師大微積分乙 113-102 期中考歷屆

▶ 計算極限問題

1. (師大微乙 (一)113 暑 #1) Find the limit if it exists. Notice that L'Hôpital's Rule is forbidden.

   (a) $\lim_{x\to 0} \frac{\tan x+2x}{e^x+x}$

   (b) $\lim_{h\to 0} \frac{2h}{\sqrt{5h+4}-2}$

   (c) $\lim_{x\to 0} \frac{\sin(4x)}{\sin(5x)\cos(6x)}$

   (d) $\lim_{t\to\infty} \frac{2-t}{2t+\cos t}$

   (e) $\lim_{x\to 1} \frac{\cos\left(\pi x-\frac{\pi}{2}\right)}{x-1}$

2. (師大微乙 (一)113#1) Find the limits. (Write ∞ or $-\infty$ where appropriate). Note that L'Hopital's Rule is forbidden. 計算以下函數的極限, 必要時請寫 ∞ 或 $-\infty$. 不可使用羅必達

   (a) $\lim\limits_{x\to 4} \dfrac{x-4}{x^2-5x+4}$

   (b) $\lim\limits_{x\to 0} \dfrac{\sqrt{7}-\sqrt{3x^2+x+7}}{x}$

   (c) $\lim\limits_{x\to 0^-} \dfrac{|\sin 2x|}{5x}$

   (d) $\lim\limits_{x\to\infty} \dfrac{3x^3+5x^2+9}{5x^3+x+1}$

   (e) $\lim\limits_{x\to -1} \dfrac{x^{10}-1}{x+1}$

   (f) $\lim\limits_{x\to 5^-} \dfrac{3x}{2x-10}$

3. (師大微乙 (一)112#1) Find the limit if it exists. Note that L'Hopital's Rule is forbidden.

   (a) $\lim\limits_{x\to 0} \dfrac{x+x\sin 4x}{\sin 2x\cdot\cos 3x}$

   (b) $\lim\limits_{x\to 0} \sqrt[3]{x}\sin\left(\dfrac{1}{x}\right)$

   (c) $\lim\limits_{x\to 5} \dfrac{\frac{x}{x-4}-5}{x-5}$

   (d) $\lim\limits_{x\to\left(-\frac{4}{3}\right)} \sin\left(2\tan^{-1}(x)\right)$

4. (師大微乙 (一)111#2) Find the limit if it exists.

(a) $\lim_{x\to 0} \dfrac{\sin 3x - 3x + x^2}{\sin x \sin 2x}$

(b) $\lim_{x\to \infty} \dfrac{3x}{5x + 2\sin x}$

(c) $\lim_{x\to 0} \left(\dfrac{1}{x\sqrt{1+x}} - \dfrac{1}{x} \right)$

5. (師大微乙 (一)110#2) Find the limit if it exists. Notice that L'Hôpital's Rule is forbidden.

(a) $\lim_{x\to 2} \dfrac{\sqrt{x+7} - 3}{x - 2}$

(b) $\lim_{x\to 0} \dfrac{\tan(5x)}{\sin(3x)}$

(c) $\lim_{x\to -\infty} (x + \sqrt{x^2+3})$

(d) $\lim_{x\to \infty} \dfrac{\sqrt[3]{8x^6 + 2x^3 + 1}}{\sqrt{4x^4 + 3x + 1}}$

6. (師大微乙 (一)109#4) Find the limit (極限).

(a) $\lim_{x\to -\infty} (2x + \sqrt{4x^2+1})$

(b) $\lim_{x\to 0^+} \dfrac{\sqrt{x+1} - 1}{\sqrt{x^2+1} - \sqrt{x+1}}$

(c) $\lim_{\theta\to 0} \theta^2 \cot(7\theta) \csc(4\theta)$

(d) $\lim_{t\to 0} \ln \sqrt{(1+t)^{\frac{1}{3t}}}$

7. (師大微乙 (一)108#1) Find the limit.

