11401 微積分乙期中考評分標準

題號 1(a)

1(a) $\lim_{x\to\infty}\sin\left(\frac{\sqrt{x^2+1}}{x+2}\pi\right)$

Sol $\lim_{x\to\infty}\sin\left(\frac{\sqrt{x^2+1}}{x+2}\pi\right)=\lim_{x\to\infty}\sin\left(\frac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{2}{x}}\pi\right)$ (1分)

$=\sin\pi=0$ # (1分)

Note: 寫 when $x\to\infty$, $\frac{\sqrt{x^2+1}}{x+2}\approx\frac{\sqrt{x^2}}{x}=1$，使用"≈"不夠嚴謹，-1~2分

又或者 $\lim_{x\to\infty}\frac{\sqrt{x^2+1}}{x+2}=\lim_{x\to\infty}\frac{\sqrt{x^2}}{x}=1$，未代入 $x\to\infty$ 卻忽略 +1 和 +2，-1~2分

(i) 計算錯誤—答案正確 (-1)；答案錯誤 (-2)

(ii) 無過程—答案正確 (-1)；答案錯誤 (-5)

(iii) "=" 後沒寫 $\lim_{x\to a}$ (-2)，同理 $\sin(x)$ (-2)

(iv) 出現 ∞ 在算式內 (-1)

(v) For $\sin(-\pi)\le f(x)\le\sin\pi$，無法使用 Squeeze Thm,

The value of $f(x)$ can be :

Note that $\lim_{x\to\infty}(-1)=-1$, $\lim_{x\to\infty}(1)=1$ (依錯誤程度-2~5分)

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題號 1(b)

1(b) $\lim_{x\to2}\frac{e^x-e^2}{x-2}$

Sol 1 Let $f(x)=e^x$, +1

by alternative formula for the derivative, $f'(z)=\lim_{x\to z}\frac{e^x-e^z}{x-z}=e^z$ +3

For $z=2$, $\lim_{x\to2}\frac{e^x-e^2}{x-2}=f'(2)=e^2$ # +1

Sol 2 (+4分)

Using taylor expansion, but will have infinite term all closed to zero,

Confuse it will be zero when $\lim_{x\to\infty}\lim_{y\to0}xy\neq0$.

(i) 計算錯誤—答案正確 (-1)；答案錯誤 (-2)

(ii) 無過程—答案正確 (-4)：會有使用 L'Hopital 的疑慮；答案錯誤 (-5)

(iii) "=" 後沒寫 $\lim_{x\to a}$ (-2)

(iv) 使用 L'Hopital (-5)

(v) Ans：e^x 斟酌扣分 (-1~-2分)

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題號 1(c)(d)

(c) 1分, $\tan x=\frac{\sin x}{\cos x}$

$\lim_{x\to0}\frac{\sin3x}{\tan5x}=\lim_{x\to0}\frac{\sin3x}{\sin5x}\cos5x=\lim_{x\to0}\frac{x}{x}\cdot\frac{\sin3x}{\sin5x}\cos5x=\lim_{x\to0}\frac{\sin3x}{x}\cdot\frac{x}{\sin5x}\cos5x=\lim_{x\to0}\frac{3}{5}\cdot\frac{\sin3x}{3x}\cdot\frac{5x}{\sin5x}\cos5x$ (3分)

$=\frac{3}{5}\cdot1\cdot1\cdot1=\frac{3}{5}$ (1分) (note $\lim_{x\to0}\frac{\sin x}{x}=1$, $\lim_{x\to0}\frac{x}{\sin x}=1$)

使用 $\lim_{x\to0}\frac{\sin3x}{\sin5x}=\frac{3}{5}$, $\lim_{x\to0}\frac{\sin3x}{x}=3$, $\lim_{x\to0}\frac{x}{\sin5x}=\frac{1}{5}$, $\lim_{x\to0}\frac{5x}{\tan5x}=1$ 皆給對，但最後算錯給全錯

如果有使用 $\lim_{x\to0}\frac{\sin3x}{3x}=1$ 或 $\lim_{x\to0}\frac{5x}{\sin5x}=1$，但答案錯誤給1分，有 $\tan x=\frac{\sin x}{\cos x}$ 再拿1分

過程中 lim 沒寫，如 $\lim_{x\to0}\frac{\sin3x}{\tan5x}=\frac{\sin3x}{\sin5x}\cos5x=\cdots$，最高只能拿3分，這件事適用於下題

