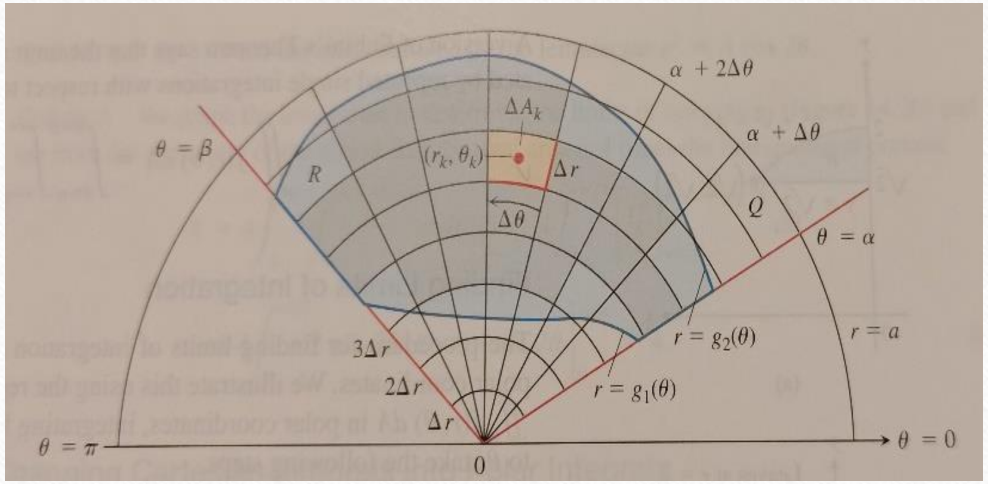
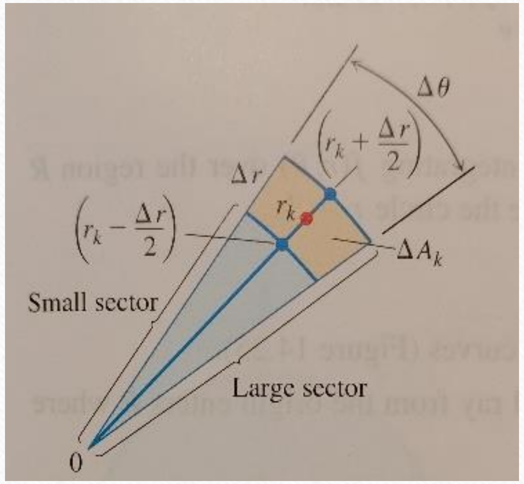
14-4 Double Integrals in Polar Form

師大工教一

Suppose that a function $f(r,\theta)$ is defined over a region R that is bounded by the rays $\theta=\alpha$ and $\theta=\beta$ and by the continuous curves $r=g_1(\theta)$ and $r=g_2(\theta)$. Suppose also that $0\le g_1(\theta)\le g_2(\theta)\le a$. Then R lies in Q defined by $0\le r\le a, \alpha\le\theta\le\beta$.





Consider one polar rectangle whose area is ΔA_k, by figure,

$\Delta A_k = \frac{1}{2}\left(r_k + \frac{\Delta r}{2}\right)^2 \Delta\theta - \frac{1}{2}\left(r_k - \frac{\Delta r}{2}\right)^2 \Delta\theta = r_k \Delta r \Delta\theta$. Thus, the Riemann sum

$S_n = \sum_{k=1}^{n} f\left(r_k, \theta_k\right) \Delta A_k = \sum_{k=1}^{n} f\left(r_k, \theta_k\right) r_k \Delta r \Delta\theta$. Then,

$\lim_{n\to\infty} S_n = \iint_R f\left(r,\theta\right) dA = \iint_R f\left(r,\theta\right) r dr d\theta$.

Fubini's Theorem: $\iint_R f(r,\theta)\,dA = \int_{\theta=\alpha}^{\theta=\beta}\int_{r=g_1(\theta)}^{r=g_2(\theta)} f(r,\theta)\,r\,dr\,d\theta$.

Ex3(p817) Evaluate $\iint_R e^{x^2+y^2}\,dydx$, where R is the semicircle region bounded by the $x-$axis and the curve $y=\sqrt{1-x^2}$.

Ex4(p817) Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \left(x^2 + y^2\right) dydx$.

Ex5(p818) Find the volume of the solid region bounded above by the paraboloid $z = 9 - x^2 - y^2$ and below by the unit circle in the $xy -$ plane.

HW14-4

- **HW: 10,12,21,28,31**