4-8 Antiderivatives

師大工教一

Finding Antiderivatives

Definition: A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Ex1(p289) Find an antiderivative of each of the following functions.

(a) $f(x) = 2x$ (b) $g(x) = \cos x$ (c) $h(x) = \frac{1}{x} + 2e^{2x}$

$F(x) = x^2$ $G(x) = \sin x$ $H(x) = \ln|x| + e^{2x}$

Theorem: If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x)+C$ where C is an arbitrary constant.

Theorem: If $F'(x) = G'(x)$, $\forall x \in I$,
then $F(x) = G(x) + C$, $x \in I$.

Ex2(p290) Find an antiderivative of $f(x) = 3x^2$ that satisfies $F(1) = -1$.

$F(x) = x^3 - 2$

Table 4.2 Antiderivative formulas

Recite the formulas, or at least know how to derive it!

Ex3(p208) Find the general antiderivative of each of the following functions,

(a) $f(x) = x^5$

$F(x) = \frac{1}{6}x^6 + C$

(b) $g(x) = \dfrac{1}{\sqrt{x}}$

$G(x) = 2x^{\frac{1}{2}} + C$

(c) $h(x) = \sin 2x$

$H(x) = -\frac{1}{2}\cos 2x + C$

(d) $i(x) = \cos\dfrac{x}{2}$

$I(x) = 2\sin\frac{x}{2} + C$

(e) $j(x) = e^{-3x}$

$J(x) = -\frac{1}{3}e^{-3x} + C$

(f) $k(x) = 2^x$

$K(x) = \frac{2^x}{\ln 2} + C$

Initial Value Problems and Differential Equations

Ex(#98, p297) Solve the initial value problem $\dfrac{ds}{dt} = \cos t + \sin t, \quad s(\pi) = 1.$

$$S(t) = \sin t - \cos t + C$$

$$C = 0$$

$$S(t) = \sin t - \cos t$$

反導函數 = 不定積分

Indefinite Integrals

Definition The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x; it is denoted by $\int f(x)\,dx$. The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integrsl, and x is the **variable of integration**.

$\int f(x)dx = F(x) + C$

$\int$: integral sign 積分符號

f : integrand 被積分函數

Ex(#70,p296) $\displaystyle\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta$

$$= \int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C$$

$$= \int \frac{1}{1-\sin^2\theta}\,d\theta$$

$$= \int \frac{1}{\cos^2\theta}\,d\theta$$

HW4-8

- **HW: 26,29,39,46,52,67,76,78**