# 11.6 Cylinders and Quadric Surfaces

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師大工教一

Cylinders

A **cylinder** is a surface that is generated by moving a line that is parallel to the given line. The curve is called a **generating curve** for the cylinder.
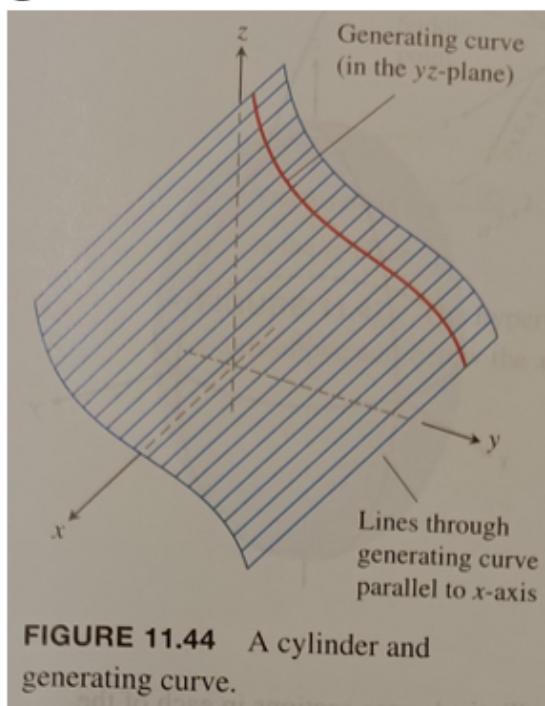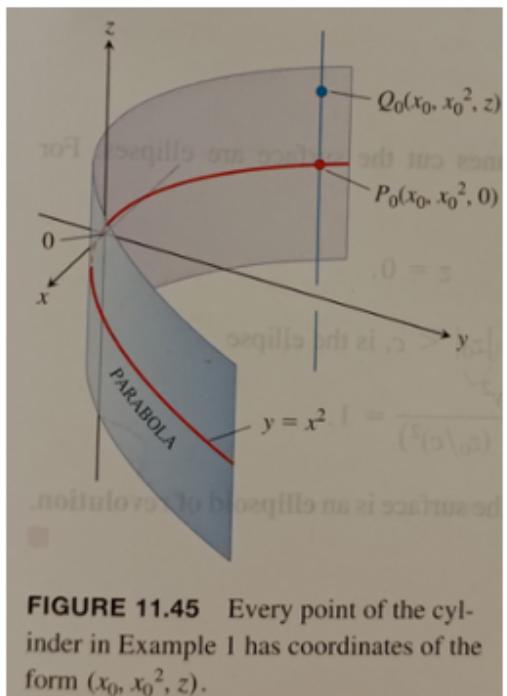
FIGURE 11.44 A cylinder and generating curve.

Ex1(p669) Find an equation for the cylinder made by the lines parallel to the $z-$axis that pass through the parabola $y = x^2, z = 0$.

FIGURE 11.45 Every point of the cyl-
inder in Example 1 has coordinates of the
form $(x_0, x_0{}^2, z)$.

Quadric Surface

A quadric surface is the graph in space of second-degree equation in x, y, z.
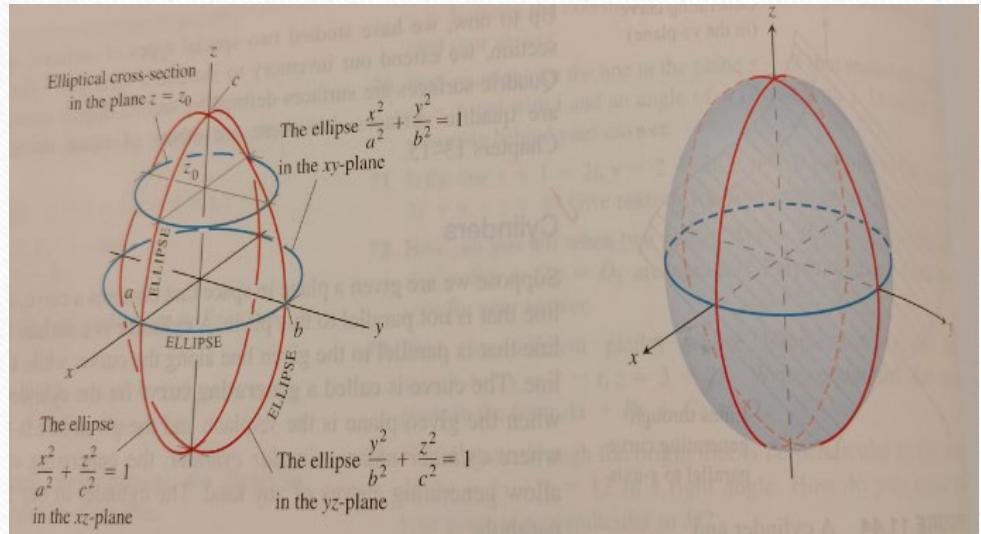
We first focus on quadric surfaces given by the equation

