14-5 Triple Integrals in Rectangular Coordinates

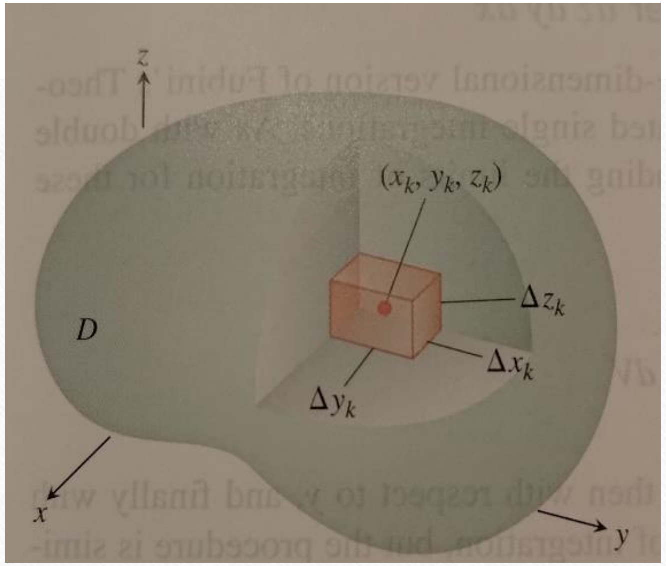
師大工教一

Triple Integrals

Let $F(x, y, z)$ be defined on a closed bounded region D in space. We

partition a rectangular boxlike region containing D into rectangular cells by planes parallel to the coordinate axes. The volume of k th cell is

$\Delta V_k = \Delta x_k \Delta y_k \Delta z_k$.



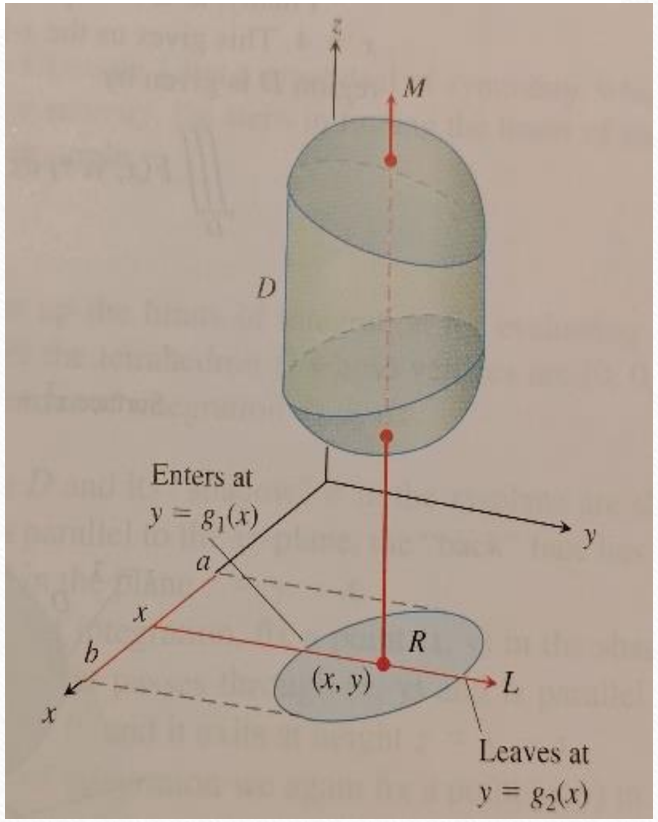
Riemann sum $S_n = \sum_{k=1}^{n} F\left(x_k, y_k, z_k\right) \Delta V_k$. Let $\|P\|$ be the largest value among $\Delta x_k, \Delta y_k, \Delta z_k$. As $\|P\| \to 0$ and the number of cells n goes to ∞, the sums S_n approach a limit. We call the limit the **triple integral of** F **over** D and write

$$\lim_{n\to\infty} S_n = \iiint_D F\left(x, y, z\right) dV \quad \text{or} \quad \lim_{\|P\|\to 0} S_n = \iiint_D F\left(x, y, z\right) dxdydz .$$