(d) $\lim_{x\to\infty}\frac{\cos x}{\sin x+x}=\lim_{x\to\infty}\frac{\frac{\cos x}{x}}{\frac{\sin x}{x}+1}$

$-1\le\cos x\le1$

$\Rightarrow-\frac{1}{|x|}\le\frac{\cos x}{x}\le\frac{1}{|x|}$

$\because\lim_{x\to\infty}-\frac{1}{|x|}=0$, $\lim_{x\to\infty}\frac{1}{|x|}=0$

By squeeze theorem

$\lim_{x\to\infty}\frac{\cos x}{x}=0$

$-1\le\sin x\le1$

$\Rightarrow-\frac{1}{|x|}\le\frac{\sin x}{x}\le\frac{1}{|x|}$

$\because\lim_{x\to\infty}-\frac{1}{|x|}=0$, $\lim_{x\to\infty}\frac{1}{|x|}=0$

By squeeze theorem

$\lim_{x\to\infty}\frac{\sin x}{x}=0$

至少要寫一個，才算有使用夾擠

$\Rightarrow\lim_{x\to\infty}\frac{\frac{\cos x}{x}}{\frac{\sin x}{x}+1}=\frac{0}{0+1}=0$ (1分)

未使用夾擠最高給3分，如" $\lim_{x\to\infty}\frac{\frac{\cos x}{x}\to0}{\frac{\sin x}{x}+1}=0$ "等

夾擠有明顯問題，如" $-\infty\le\lim_{x\to\infty}\frac{\cos x}{\sin x+1}\le\infty$ "等，最高給4分

未使用夾擠：*有寫到如"$-1\le\sin x\le1$"，"$-1\le\cos x\le1$"，"$0\le|\cos x|\le1$"，"$0\le|\sin x|\le1$"等說明 $\sin x$ 與 $\cos x$ 的範圍，但答案正確給4分，但答案錯誤給1分

有夾擠，但答案錯誤給1分

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題號 2.(a).(b)

$x^2+y^2=\sin(x+y)$

$\Rightarrow\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(\sin(x+y))$ 嘗試隱微分 2分

$\Rightarrow2x+2y\cdot y'=\cos(x+y)\cdot(1+y')$

$\Rightarrow y'(2y-\cos(x+y))=\cos(x+y)-2x$

$\Rightarrow y'=\frac{\cos(x+y)-2x}{2y-\cos(x+y)}$

共2分，微分錯一個或計算錯扣0.5

slope of tangent line of C at (0,0)

is $y'\big|_{(x,y)=(0,0)}=\frac{\cos0-0}{0-\cos0}=-1$ 值1分 (微分錯但值對不給分)

$\Rightarrow$ (a) tangent line : $y=-x$ 方程式1分

Let slope of normal line of C at (0,0) is m.

We have $m\times(-1)=-1$ 2分 $\Rightarrow m=1$ 值1分

$\Rightarrow$ (b) normal line : $y=x$ 方程式1分

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題號 3(a)(b)

(a)

$y=x^4+2025x^3+114$

$\Rightarrow y'=4x^3+6075x^2+0$ (各個部份1分)

(b)

$y=\sin(100^{x^3+5})$

Let $u=100^{x^3+5}$ $\Rightarrow$ $y=\sin u$

Let $v=x^3+5$ $\Rightarrow$ $u=100^v$

Then $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}$

(1) $\frac{dy}{du}=\cos u=\cos(100^v)=\cos(100^{x^3+5})$ 1分 1分 有寫 cos 就有一分

(2) $\frac{du}{dv}=\frac{d}{dv}(100^v)=100^v\cdot\ln100=2\ln10\cdot100^v=2\ln10\cdot100^{x^3+5}$ 1分 1分

(3) $\frac{dv}{dx}=3x^2$ 1分

上述 (1),(2)，若無將變數換回 x 之式子，則扣1分.