(a) $\lim_{x\to 0} \dfrac{1 - e^{-x}}{e^{2x} - 1}$

(b) $\lim_{x\to 0} \dfrac{\tan^3(2x) + x^4 \sin \frac{1}{x}}{x^3}$

(c) $\lim_{x\to \infty} (2x^2 + 3x + 1) \sin \left(\dfrac{1}{5x^2 + 4x + 2} \right)$

8. (師大微乙 (一)107#1) Find the limit (極限).

(a) $\lim_{\theta\to 0^+} \operatorname{arccot} \left(\dfrac{1}{\theta} \right) \sin^2 \left(\dfrac{1}{\theta} \right)$

(b) $\lim_{x\to 1} \dfrac{\sqrt{2+x} - \sqrt{3}}{1 - x}$

(c) $\lim_{x\to \infty} \dfrac{4 - 5x^{2/3}}{2x^{4/5} + 3}$

(d) $\lim_{z\to 0} \dfrac{5 - 5e^{2z}}{1 - e^z}$

(e) $\lim_{x\to 0} 6x\cot(3x)$

9. (師大微乙 (一)106 本部 #2) 計算下列極限值。

(a) $\lim_{\theta\to 0}\theta\sin\left(\frac{1}{\theta}\right)$

(b) $\lim_{x\to 7}\frac{\sqrt{7x}-7}{7-x}$

(c) $\lim_{t\to-\infty}\frac{5-4t^3}{3t^2+2}$

10. (師大微乙 (一)105 本部 #2) Find the limit.

(a) $\lim_{x\to\infty}\frac{5-2x^{3/2}}{3x^2-4}$

(b) $\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}$

(c) $\lim_{\theta\to 0}\theta^2\cos\left(\frac{1}{\theta}\right)$

(d) $\lim_{t\to 0}\frac{1-e^{-t}}{e^t-1}$

11. (師大微乙 (一)104#1) Find the limit:

(a) $\lim_{x\to 9}\frac{9x-x^2}{3-\sqrt{x}}$

(b) $\lim_{x\to\infty}\frac{5x^4+2x-7}{2x^4-3x^3-x+6}$

(c) $\lim_{x\to\infty}e^{1/x}\cos\left(\frac{1}{x}\right)$

(d) $\lim_{x\to 0}\frac{\tan^2(3x)}{x\sin(5x)}$

(e) $\lim_{x\to 0^-}\left(x^2-\frac{1}{x}\right)$

12. (師大微乙 (一)103#1)

(a) $\lim_{x\to\infty}(\sqrt{x+12}-\sqrt{x+5}) = ?$

(b) $\lim_{x\to 0}\frac{\tan(5x)}{\sin(2x)\cos(3x)} = ?$

(c) $\lim_{x\to 0}\frac{4(x-\sin(x))}{x^3} = ?$

(d) If $\lim_{x\to 1}\frac{f(x)-1}{x-1} = 3$, find $\lim_{x\to 1}\frac{f(x)}{x-2} = ?$

13. (師大微乙 (一)102#1) 下列各極限是否存在？若存在試求其極限：

(a) $\lim_{x\to 3} \dfrac{x^3-9x}{\sqrt{x^2+7}-4}$

(b) $\lim_{x\to\infty} \dfrac{x^2}{e^{2x}}$

(c) $\lim_{x\to 0} \dfrac{x^2+\sin x}{e^{2x}}$

(d) $\lim_{x\to\infty} \dfrac{2x^3+7}{x^3+3x^2-7}$

(e) $\lim_{x\to\infty} \dfrac{\log_2 x}{\log_3(x+3)}$

(f) $\lim_{x\to 0} \dfrac{\sin 5x}{\sin 3x \cos 2x}$

14. (師大微乙 (一)102#1, Bonus) 求極限 $\lim_{x\to 0} x \sin\frac{1}{x} =?$

▶ **尋找漸進線問題**

15. (師大微乙 (一)113 暑 #5) Consider a function $f(x) = \dfrac{x^2-4}{x^4-16}$.

(a) Find the domain of the function f.

(b) Does the graph of f have vertical asymptote(s)? Given your reason!