$Ax^2 + By^2 + Cz^2 + Dz = E$, where A, B, C, D, E are constants.
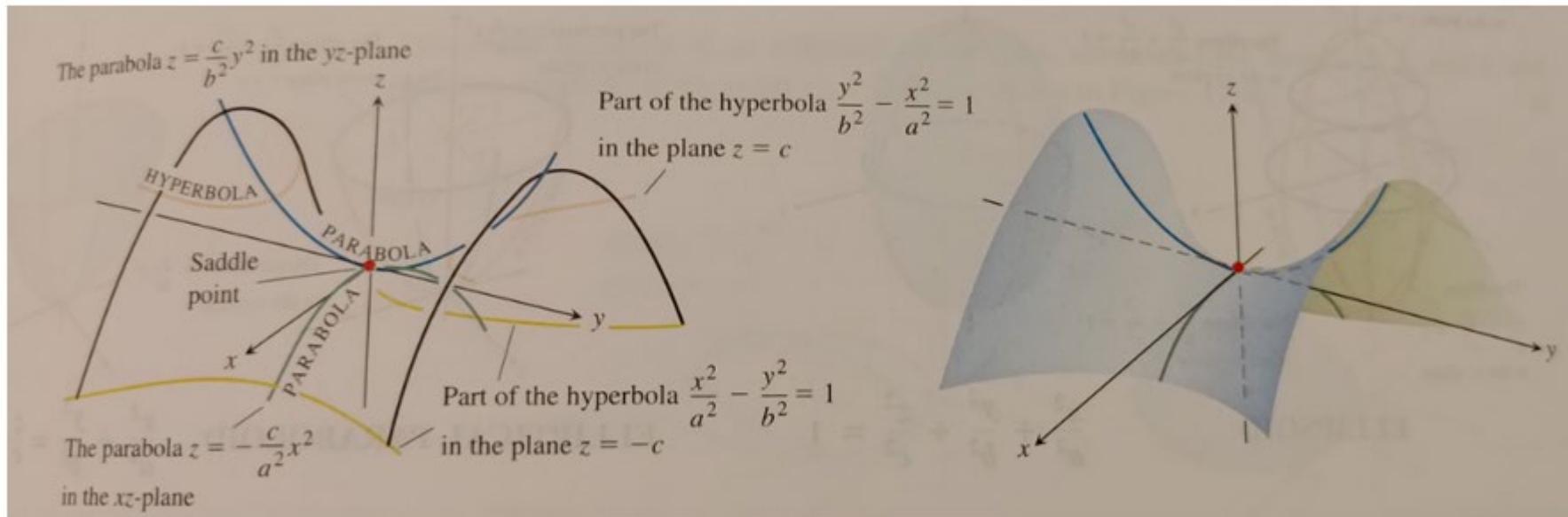
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Ex2(p670) The **ellipsoid** $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

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Ex3(p670) The **hyperbolic parabolid** $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = \dfrac{z}{c}$, $\quad c > 0$.

The parabola $z = \frac{c}{b^2}y^2$ in the yz-plane

HYPERBOLA

Saddle point

PARABOLA

PARABOLA

x

y

Part of the hyperbola $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$ in the plane $z = c$

Part of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ in the plane $z = -c$

The parabola $z = -\dfrac{c}{a^2}x^2$ in the xz-plane

Note: **saddle point**.

