Wrap up session and open questions

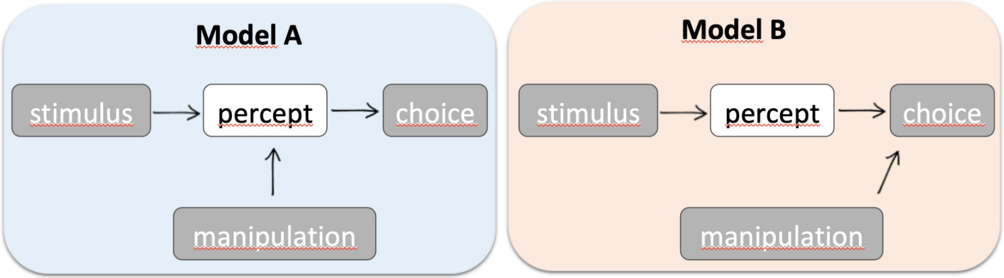
BAMB! Summer School

Day 8 - 26 July 2023

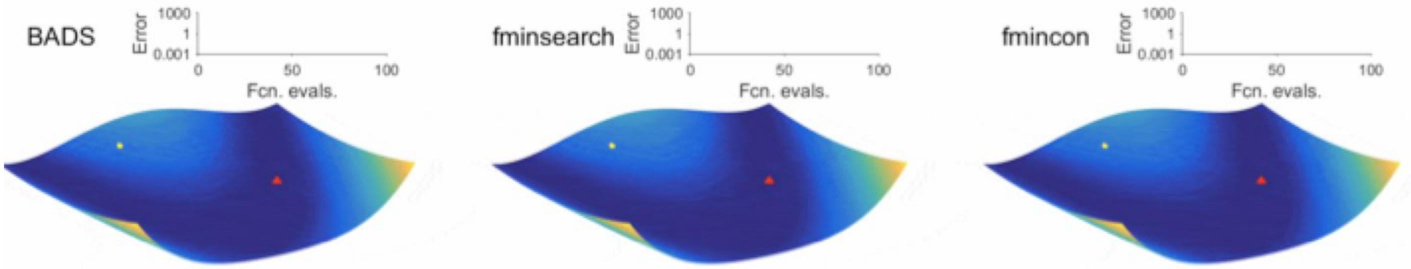
What have we learnt?

Day 1 What is a model (1A)

Model definition & simulations

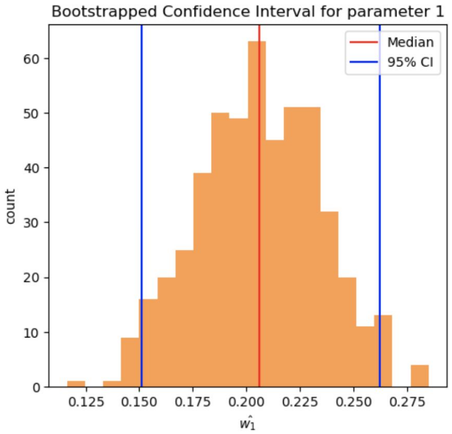


Maximum likelihood

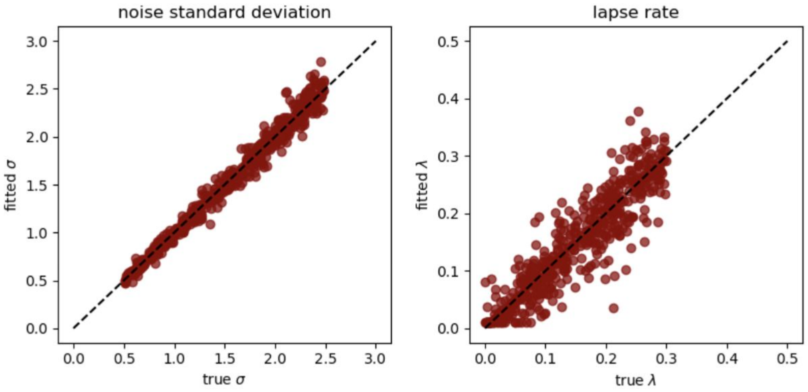


Day 1 What is a model (1B)

