Tutorial 2: Drift-diffusion models

BAMB! Summer School
Tutorial 2

Tutorial overview

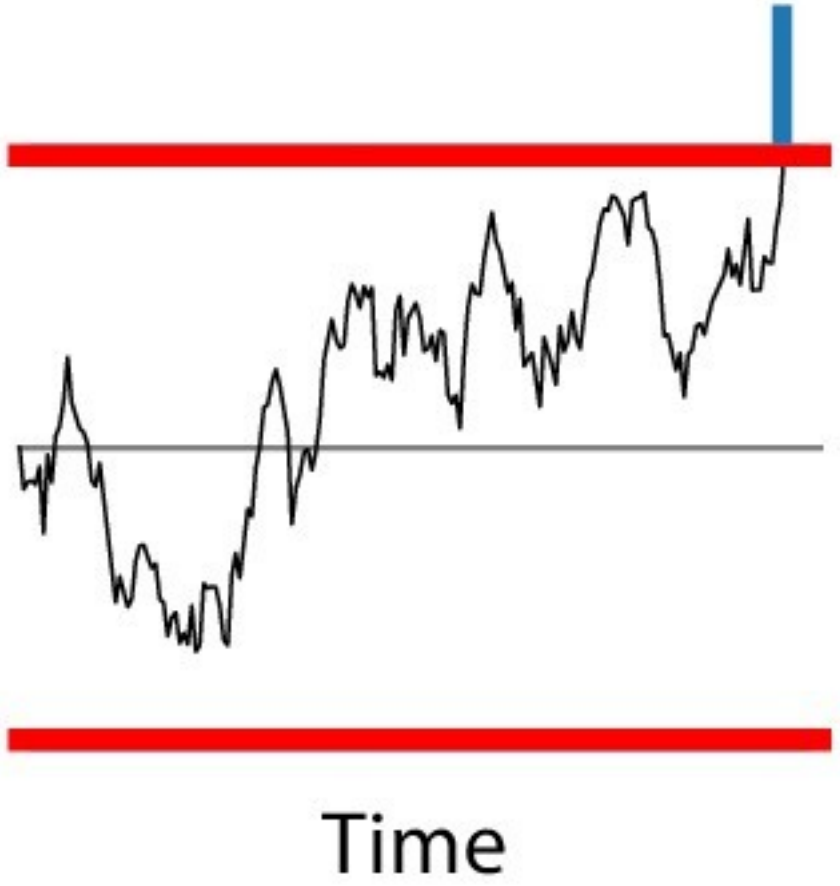
- Hour 1: Simulating the DDM by hand
  - Construct a DDM from first principles
- Hour 2: Simulating the DDM using PyDDM
  - Use efficient and higher-accuracy methods to perform simulations
- Hour 3: Fitting the DDM to data
  - Use PyDDM to fit the DDM to monkey random dot motion data
- Hour 4: Generalized drift diffusion models (GDDMs)
  - Create variants of the DDM which are specialized to specific tasks or encapsulate distinct strategies

Hour 1: Simulating the DDM by hand

- Basic algorithm
  - 1. Set x to starting point
  - 2. Set:

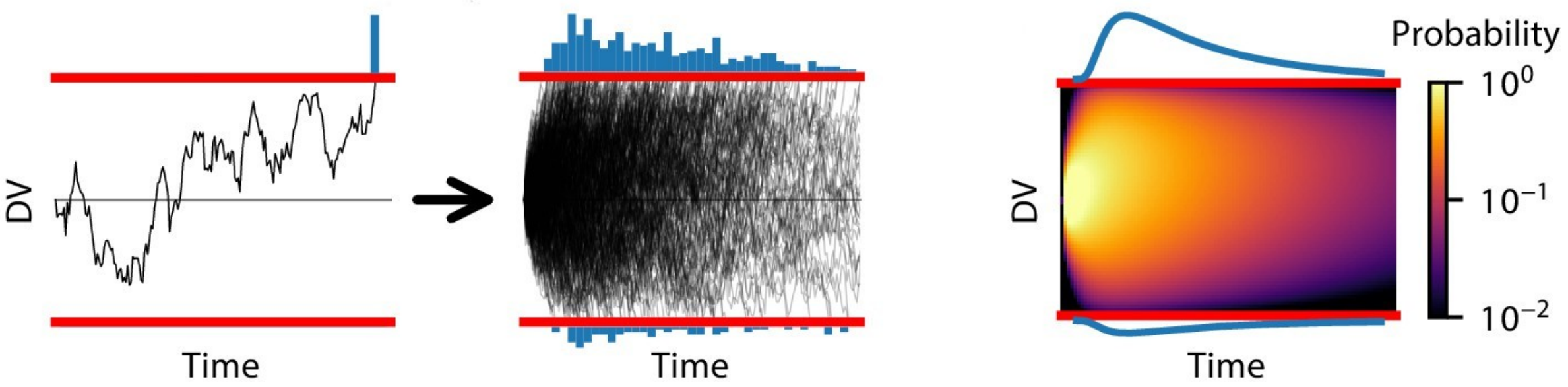
$$x_{t+1} = x_t + [\text{drift}]\Delta t + [\text{noise}] z_t \sqrt{\Delta t}$$
$$z_i \sim N(0,1)$$

