分部積分法

8-1 Integration by Parts

師大工教一

Integration by Parts: Indefinite Integrals

$$\frac{d}{dx}\left(u(x)v(x)\right) = u'(x)v(x) + u(x)v'(x)$$

$$\Rightarrow \int \frac{d}{dx}\left(u(x)v(x)\right)dx = \int \left[u'(x)v(x) + u(x)v'(x)\right]dx$$

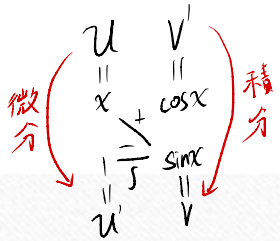
$$\Rightarrow \int \frac{d}{dx}\left(u(x)v(x)\right)dx = \int u'(x)v(x)dx + \int u(x)v'(x)dx$$

$$\Rightarrow \int u(x)v'(x)dx = \int \frac{d}{dx}\left(u(x)v(x)\right)dx - \int u'(x)v(x)dx$$

Formula: $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$

Differential Version: $\int udv = uv - \int vdu$

Ex1(p456) Find $\int x\cos x\,dx$.



$= x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$

Note: What if choose u,v wrong?

Ex2(p457) Evaluate $\int \ln x \, dx$.

$$\text{origin} = x\ln x - \int \frac{1}{x} \cdot x \, dx$$

$$= x\ln x - x + C$$

$\ln x$ | 1

$\frac{1}{x}$ | x

Ex3(p457) Find $\int x^2 e^x dx$.

$$\begin{aligned} \text{orgn} &= x^2e^x - 2xe^x + 2e^x - \int 0\,dx \\ &= x^2e^x - 2xe^x + 2e^x + C \end{aligned}$$

x^2 e^x (+)
$2x$ e^x (−)
2 e^x (+)
0 e^x (−) $\int$

Ex4(p458) Evaluate $\int e^x \cos x \, dx$.

$\int e^x \cos x \, dx$ or $\int e^{ax} \sin bx \, dx$

一定要做兩次

$$\text{original} = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$\cos x$	$+ \; e^x$
$-\sin x$	$- \; e^x$
$-\cos x$	$+ \; \int \; e^x$

$$2\int e^x \cos x \, dx = e^x \cos x + e^x \sin x + C$$

$$\int e^x \cos x \, dx = \frac{e^x \cos x + e^x \sin x + C}{2}$$

Integration by Parts: Definite Integrals

$$\int_a^b f(x)g'(x)\,dx = f(x)\,g(x)\Big|_a^b - \int_a^b f'(x)g(x)\,dx$$

Ex6(p459) Find the area of the region bounded by the curve $y = xe^{-x}$ and the $x-$axis from $x=0$ to $x=4$.

$$A = \int_0^4 xe^{-x}\,dx$$

$$= -xe^{-x}\Big|_0^4 + \int_0^4 e^{-x}\,dx$$

$$= -4e^{-4} + \left(-e^{-x}\Big|_0^4\right)$$

$$= -4e^{-4} - e^{-4} + 1$$

$$= -5e^{-4} + 1 \quad \text{※}$$

	e^{-x}
x	+
1	$-e^{-x}$

HW8-1

- **HW:1,4,5,8,15,21,35,45,58.**

Ex(102, 1(7)) $\int x^2 e^{-x}\,dx$

x^2 (+) e^{-x}
$2x$ (−) $-e^{-x}$
2 (+) e^{-x}
0 (∫) $-e^{-x}$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} - \int 0\,dx$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$ ✱

Ex(102,1(8)) $\int \sin\sqrt{x}\,dx$

$u = \sqrt{x}$
$x = u^2$
$dx = 2u\,du$

$= \int \sin u \cdot 2u\,du$

u (+) $\sin u$
1 (∫) $-\cos u$

$= 2\int u\sin u\,du$
$= 2(-u\cos u + \int \cos u\,du)$
$= -2u\cos u + 2\sin u + C$
$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

Ex(103,1(f)) $\int \ln x\,dx$

Ex(103,1(g)) $\int \dfrac{\ln x}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$= \int u\,du$
$= \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C$