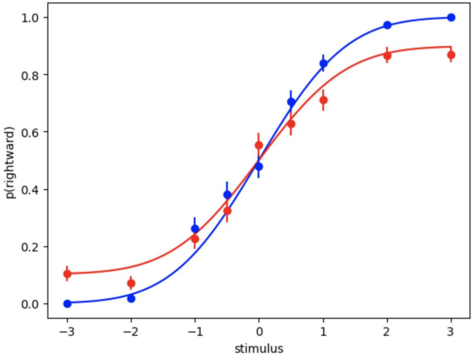
Bootstrapping



Parameter recovery

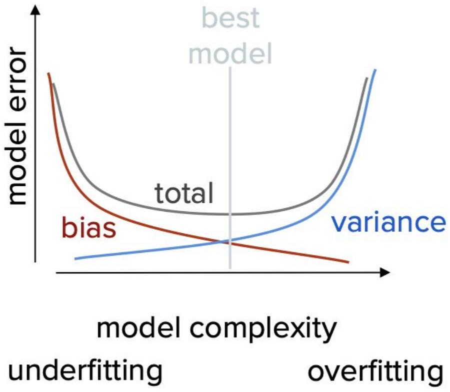


Model validation

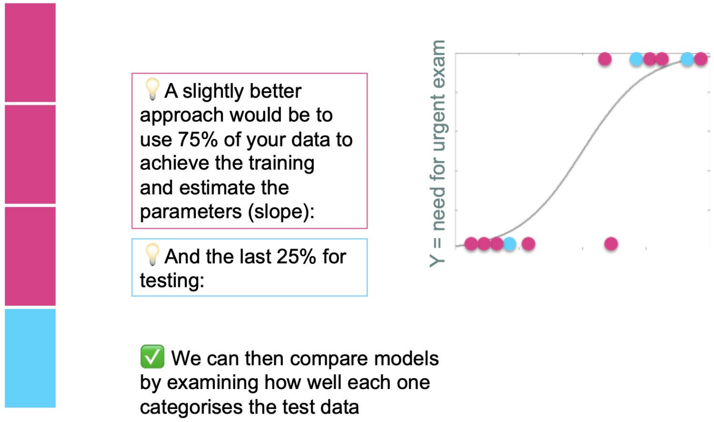


Day 1 What is a model (1C)

