THE UNIVERSITY OF MELBOURNE

Robotics Systems MCEN90028
Assignment 2: Robot Jacobian

Assignment Group 20

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1 Introduction

This report outlines the differential kinematics for the 5 degrees of freedom robotic manipulator, affectionately known as "Jenghis-Khan". An additional frame is defined for the wrist joint, and the jacobian is derived for this frame with respect to the joint velocities. The derivation of components used in the jacobian calculation is shown here, as well as the final jacobian calculated. This jacobian is then verified by calculation of partial derivatives.

The jacobian is a useful tool in motion planning, as it allows the designer a convenient tool for transforming velocities from joint space into task space. The inverse of the jacobian may be used for the reverse operation, calculating joint velocity commands to achieve a task space trajectory.

2 Derivation of Jacobian Matrix

2.1 Definition of Wrist Frame

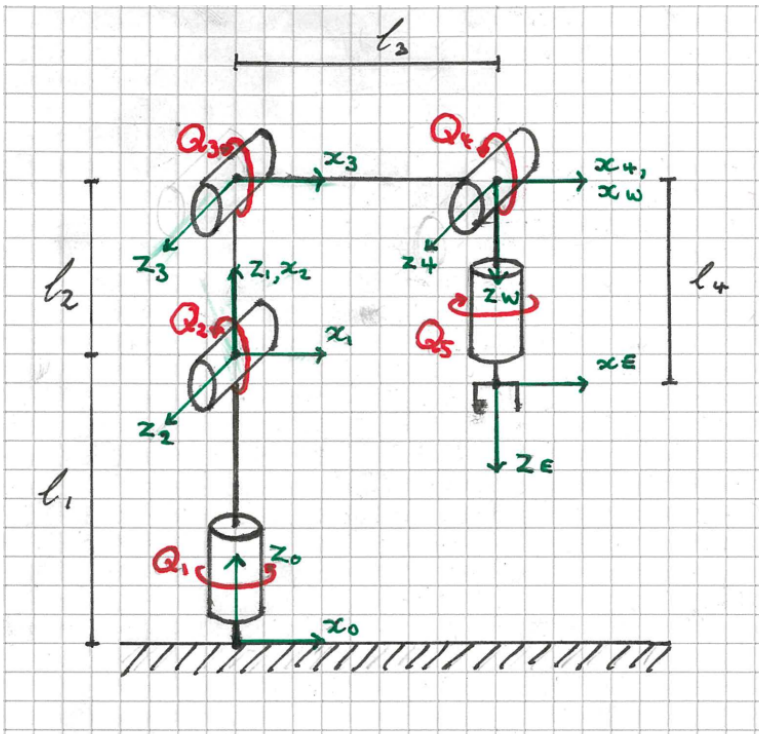

Figure 1: Kinematics Frame Definition

To calculate the robot jacobian, the forward kinematics are augmented with a wrist frame. The wrist frame is defined at the intersection of the z_4 and z_5 axis, with the same orientation as the end effector. Figure 1, shows the frame definition with this additional frame.

Building on the kinematic definition outlined in assignment 1, the DH-table for the robot has been augmented with an additional line to represent the wrist frame, this is shown in table 1.

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	L_1	q_1
2	0	90°	0	q_2 + 90°
3	L_2	0	0	q_3 - 90°
4	L_3	0	0	q_4
E	0	90°	L_4	q_5
W	0	90°	0	q_5

Table 1: Augmented DH table, showing wrist frame

The transformation from frame 4 to the wrist frame is hence defined as:

$$ {}^{4}_{W}T = \begin{bmatrix} cos(q_5) & -sin(q_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ sin(q_1) & cos(q_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2.2 Jacobian Definition

Jenghis Khan has only revolute joints, so each column vector component of the Robot jacobian can be derived using the same expression. To calculate the Jacobian with respect to the wrist frame defined in 2.1 the Jacobian can be represented as:

$$ \begin{bmatrix} \dot{x} \\ \omega \end{bmatrix} = \begin{bmatrix} J_V \\ J_\omega \end{bmatrix} \dot{Q} $$

