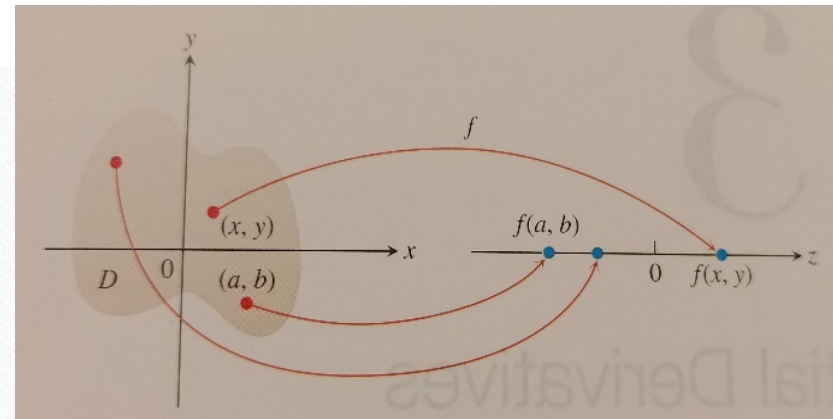# 13-1 Functions of Several Variables

師大工教一

Definition Suppose D is a set of n-tuples of real numbers $(x_1, x_2, \cdots, x_n)$. A

real-valued function f on D is a rule that assigns a real number

$w = f(x_1, x_2, \cdots, x_n)$ to each element in D. The set D is the function's

domain. The set of $w-$values taken on by f is the function's **range**. The

symbol w is the **dependent variable** of f, and f is said to be a function

of the n **independent variables** x_1 to x_n.

$f : D(\subset \mathbb{R}^n) \longrightarrow \mathbb{R}$

$w = f(x_1, x_2, \cdots, x_n)$ $n-$變數函數

$D = $ domain 定義域

$f(D) = $ range 值域

$w :$ dependent variable 相関変数

$x_1, \cdots, x_n :$ independent variable (獨立)変数

Domains and Ranges

Ex1:

Functions	Domain	Range
$z = \sqrt{y - x^2}$	$y \geq x^2$	$[0, \infty)$
$z = \dfrac{1}{xy}$	$xy \neq 0$	$(-\infty, 0) \cup (0, \infty)$
$z = \sin xy$	Entire plane	$[-1, 1]$

