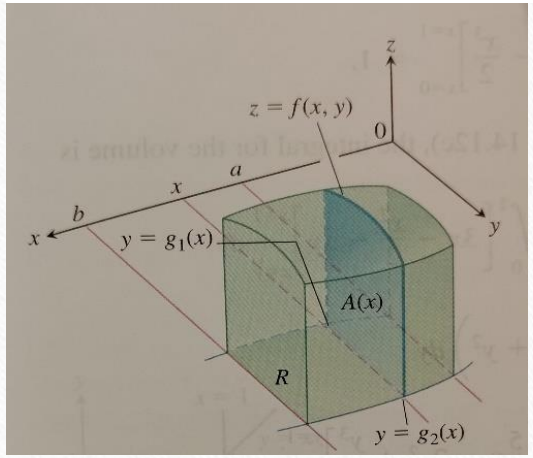
# 14-2 Double Integrals over General Regions

師大工教一

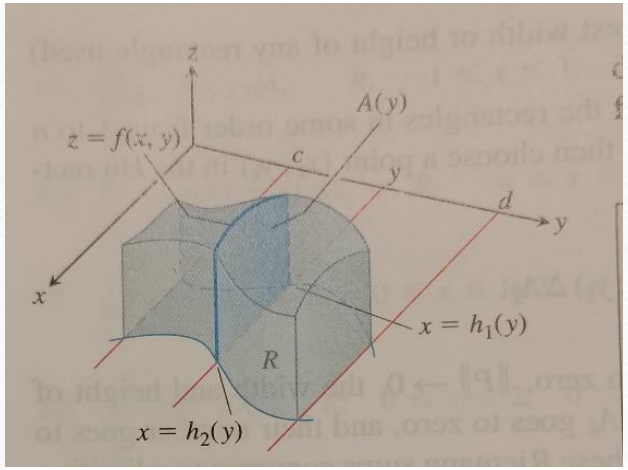
Q: If $R$ is a region enclosed by $a \le x \le b$, $g_1(x) \le y \le g_2(x)$, how can we get the volume or the double integral $\iint_R f(x,y)\,dA$?




As the above figure shows, $A(x) = \int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\,dy$. Thus,

$$V = \int_a^b A(x)\,dx = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dydx.$$

Q: If $R$ is a region enclosed by $c \le y \le d$, $h_1(y) \le x \le h_2(y)$, how can we get the volume or the double integral $\iint_R f(x,y)\,dA$?




As the above figure shows, $A(y) = \int_{x=h_1(y)}^{x=h_2(y)} f(x,y)\,dx$. Thus,

$$V = \int_c^d A(y)\,dy = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy .$$

**Theorem 2—Fubini's Theorem (Stronger Form)**

Let $f$ be continuous on a region $R$.

1. If $R$ is a region enclosed by $a \le x \le b$, $g_1(x) \le y \le g_2(x)$, with $g_1, g_2$ continuous on $[a,b]$, then $\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$.

2. If $R$ is a region enclosed by $c \le y \le d$, $h_1(y) \le x \le h_2(y)$, with $h_1, h_2$ continuous on $[c,d]$, then $\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$.

Ex1(p804) Find the volume of the right prism(角柱) whose base is the triangle in the $xy-$plane bounded by the $x-$axis and the lines $y=x$ and $x=1$ and whose top lies in the plane $z=f(x,y)=3-x-y$.

Ex2(p805) Calculate $\iint_R \frac{\sin x}{x} dA$ where $R$ is the triangle in the $xy-$plane bounded by the $x-$axis, the line $y=x$, and the line $x=1$.

## Properties of Double Integrals

If $f(x,y)$ and $g(x,y)$ are continuous on the bounded region $R$, then the following properties holds.

1. *Constant Multiple:* $\iint_R cf(x,y)\,dA = c\iint_R f(x,y)\,dA$

2. *Sum and Difference:* $\iint_R \left(f(x,y) \pm g(x,y)\right)dA = \iint_R f(x,y)\,dA \pm \iint_R g(x,y)\,dA$

3. *Domination:*

(a) $\iint_R f(x,y)\,dA \geq 0$ if $f(x,y) \geq 0$ on $R$

(b) $\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$ if $f(x,y) \geq g(x,y)$ on $R$

4. *Additivity:* If $R$ is the union of two nonoverlapping regions $R_1$ and $R_2$, then

$$\iint_R f(x,y)\,dA = \iint_{R_1} f(x,y)\,dA + \iint_{R_2} f(x,y)\,dA$$

Ex4(p807) Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region $R$ bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the $x-$axis.

楔形

# HW14-2

- **HW: 9,19,26,47,48,58.**

102 年#4 Evaluate the following iterated integrals.

(b) $\int_0^{\frac{\pi}{4}} \int_y^{\frac{\pi}{4}} \frac{\sec^2 x}{x} dxdy$

103 年#1 Evaluate the following iterated integrals.

(b) $\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{2\pi \sin\left(\pi x^2\right)}{x^2} dxdy$

104 年#3 計算下列疊代積分(逐次積分)(iterated integral):

(a) $\int_0^2 \int_x^2 2y^2 \sin(xy)\, dydx$