3-3 Differentiation Rules

師大工教一

1. $\dfrac{d}{dx}(c) = 0$ 2. $\dfrac{d}{dx}(x) = 1$ 3. n is any real number, $\dfrac{d}{dx}\left(x^n\right) = n\,x^{n-1}$

Ex1(p148): Differentiate the following powers of x.

(a) x^3 (b) $x^{2/3}$ (c) $x^{\sqrt{2}}$ (d) $\dfrac{1}{x^4}$ (e) $x^{-4/3}$ (f) $\sqrt{x^{2+\pi}}$

(a) $\Downarrow$ $3x^2$

(d) $-4x^{-5}$

(e) $-\frac{4}{3}x^{-\frac{7}{3}}$

(f) $\left(x^{2+\pi}\right)^{\frac{1}{2}}$

$= x^{1+\frac{\pi}{2}}$

$\Downarrow$

$\frac{2+\pi}{2}\,x^{\frac{\pi}{2}}$

微分的線性性質

4. $\dfrac{d}{dx}[cu] = c\dfrac{du}{dx}$

5. $\dfrac{d}{dx}(u \pm v) = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

Ex3(p150) Find the derivative of the polynomial $y = x^3 + \dfrac{4}{3}x^2 - 5x + 1$

$$\frac{dy}{dx} = 3x^2 + \frac{8}{3}x - 5$$

6. $\frac{d}{dx}\left(e^x\right)=e^x$

7.The Product Rule: $\frac{d}{dx}(u\,v)=\frac{du}{dx}\cdot v+u\cdot\frac{dv}{dx}$

Ex6(p152): Find the derivative of (a) $y=\frac{1}{x}\left(x^2+e^x\right)$ (b) $y=e^{2x}$

$= (e^x)^2$

(a) $\Rightarrow -x^{-2}(x^2+e^x)+\frac{1}{x}(2x+e^x)$

$= -1-x^{-2}e^x+2+\frac{e^x}{x}$

$= \frac{e^x}{x}-x^{-2}e^x+1$

(b) $\frac{dy}{dx}=2e^x$

8.The Quotient Rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{\frac{du}{dx}\cdot v-u\cdot\frac{dv}{dx}}{v^2}$

Ex7(p153) Find the derivative of (a) $y=\frac{t^2-1}{t^3+1}$ (b) $y=e^{-x}$ $\frac{1}{e^x}$

(a) $\frac{2t(t^3+1)-(t^2-1)3t^2}{(t^3+1)^2}$

$=\frac{2t^4+2t-3t^4+3t^2}{t^6+2t^3+1}=\frac{-t^4+3t^2+2t}{t^6+2t^3+1}$

(b) $f'(x)=\frac{-1\times\cancel{e^x}}{e^{\cancel{2}x}}$

$=\frac{-1}{e^x}$

Ex8(p154) Find the derivative of $y = \dfrac{(x-1)(x^2-2x)}{x^4}$.

(cf: use quotient rule or not)

$$= \frac{(x-1)(x-2)}{x^3} = \frac{x^2-3x+2}{x^3} = \frac{1}{x} - \frac{3}{x^2} + \frac{2}{x^3}$$

$$\frac{dy}{dx} = \frac{(2x-3)(\cancel{x^3}^{\,1}) - (x^2-3x+2)(3\cancel{x^2})}{x^{\cancel{6}\,4}} = \frac{2x^2-3x-3x^2+9x-6}{x^4} = \frac{-x^2+6x-6}{x^4}$$

Higher-Order Derivatives

Newton: $f(x) \to f'(x) \to f''(x) \to f'''(x) \to f^{(4)}(x) \to \cdots \to f^{(n)}(x) \to \cdots$

Leibniz: $y \to \dfrac{dy}{dx} \to \dfrac{d^2 y}{dx^2} \to \dfrac{d^3 y}{dx^3} \to \cdots \to \dfrac{d^n y}{dx^n} \to \cdots$

HW3-3

- **HW:3,10,15,18,30,45,53,57,69**