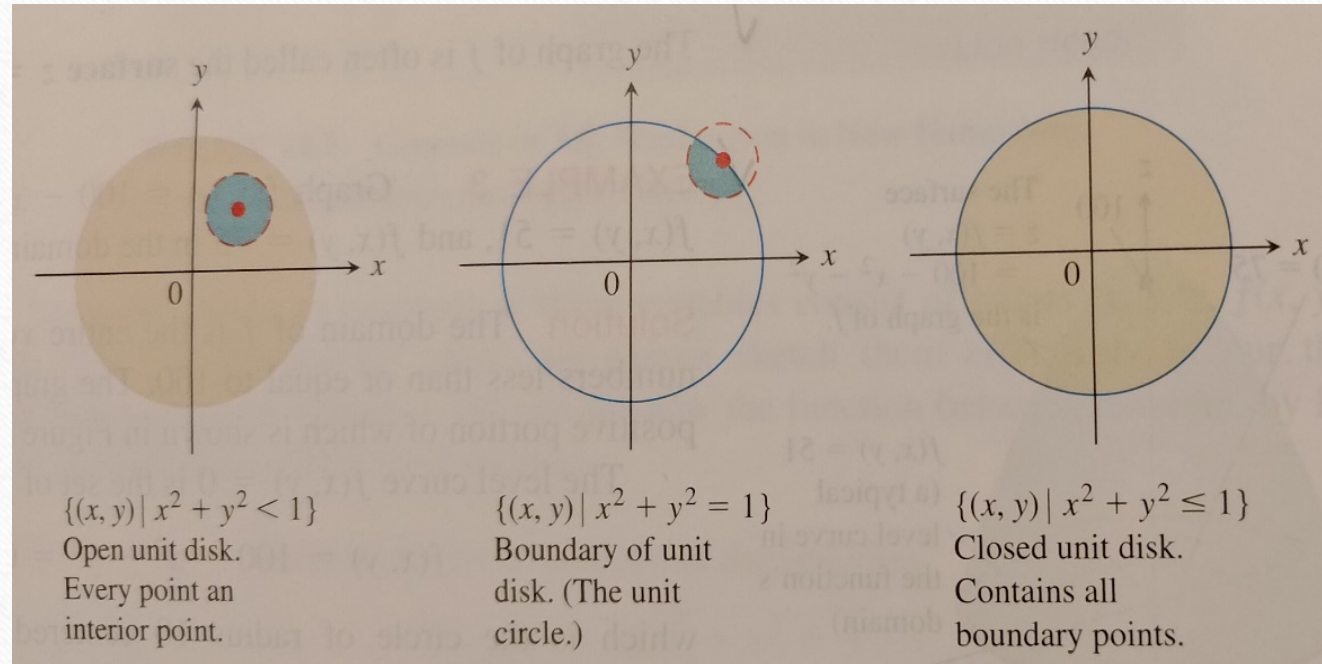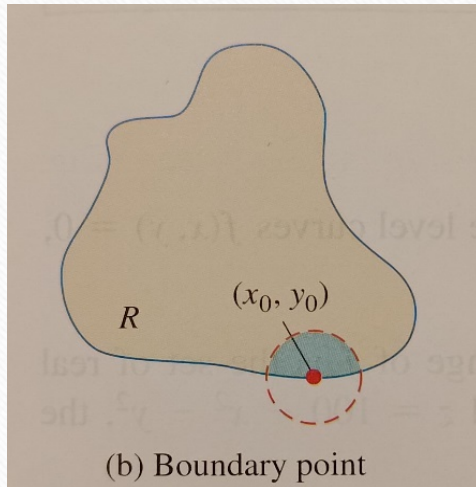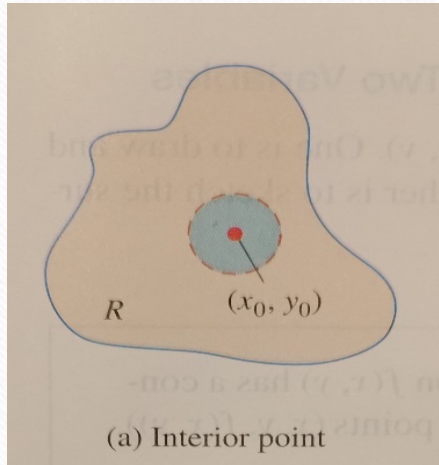| Functions	Domain	Range
$w = \sqrt{x^2 + y^2 + z^2}$	Entire space	$[0, \infty)$
$w = \dfrac{1}{x^2 + y^2 + z^2}$	$(x, y, z) \neq (0, 0, 0)$	$(0, \infty)$
$w = xy \ln z$	Half space $z > 0$	$(-\infty, \infty)$

Functions of Two Variables

Definition A point (x_0, y_0) in a region (set) R in the xy – plane is an **interior**

point of R if it is the center of a disk of positive radius that lies entirely in R.

A point (x_0, y_0) is a **boundary point** of R if every disk centered at (x_0, y_0)

contains points that lie outside of R as well as points that lies in R.

　　The interior points of a region, as a set, make up the **interior** of the

region. The boundary points make up its **boundary**. A region is **open** if it

consists entirely of interior points. A region is **closed** if it contains all its

boundary points.

(a) Interior point

(b) Boundary point

$\{(x, y) \mid x^2 + y^2 < 1\}$
Open unit disk.
Every point an
interior point.

$\{(x, y) \mid x^2 + y^2 = 1\}$
Boundary of unit
disk. (The unit
circle.)

$\{(x, y) \mid x^2 + y^2 \le 1\}$
Closed unit disk.
Contains all
boundary points.