(c) Does the graph of f have horizontal asymptote(s)? Given your reason!

16. (師大微乙 (一)113#2) Find the horizontal and vertical asymptote of the function. 求以下函數的水平漸近線和垂直漸近線

$$f(x) = \frac{1}{x^2-9} + 3$$

17. (師大微乙 (一)112#6) For the function $f(x) = \dfrac{x^3+3x^2+3x+1}{x^2-2x-3}$, find the vertical asymptotes(鉛直漸近線), oblique asymptotes(斜漸近線) for f.

18. (師大微乙 (一)111#8) Let $f(x) = \dfrac{2x^2+6}{x+1}$. Find all asymptotes(漸進線).

19. (師大微乙 (一)109#5) For $f(x) = \dfrac{2x^3-6x^2+4x}{x^2-1}$, describe the domain (定義域) of f, find the vertical asymptotes(鉛直漸近線), oblique asymptotes(斜漸近線) for f.

20. (師大微乙 (一)108#6) Let $f(x) = \dfrac{x}{(x-\sqrt{x^2+x})(x+1)}$. Find the domain(定義域) of the function f, find the vertical and horizontal asymptotes (水平及鉛直漸近線) of the graph of f.

21. (師大微乙 (一)107#4) For the real-valued function (實值函數) $f(x) = \dfrac{x^3}{x^2-1}$. Find the vertical asymptotes (垂直漸近線), slant asymptotes (斜漸近線) for f.

22. (師大微乙 (一)106 本部 #3) 對於實值函數 $f(x) = \dfrac{-x}{x^2-4}$, 找出上述函數圖形所有的水平漸近線、垂直漸近線 (或稱鉛直漸近線).

23. (師大微乙 (一)105 本部 #3) For $f(x) = \dfrac{x^2+2}{x^2-2}$. Find all horizontal asymptotes (水平漸近線), vertical asymptotes (垂直漸近線) for f.

24. (師大微乙 (一)104#2) Find the horizontal, vertical and slant asymptotes (水平、垂直和斜漸近線) of the graph of $f(x) = \dfrac{x^3-1}{2x^2-4}$.

25. (師大微乙 (一)103#4) Let $f(x) = \dfrac{2x^3}{x^2-1}$. Find horizontal asymptotes (水平漸近線), vertical asymptotes (垂直漸近線) and slant line asymptotes (斜漸近線) of $f(x)$.

► $\varepsilon - \delta$ **定義加分題**

26. (師大微乙 (一)112#8, Bonus) Use the $\varepsilon-\delta$ definition to show that $\lim\limits_{x\to 2}(7-3x) = 1$.

27. (師大微乙 (一)110#9, Bonus)

    (a) State the definition of limit $\lim\limits_{x\to x_0} f(x) = L$.

    (b) Prove that $\lim\limits_{x\to 2}(3x+1) = 7$ by the definition of limit.

28. (師大微乙 (一)106#9, Bonus) 請利用極限的 $\varepsilon - \delta$ 定義證明

$$\lim_{x\to -1}(8-3x) = 11$$

29. (師大微乙 (一)104#6, Bonus) choose only one question from the following questions.

    (a) Show that $\lim\limits_{x\to 3}(8-x) = 5$, using the precise definition, $\varepsilon - \delta$, of the limit.

    (b) Show that $e = \lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}$

30. (師大微乙 (一)102#5, Bonus) 寫出極限 $\lim\limits_{x\to 2} f(x) = 5$ 的 $\varepsilon - \delta$ 的定義.

► **判斷連續或可微與計算導函數問題**

31. (師大微乙 (一)113 暑 #2)

(a) Given $y = \frac{1}{\cot x} + \left(\frac{1+3x}{3x}\right)(3-x) + \sqrt{x^7} + \ln 2$. Find the derivative $\frac{dy}{dx}$.

(b) Given $y = \pi^{\sqrt{x}} + x^{\sqrt{\pi}} + \left(\frac{\sin x}{1+\cos x}\right)^2$. Find the derivative $\frac{dy}{dx}$.

