

3-5 Derivatives of Trigonometric Functions

師大工教一

$$1. \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Ex1(p167): Find the derivatives of the following functions:

$$(a) y = x^2 - \sin x$$

$$(b) y = e^x \sin x$$

$$(c) y = \frac{\sin x}{x}$$

$$y' = 2x - \cos x$$

$$y' = e^x \sin x + e^x \cos x$$

$$y' = \frac{\cos x \cdot x - \sin x}{x^2}$$

$$2. \frac{d}{dx}(\cos x) = -\sin x$$

Ex2(p167): Find the derivatives of the following functions:

$$(a) y = 5e^x + \cos x$$

$$y' = 5e^x - \sin x$$

$$(b) y = \sin x \cos x$$

$$\begin{aligned} y' &= \cos^2 x - \sin^2 x \\ &= \cos 2x \end{aligned}$$

$$(c) y = \frac{\cos x}{1 - \sin x}$$

$$\begin{aligned} y' &= \frac{-\sin x(1 - \sin x) + \cos x}{(1 - \sin x)^2} = \frac{-\sin x + \sin^2 x + \cos^2 x}{(1 - \sin x)^2} \\ &= \frac{1 - \sin x}{(1 - \sin x)^2} = \frac{1}{1 - \sin x} \end{aligned}$$

$$3. \frac{d}{dx}(\tan x) = \sec^2 x$$

$$4. \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$5. \frac{d}{dx}(\sec x) = \sec x \cdot \tan x$$

$$6. \frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$$

Ex5(p169): Find $\frac{d}{dx}(\tan x)$.

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Ex6(p170): Find $\lim_{x \rightarrow 0} \frac{\sqrt{2 + \sec x}}{\cos(\pi - \tan x)}.$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2 + \sec x}}{\cos(\pi - \tan x)} = \frac{\sqrt{3}}{\cos \pi} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

HW3-5

- HW: 3,6,7,12,17,34,39,52,53