With the jacobian with respect to a single revolute joint expressed in the inertial frame as:

$$ \begin{bmatrix} J_{vi} \\ J_{\omega i} \end{bmatrix} = \begin{bmatrix} {}^{0}\hat{z}_i \times^{0} r_{iW} \\ {}^{0}\hat{z}_i \end{bmatrix} $$

Where:

${}^{0}\hat{z}_i$ is the unit z vector for frame i expressed in frame 0

${}^{0}r_{iW} =^{0} r_W -^{0} r_i$ is the position of frame W with respect to frame i expressed in frame 0

2.3 Derivation of Components

The components of the Jacobian are obtained and rotated into the inertial frame 0 such that:

$$ {}^{0}\hat{z}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ {}^{0}r_{1W} = \begin{bmatrix} cos(q_1) \cdot (L_3 cos(q_2 + q_3) - L_2 sin(q_2)) \\ sin(q_1) \cdot (L_3 cos(q_2 + q_3) - L_2 sin(q_2)) \\ (L_3 sin(q_2 + q_3) + L_2 cos(q_2) \end{bmatrix} $$

$$ {}^{0}\hat{z}_2 = \begin{bmatrix} sin(q_1) \\ -cos(q_1) \\ 0 \end{bmatrix} $$

$$^{0}r_{2W} = \begin{bmatrix} cos(q_1) \cdot (L_3cos(q_2+q_3) - L_2sin(q_2)) \\ sin(q_1) \cdot (L_3cos(q_2+q_3) - L_2sin(q_2)) \\ (L_3sin(q_2+q_3) + L_2cos(q_2) \end{bmatrix}$$

$$^{0}\hat{z_3} = \begin{bmatrix} sin(q_1) \\ -cos(q_1) \\ 0 \end{bmatrix}$$

$$^{0}r_{3W} = \begin{bmatrix} cos(q_1) \cdot L_3cos(q_2+q_3) \\ sin(q_1) \cdot L_3cos(q_2+q_3) \\ L_3sin(q_2+q_3) \end{bmatrix}$$

$$^{0}\hat{z_4} = \begin{bmatrix} sin(q_1) \\ -cos(q_1) \\ 0 \end{bmatrix}$$

$$^{0}r_{4W} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the constraint $q_4 = -(q_2 + q_3)$ derived in the previous report:

$$^{0}\hat{z_5} = \begin{bmatrix} sin(q_1+q_2+q_3+q_4)/2 + sin(q_2+q_3+q_4-q1)/2 \\ cos(q_2+q_3+q_4-q_1)/2 - cos(q_1+q_2+q_3+q_4)/2 \\ -cos(q_2+q_3+q4) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$^{0}r_{5W} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2.4 Jacobian Matrix

Using the components calculated in section 2.3, the Jacobian is calculated:

$$J = \begin{bmatrix} -S_1(L_3C_{23} - L_2S_2) & -C_1(L_3S_{23} + L_2C_2) & -L_3S_{23}C_1 & 0 & 0 \\ C_1(L_3C_{23} - L_2S_2) & -S_1(L_3S_{23} + L_2C_2) & -L_3S_{23}S_1 & 0 & 0 \\ 0 & L_3C_{23} - L_2S_2 & L_3C_{23} & 0 & 0 \\ 0 & S_1 & S_1 & S_1 & 0 \\ 0 & -C_1 & -C_1 & -C_1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Where:

S_i abbreviates $sin(q_i)$

C_i abbreviates $cos(q_i)$

C_{ij} abbreviates $cos(q_i + q_j)$

L_2 is defined as the length from joint 2 to joint 3

L_3 is defined as the length from joint 3 to joint 4

3 Verification of Jacobian Matrix

3.1 Partial Derivatives

To verify the Jacobian matrix, the velocity of the wrist frame will be calculated in the inertial frame using partial derivatives. The vector r_{0W} is first calculated using the transformation matrix from frame 4 to frame 0 (${}^0_4T$).