$2\ln10$ 可寫 $\ln100$ 也可寫 $\log100$ (此處 $\log100$ 預設為 $\log_e100$)

Therefore $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}=\cos(100^{x^3+5})\cdot2\ln10\cdot100^{x^3+5}\cdot(3x^2)$

若無寫出最終之 $\frac{dy}{dx}$，則扣1分

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題號 3(c)

解法1：

$\ln y=\frac{1}{2}\ln(x^3+3)+4\ln(x+4)-2\ln(x^{\frac{1}{3}}+3x)$ → +1

$\Rightarrow\frac{1}{y}\cdot y'=\frac{3x^2}{2(x^3+3)}+\frac{4}{x+4}-\frac{2(\frac{1}{3}x^{-\frac{2}{3}}+3)}{x^{\frac{1}{3}}+3x}$ → +2

$\Rightarrow y'=\frac{\sqrt{x^3+3}\,(x+4)^4}{(x^{\frac{1}{3}}+3x)^2}\cdot\left[\frac{3x^2}{2(x^3+3)}+\frac{4}{x+4}-\frac{2(\frac{1}{3}x^{-\frac{2}{3}}+3)}{x^{\frac{1}{3}}+3x}\right]$ → +2

解法2：

$$y'=\frac{\overbrace{\left[\frac{1}{2}\cdot\frac{3x^2}{\sqrt{x^3+3}}(x+4)^4+4(x+4)^3\cdot\sqrt{x^3+3}\right]}^{+2}\cdot(x^{\frac{1}{3}}+3x)^2-\overbrace{2(x^{\frac{1}{3}}+3x)(\tfrac{1}{3}x^{-\frac{2}{3}}+3)\sqrt{x^3+3}\,(x+4)^4}^{+2}}{(x^{\frac{1}{3}}+3x)^4}$$

$$=\frac{\sqrt{x^3+3}\,(x+4)^4}{(x^{\frac{1}{3}}+3x)^2}\cdot\left[\frac{3x^2}{2(x^3+3)}+\frac{4}{x+4}-\frac{2(\frac{1}{3}x^{-\frac{2}{3}}+3)}{x^{\frac{1}{3}}+3x}\right]$$

or

$$\frac{(x+4)^3(11x^3+12x^2+24)}{2\sqrt{x^3+3}\,(x^{\frac{1}{3}}+3x)^2}-\frac{2(\frac{1}{3}x^{-\frac{2}{3}}+3)\sqrt{x^3+3}\,(x+4)^4}{(x^{\frac{1}{3}}+3x)^3}$$ +1

P.S.

1. 沒括弧 -1
2. 計算錯誤 -1
3. 解法1 時 y 沒代換回來 -1

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題號 3(d)

解法1：

$y'=\frac{1}{1+\sin^2x}\cdot(\sin x)'$ → +3

$=\frac{\cos x}{1+\sin^2x}$ → +2

解法2：

$\tan y=\tan(\tan^{-1}(\sin x))=\sin x$ → +2

$\Rightarrow\sec^2y\cdot y'=\cos x$ → +1

$\Rightarrow(\sqrt{1+\sin^2x})^2\cdot y'=\cos x$ → +1

$\Rightarrow y'=\frac{\cos x}{1+\sin^2x}$ → +1

P.S.

只對 chain rule or 反三角微分 +2

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題號 4(a)(b)

4 $f(x)=\sin x+3x+5$

(a) $f(0)=0+0+5=5$ 5分 (全對才給分)

(b) 法一：

$f'(x)=\cos x+3>0$ 1分 (on the interval $I=(\frac{-\pi}{2},\frac{\pi}{2})$),

$f(x)$ is strictly increasing 3分 (on the interval $I=(\frac{-\pi}{2},\frac{\pi}{2})$), (so $f(x)$ is one-to-one.)

$f(x)$ has an inverse function on the interval $I=(\frac{-\pi}{2},\frac{\pi}{2})$. 1分

扣分標準：

(i) 只有寫遞增或 $f'(x)>0$，最多得3分

(ii) 結論明顯與題目無關者，最多得4分。

法二：使用 Rolle's Theorem

Suppose $f(a)=f(b)$. WLOG (不失一般性), $\frac{-\pi}{2}<a<b<\frac{\pi}{2}$

$f(x)$ is continuous on the closed interval $[a,b]$ (or $[\frac{-\pi}{2},\frac{\pi}{2}]$) and differentiable on the open interval (a,b) (or $(\frac{-\pi}{2},\frac{\pi}{2})$). By Rolle's Theorem, (because $f(a)=f(b)$) $\exists c\in(a,b)$, such that $f'(c)=0$

but $f'(x)=\cos x+3>0$ $\forall x\in(\frac{-\pi}{2},\frac{\pi}{2})$ ($\rightarrow\leftarrow$)

Hence if $a\neq b$ then $f(a)\neq f(b)$ $\because f(x)$ is one-to-one.