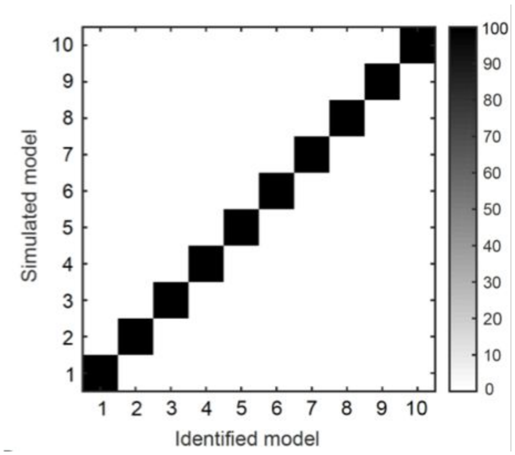
Model selection



Cross validation

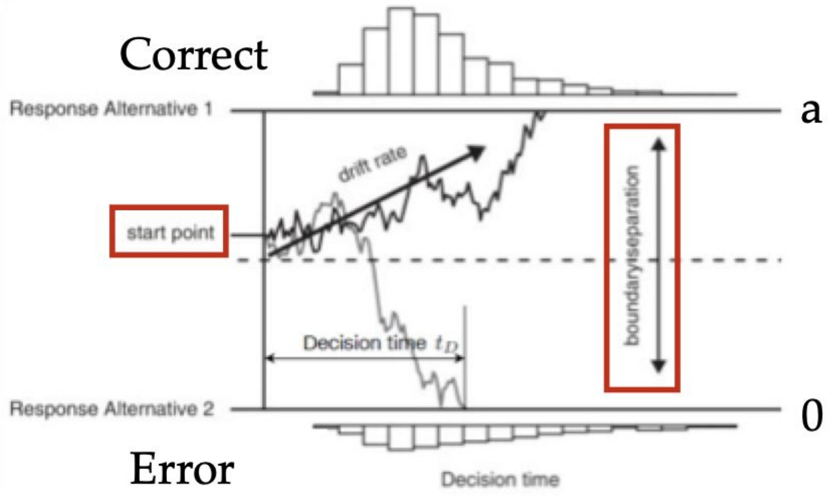


Model recovery

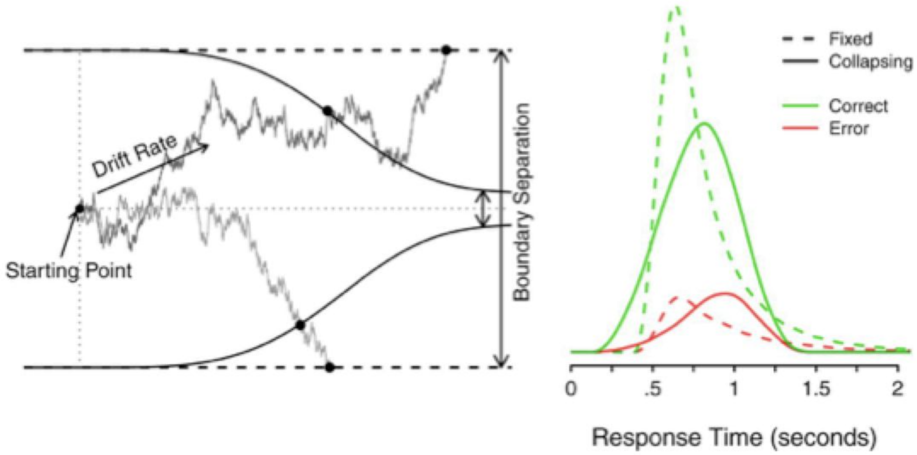


Day 2 Drift-diffusion models

Basic structure …



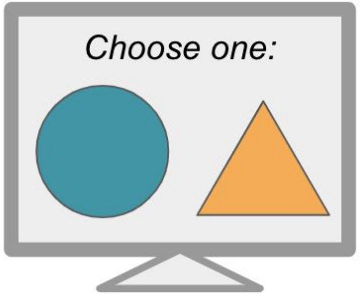
… Lots of extensions eg. collapsing bounds



Day 3 RL

Reinforcement Learning as a cognitive model

```
Value(s)  += α * RPE
RPE = r - Value(s)
```

Goal	Reward	Algorithm
	+1	a = [F, H] s = [] r = [0, +1] *V(a|s) += α * RPE* *RPE = r - V(a|s)* *p(a|s) = softmax(β * V(a|s))*
	+1	a = [→, ←] s = [] r = [0, +1] *V(a|s) += α * RPE* *RPE = r - V(a|s)* *p(a|s) = softmax(β * V(a|s))*

Reasons for cognitive models:

- Process models
- Precise
- Quantifiable
- Generate predictions
- Optimality, complexity
- Statistical methods

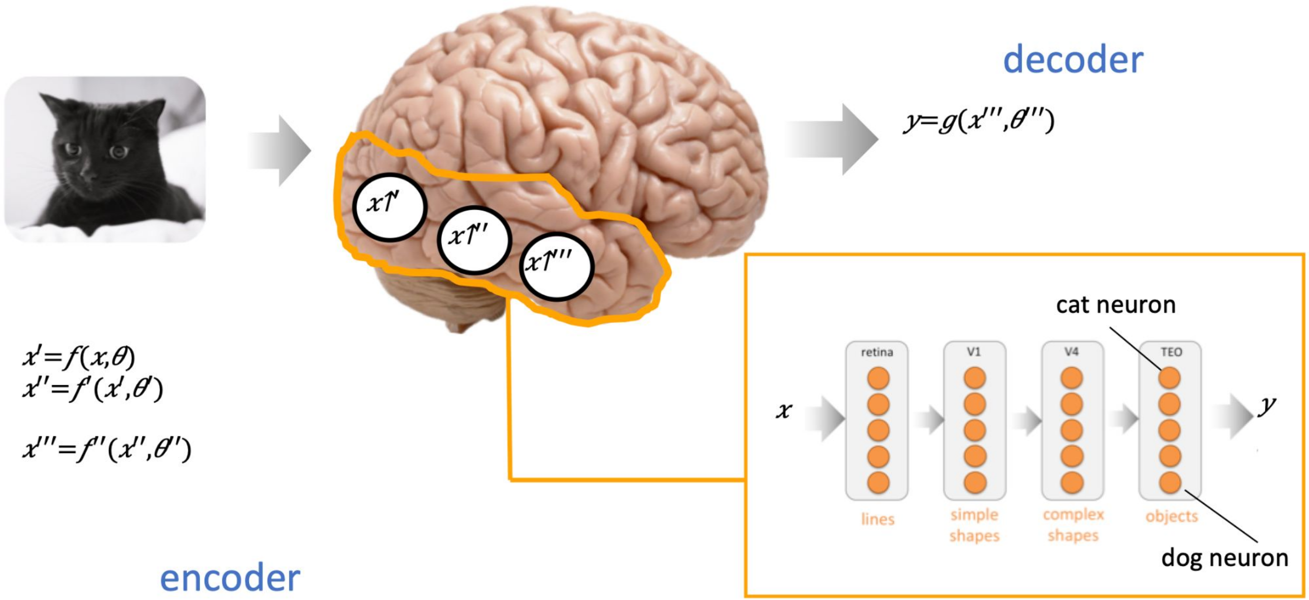
[Daw, 2011. *DM,A&L: A&P*]
[Wilson & Collins, 2019. *eLife*]