(c) Given $y = \frac{(2-x)^{\frac{3}{2}}}{(1-x)^2\sqrt{1-x^2}}$. Find the value of $\frac{dy}{dx}$ at $x = 0$.

(d) Given $f(x) = x^2 + 4x - 5$, $x < -2$. Find the value of $(f^{-1})'(0)$.

(e) Given $f(x) = \frac{(x-1)(x-2)(x-3)(x-4)}{(x-5)(x-6)(x-7)}$. Find the value of $f'(2)$.

32. (師大微乙 (一)113 暑 #4) Let $f(x) = \begin{cases} x^2 - x, & x \le -1 \\ -2x^2 - 7x + a, & x > -1 \end{cases}$ be a continuous function on $\mathbb{R}$.

(a) Find the value of a.

(b) Use the definition of the derivative to determine whether (是否) f is differentiable at $x = -1$.

33. (師大微乙 (一)113 暑 #7) Suppose f is a function of x that satisfies the following two conditions

$$f(a+b) = f(a) + f(b) + a^2b + ab^2 \quad \forall a, b \in \mathbb{R} \quad \text{and} \quad \lim_{x\to 0} \frac{f(x)}{x} = 2025$$

(a) Find the value of $f(0)$.

(b) Find the value of $f'(0)$.

(c) Find $f'(x)$.

34. (師大微乙 (一)113#3) Find the derivatives of the following functions dy/dx. 計算以下函數的微分.

(a) $y = \dfrac{e^{2x}}{4x-3}$

(b) $y = \sqrt{2x^3 - 5x}$

(c) $y = (\sec x)^3$

(d) $y = \arctan|x|, \quad x \neq 0$

(e) $y = \sqrt[5]{\dfrac{x(x-1)}{(x-2)(x-3)}}, \quad x > 3$

(f) $y = \log_5 x^2$

35. (師大微乙 (一)113#4) Use implicit differentiation to find dy/dx. 使用隱函數微分計算 dy/dx.

$$x^3 + y^3 = \sin(xy)$$

36. (師大微乙 (一)112#2) Find the derivatives $\frac{\mathrm{d}y}{\mathrm{d}x}$ ：

(a) $y = \left(3x^2 - 2x + 1\right) e^{x^2}$

(b) $y = \sin\left(\cos\left(2x + 3\right)\right)$

(c) $y = \dfrac{\sqrt{(x^2+1)(x-1)}}{\sqrt[3]{x^2} \cdot \sqrt[4]{(x+1)^3} \cdot \sqrt[5]{(x+2)^5}}$ (Hint: Use logarithmic differentiation.)

37. (師大微乙 (一)112#7) Let $f(x) = x\cos x$, find $f^{(112)}(x)$.

38. (師大微乙 (一)111#3) Find the derivatives $\frac{\mathrm{d}y}{\mathrm{d}x}$：

(a) $y = \sec^{-1}\left(\sqrt{x^2+4}\right) + \cot\left(3x\right)$

(b) $y = \ln\left(e^{2x} + e^{\sin 3x}\right)$

(c) $y = (\sin x)^{\sqrt{x}}, \quad x > 0$

39. (師大微乙 (一)111#9) Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$

(a) Use the definition of continuity to show that f is continuous at $x = 0$.

(b) Is f differentiable at $x = 0$? Explain it.

40. (師大微乙 (一)110#3)

(a) Write the definition of the derivative $f'(x_0)$ of a function f at a point x_0.

(b) Show that $f(x) = |x|$ is continuous but not differentiable at $x_0 = 0$.

41. (師大微乙 (一)110#4) Finding the derivative $\dfrac{dy}{dx}$. (15 pts)

(a) $y = (\sin x)^{\cos x}$

(b) $y = \ln\left(\dfrac{e^x - 1}{e^x + 1}\right)$

(c) $y = \arctan\dfrac{x}{2} - \dfrac{1}{2(x^2+4)}$

42. (師大微乙 (一)109#2) Let $f(x) = x\cos(1/x)$ for $x \neq 0$.