鞍点

General Quadric Surfaces

The general equation of second degree in three variables x, y, z is
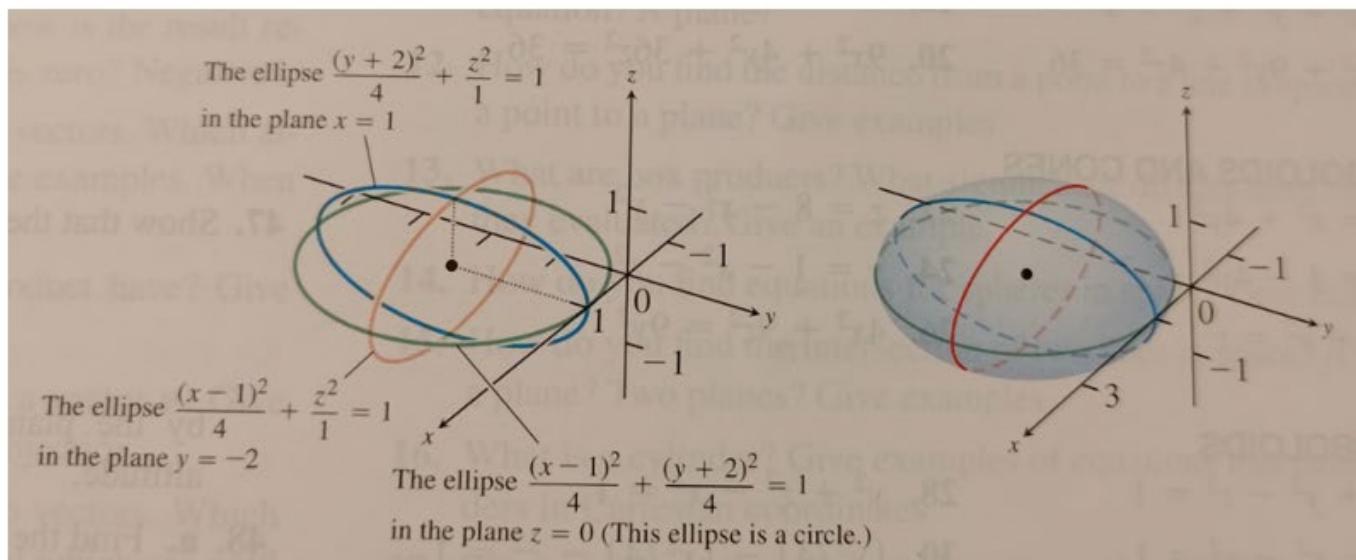
$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0.$$