  - 3. Check if x crosses a boundary. If so, you are done
  - 4. Otherwise, go to (2)



Hour 2: Simulating the DDM using PyDDM

- Use more efficient methods to simulate the probability distribution of a trajectory's position instead of one trial at a time



DDM libraries

	PyDDM	HDDM	EZ-Diffusion	CHaRTr	DMAT	fast-dm
Language	Python3	Python2/3	Matlab, R, Javascript, or Excel	Requires both R and C	Matlab	Command line
Solver	Fokker-Planck, analytical	Analytical numerical hybrid	None	None (Monte Carlo)	Analytical numerical hybrid	Fokker-Planck
Task parameters						
Time dependence of drift/noise	Any function	Constant	Constant	Any function	Constant	Constant
Position dependence of drift/noise	Any function	Constant	Constant	Any function	Constant	Constant
Bounds	Any function	Constant	Constant	Any function	Constant	Constant
Parameter dependence on task conditions	Any relationship for any parameter	Regression model	Categorical	Categorical	Linear	Categorical
Across-trial variability						
Across-trial drift variability	Slow discretization (via extension)	Normal distribution	None	Any distribution	Normal distribution	Normal distribution
Across-trial starting point variability	Any distribution	Uniform distribution	None	Any distribution	Uniform distribution	Uniform distribution
Across-trial non-decision variability	Any distribution	Uniform distribution	None	Any distribution	Uniform distribution	Uniform distribution
Model simulation and fitting						
Hierarchical fitting	No	Yes	No	No	No	No
Fitting methods	Any numerical (default: differential evolution)	MCMC	Analytical	Any numerical	Nelder-Mead	Nelder-Mead
Objective function	Any function (default: likelihood)	Likelihood	Mean/stdev RT and P(correct)	Any sampled (e.g. quantile maximum likelihood)	Quantile maximum likelihood or chi-squared	Likelihood, chi-squared, Kolmogorov-Smirnov
Mixture model	Any distribution(s)	Uniform	None (extendable)	None	Uniform and undecided guesses	Uniform

How PyDDM works:

- Construct a Model from its components
- Model components:
  - Drift rate
  - Noise
  - Bound
  - Initial Condition
  - “Overlay” (diffusion-independent processes, e.g., non-decision time)

Many model components are built-in:

