單調的

4-3 Monotonic Functions and the First Derivative Test

一階導數判別法

師大工教一

Increasing Functions and Decreasing Functions

Corollery 3 Suppose that f is continuous on $[a,b]$ and differentiable on

$f'(x)>0$, on (a,b)

$\Rightarrow f$ is increasing (递增, $\nearrow$) on $[a,b]$

(a,b).

$f'(x)<0$, on (a,b)

$\Rightarrow f$ is decreasing (递减, $\searrow$) on $[a,b]$

If $f'(x)>0$ at each point $x\in(a,b)$, then f is increasing on $[a,b]$.

If $f'(x)<0$ at each point $x\in(a,b)$, then f is decreasing on $[a,b]$.

Note: A function that is either increasing on a ninterval or decreasing on an interval is said to be **monotonic** on the interval.

Ex1(p247) Find the critical points of $f(x)=x^3-12x-5$ and identify the open intervals on which f is increasing and those on which f is decreasing.

$f'(x)=3x^2-12 \overset{let}{=} 0$

$x^2=4$

$x=\pm 2$, (critical points)

x	-2	2
$f'(x)$	0	0

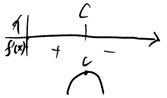
increasing ($\nearrow$) : $(-\infty,-2)$, $(2,\infty)$

decreasing ($\searrow$) : $(-2,2)$

First Derivative Test for Local Extrema

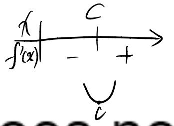
Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right.

1. if f' change from positive to negative at c, then f has a local maximum at c.



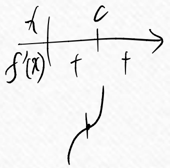
∴ f(c) is a local maximum value.

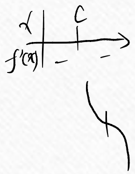
2. if f' change from negative to positive at c, then f has a local minimum at c.



∴ f(c) is a local minimum value.

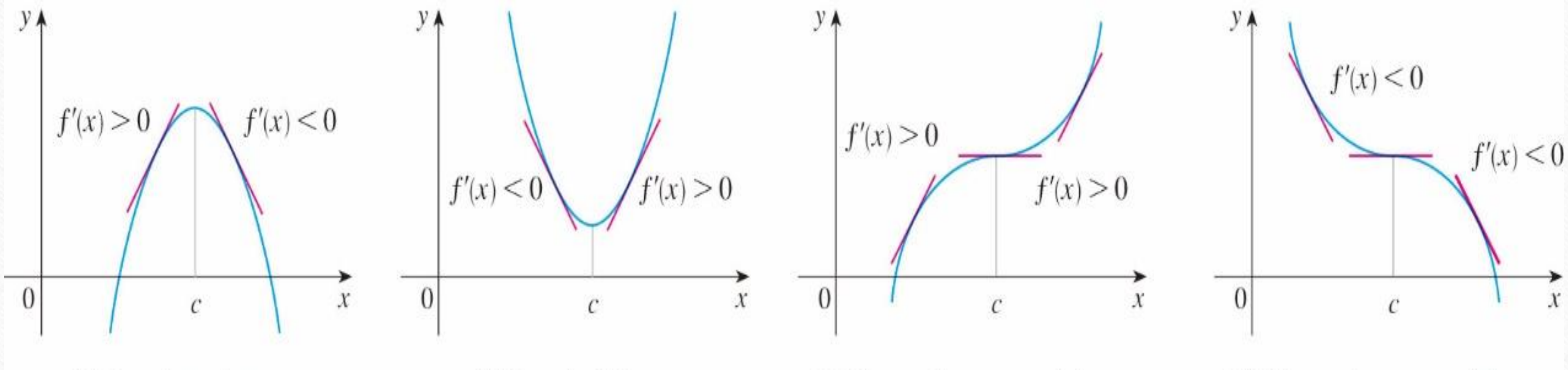
3. if f' does not change sign at c (that is, f' is positive on both sides of c or negative on both sides), then f has no local extremum at c.