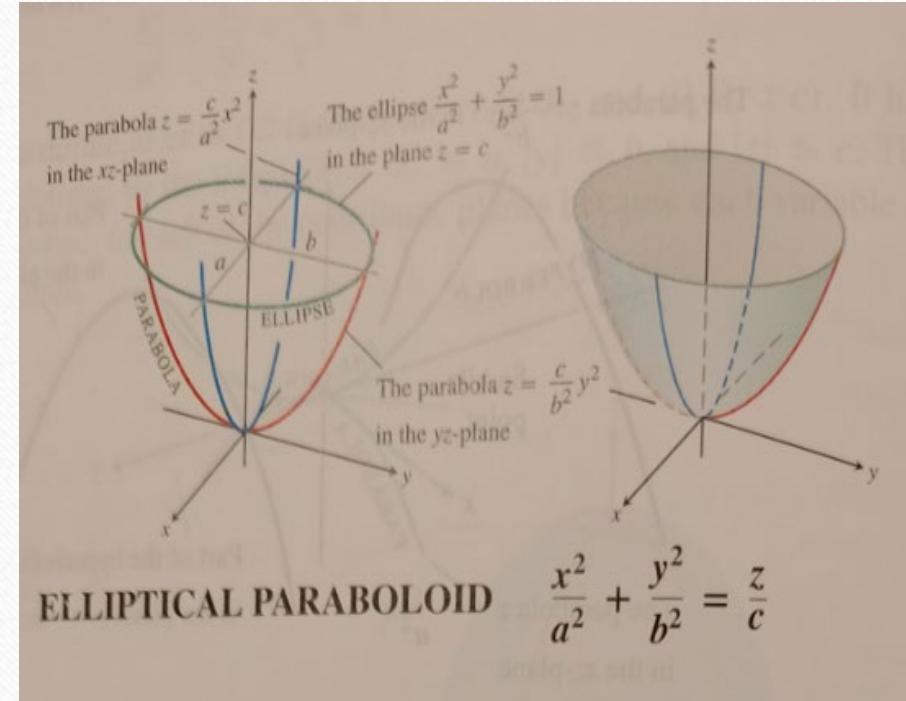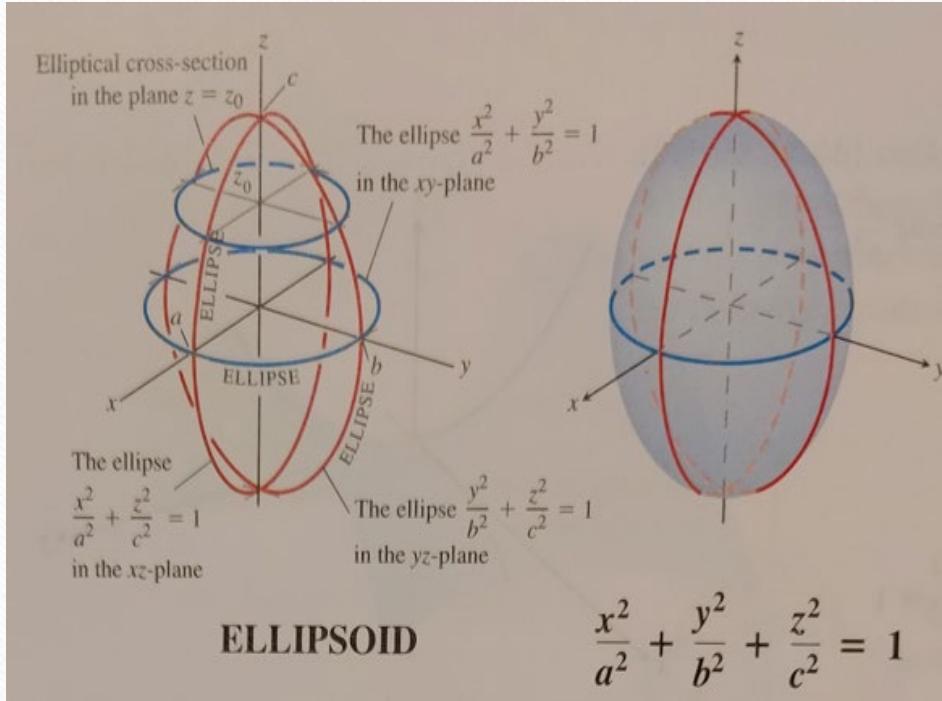
Ex4(p671) Identify the surface given by equation $x^2 + y^2 + 4z^2 - 2x + 4y + 1 = 0$.
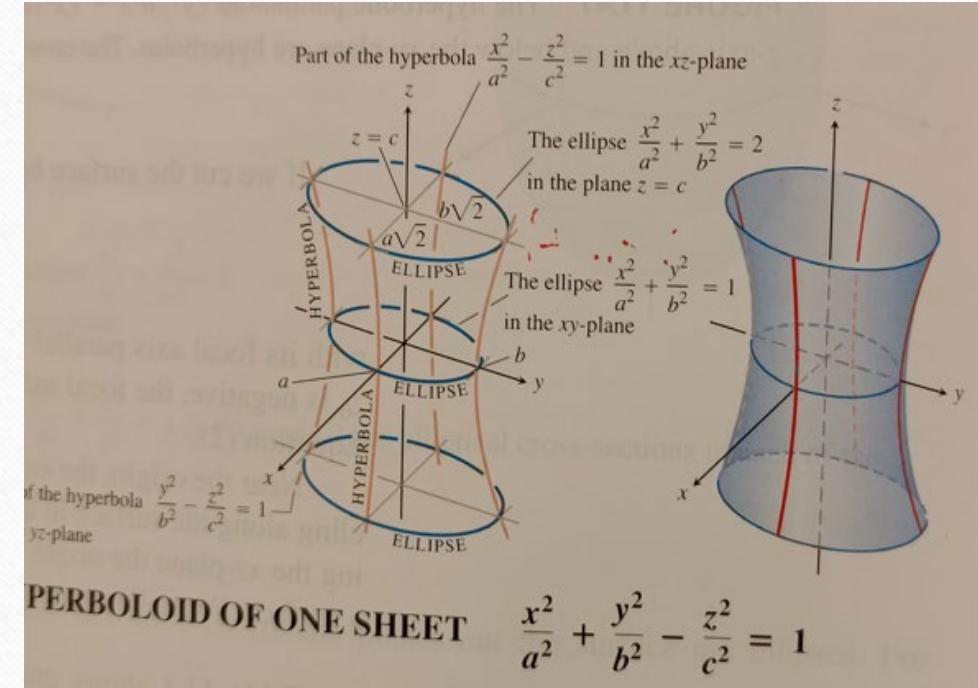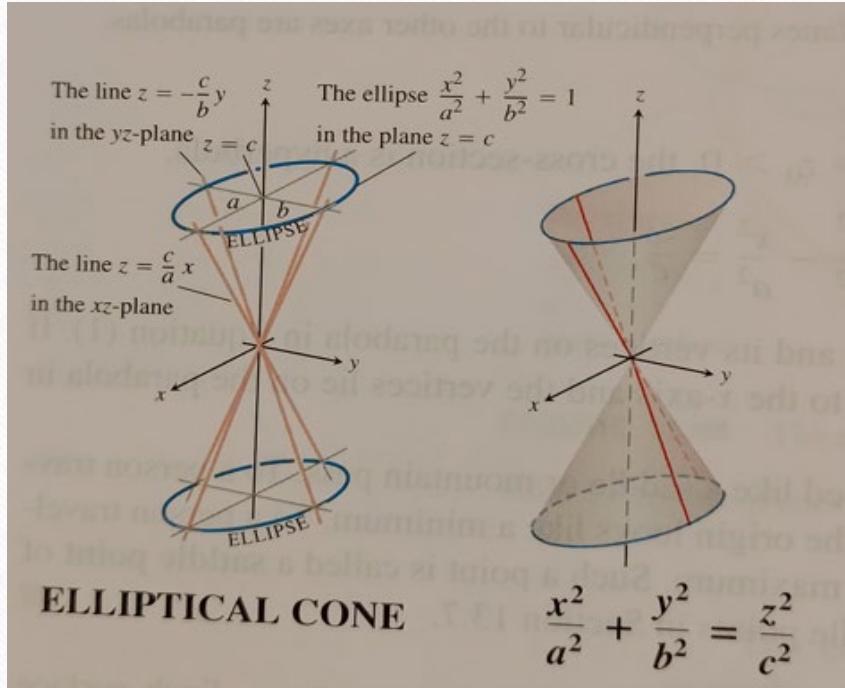