Day 4 RNN

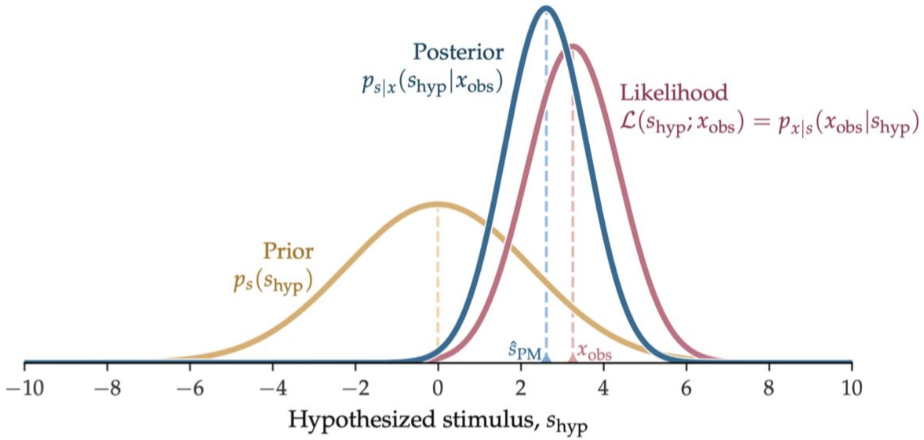
Theories of representation learning for sensory neocortex

Accurate but fragile:

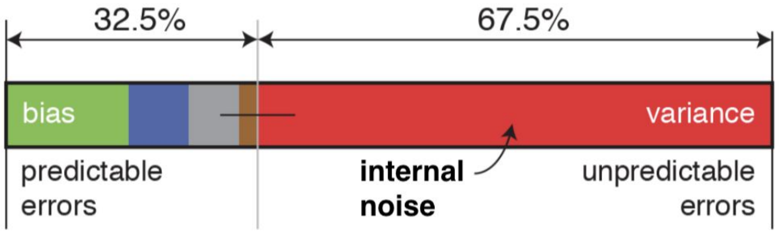
- **vulnerable to seemingly innocuous changes in inputs**
- **error patterns different from biological systems**



Day 6 Bayesian models



Bayes-optimal models can be used to quantify and qualify suboptimal human inferences



Day 7 Latent variables

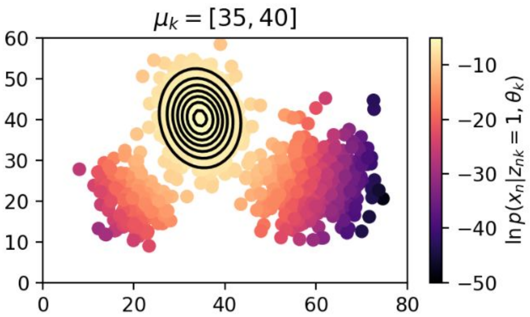
Expectation Maximization

The expectation maximization (EM) algorithm

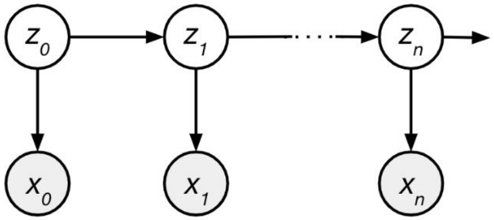
0. **Initialization:** Choose θ^0.
1. **E step:** $\mathbb{E}[\boldsymbol{z}]$ under posterior $p(\boldsymbol{z}|\boldsymbol{x}, \theta^i)$
2. **M step:** Update $\boldsymbol{\theta}^{i+1}$ by maximizing $\mathbb{E}_{\boldsymbol{z}}[\ln p(\boldsymbol{x}, \boldsymbol{z}|\theta)]$

Alternate 1. and 2. until convergence.

