3-9 Inverse Trigonometric Functions

師大工教一

Definition:

1. $y=\sin^{-1}x$ is the number in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ for which $\sin y=x$.

2. $y=\cos^{-1}x$ is the number in $[0,\pi]$ for which $\cos y=x$.

3. $y=\tan^{-1}x$ is the number in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ for which $\tan y=x$.

4. $y=\cot^{-1}x$ is the number in $(0,\pi)$ for which $\cot y=x$.

5. $y=\sec^{-1}x$ is the number in $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$ for which $\sec y=x$.

6. $y=\csc^{-1}x$ is the number in $\left[\frac{-\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$ for which $\csc y=x$.

The Derivative of $y = \sin^{-1} x$

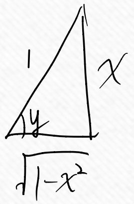
Derive $\left(\sin^{-1} x\right)' = \dfrac{1}{\sqrt{1-x^2}}$.

$$y = \sin^{-1} x$$

$$\sin y = \sin\left(\sin^{-1} x\right) = x$$

$$\cos y \cdot y' = 1$$

$$y' = \frac{1}{\cos y}$$



$$y' = \frac{1}{\sqrt{1-x^2}}$$

Chain Rule: $\left(\sin^{-1}\left(f(x)\right)\right)' = \dfrac{1}{\sqrt{1-\left(f(x)\right)^2}} \cdot f'(x)$.

Ex2:Find $\dfrac{d}{dx}\left(\sin^{-1} x^2\right)$.

$$= \frac{1}{\sqrt{1-x^4}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

The Derivative of $y = \tan^{-1} x$

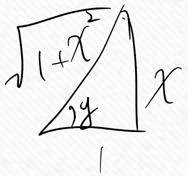
Derive $\left(\tan^{-1} x\right)' = \dfrac{1}{1+x^2}$.

$y = \tan^{-1} x$

$\tan y = x$

$\sec^2 y \cdot y' = 1$

$y' = \dfrac{1}{\sec^2 y}$



$y' = \dfrac{1}{1+x^2}$

Chain Rule: $\left(\tan^{-1}\left(f\left(x\right)\right)\right)' = \dfrac{1}{1+\left(f\left(x\right)\right)^2} \cdot f'\left(x\right)$.

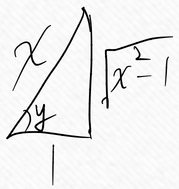
The Derivative of $y = \sec^{-1} x$

Derive $\left(\sec^{-1} x\right)' = \dfrac{1}{|x|\sqrt{x^2-1}}$.

$\sec y = x = \dfrac{1}{\cos y}$

$\sec y \cdot \tan y \cdot y' = 1$

$$y' = \frac{1}{\sec y \cdot \tan y} = \begin{cases} \dfrac{1}{x\sqrt{x^2-1}}, & x > 0 \\[2ex] \dfrac{1}{-x\sqrt{x^2-1}}, & x < 0 \end{cases} = \frac{1}{|x|\sqrt{x^2-1}}$$



$\boxed{\sec^{-1} x \ , \ [0, \tfrac{\pi}{2}), (\tfrac{\pi}{2}, \pi]}$

inverse function 要有 one to one
∴只討論 $x>0$ 的情況

Chain Rule: $\left(\sec^{-1}\left(f\left(x\right)\right)\right)' = \dfrac{1}{\left|f\left(x\right)\right|\sqrt{f\left(x\right)^2-1}} \cdot f'\left(x\right)$.

Ex3: Find $\dfrac{d}{dx}\left(\sec^{-1}\left(5x^4\right)\right)$.

$$= \frac{1}{|5x^4|\sqrt{25x^8-1}} \cdot 20x^3 = \frac{20x^3}{|5x^4|\sqrt{25x^8-1}}$$

Inverse Function-Inverse Cofunction Identities

1. $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$

2. $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$

3. $\csc^{-1} x = \frac{\pi}{2} - \sec^{-1} x$

Chain Rule:

1. $\left(\cos^{-1}\left(f(x)\right)\right)' = \dfrac{-1}{\sqrt{1-\left(f(x)\right)^2}} \cdot f'(x)$