$$x^2 + y^2 + 4z^2 - 2x + 4y + 1 = 0$$
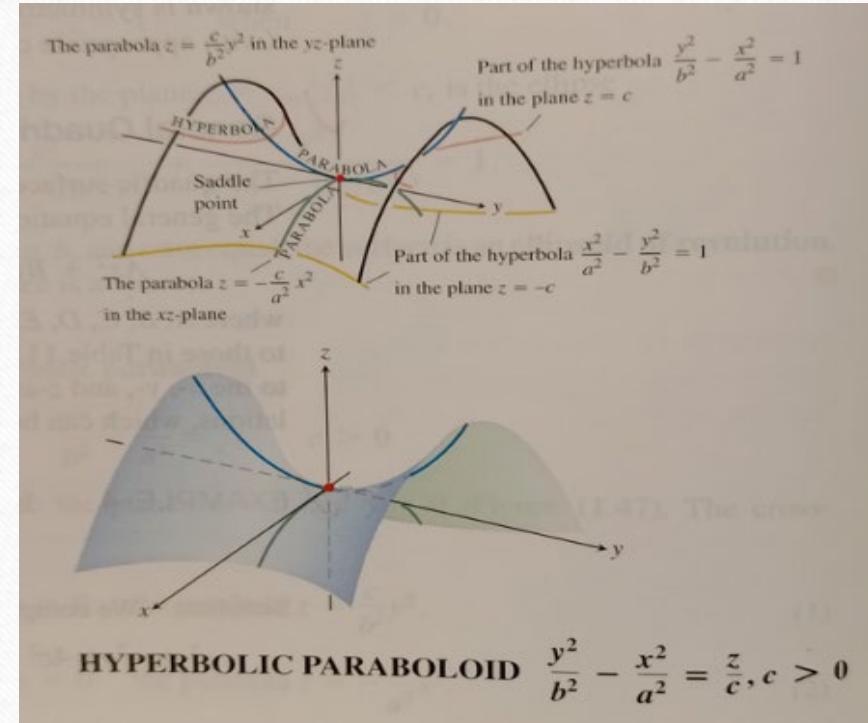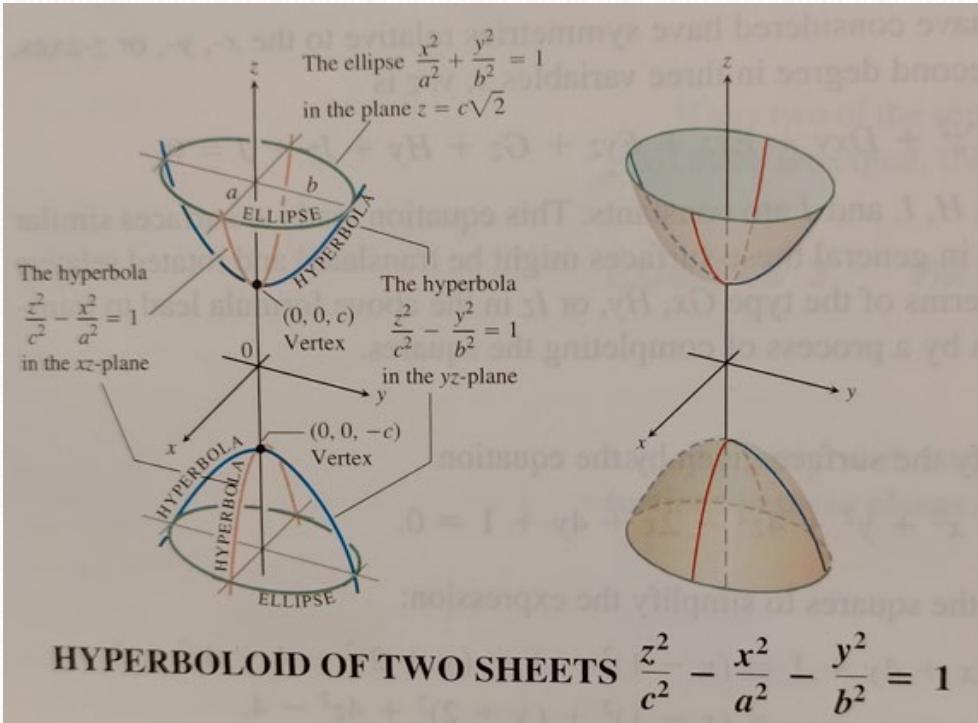
$$(x-1)^2 + (y+2)^2 + 4z^2 = 4$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{4} + \frac{z^2}{1} = 1$$