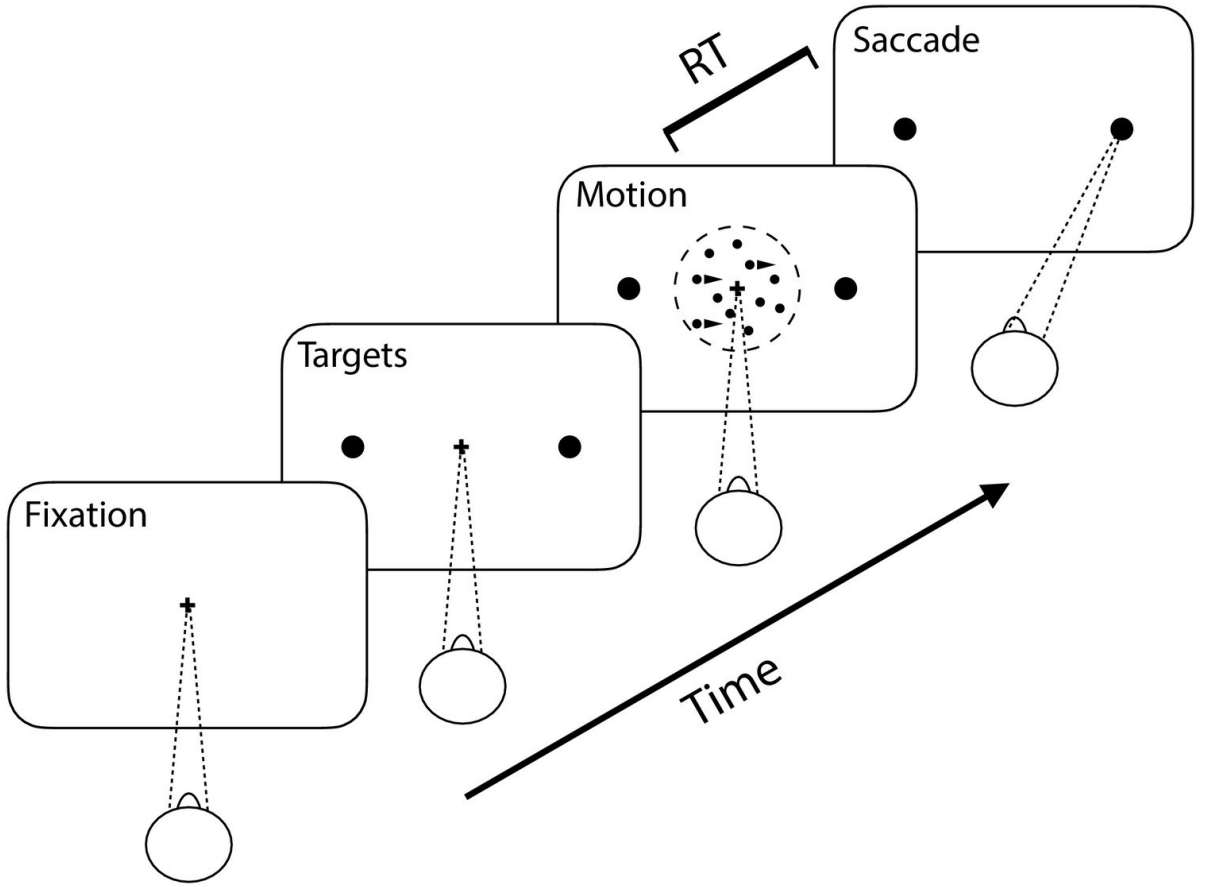
- For a simple DDM:
  - Drift rate = DriftConstant
  - Noise = NoiseConstant
  - Bound = BoundConstant
  - Initial Condition = ICPointSourceCenter
  - “Overlay” = OverlayNonDecision

Parameters and conditions

- Parameters: Have the same value for the entire dataset
  - E.g. bound height
- Conditions: May change from trial to trial
  - E.g. strength of motion coherence

Hour 3: Fitting the DDM to data

- Dataset: Monkeys performing the random dot motion task (Roitman and Shadlen, 2002)
- Several levels of motion coherence



https://shadlenlab.columbia.edu/resources/RoitmanDataCode.html

Multiple motion coherences in PyDDM

- How to make drift rate depend on the “coherence” condition?
- Create a new type of drift rate by creating a subclass of Drift
  - See PyDDM quickstart guide or PyDDM cookbook for an example

```
class DriftThatDependsOnSomething(pyddm.Drift):
    name = “drift that depends on something”
    required_conditions = [“the_thing_it_depends_on”]
    required_parameters = [“param”]
    def get_drift(self, x, t, conditions, **_):
        return conditions[“the_thing_it_depends_on”] * self.param
```

Hour 4: Generalized DDMs (GDDMs)

- Construct a Model from its components – all components can depend on conditions
- Model components:
  - Drift rate can also depend on position (x) and time
  - Noise can also depend on position and time
  - Bound can also change over time
  - Initial Condition can be any probability distribution
  - "Overlay" can be anything which modifies the RT distributions
- Some are built-in so you don't have to make them yourself!

Example GDDMs