area of the surface generated by revolving the curve $x = 2\sqrt{8-y}$, $0 \le y \le 5$, about y-axis.

50. (師大微乙 (一)112#3) Let $\mathcal{R}$ be the region bounded by the curves $x^3 - x + y = 0$, $y = 0$, $x = 0$ and $1 - x = 0$. Use the Shell Method to find the volume of the solid generated by revolving $\mathcal{R}$ about the y-axis.

51. (師大微乙 (一)112#6) Let Ω be the region bounded by the curves $y = x^2$ and $y = x + 2$.

    (a) Find the area of the plane region Ω.

    (b) Use the Washer Method to find the volume of the solid generated by revolving Ω about the x-axis.

52. (師大微乙 (一)111#2) Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the line $y = x + 2$ about the line $x = 2$.

53. (師大微乙 (一)110#3) 計算由 $y = 2 - x^2$, $x = 0$, $x = 1$ 和 $y = 0$ 所圍的區域，繞 y-軸所得旋轉體的體積. (Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 2 - x^2$, $x = 0$, $x = 1$ and $y = 0$ about y-axis.)

54. (師大微乙 (一)109#4) Let D be the region bounded by the graphs of $y = x^2+1$, $x = 0$, $y = x$ and $x = 1$.

    (a) Find the volume of the solid generated by revolving the region D about the y-axis.

    (b) Find the volume of the solid generated by revolving the region D about the line $y = -1$.

55. (師大微乙 (一)108#5) Let Ω be the plane region bounded by the graphs of $x + y^2 - 6y + 5 = 0$ and $x - y + 1 = 0$.

    (a) Find the area of the region Ω.

    (b) Find the volume of the solid formed by revolving Ω about the x-axis.

    (c) Find the volume of the solid formed by revolving Ω about the y-axis.

56. (師大微乙 (一)107#4) Let D be the plane region bounded by $y = -\sqrt{x+2}$, $y = x$, $y = 0$. Plot the region D and use the shell method to find the volume of the solid generated by revolving the region D about the x-axis.

57. (師大微乙 (一)106#2) 求由 $y = 4x^2$, $x = 0$ 及 $y = 4$ 在第一象限 (first quadrant) 所圍成的封閉區域對 x 軸旋轉所得到的旋轉體體積 (volume).

58. (師大微乙 (一)105#3) 給定由曲線 $y = x\sqrt{x+1}$ 與 x 軸所圍成的區域，試做下列各題：

    (a) 求該封閉區域的面積。

    (b) 求該區域對 x 軸旋轉所得到的旋轉體體積。

    (c) 求該區域對 y 軸旋轉所得到的旋轉體體積。

59. (師大微乙 (一)104#2) 函數 $y = \ln x$ 和 x-軸在區間 $1 \le x \le e^3$ 所圍出區域.

    (a) 繞 x-軸旋轉的體積.

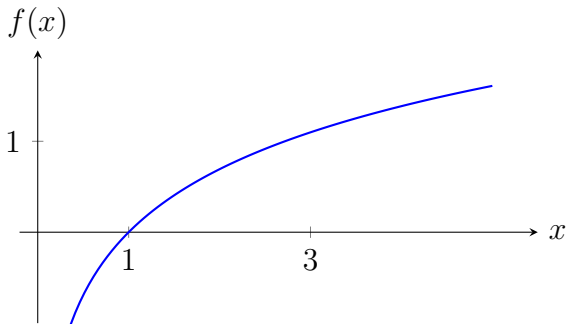
    (b) 繞 y-軸旋轉的體積.

60. (師大微乙 (一)104#7) 函數 $y = \ln x$ 和 x-軸在區間 $1 \le x \le e^3$ 所圍出區域，繞 $x = 1$ 旋轉的旋轉體積.

61. (師大微乙 (一)103#2) 求兩條曲線 $y = 7 - 2x^2$ 和 $y = x^2 - 2$ 所圍出區域的面積.

62. (師大微乙 (一)103#3) 令曲線 $y = \sin x$ 和 x-軸在區間 $0 \le x \le \frac{\pi}{2}$ 所圍出的區域，繞 x-軸旋轉所得出的實體為 V，求 V 的體積.

63. (師大微乙 (一)102#2) 求函數 $y = \ln x$ 與 x-軸在 $1 \le x \le 3$ 圍出的區域面積 (如圖).



64. (師大微乙 (一)102#2) 令曲線 $y = \sin x$ 和 x-軸在區間 $0 \le x \le \pi$ 所圍出的區域，繞 y-軸旋轉所得的旋轉體為 V. 求旋轉體 V 的體積.

65. (師大微乙 (一)102#7) 曲線 $y = x^2$ 和 $y = 4$ 圍出的區域繞 $y = 4$ 旋轉. 求旋轉體體積.

▶ **計算弧長或表面積**

66. (師大微乙 (一)112#1) Find the area of the surface generated by revolving the following curve
$$x = 2\sqrt{4 - y}, \quad 0 \le y \le \frac{15}{4}$$
about the y-axis.

67. (師大微乙 (一)110#4) 求曲線 $y^2 = \frac{4}{9}(x + 1)^3$ 由 $x = 2$ 到 $x = 7$ 的弧長.

68. (師大微乙 (一)109#3) Find the arc length the graph of $y = \frac{1}{2}(e^x + e^{-x})$, $0 \le x \le 2$.

69. (師大微乙 (一)107#3) Find the arc length of the graph of the function
$$y = \frac{x^5}{10} + \frac{1}{6x^3}, \quad 1 \le x \le 3.$$

70. (師大微乙 (一)106#3) 求曲線 $y = \int_{\pi/4}^{x} \cot t \, \mathrm{d}t$ 在 $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ 範圍內的長度 (arc length).

71. (師大微乙 (一)105#4) 求線段 $y = 1 - \frac{x^2}{4}$, $0 \le x \le 2$ 對 y 軸旋轉所得到的旋轉體表面積.

72. (師大微乙 (一)104#3) 線段 $y = 2\sqrt{x}$, $1 \le x \le 2$ 對 x-軸旋轉的表面積.

73. (師大微乙 (一)103#4) 求曲線 $y = \int_{0}^{x} \tan t \, \mathrm{d}t$, $0 \le x \le \frac{\pi}{4}$ 的長度.

74. (師大微乙 (一)102#4) 求曲線 $y = \int_{0}^{x} \sqrt{(\cos 4t)} \, \mathrm{d}t$, $0 \le x \le \frac{\pi}{4}$ 的長度.

► **計算平均值**

75. (師大微乙 (一)113 暑 #4) Find the average value of $g(x) = |x| - 1$ on the interval $[-1, 3]$.

76. (師大微乙 (一)112#5) Find the average value of $g(x) = \dfrac{1}{e^x + e^{-x}}$ on the interval $[-\ln 3, 0]$.

77. (師大微乙 (一)108#1) Find the average vale of $f(x) = \dfrac{1}{x^2 - 4x + 7}$ on the interval $[1, 2]$.