Mixture Models



Hidden Markov Models

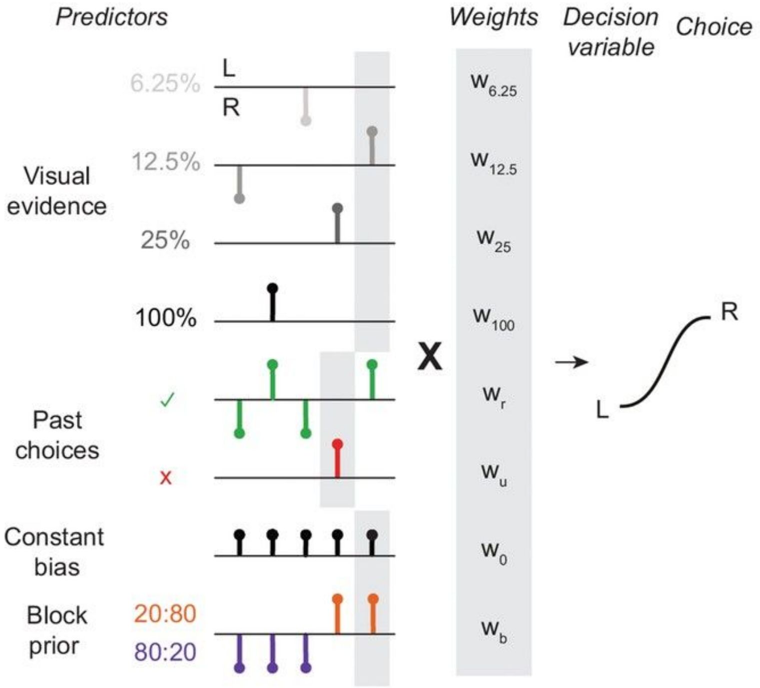


- Iterative algorithm
- Joint inference of posteriors and parameter estimation
- E and/or M step can also be numerical

- Cluster complex data into simpler patterns (*classes* [MM] / *states* [HMM])
- Inference of *latent variables* and *observation models*
- Observation models can be more complex than Gaussian (e.g. linear model [MM/HMM], differential equation [HMM]…)

Regression analyses - mechanistic-free modelling

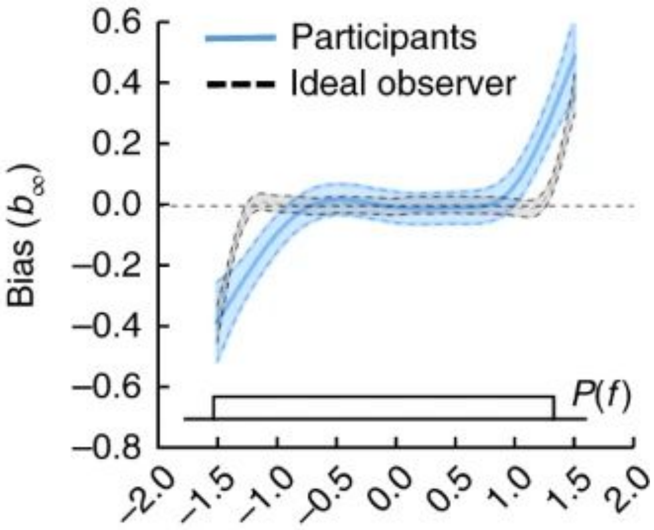
Generalized Linear Models (GLMs)



(International Brain Lab, eLife, 2021)

Generalized Additive Models (GAMs)