(a) Show that f has a removable discontinuity (可移除不連續點) at $x = 0$.

(b) Is f differentiable (可微分) at $x = 0$ when $f(0)$ is defined as in the part (a)? Give your reasons.

43. (師大微乙 (一)108#2) Find the derivatives $\frac{\mathrm{d}y}{\mathrm{d}x}$：

(a) $y = \ln\left(2x + \sqrt{x+1}\right), \quad x > 0$

(b) $y = \cos\left(\arctan e^{3x}\right)$

(c) $y = x^{2/x}, \quad x > 0$

44. (師大微乙 (一)107#5) Consider the real-valued function f defined by:

$$f(x)=\begin{cases} x\cdot \operatorname{arcsec}\left(\frac{1}{x}\right) & \text{if } -1\le x<0 \text{ or } 0<x\le 1,\\ 0 & \text{if } x=0.\end{cases}$$

(a) Is f continuous (連續) on the closed interval $[-1,1]$? Give your reasons.

(b) Find the first derivative (一階導函數) of f for $-1<x<0$ or $0<x<1$.

(c) Is f differentiable (可微分) at $x=0$? Give your reasons.

45. (師大微乙 (一)106 本部 #4) 假設一實值函數 h 定義為

$$h(x)=\begin{cases} x\arctan\left(\frac{1}{x}\right), & x\neq 0,\\ 0, & x=0.\end{cases}$$

(a) 函數 h 是否在 $x=0$ 處可微分？若是，請求出 $h'(0)$.

(b) 計算函數 h 在 $x\neq 0$ 處的一階導數.

46. (師大微乙 (一)106 本部 #5) 對於函數 $f(x)=4-\dfrac{3}{x}$, 試求一實數 $c\in(1,3)$ 滿足

$$f'(c)=\frac{f(3)-f(1)}{3-1}.$$

47. (師大微乙 (一)106 本部 #8) 試求一實數 k, 使得 $x=0$ 為下列函數 g 的一個可移除不連續點.

$$g(x)=\begin{cases} \dfrac{\tan(kx)}{6x}, & x>0,\\ \dfrac{2\sin x+k-e^{5x}}{k^2x+5}, & x<0\end{cases}$$

48. (師大微乙 (一)105 本部 #7) Consider a real-valued function (實值函數) defined by:

$$g(x)=\begin{cases} x\sin\left(\frac{1}{x}\right), & x\neq 0,\\ 0, & x=0.\end{cases}$$

(a) Is g differentiable (可微分) at $x=0$? If yes, find $g'(0)$.

(b) Find the derivative of g for all $x\neq 0$.

49. (師大微乙 (一)105#4) Find the constant a such that the function

$$f(x)=\begin{cases} \dfrac{\sin 2x}{3x}, & x<0,\\ \dfrac{1}{6}a-7x, & x\ge 0\end{cases}$$

is everywhere continuous (處處連續).

50. (師大微乙 (一)104#3) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

(a) $y = 2^x\left(1 - \dfrac{4}{x+3}\right)$

(b) $y = \left(\ln\dfrac{3}{x}\right)^{x/3}$

(c) $y = e^{3x}\sin\left(\cos^2(3x)\right)$

(d) $y = \sqrt{2+\sqrt{2+\sqrt{x}}}$

(e) $\ln(xy) - e^{x+y} + x^2y = 1$

51. (師大微乙 (一)103#2) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

(a) $y = \left(\dfrac{\cos x}{1+\sin x}\right)^2$

(b) $x\cdot\sin(5y) = y\cdot\cos(5x) + \dfrac{1}{2}$

(c) $y = \ln\left(|\tan^{-1}(x^3)|\right)$

(d) $y = \sqrt[3]{\dfrac{x^5(x+1)^{16}}{(x+2)}}$

52. (師大微乙 (一)102#2) 試求下列各函數的導函數：

(a) $y = \dfrac{\sin x - x}{x^3}$

(b) $y = e^{2x}x^5\sin x$

(c) $y = \cos(x^{\sqrt{3}})$

(d) $y = (\cos x)^{\sqrt{3}}$

(e) $y = (\sqrt{3})^{\sin x} + \sqrt{3}\tan x + \sec(\sqrt{3x})$

(f) $y = \dfrac{(x+2)^5(e^x-1)^2}{x^3(2x-1)^3}$

▶ 反函數與其微分問題

53. (師大微乙 (一)112#4) Given a function $f(x) = x^3 - 3x + 2$, $x > 1$.

