4-5 Indeterminate Forms and L’Hôpital’s Rule

師大工教一

Indeterminate Form $0/0$

Theorem 6—L'Hôpital's Rule

Suppose that $f(a) = g(a) = 0$, that f, g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

assuming that the limit on the right side of the equation exists.

Ex1(p265) Find the following limit: (b) $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ (d) $\lim_{x\to 0}\frac{x-\sin x}{x^3}$

(b) origin $\left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}}}{1} = \frac{1}{2}$

(d) $\left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{1-\cos x}{3x^2} = \lim_{x\to 0}\frac{\sin x}{6x} = \lim_{x\to 0}\frac{\cos x}{6} = \frac{1}{6}$

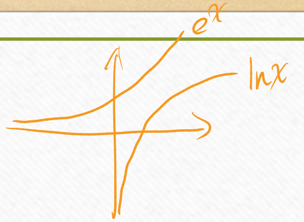
103 1 (c) (5 points) $\lim_{x\to 0}\frac{4(x-\sin(x))}{x^3} = ?$ $= \lim_{x\to 0}\frac{4(1-\cos x)}{3x^2} = \lim_{x\to 0}\frac{4(\sin x)}{6x} = \lim_{x\to 0}\frac{4\cos x}{6} = \frac{2}{3}$

Ex2(p266) Find $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x+x^2}$. (Be careful. Check the limit after every applications.)

$$\lim_{x\to 0}\frac{1-\cos x}{x+x^2}=\lim_{x\to 0}\frac{\sin x}{1+2x}=0$$

Indeterminate Forms $\infty/\infty, \infty \cdot 0, \infty - \infty$

Ex4(p266) Find the limit (b) $\lim\limits_{x\to\infty} \frac{\ln x}{2\sqrt{x}}$



$\lim\limits_{x\to\infty} e^x = \infty$

$\lim\limits_{x\to\infty} \ln x = \infty$

$\lim\limits_{x\to-\infty} e^x = \lim\limits_{x\to\infty} e^{-x} = 0$

$\lim\limits_{x\to 0^+} \ln x = -\infty$

(b) $\lim\limits_{x\to\infty} \frac{\ln x}{2\sqrt{x}} \left(\frac{\infty}{\infty}\right) = \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{\cancel{2}\frac{1}{\cancel{2}\sqrt{x}}} = \lim\limits_{x\to\infty} \frac{\sqrt{x}}{x} = \lim\limits_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{1} = 0$

109 4 (d) $\lim\limits_{t\to 0} \ln\sqrt{(1+t)^{\frac{1}{3t}}} = \lim\limits_{t\to 0} \ln(1+t)^{\frac{1}{6t}} = \lim\limits_{t\to 0} \frac{1}{6t} \ln(1+t) \left(\frac{0}{0}\right) = \lim\limits_{t\to 0} \frac{\frac{1}{1+t}}{6} = \frac{1}{6}$

Ex5(p267) Find the limit (b) $\lim_{x\to 0^+}\sqrt{x}\ln x$ $(0\cdot(-\infty))$ $= \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{\sqrt{x}}} = \lim_{x\to 0^+}\frac{\ln x}{x^{-\frac{1}{2}}} = \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{2}x^{-\frac{3}{2}}} = \lim_{x\to 0^+}\frac{-2x^{\frac{3}{2}}}{x} = 0$

111 2

(c) $\lim_{x\to 0}\left(\frac{1}{x\sqrt{1+x}} - \frac{1}{x}\right) = \lim_{x\to 0}\frac{1-\sqrt{1+x}}{x\sqrt{1+x}}$ $\left(\frac{0}{0}\right)$ $= \lim_{x\to 0}\frac{-\frac{1}{2\sqrt{1+x}}}{\sqrt{1+x}+\frac{x}{2\sqrt{1+x}}} = \lim_{x\to 0}\frac{-1}{2(1+x)+x} = -\frac{1}{2}$

However 就是要變成 $\frac{0}{0}$ or $\frac{\infty}{\infty}$，再羅必達

Indeterminate Powers

If $\lim\limits_{x\to a}\ln f(x)=L$, then $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} e^{\ln f(x)}=e^{L}$. Here a may be either finite or infinite.

Ex7(p268) Find the limit $\lim\limits_{x\to 0^+}(1+x)^{\frac{1}{x}}$

$$=\lim_{x\to 0^+}\ln(1+x)^{\frac{1}{x}}$$

$$=\lim_{x\to 0^+}\frac{1}{x}\ln(1+x)\quad(\infty\cdot 0)$$

$$=\lim_{x\to 0^+}\frac{\ln(1+x)}{x}\quad\left(\frac{0}{0}\right)=\lim_{x\to 0^+}\frac{\frac{1}{1+x}}{1}=1\qquad e^{1}=e\ \#$$

105分 1 (d) $\lim\limits_{x\to 0}(1+3x)^{\frac{1}{x}}$. $\to \lim\limits_{x\to 0}\frac{1}{x}\ln(1+3x)\ (\infty\cdot 0)=\lim\limits_{x\to 0}\frac{\ln(1+3x)}{x}\left(\frac{0}{0}\right)=\lim\limits_{x\to 0}\frac{\frac{1}{1+3x}}{1}=1$

$$e^{1}=e\ \#$$

HW4-5

- **HW:5,16,21,30,35,37,53,58,59,71**