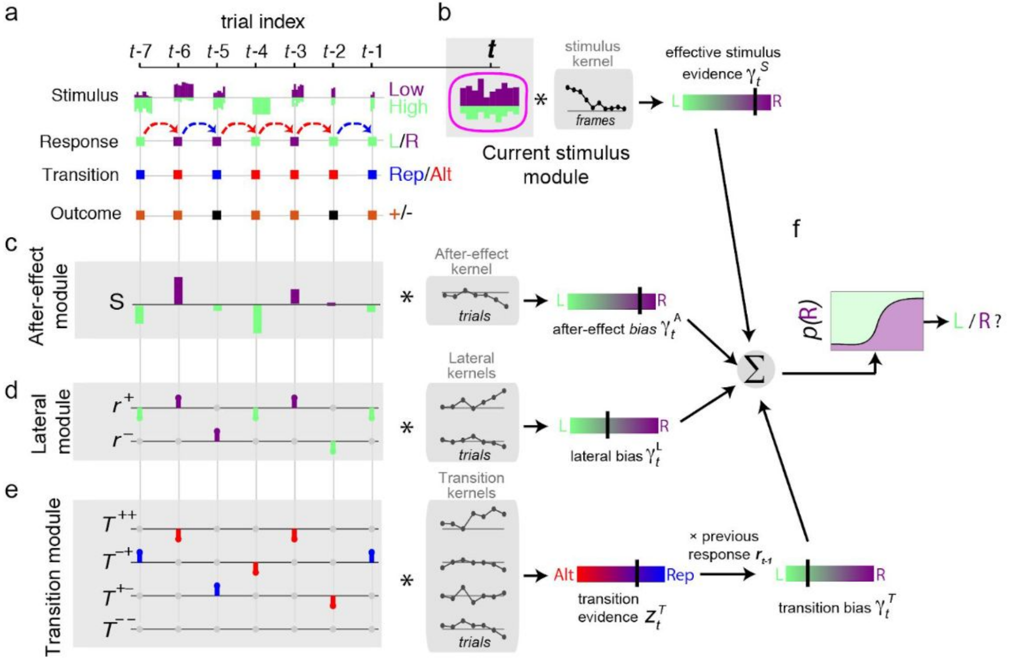
fits arbitrary function of regressor



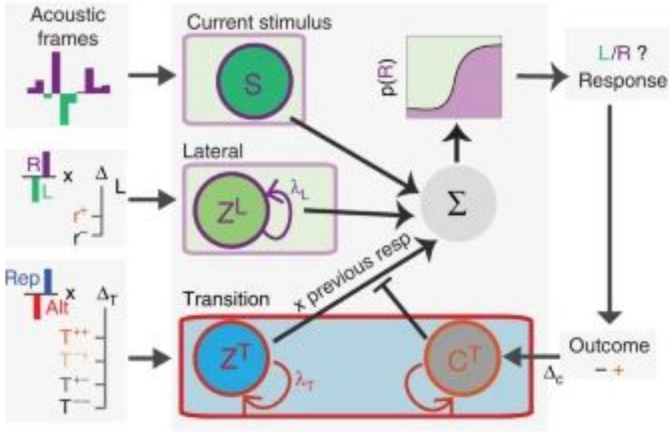
(Lieder et al, Nat Neuro, 2019)

From mechanistic-free to mechanistic model

logistic regression model



RL model



Hermoso-Mendizabal et al., ,Nat Comms (2019)

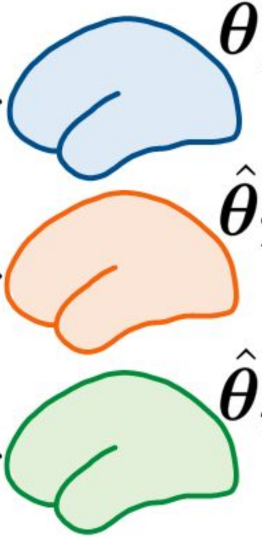
Population-level analyses

Subject 1: $(\boldsymbol{X}_1, \boldsymbol{Y}_1) \longrightarrow$ $\hat{\boldsymbol{\theta}}_1$

Subject 2: $(\boldsymbol{X}_2, \boldsymbol{Y}_2) \longrightarrow$ $\hat{\boldsymbol{\theta}}_2$

....

Subject m: $(\boldsymbol{X}_m, \boldsymbol{Y}_m) \longrightarrow$ $\hat{\boldsymbol{\theta}}_m$

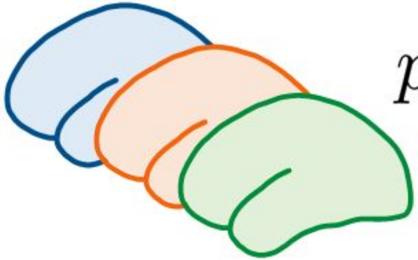


We want to infer from a sample of subjects conclusions about general population(s)