Definition A region in the plane is **bounded** if it lies inside a disk of finite radius. A region is **unbounded** if it is not bounded.
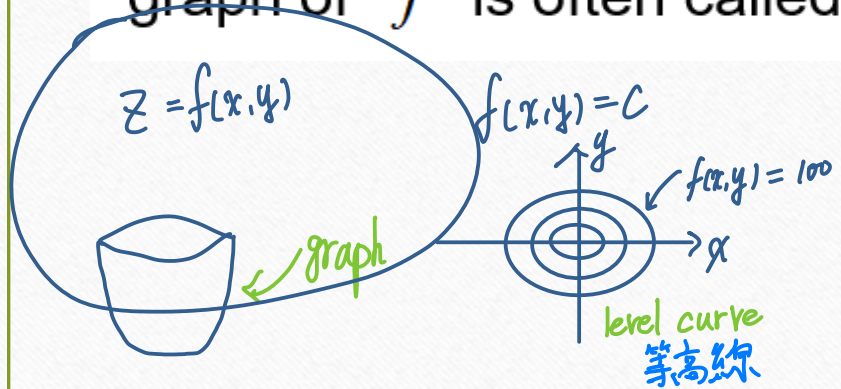
Ex2(p717) Describe the domain of the function $f(x,y)=\sqrt{y-x^2}$.
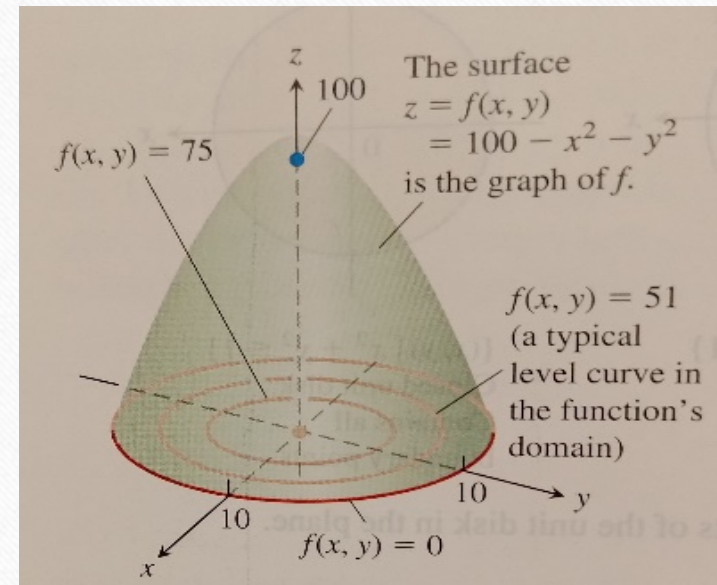
$$y-x^2 > 0 \quad , \quad y \geq x^2$$
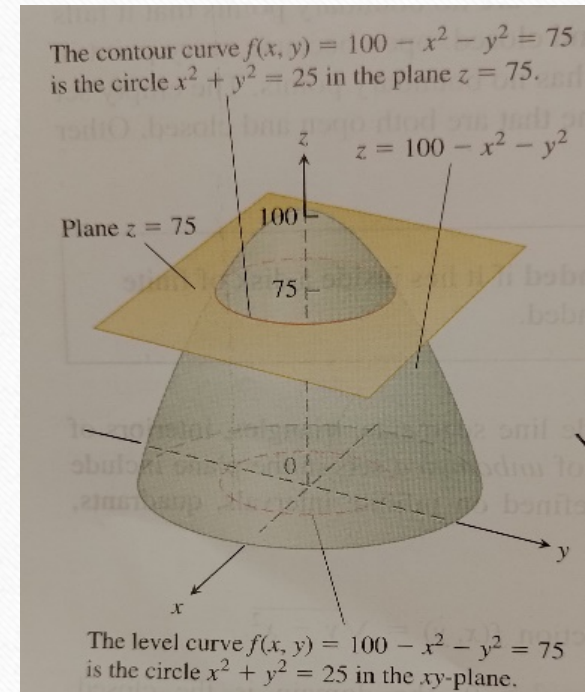
$$y = x^2$$

Domain $= D$ closed unbounded

Definition The set of points in the plane where a function $f(x,y)$ has a constant value $f(x,y)=c$ is called a **level curve**(等高線) of f. The set of all points $(x,y,f(x,y))$ in the domain of f, is called the **graph** of f. The graph of f is often called the **surface** $z=f(x,y)$.

$z=f(x,y)$ $f(x,y)=c$

$f(x,y)=100$

graph

level curve
等高線

Ex3(p718) Graph $f(x,y) = 100 - x^2 - y^2$ and plot the level curves $f(x,y) = 0$,

