5-6 Definite Integral Substitutions and the Area Between Curves

師大工教一

Theorem 7-Substitution in Definite Integrals

If g' is continuous on the interval $[a,b]$ and f is continuous on the range

of $g(x)=u$, then $\int_a^b f(g(x))\cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Ex2(p359) Evaluate the following definite integrals:

(a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot\theta \csc^2\theta\, d\theta$ (b) $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \tan\theta\, d\theta$

Let $u=\cot\theta$

$du=-\csc^2\theta\, d\theta$

$-du=\csc^2\theta\, d\theta$

origin $=\int_1^0 u(-du)$

$=-\int_1^0 u\,du=\int_0^1 u\,du$

$=\frac{u^2}{2}\Big|_0^1=\frac{1}{2}$

$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin\theta}{\cos\theta}\,d\theta$

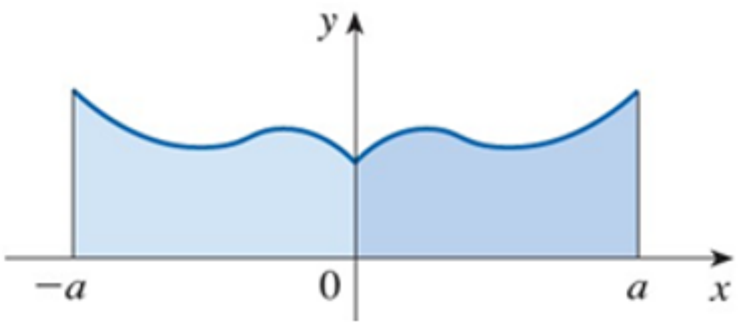
Let $u=\cos\theta$

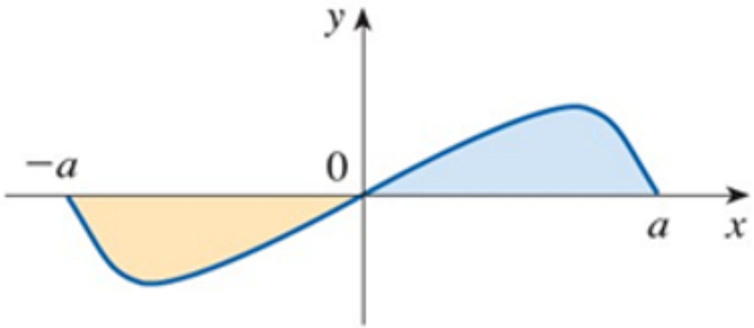
$du=-\sin\theta\, d\theta$

$-du=\sin\theta\, d\theta$

$=\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \frac{-du}{u}=0$

Definite Integrals of Symmetric Functions





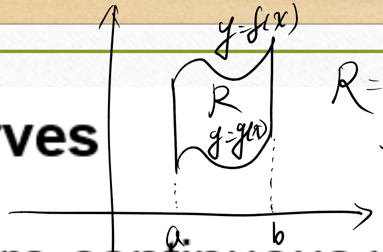
Theorem 8 Let f be continuous on the symmetric interval $[-a, a]$.

(a) If f is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

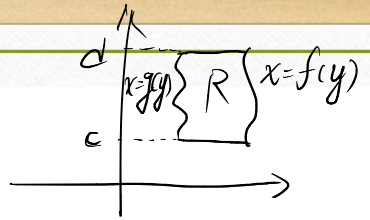
(b) If f is odd, then $\int_{-a}^{a} f(x)\,dx = 0$.

Ex3(p360) Evaluate $\int_{-2}^{2}\left(x^4-4x^2+6\right)dx$.

Area Between Curves



$R = \int_a^b (f(x) - g(x))\,dx$



$R = \int_c^d (f(y) - g(y))\,dy$

Definition If f, g are continuous with $f(x) \geq g(x)$ throughout $[a,b]$, then

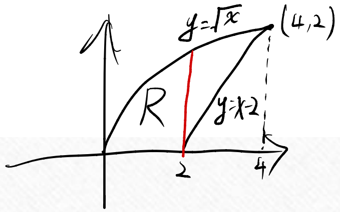
the **area of the region between the curve** $y = f(x)$ **and** $y = g(x)$ **from** a

to b is the integral of $(f - g)$ from a to b: $A = \int_a^b \left(f(x) - g(x)\right)dx$.

Ex6(p362) Find the area of the region in the first quadrant that is bounded

above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$. (Figure

5.30)



$$\int_0^2 \sqrt{x}\,dx + \int_2^4 [\sqrt{x} - (x-2)]\,dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}}\Big|_0^2\right] + \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + 2x\Big|_2^4\right]$$

$$= \cancel{\frac{4\sqrt{2}}{3}} + \left[\left(\frac{16}{3} \cancel{-8+8}\right) - \left(\cancel{\frac{4\sqrt{2}}{3}} - 2 + 4\right)\right] = \frac{16}{3} - 2 = \frac{10}{3}$$

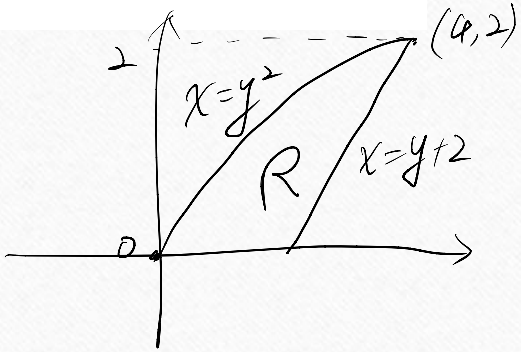
Integration with Respect to y

Ex7(p363) Find the area of the region in Exmple 6 by integrating with respect to y. (region in the first quadrant that is bounded above by $y=\sqrt{x}$ and below by the $x-$axis and the line $y=x-2$).

$$\text{Area} = \int_0^2 \left[(y+2)-(y^2)\right]dy$$

$$= \frac{1}{2}y^2+2y-\frac{1}{3}y^3 \Big|_0^2$$

$$= \left(2+4-\frac{8}{3}\right) = \frac{10}{3}$$



HW5-6

- **HW: 2,5,9,13,29,39,48,51,55,60,82,85,106**