$\therefore f(x)$ has an inverse function on the interval $I=(\frac{-\pi}{2},\frac{\pi}{2})$

扣分標準：依 Rolle's Theorem 完整性酌量扣分

法三：使用 Mean Value Theorem

Suppose $f(a)=f(b)$. WLOG, $\frac{-\pi}{2}<a<b<\frac{\pi}{2}$

$f(x)$ is continuous on the closed interval $[a,b]$ (or $[\frac{-\pi}{2},\frac{\pi}{2}]$) and differentiable on the open interval (a,b) (or $(\frac{-\pi}{2},\frac{\pi}{2})$). By Mean Value Theorem, $\exists c\in(a,b)$. $f'(c)=\frac{f(b)-f(a)}{b-a}$ (or $f(b)-f(a)=f'(c)(b-a)$)

$\because f'(x)=\cos x+3>0$ $\forall x\in(\frac{-\pi}{2},\frac{\pi}{2})$ $\therefore\frac{f(b)-f(a)}{b-a}>0$ i.e. $f(b)>f(a)$ ($\rightarrow\leftarrow$)

If $a\neq b$ then $f(a)\neq f(b)$. Hence $f(x)$ has an inverse function on $I=(\frac{-\pi}{2},\frac{\pi}{2})$

扣分標準：依 Mean Value Theorem 完整性酌量扣分

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題號 4(c)

$f(x)=\sin x+x+5$

by (a) : $f(0)=\sin0+0+5=5$

$\Rightarrow f^{-1}f(0)=f^{-1}(5)$ +2

$\Rightarrow f^{-1}(5)=0$ # +3

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題號 4(d)

by Thm. $\frac{d}{dx}f^{-1}(x)=\frac{1}{f'(f^{-1}(x))}$. by (c) $f^{-1}(5)=0$ +2

$\frac{d}{dx}f^{-1}(5)=\frac{1}{f'(f^{-1}(5))}=\frac{1}{f'(0)}$ (+2)

二擇一最多加2分

$f'(x)=\cos x+3$, $f'(0)=4$ +1

$\Rightarrow\frac{d}{dx}f^{-1}(x)=\frac{1}{4}$ # +2

114-1 微積分乙期中考評分標準第五題 Qusetion 5

Method 1

Claim: Existence of solution

Note that

- $f(0)=e^0+0-4=1-4=-3<0$ 【+1 pt】
- $f(4)=e^4+4-4=e^4>0$ 【+1 pt】

Also, since f is a continuous function 【+1 pt】, by Intermediate Value Theorem 【+0.5 pt】

$\exists\,c\in(0,4)\subseteq(-\infty,\infty)$ such that $f(c)=0$ 【+0.5 pt】

Claim: Uniqueness of the solution

Suppose that $f(a)=0=f(b)$ 【+1 pt】, where $a,b\in(-\infty,\infty)$ and $a<b$. Since f is a continuous function and differentiable function 【+1 pt】 and $f(a)=0=f(b)$, by Rolle's Theorem 【+0.5 pt】 we have

$\exists\,c\in(a,b)$ s.t. $f'(c)=0$ 【+0.5 pt】

But it is clear that $\forall x\in\mathbb{R}$

$f'(x)=e^x+1>0$ 【2 pts】

This leads to a contradiction. $\rightarrow\leftarrow$

Therefore, $f(x)=e^x+x-4$ has exact one root on $(-\infty,\infty)$. 【+1 pt】 □

Remark (評分細節 Detailed Grading Criteria).

- 黃底字和綠底字為必寫內容，若沒寫將扣分。此外, 書寫邏輯錯誤將視錯誤嚴重性斟酌扣分。The parts highlighted in yellow and green are **required**. Points will be deducted if they are missing. In addition, points may be deducted depending on the severity of any **logical errors** in the writing.
- 若直接寫 $f(0)<0$ 而未加以說明原因者會扣分, 此部分至多扣 1 分; 另外 $\pm\infty$ 不是數字, 若寫出 $f(\pm\infty)$ 一律不給分。If a student writes $f(0)<0$ without providing a justification, up to **1 point** will be deducted. In addition, since $\pm\infty$ are **not numbers**, any expression such as $f(\pm\infty)$ will receive **no credit**.
- 最後結論必寫. 沒寫一定會扣分: 若前面解釋理由不合理則結論的 1 分將不給。The **final conclusion** is required; failure to include it will **result in a deduction**. If the reasoning leading to the conclusion is **unjustified or incorrect**, the **1 point** for the conclusion will not be awarded.
- 「存在性」和「唯一性」順序顛倒寫至多 7 分，過程有缺漏者亦會斟酌扣分。If the order of **existence** and **uniqueness** is reversed, a maximum of **7 points** will be awarded. In addition, any **incomplete reasoning or missing steps** will result in partial deductions as appropriate.

