Cheat sheet for GMM and HMM tutorials (BAMB! '24)

Heike Stein

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1 Probabilities

Product rule (1) and *symmetry property* (2) to re-express the joint distribution

$$p(\boldsymbol{a}, \boldsymbol{b}) = p(\boldsymbol{a}|\boldsymbol{b})p(\boldsymbol{b}) \tag{1}$$
$$= p(\boldsymbol{b}|\boldsymbol{a})p(\boldsymbol{a}) \tag{2}$$

Marginalization of $\boldsymbol{b}$ (3): We account for all its possible values (where $\boldsymbol{b}$ is assumed to be a discrete, i.e. categorical variable. Else, the sum is replaced by an integral over $\boldsymbol{b}$).

$$p(\boldsymbol{a}) = \sum_{\boldsymbol{b}} p(\boldsymbol{a}, \boldsymbol{b}) \tag{3}$$

Bayes theorem (4) to calculate the posterior $p(\boldsymbol{b}|\boldsymbol{a})$ from the prior $p(\boldsymbol{b})$ and the likelihood $p(\boldsymbol{a}|\boldsymbol{b})$, normalized by the marginal likelihood $p(\boldsymbol{a})$. Then product rule and marginalization (5) to re-express the same equation.

$$p(\boldsymbol{b}|\boldsymbol{a}) = \frac{p(\boldsymbol{a}|\boldsymbol{b})p(\boldsymbol{b})}{p(\boldsymbol{a})} \tag{4}$$
$$= \frac{p(\boldsymbol{a}, \boldsymbol{b})}{\sum_{\boldsymbol{b}} p(\boldsymbol{a}, \boldsymbol{b})} \tag{5}$$

In the tutorial, you will be working a lot with log probabilities, according to the *log product rule*

$$\ln(p(\boldsymbol{a})p(\boldsymbol{b})) = \ln p(\boldsymbol{a}) + \ln p(\boldsymbol{b}) \tag{6}$$

2 Expected value

The *expected value* of variable $\boldsymbol{a}$ under the distribution $p(\boldsymbol{b})$ is given by

$$\mathbb{E}_{\boldsymbol{b}}[\boldsymbol{a}] = \sum_{\boldsymbol{b}} p(\boldsymbol{b})\boldsymbol{a} \tag{7}$$

The expected value can be understood as a weighted average, where variable $\boldsymbol{a}$ is weighted by the probabilities $p(\boldsymbol{b})$ for each different value of $\boldsymbol{b}$ (and summed over all possible values of $\boldsymbol{b}$). As for marginalization, variable $\boldsymbol{b}$ is assumed to be discrete. If $\boldsymbol{b}$ is continuous, the sum is replaced by an integral.