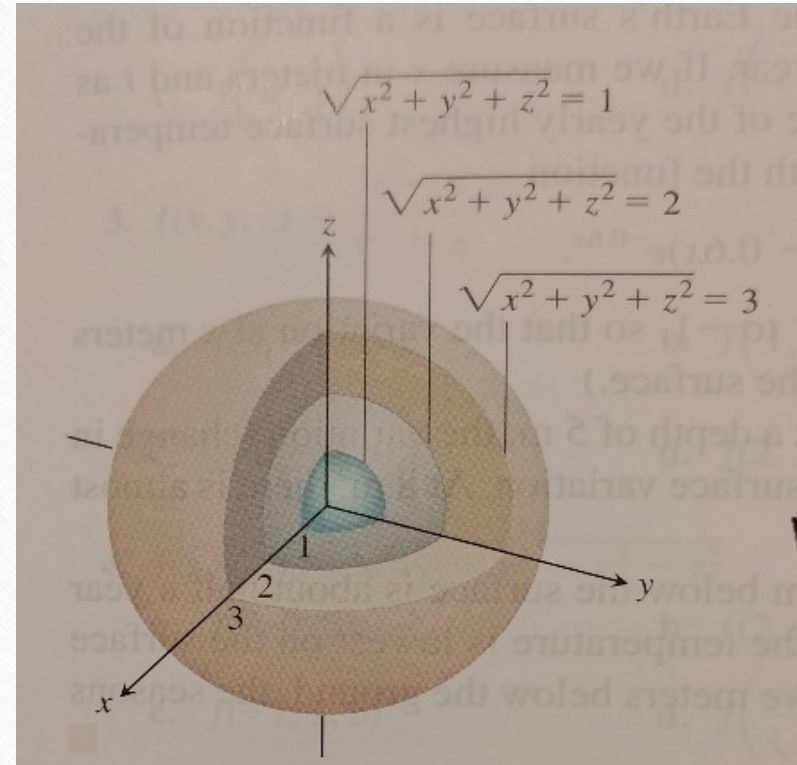$f(x,y) = 51$, $f(x,y) = 75$ in the domain of f in the plane.

The curve in space in which the plane $z = c$ cut the surface $z = f(x, y)$ is

called the contour curve(輪廓曲線, 等值曲線).

The contour curve $f(x, y) = 100 - x^2 - y^2 = 75$
is the circle $x^2 + y^2 = 25$ in the plane $z = 75$.

$z = 100 - x^2 - y^2$

Plane $z = 75$

The level curve $f(x, y) = 100 - x^2 - y^2 = 75$
is the circle $x^2 + y^2 = 25$ in the xy-plane.

Functions of Three Variables

Definition The set of points (x, y, z) in space where a function of three

independent variables has a constant value $f(x, y, z) = c$ is called a **level**
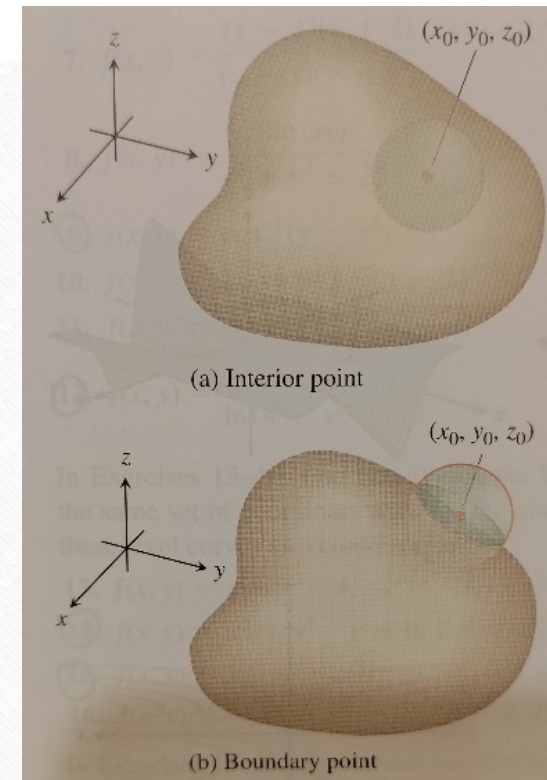
surface of f.

Ex4(p719) Describe the level surface of the function $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$.

$\sqrt{x^2 + y^2 + z^2} = 1$

$\sqrt{x^2 + y^2 + z^2} = 2$

$\sqrt{x^2 + y^2 + z^2} = 3$

Definition A point (x_0, y_0, z_0) in a region R in space is an **interior point** of R if it is the center of a solid ball that lies entirely in R. A point (x_0, y_0, z_0) is an **boundary point** of R if every solid ball centered at (x_0, y_0, z_0) contains points that lie outside of R as well as points that lie inside R. The **interior** of R is the set of interior points of R. The **boundary** of R is the set of boundary points of R.

A region is **open** if it consists entirely of interior points. A region is **closed** if it contains its entire boundary.

A region is **bounded** if it lies inside a solid ball of finite radius. A region is **unbounded** if it is not bounded.

(a) Interior point

(b) Boundary point

HW13-1

- **HW: 5,9,12,14,15,55,58.**