114-1 微積分乙期中考評分標準第五題 Qusetion 5

Method 2

Claim: Existence of solution

Note that

- $f(0)=e^0+0-4=1-4=-3<0$ 【+1 pt】
- $f(4)=e^4+4-4=e^4>0$ 【+1 pt】

Also, since f is a continuous function 【+1 pt】, by Intermediate Value Theorem 【+0.5 pt】

$\exists\,c\in(0,4)\subseteq(-\infty,\infty)$ such that $f(c)=0$ 【+0.5 pt】

Claim: Uniqueness of the solution

Note that since $\forall x\in\mathbb{R}$

$f'(x)=e^x+1>0$ 【+2 pt】

Hence, f is increasing on $\mathbb{R}$ 【+3 pt】, this implies f is one-to-one function on $\mathbb{R}$.

Therefore, $f(x)=e^x+x-4$ has exact one root on $(-\infty,\infty)$. 【+1 pt】 □

Remark.

- 黃底字和綠底字為必寫內容，若沒寫將扣分。此外, 書寫邏輯錯誤將視錯誤嚴重性斟酌扣分。The parts highlighted in yellow and green are **required**. Points will be deducted if they are missing. In addition, points may be deducted depending on the severity of any **logical errors** in the writing.
- 若直接寫 $f(0)<0$ 而未加以說明原因者會扣分, 此部分至多扣 1 分; 另外 $\pm\infty$ 不是數字, 若寫出 $f(\pm\infty)$ 一律不給分。If a student writes $f(0)<0$ without providing a justification, up to **1 point** will be deducted. In addition, since $\pm\infty$ are **not numbers**, any expression such as $f(\pm\infty)$ will receive **no credit**.
- 最後結論必寫. 沒寫一定會扣分: 若前面解釋理由不合理則結論的 1 分將不給。The **final conclusion** is required; failure to include it will **result in a deduction**. If the reasoning leading to the conclusion is **unjustified or incorrect**, the **1 point** for the conclusion will not be awarded.
- 「存在性」和「唯一性」順序顛倒寫至多 7 分，過程有缺漏者亦會斟酌扣分。If the order of **existence** and **uniqueness** is reversed, a maximum of **7 points** will be awarded. In addition, any **incomplete reasoning or missing steps** will result in partial deductions as appropriate.
- 呈上點, 所謂的不可顛倒寫就是指在書寫的第一部份一定要說明解存在性 (也就是使用中間值定理說明), 若先求微分說明再說明恆遞增將即視為先證明唯一性. 因為這類型證明唯一一根的題目就是需要先證明解的存在性再證明解的唯一性，請務必了解其邏輯順序。"Not to be written in reverse order" means that the first part must show existence (using the Intermediate Value Theorem). If differentiation and monotonicity are shown first, it will be treated as proving uniqueness first. For problems of this kind, the correct logical order is: existence → uniqueness.

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題號 6(a),(b)

6(a) $f(x)=\frac{x^3-3x^2+3x-1}{x^2+x-2}=\frac{(x-1)^3}{(x+2)(x-1)}$, $x\neq-2,1$ 1分

* $\lim_{x\to1}f(x)=\lim_{x\to1}\frac{(x-1)^3}{(x+2)(x-1)}=\lim_{x\to1}\frac{(x-1)^2}{(x+2)}=\frac{0}{3}=0\neq\pm\infty$ 1分

* $\lim_{x\to-2^-}f(x)=\lim_{x\to-2^-}\frac{(x-1)^3}{(x+2)(x-1)}=\lim_{x\to-2^-}\frac{(x-1)^2}{(x+2)}=-\infty$

$\lim_{x\to-2^+}f(x)=\lim_{x\to-2^+}\frac{(x-1)^3}{(x+2)(x-1)}=\lim_{x\to-2^+}\frac{(x-1)^2}{(x+2)}=\infty$) 2分