2. $\left(\cot^{-1}\left(f(x)\right)\right)' = \dfrac{-1}{1+\left(f(x)\right)^2} \cdot f'(x)$

3. $\left(\csc^{-1}\left(f(x)\right)\right)' = \dfrac{-1}{\left|f(x)\right|\sqrt{f(x)^2-1}} \cdot f'(x)$

HW3-9

- **HW: 10,13,15,21,31,33**

105分 3. (14 points) Find the derivatives $\frac{dy}{dx}$

(a) $y = e^{(\ln|5x| + (\cos x)^2)^2}, \quad x \neq 0,$

V (b) $y = \arctan(\sqrt{1 - x^2}), \quad |x| < 1.$

(b) $y' = \frac{1}{1+(1-x^2)} \cdot \left[(1-x^2)^{\frac{1}{2}}\right]' = \frac{1}{2-x^2}\left(\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x)\right) = \frac{-1}{2-x^2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot x = \frac{-x}{(2-x^2)(\sqrt{1-x^2})}$

108 2. (24 points) Find the derivatives $\frac{dy}{dx}$

(a) $y = \ln(2x + \sqrt{x} + 1), \; x > 0,$

V (b) $y = \cos(\arctan e^{3x}), \qquad$ (note : $\arctan x = \tan^{-1} x$)

V (c) $y = x^{2/x}, \; x > 0.$

(b) $y' = -\sin(\arctan e^{3x}) \cdot \frac{1}{1+e^{6x}} \cdot (e^{3x})'$

$= -\sin(\arctan e^{3x}) \cdot \frac{3e^{3x}}{1+e^{6x}} = \frac{-3e^{3x}\sin(\arctan e^{3x})}{1+e^{6x}}$

(c) $y' = (x^{\frac{2}{x}})'$

$\ln y = \frac{2}{x}\ln x$

$\frac{y'}{y} = \frac{2 \cdot \frac{1}{x} \cdot x - 2\ln x \cdot 1}{x^2}$

$y' = x^{\frac{2}{x}}\left[\frac{2-2\ln x}{x^2}\right]$

Differentials

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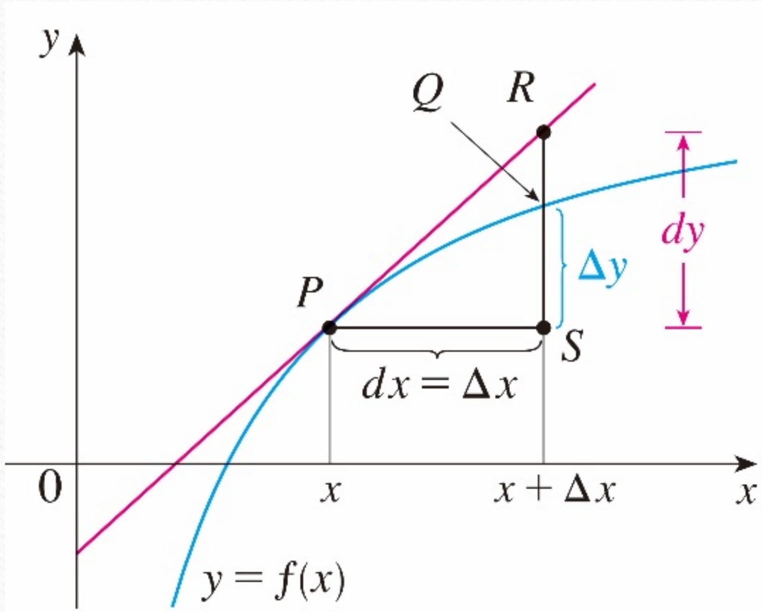
Definition: Let $y = f(x)$ be a differentiable function. The differential dx is an independent variable. The differential dy is $dy = f'(x)dx$.



Ex4(p214)

(a) Find dy if $y = x^5 + 37x$.

(b) Find the value of dy when $x = 1$ and $dx = 0.2$.

(a) $dy = f'(x)\,dx$

$= (5x^4 - 37)\,dx$

(b) $dy\Big|_{\substack{x=1,\\ dx=0.2}} = (5-37)\times 0.2 = -32\times 0.2 = -6.4$ #