Summary statistics approach

$$\{\hat{\boldsymbol{\theta}}_j\} \longrightarrow p(\boldsymbol{\theta}|\boldsymbol{Y})$$

$$\{(\boldsymbol{X}_j, \boldsymbol{Y}_j)\} \longrightarrow$$



$$p(\boldsymbol{\theta}|\boldsymbol{Y})$$

Mixed models

(Generalized Linear Mixed Models (GLMM), HDDM

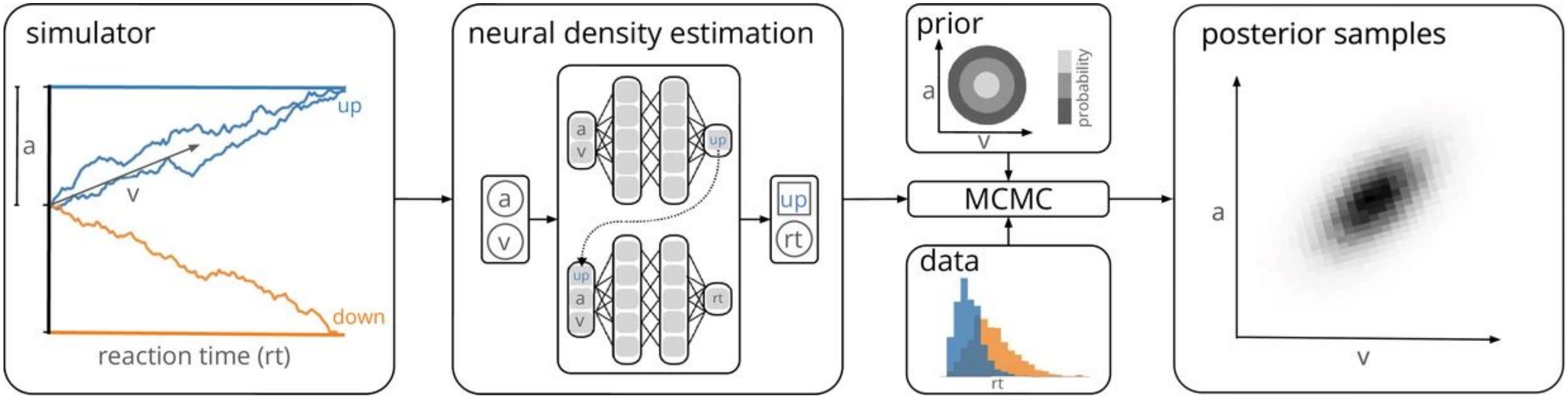
Knoblauch & Maloney, JVis (2008)

Computing the likelihood

- **direct access**:
  - psychometric curve (with/without lapses) [Wichmann&Hill, 2001]
  - generalized linear model
  - standard DDM
  - RL model with deterministic value update
- **numerical approximation/integration**:
  - generalized DDM [Shinn et al, eLife 2001]
- **expectation-maximization**: for latent variable model
  - HMM, Gaussian mixtures
  - RL model with stochastic value update, Kalman filter
  - GLMM

Computing the likelihood (cont'd)

- **simulation-based inference**:

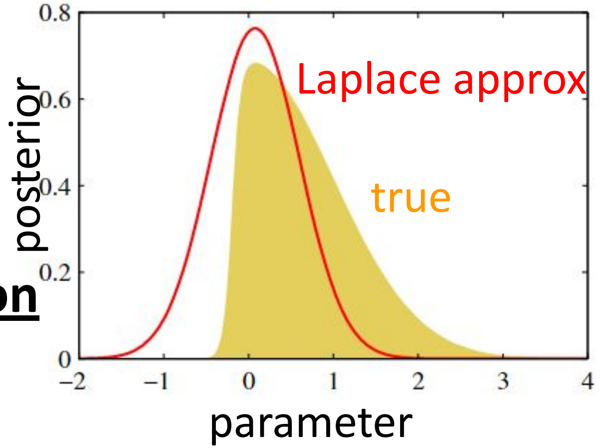


Boelts et al., eLife (2022)