(a) Show that f is a one-to-one function on the interval $(1,\infty)$.

(b) Let f^{-1} denote the inverse function of f, find $f^{-1}(4)$.

(c) Find the derivative $(f^{-1})'(4)$.

54. (師大微乙 (一)111#1) Evaluate $\sin(2\sin^{-1}(-\frac{3}{5}))$.

55. (師大微乙 (一)111#5) Let $f(x) = x - \pi + \cos x$.

(a) Show that f has an inverse on $(-2\pi, 2\pi)$.

(b) Find $(f^{-1})'(-1)$.

56. (師大微乙 (一)110#1) Given the function $f(x) = x^4 + 4x^2 + 5$, $x > 0$.

(a) Show that f is one to one.

(b) Find the inverse function f^{-1}.

(c) Find the derivative $(f^{-1})'(10)$.

57. (師大微乙 (一)108#4) Given a function $f(x) = x^3 - \frac{4}{x}$, $x > 0$.

(a) Show that $f(x)$ is a one-to-one function.

(b) Let f^{-1} denote the inverse function of f. Find the derivative $(f^{-1})'(6)$.

▶ 切線問題

58. (師大微乙 (一)113 暑 #3) Find an equation of the tangent line to the graph of

$$3\tan^{-1}(2x) + \sin^{-1} y = \frac{11}{12}\pi$$

at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

59. (師大微乙 (一)112#3) Find an equation of tangent line to the graph of $\ln(xy) = \sin(x - y)$ at the point $(1, 1)$.

60. (師大微乙 (一)111#4) Given $x\tan^{-1}(x^2) = e^y$. Find the tangent line at point $(1, \ln\frac{\pi}{4})$.

61. (師大微乙 (一)110#5) Given $x^3 + ye^{x+y} + y^3 = 2$. Find the tangent line at point $P = (0, 1)$.

62. (師大微乙 (一)109#6) Find an equation of the tangent line (切線) to the graph of

$$3e^{x+y} + \ln(x/y^2) - \arcsin(x + y) = 3$$

at the point $(1, -1)$.

63. (師大微乙 (一)108#3) Find the tangent line to the graph of the equation at the given point

$$(x^2 + y^2)^2 = 4x^2y, \quad (1, 1).$$

64. (師大微乙 (一)107#6) Use the Logarithmic Differentiation (對數微分法) to find an equation of the tangent line (切線方程式) to the graph of $y = \dfrac{(x+2)^2}{\sqrt{x^2+1}}$ at the point $(0, 4)$.

65. (師大微乙 (一)106#6) 試求 $x^y = y^x$ 在點 $(1, 1)$ 處的切線方程式.

66. (師大微乙 (一)105 本部 #6) Find an equation of the tangent line (切線) to the graph of
$$x^2 + xy + y^2 = 4$$
at the point $(2, 0)$.

67. (師大微乙 (一)102#3) 試求曲線 $y = \sin\left(\frac{\pi y}{x}\right)$ 在點 $P(2, 1)$ 的切線方程式。

▶ **極值與凹凸性分析問題**

68. (師大微乙 (一)113 暑 #6) Find the absolute maximum and minimum of $f(x) = 10x(2 - \ln x)$ on $[1, e^2]$.

69. (師大微乙 (一)113#6) Find the open intervals on which the function f is increasing and those on which is decreasing. 求以下函數遞增的區間和遞減的區間
$$f(x) = (x^2 - 2)e^{2x}.$$

70. (師大微乙 (一)112#6) For the function $f(x) = \dfrac{x^3 + 3x^2 + 3x + 1}{x^2 - 2x - 3}$, answer the following questions.
    (a) Determine the open intervals on which f is increasing(遞增) or decreasing(遞減).
    (b) Find the relative extrema(相對極值) for f.
    (c) Determine the open intervals on which the graph of f is concave upward(凹向上) or concave downward(凹向下).