$\Rightarrow$ vertical asymptotes : $x=-2$ 1分

- 只寫答案 $x=-2$，沒檢查 $\lim_{x\to1}f(x)$, $\lim_{x\to-2^+}f(x)$ ($\lim_{x\to-2^-}f(x)$)：得3分
- 只少 $\lim_{x\to1}f(x)$，只少 $\lim_{x\to-2^+}f(x)$，或寫了 $\lim_{x\to-2}$ 者：扣1分
- 答案寫了 $x=1$、$x=-2$ 者：得2分

6(b) * $\Rightarrow f(x)=x-4+\frac{9x-9}{x^2+x-2}$ 2分

$\because\lim_{x\to\infty}[f(x)-(x-4)]=\lim_{x\to\infty}\left[(x-4)+\frac{9x-9}{x^2+x-2}-(x-4)\right]=\lim_{x\to\infty}\frac{9(x-1)}{(x+2)(x-1)}=0$

* 且 $\lim_{x\to-\infty}[f(x)-(x-4)]=\lim_{x\to-\infty}\left[(x-4)+\frac{9x-9}{x^2+x-2}-(x-4)\right]=0$) 2分

$\therefore$ slant asymptote : $y=x-4$ 1分

- 有用長除法，答案 $y=x-4$，沒有 $\lim_{x\to\pm\infty}[f(x)-(x-4)]$：得3分
- 答案正確，有 $\lim_{x\to\infty}$，沒 $\lim_{x\to-\infty}$：扣1分，得4分
- 使用「$y=mx+b$ 求 $m=\lim_{x\to\pm\infty}\frac{f(x)}{x}$，$b=\lim_{x\to\pm\infty}f(x)-mx$」此方法：m 得2分，b 得2分，只考慮 $x\to\infty$ 扣1分

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題號 6(c)

For $x\in dom(f)$, we have $f(x)=\frac{(x-1)^2}{x+2}$

$\therefore f'(x)=\frac{2(x-1)(x+2)-(x-1)^2\cdot1}{(x+2)^2}$

$=\frac{(x-1)[2x+4-(x-1)]}{(x+2)^2}$

$=\frac{(x-1)(x+5)}{(x+2)^2}$ +1

要討論到

Note that $(x+2)^2>0$, $\forall x\in dom(f)$

- $f'(x)>0$ $\Rightarrow(x-1)(x+5)>0$ $\Rightarrow x>1$ or $x<-5$ +1
- $f'(x)<0$ $\Rightarrow(x-1)(x+5)<0$ $\Rightarrow-5<x<1\setminus\{x=-2\}$ +1

Hence,

increasing : $(-\infty,-5)\cup(1,\infty)$ +1

decreasing : $(-5,-2)\cup(-2,1)$ +1

- 寫 $f'(x)$：+ (−5) − (1) + 加1分
- 寫 + (−5) − (1) +，但有提 $f(-2)$ 不存在，加2分
- 寫 + (−5) − (−2) − (1) + 加2分

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題號 6(d)

When $f'(x)=\frac{(x-1)(x+5)}{(x+2)^2}=0$

then $x=1$ or $x=-5$. (note that $x=1$ is not in the domain of f)

$f(-5)=\frac{(-6)^2}{-3}=-12$ is a local maximum of f. +5

- local maximum 寫成 local minimum 扣1分
- 算出 local minimum 或多餘的答案扣1分
- 知道 $x=-5$ 為 local maximum，但沒算出來，加2分
- 寫 $\lim_{x\to-5}f(x)=-12$ 全扣

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題號 7

Let $f(x)=\sin x$

$\because f'(x)=\cos x$ (1)

$\therefore-1\le f'(c)\le1$ $\forall c\in\mathbb{R}$ $\Rightarrow|f'(c)|\le1$ $\forall c\in\mathbb{R}$ (1)

(H1) $f(x)$ is conti. on $\mathbb{R}$.

(H2) $f(x)$ is diff. on $\mathbb{R}$ } (1)

By M.V.T, $\forall x,y\in\mathbb{R}$, $\exists c\in(x,y)$ s.t.

$f'(c)=\frac{f(x)-f(y)}{x-y}=\frac{\sin x-\sin y}{x-y}$ (1)

$\Rightarrow|f'(c)|=\left|\frac{\sin x-\sin y}{x-y}\right|\le1$ $\forall x,y\in\mathbb{R}$

$\Rightarrow|\sin x-\sin y|\le|x-y|$ $\forall x,y\in\mathbb{R}$ } (1)

*若有寫出

$0\le|\sin x-\sin y|\le2$ (1)

*此解未用MVT至多2分

*有討論 $x=y$ (1)