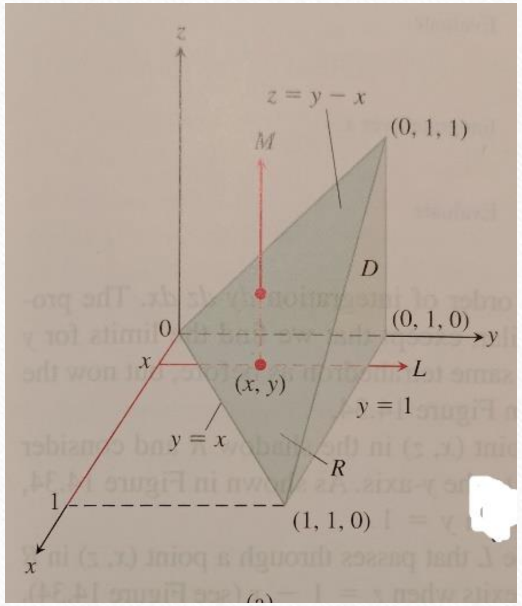
Definition The volume of a closed, bounded region D in space is $V = \iiint\limits_D dV$.

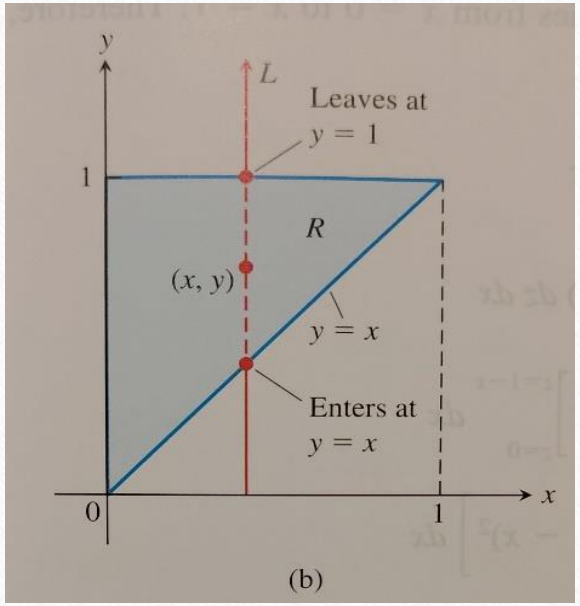
Finding Limits of Integration in the Order $dz\,dy\,dx$

$$\int_{x=a}^{x=b} \int_{y=g_1(x)}^{y=g_2(x)} \int_{z=f_1(x,y)}^{z=f_2(x,y)} F(x,y,z)\,dz\,dy\,dx$$

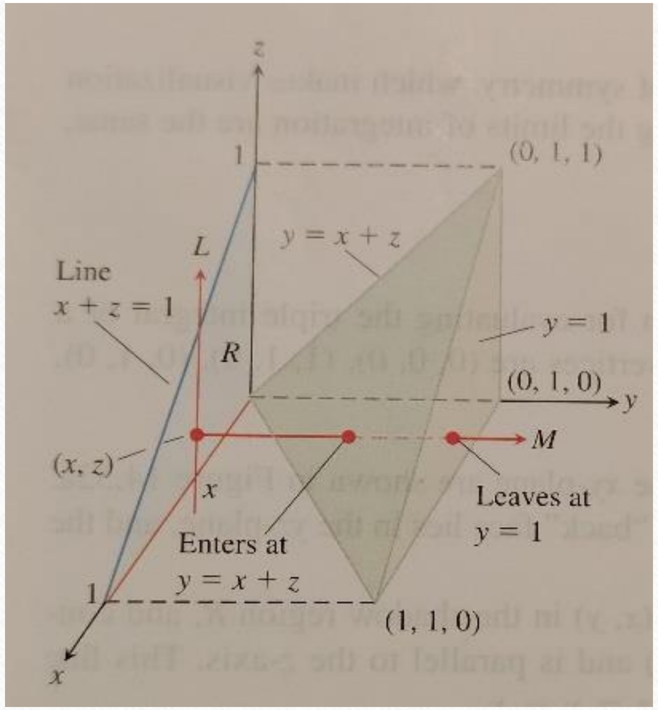


Ex2(p825) Set up the limits of integration for evaluating the triple integral of a function $F(x,y,z)$ over the tetrahedron D whose vertices are $(0,0,0),(1,1,0),(0,1,0),(0,1,1)$. Use the order of integration $dz\,dy\,dx$.





Ex3(p826) Find the volume of the tetrahedron D from Example 2 by integrating $F(x, y, z) = 1$ over the region using the order $dz\,dy\,dx$ Then do the same calculation using the order $dy\,dz\,dx$.



Ex4(p827) Find the volume of the region D enclosed by the surface $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

Average value of F **over** $D = \dfrac{1}{\text{volume of } D} \iiint\limits_{D} F\,dV$.

HW14-5

- **HW:5,6,8,11,23,32**