71. (師大微乙 (一)111#7) Find all critical points and all local extrema (relative extrema) of the function
$$f(x) = x^{1/3}(x^2 - 7),$$
verify your answer by the first or the second derivative test.

72. (師大微乙 (一)111#8) Let $f(x) = \dfrac{2x^2 + 6}{x + 1}$.
    (a) Determine the open intervals on which f is increasing(遞增) or decreasing(遞減).
    (b) Determine the open intervals on which f is concave upward(凹向上) or concave downward(凹向下).

73. (師大微乙 (一)110#8) Given a function $f(x) = x^4 - 2x^2$ on the interval $[-2, 3]$.
    (a) Find open intervals where f is increasing or decreasing.
    (b) Find the relative extrema of f.

(c) Determine open intervals where f is concave upward or downward.

74. (師大微乙 (一)109#3) Let $g(x) = 5e^x - e^{2x} + 1$ for all $x \in \mathbb{R}$. Use the **Second Derivative Test** to find the relative extrema (相對極值) of g.

75. (師大微乙 (一)109#5) For $f(x) = \dfrac{2x^3 - 6x^2 + 4x}{x^2 - 1}$, answer the following questions.

(a) Find the critical numbers (臨界數) of f.

(b) Determine the open intervals on which f is increasing (遞增) or decreasing (遞減).

(c) Determine the open intervals on which the graph of f is concave upward (凹向上) or concave downward (凹向下).

76. (師大微乙 (一)108#5) Consider the function

$$f(x) = x^{2/3}(x^2 - 1).$$

(a) Find the open intervals on which f is increasing (遞增) or decreasing (遞減).

(b) Find the open intervals on which the graph of f is concave upward (凹向上) or concave downward (凹向下).

(c) Locate the point of inflection (反曲點) if it exists.

77. (師大微乙 (一)107#4) For the real-valued function (實值函數) $f(x) = \dfrac{x^3}{x^2 - 1}$, answer the following questions.

(a) Determine the open intervals on which f is increasing (遞增) or decreasing (遞減).

(b) Find the relative extrema (相對極值) for f.

(c) Determine the open intervals on which the graph of f is concave upward (凹向上) or concave downward (凹向下).

(d) Find the points of inflection (反曲點) of f.

78. (師大微乙 (一)106#3) 對於實值函數 $f(x) = \dfrac{-x}{x^2 - 4}$, 試回答下列問題.

(a) f 的函數圖形在那些開區間上是遞增或遞減?

(b) f 的函數圖形在那些開區間上是凹口向上或凹口向下?

(c) 找出函數 f 所有的反曲點.

79. (師大微乙 (一)106#7) 找出函數 $f(x) = x\ln(x + 3)$ 在閉區間 $[0, 3]$ 上的絕對極值.

80. (師大微乙 (一)105#1) Find the absolute extrema (絕對極值) of $f(x) = 5e^x - e^{2x}$ on the closed interval $[-1, 2]$.

81. (師大微乙 (一)105#3) For $f(x) = \dfrac{x^2+2}{x^2-2}$, answer the following questions.

    (a) Determine the open intervals on which f is increasing (遞增) or decreasing (遞減).

    (b) Find all relative extrema (相對極值) for f.

    (c) Determine the open intervals on which the graph of f is concave upward (凹向上) or concave downward (凹向下).

    (d) Find all points of inflection (反曲點) of the graph of f.

82. (師大微乙 (一)104#4) $f(x) = 2x + \dfrac{1}{x}$

    (a) (5 points) Find all the critical points (臨界點) of $f(x)$.

    (b) (10 points) Find the interval(s) on which $f(x)$ is increasing or decreasing.

    (c) (5 points) Find the extreme values.