$$ {}^0_4T = {}^0_1T {}^1_2T {}^2_3T {}^3_4T $$

$$ {}^0_4T = \begin{bmatrix} S_1S_5 + C_5C_1 & C_5S_1 - S_5C_1 & 0 & C_1(L_3C_{23} - L_2S_2) \\ C_5S_1 - C_1S_5 & -C_1C_5 - S_5S_1 & 0 & S_1(L_3C_{23} - L_2S_2) \\ 0 & 0 & -1 & L_1 + L_3S_{23} + L_2C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The vector r_{0W} can be found in column 4 of ${}^0_4T$. The first three elements of this column correspond to the x, y and z components of r_{0W} respectively.

$$ \begin{aligned} x &= C_1(L_3C_{23} - L_2S_2) \\ y &= S_1(L_3C_{23} - L_2S_2) \\ z &= L_1 + L_3S_{23} + L_2C_2 \end{aligned} $$

Taking partial derivatives of each of the above equations with respect to the generalised coordinates q_1, q_2 and q_3 (q_4 and q_5 are omitted as x, y and z are independent of these and hence all equal 0) generates the following:

$$ \begin{aligned} \frac{\partial x}{\partial q_1} &= [-S_1(L_3C_{23} - L_2S_2)]\dot{q_1} & \frac{\partial x}{\partial q_2} &= [-C_1(L_3S_{23} + L_2C_2)]\dot{q_2} & \frac{\partial x}{\partial q_3} &= [-L_3S_{23}C_1]\dot{q_3} \\ \frac{\partial y}{\partial q_1} &= [C_1(L_3C_{23} - L_2S_2)]\dot{q_1} & \frac{\partial y}{\partial q_2} &= [-S_1(L_3S_{23} + L_2C_2)]\dot{q_2} & \frac{\partial y}{\partial q_3} &= [-L_3S_{23}S_1]\dot{q_3} \\ \frac{\partial z}{\partial q_1} &= 0 & \frac{\partial z}{\partial q_2} &= [L_3C_{23} - L_2S_2]\dot{q_2} & \frac{\partial z}{\partial q_3} &= [L_3C_{23}]\dot{q_3} \end{aligned} $$

These partial derivatives match the top three rows of the Jacobian Matrix, hence validating it mathematically.

3.2 Numerical Verification

As another point of validation, a state was tested to see if hand calculated velocity values matched those generated using the Jacobian Matrix. The state that was tested was with the robot in its home configuration (refer to Figure 1 and with $\dot{q_1} = 10rad/s$ (all other $\dot{q_i} = 0$). Note, $L_3 = 200mm$. The velocity is calculated below:

$$ \begin{aligned} \dot{r}_{0W} &= \omega \times r_{0W} \\ \dot{r}_{0W} &= \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \times \begin{bmatrix} 0.2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} m/s \end{aligned} $$

Calculating using the Jacobian Matrix equates to the following:

$$\begin{aligned}
\dot{r}_{0W} &= \begin{bmatrix} -S_1(L_3C_{23} - L_2S_2) & -C_1(L_3S_{23} + L_2C_2) & -L_3S_{23}C_1 & 0 & 0 \\ C_1(L_3C_{23} - L_2S_2) & -S_1(L_3S_{23} + L_2C_2) & -L_3S_{23}S_1 & 0 & 0 \\ 0 & L_3C_{23} - L_2S_2 & L_3C_{23} & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&= -10sin(q_1)(L_3cos(q_2 + q_3) - L_2sin_(q_2) \\
&= \begin{bmatrix} -10sin(0)(0.2cos(0) - 0.2sin_(0)) \\ 10cos(0)(0.2cos(0) - 0.2sin_(0)) \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} m/s
\end{aligned}$$

This matches the velocity vector calculated using a manual calculation.

4 Conclusion

This report has outlined the process for generating the Jacobian Matrix for the use of calculating velocities in the task space based on velocities in the joint space. By using this method, the solving method has been reduced from calculating many partial derivatives which can be computationally expensive to using relatively simple matrix multiplication. The next step will be to take the inverse of the Jacobian to generate joint velocity commands to control a task space trajectory.