The ellipse $\frac{(y+2)^2}{4} + \frac{z^2}{1} = 1$ in the plane $x = 1$

The ellipse $\frac{(x-1)^2}{4} + \frac{z^2}{1} = 1$ in the plane $y = -2$

The ellipse $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{4} = 1$ in the plane $z = 0$ (This ellipse is a circle.)

ELLIPSOID $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

Elliptical cross-section in the plane $z = z_0$

The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the xy-plane

The ellipse $\dfrac{x^2}{a^2} + \dfrac{z^2}{c^2} = 1$ in the xz-plane

The ellipse $\dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ in the yz-plane

ELLIPTICAL PARABOLOID $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{z}{c}$

The parabola $z = \dfrac{c}{a^2}x^2$ in the xz-plane

The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the plane $z = c$

The parabola $z = \dfrac{c}{b^2}y^2$ in the yz-plane

The line $z = -\dfrac{c}{b}y$ in the yz-plane

The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the plane $z = c$

$z = c$

ELLIPSE

The line $z = \dfrac{c}{a}x$ in the xz-plane

ELLIPSE

ELLIPTICAL CONE $\qquad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{z^2}{c^2}$

Part of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{z^2}{c^2} = 1$ in the xz-plane

$z = c$

The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 2$ in the plane $z = c$

$b\sqrt{2}$

$a\sqrt{2}$

ELLIPSE

HYPERBOLA

The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the xy-plane

b

ELLIPSE

a

HYPERBOLA

ELLIPSE

f the hyperbola $\dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$ yz-plane

ELLIPSE

PERBOLOID OF ONE SHEET $\qquad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$

The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the plane $z = c\sqrt{2}$

ELLIPSE

The hyperbola $\dfrac{z^2}{c^2} - \dfrac{x^2}{a^2} = 1$ in the xz-plane

HYPERBOLA

$(0, 0, c)$ Vertex

The hyperbola $\dfrac{z^2}{c^2} - \dfrac{y^2}{b^2} = 1$ in the yz-plane

$(0, 0, -c)$ Vertex

HYPERBOLA

HYPERBOLA

ELLIPSE

HYPERBOLOID OF TWO SHEETS $\dfrac{z^2}{c^2} - \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

The parabola $z = \dfrac{c}{b^2}y^2$ in the yz-plane

Part of the hyperbola $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$ in the plane $z = c$

HYPERBOLA

PARABOLA

Saddle point

PARABOLA

The parabola $z = -\dfrac{c}{a^2}x^2$ in the xz-plane

Part of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ in the plane $z = -c$

HYPERBOLIC PARABOLOID $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = \dfrac{z}{c}, c > 0$

HW11-6

- **HW:15,19,21,25,27,29**