✗ local max.

✗ local min.



(a) Local maximum (b) Local minimum (c) No maximum or minimum (d) No maximum or minimum

Ex3 Find the critical points of $f(x)=\left(x^2-3\right)e^x$. Identify the open intervals on which f is increasing and those on which it is decreasing. Find the function's local and absolute extreme values.

$$f'(x) = 2xe^x + (x^2-3)e^x$$
$$= e^x(x^2+2x-3)$$
$$= e^x(x+3)(x-1) \overset{\text{let}}{=} 0$$

$x=-3, 1$ (critical point)

x		-3		1	
$f'(x)$	$+$	0	$-$	0	$+$

$\nearrow$: $(-\infty,-3)$, $(1,\infty)$

$\searrow$: $(-3,1)$

$f(-3)=6\cdot e^{-3}$ is a local maximum value.

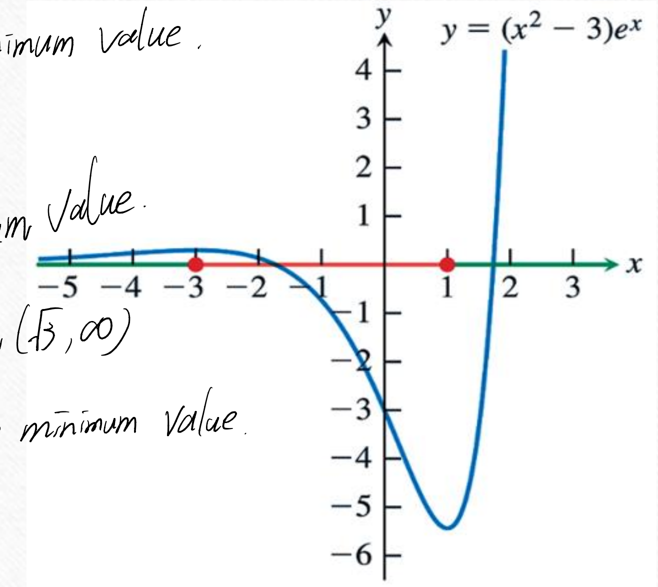
$f(1)=-2e$ is a local minimum value.

$\lim_{x\to\infty}(x^2-3)e^x=\infty$

No Absolute Maximum value.

$\because f(x)>0$ when $x\in(-\infty,-\sqrt{3}),(\sqrt{3},\infty)$

$\therefore f(1)=-2e$ is the Absolute minimum value.



Ex(107 考古題#4(a)) For the real-valued function(實值函數) $f(x)=\dfrac{x^3}{x^2-1}$, …

(a) Determine the open intervals on which f is increasing(遞增) or decreasing(遞減).

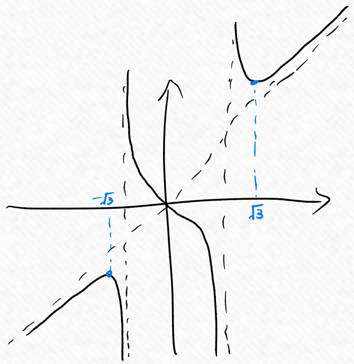
$$f'(x)=\frac{3x^2(x^2-1)-x^3(2x)}{(x^2-1)^2}=\frac{3x^4-3x^2-2x^4}{(x^2-1)^2}=\frac{x^4-3x^2}{(x^2-1)^2}$$

$$=\frac{x^2(x^2-3)}{(x^2-1)^2}=\frac{x^2(x+\sqrt{3})(x-\sqrt{3})}{(x^2-1)^2}$$

x		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
$f'(x)$	$+$		$-$		$-$		$-$		$-$		$+$

$\nearrow$: $(-\infty,-\sqrt{3})$, $(\sqrt{3},\infty)$

$\searrow$: $(-\sqrt{3},-1)$, $(-1,0)$, $(0,1)$, $(1,\sqrt{3})$



HW4-3

- **HW: 21,26,43,53,77,80**