5-2 Sigma Notation and Limits of Finite Sums

師大工教一

Limits of Finite Sums

Ex5(p321) Find the limiting value of lower sum approximations to the area of the region R below the graph $y=1-x^2$ and above the interval $[0,1]$ on the $x-$axis using equal-width rectangles whose widths approach zero and whose number approaches infinity.

$$[0,1]\rightarrow\left[0,\frac{1}{n}\right],\left[\frac{1}{n},\frac{2}{n}\right],\cdots,\left[\frac{n-1}{n},\frac{1}{n}\right]$$

$$f\left(\frac{1}{n}\right)\cdot\frac{1}{n}+f\left(\frac{2}{n}\right)\cdot\frac{1}{n}+\cdots+f\left(\frac{n}{n}\right)\cdot\frac{1}{n}=\sum_{k=1}^{n}f\left(\frac{k}{n}\right)\cdot\frac{1}{n}$$

$$\vdots$$

$$=1-\frac{2n^3+3n^2+n}{6n^3}$$

Taking the limit…

Riemann Sums

Dividing $[a,b]$ into n subintervals with endpoints $a = x_0 < x_1 < x_2 < \cdots < x_n = b$

$P = \{x_0, x_1, x_2, \cdots, x_{n-1}, x_n\}$ is called a **partition** of $[a,b]$.

$P = \{x_0, x_1, x_2, x_n\}$

The $k-$th subinterval is $[x_{k-1}, x_k]$, and the width of $k-$th subinterval is

$\Delta x_k = x_k - x_{k-1}$. In $[x_{k-1}, x_k]$, we may choose a **sample point** c_k whose value

$S_P = \sum_{k=1}^{n} f(c_k) \Delta x_k$

$f(c_k)$ as the height of the rectangle.

$S_P = \sum_{k=1}^{n} f(c_k) \Delta x_k$ is called a Riemann Sum of f on $[a,b]$.

Definition The **norm** of a partition P, written $\|P\|$, is the largest of all the subinterval width.

$\|P\| = \mathrm{Max}\{\Delta x_k \mid k=1, \cdots, n\}$

Ex6(p325) The set $P=\{0,0.2,0.6,1,1.5,2\}$ is a partition of $[0,2]$. What is $\|P\|$?

$\|P\| = 0.5$

HW5-2

- **HW: 33,35,39**