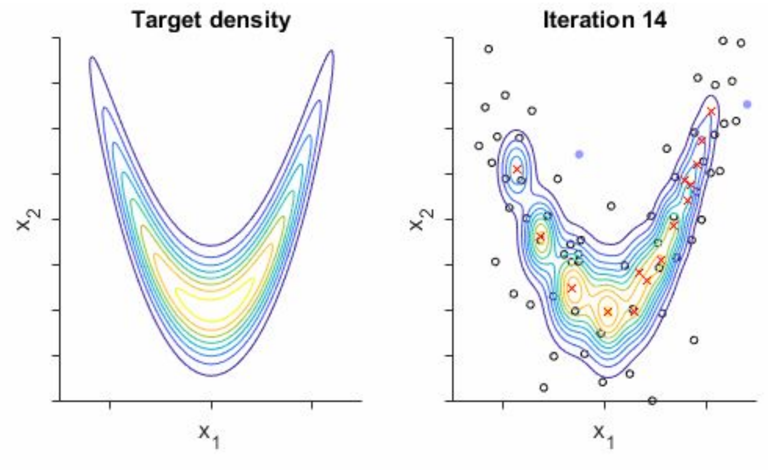
Computing the posterior

direct access: linear regression

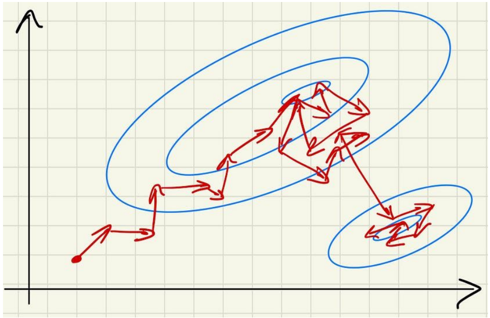
Laplace approximation



VBMC [Acerbi, NeuIPS 2018,2020]



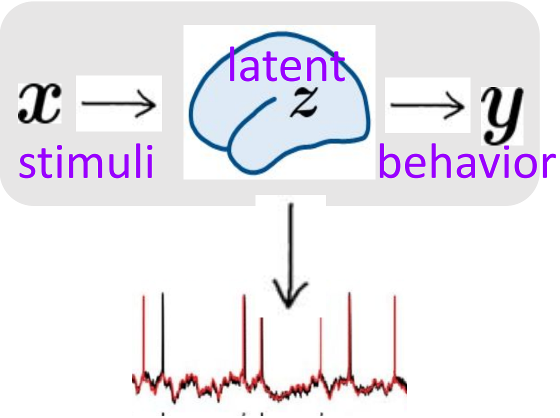
sampling methods (MCMC)



variational inference:
e.g. VBA [Daunizeau et al., PlosCB 2014]

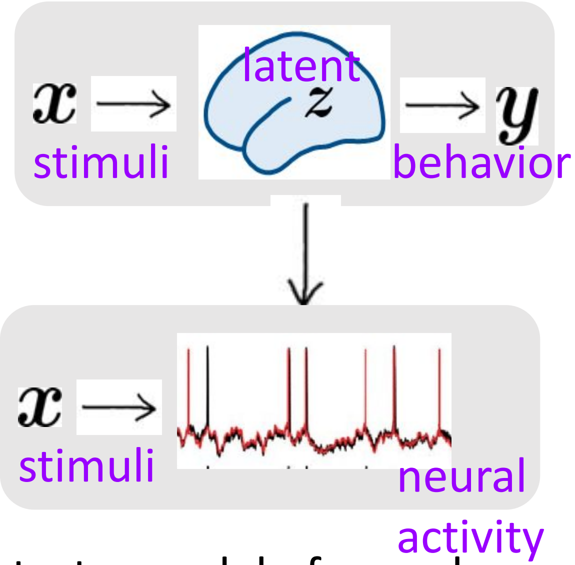
Illuminating neural analysis with behavioral modelling

model-based fMRI (brain mapping)



correlate latent variable estimated from behavioral model with neural activity

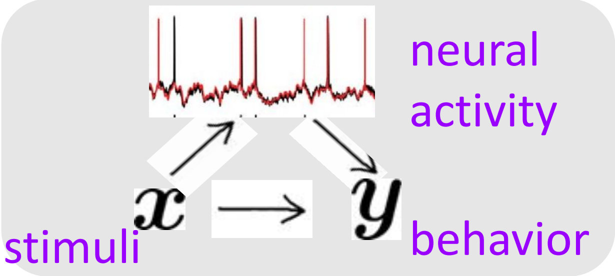
neural modelling



test a model of neural activity inspired from behavioral model

Weiss et al, Nat Comms (2021)
Hyafil et al, eLife 2023

mediation analysis



test whether the impact of variables onto behavior is mediated by neural activity

Padoa Schioppa, Neuron (2022)
El Zein et al, eLife 2015

Open questions session