2-3 The Precise Definition of a Limit

師大工教一

2-3 The Precise Definition of a Limit

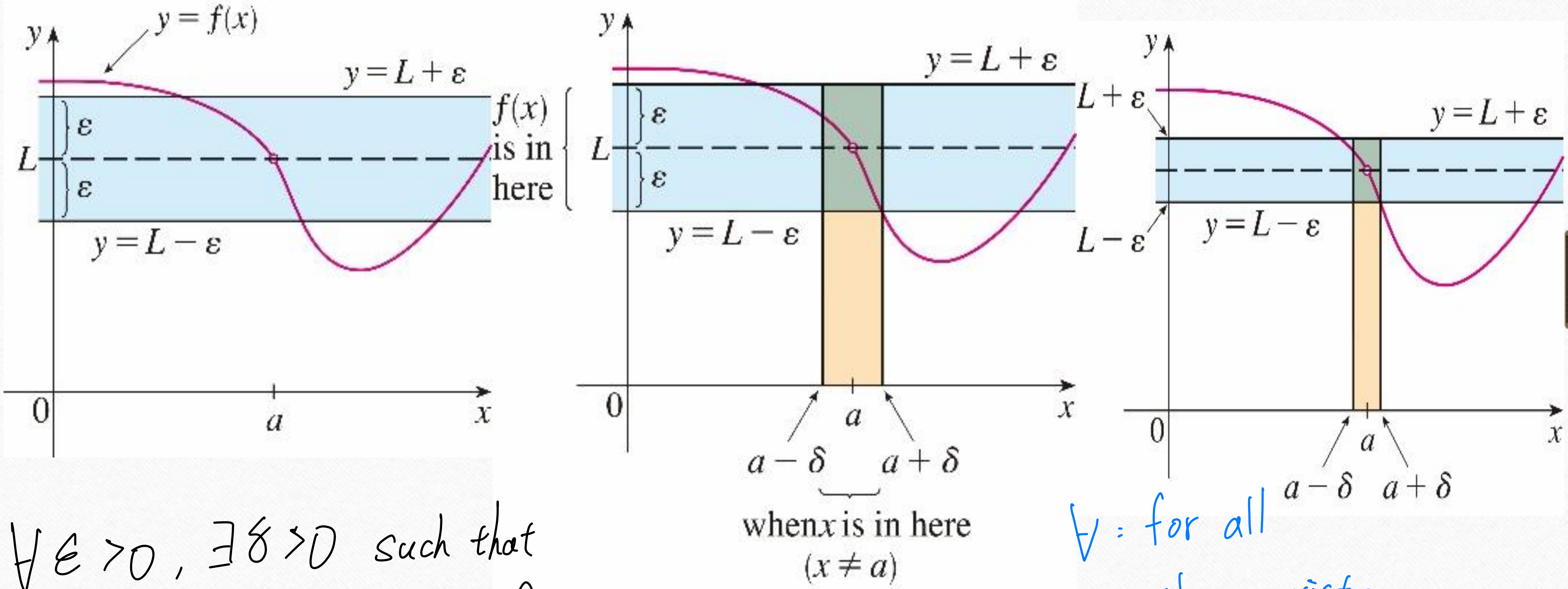
DEFINITION Let $f(x)$ be defined on an open interval about c, except possibly at c itself. We say that the **limit of $f(x)$ as x approaches c is the number L**, and write

$$\lim_{x \to c} f(x) = L,$$

if, for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that

相應的

$$|f(x) - L| < \varepsilon \quad \text{whenever} \quad 0 < |x - c| < \delta.$$



$\forall \varepsilon > 0$, $\exists \delta > 0$ such that

as $0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$\forall$: for all

$\exists$: there exists 存在

Ex2(p89) Show that $\lim_{x\to 1}(5x-3)=2$.

Analysis

$|(5x-3)-2|<\varepsilon$

$\Leftrightarrow |5x-5|<\varepsilon$

$\Leftrightarrow 5|x-1|<\varepsilon$

$\Leftrightarrow |x-1|<\frac{\varepsilon}{5}$

$\forall \varepsilon>0$

Let $\delta=\frac{\varepsilon}{5}$

As $0<|x-1|<\delta=\frac{\varepsilon}{5}$

$5|x-1|<\varepsilon$

$|5x-5|<\varepsilon$

$|(5x-3)-2|<\varepsilon$

$\therefore \lim_{x\to 1}(5x-3)=2$

HW2-3

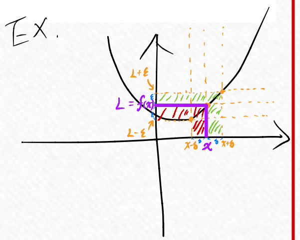
- **No HW.**

110 9. (加分題)

(a) State the definition of limit $\lim_{x \to x_0} f(x) = l$. (2 pts)

(b) Prove that $\lim_{x \to 2}(3x+1) = 7$ by the definition of limit. (3 pts)

EX.



(a) $\forall \varepsilon > 0$, $\exists \delta > 0$ such that as $0 < |x - x_0| < \delta \Rightarrow |f(x) - l| < \varepsilon$.

(b) $\lim_{x \to 2}(3x+1) = 7$

$|(3x+1) - 7| < \varepsilon$

$\Leftrightarrow |3x - 6| < \varepsilon$

$\Leftrightarrow 3|x-2| < \varepsilon$

$\Leftrightarrow |x-2| < \frac{\varepsilon}{3}$

$\forall \varepsilon > 0$, $\exists \delta = \frac{\varepsilon}{3} > 0$

such that as $0 < |x-2| < \delta = \frac{\varepsilon}{3}$

$\Rightarrow 3|x-2| < \varepsilon$

$\Rightarrow |3x-6| < \varepsilon$

$\Rightarrow |(3x+1) - 7| < \varepsilon$

$\therefore \lim_{x \to 2}(3x+1) = 7$