83. (師大微乙 (一)104#5) $f(x) = x^3 - 9x^2$

    (a) (5 points) Find all the points of inflection (反曲點) of $f(x)$.

    (b) (10 points) Find the interval(s) on which the curve $f(x)$ is concave up or down.

    (c) (5 points) Find the extreme values.

84. (師大微乙 (一)103#4) Let $f(x) = \dfrac{2x^3}{x^2-1}$. Find all critical points (臨界點) of $f(x)$.

85. (師大微乙 (一)103#5) Let $g(x) = x^3 + 2x^2 + x - 4$ on $[-1, 1]$.

    (a) (10 points) find local extreme values (區域極值) of $g(x)$.

    (b) (10 points) find the intervals $g(x)$ is concave up (凹向上) or concave down (凹向下).

    (c) (10 points) find absolute extreme values (絕對極值) of $g(x)$ on $[-1, 1]$.

86. (師大微乙 (一)102#4) 設函數 $f(x) = \frac{x^2}{x^2-4}$.

    (a) 找出函數 $f(x)$ 在哪些區間內是遞增的？

    (b) 找出函數 $f(x)$ 在哪些區間內是凹口向上的？

    (c) 找出函數 $f(x)$ 的水平和鉛直漸近線？

▶ **均值定理問題**

87. (師大微乙 (一)113 暑 #8)

    (a) Show that $|\sin x| \le |x|$ for all $x \in \mathbb{R}$. ($\star$ Hint: 分 $x = 0$ 和 $x \neq 0$ 的情況討論; 其中 $x = 0$ 的證明很簡單; $x \neq 0$ 的證明可使用均值定理)

    (b) Use the fact of (a) to find the limit $\lim_{x\to 0} \dfrac{\sin\left(x^2 \sin\frac{1}{x}\right)}{x}$.

88. (師大微乙 (一)113#5) Show that the equation $x^5 + 4x + 1 = 0$ has exactly one solution. 請證明方程式 $x^5 + 4x + 1 = 0$ 只有一個實數解.

89. (師大微乙 (一)112#5) Show that the function $g(t) = \sqrt{t + \sqrt{t+2}} - 4$ has exactly one zero in the interval $(0, \infty)$.

90. (師大微乙 (一)111#6) Prove that the equation $x + e^{x^3} = 0$ has exactly one real root.

91. (師大微乙 (一)110#7) Prove that the equation $2x - 2 - \cos x = 0$ has exactly one real solution.

92. (師大微乙 (一)109#3) Let $g(x) = 5e^x - e^{2x} + 1$ for all $x \in \mathbb{R}$. Prove that the nonlinear equation $g(x) = 0$ has *exactly one* root (零根) in the closed interval $[0, \ln 10]$.

93. (師大微乙 (一)107#3) Does the nonlinear equation (非線性方程式) $e^{-3x} - x = 0$ have a unique solution (唯一解) in the interval $[0, 1]$? Give your reasons.

94. (師大微乙 (一)105#5) Use **Intermediate Value Theorem** and **Rolle's Theorem** to prove that the equation $2x^5 + 7x - 1 = 0$ has *exactly one* real solution.

95. (師大微乙 (一)103#3) Show that the equation $\cos(x) + 1 = 2x$ has exactly one real solution.

96. (師大微乙 (一)102#5, Bonus) 敘述中間值定理 (Intermediate Values Theorem)。

► 微分量問題

97. (師大微乙 (一)110#6) Use differential to approximate $\sqrt[4]{626}$.

98. (師大微乙 (一)109#1) Use the differential to approximate (近似) the value of $\sqrt[3]{63.952}$.

99. (師大微乙 (一)107#2) Let $y = g(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ for $x > 0$.

    (a) Find the differential dy.

    (b) Use the part (a) to find the approximate value (近似值) of $g(4.16)$.

100. (師大微乙 (一)106#1) 試用 differential 估計 $\sqrt[3]{8.12}$ 的近似值.

101. (師大微乙 (一)105#8) Find the differential dy of the given function.

(a) $y = \ln\sqrt{4-x^2}$

(b) $y = \arctan(x-2)$