

# Homework #4

20171621 민대인

## 1. 목차

내용
1. 목차
2. 스크린샷
3. 간단한 소감

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localhost:8888/notebooks/Lecture\_5/ch5\_data.ipynb ☆ -

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Run Code

```
In [13]: # 리스트 5-1-(1)
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline

#데이터 생성 -----
np.random.seed(seed=1) # 난수를 고정
X_min = 4
X_max = 30
X_n = 16
X = 5 + 25*np.random.rand(X_n)
Prm_c = [170, 108, 0.2]
T = Prm_c[0] - Prm_c[1] * np.exp(-Prm_c[2] * X) / + 4 * np.random.randn(X_n)
np.savez('ch5_data.npz', X=X, X_min=X_min, X_max=X_max, X_n=X_n, T=T)
```

```
In [14]: # 리스트 5-1-(2)
print(X)
```

```
[15.42555012 23.00811234 5.00285937 12.55831432 8.66889727 7.30846487
 9.65650528 13.63901818 14.91918686 18.47041835 15.47986286 22.13048751
10.11130624 26.95293591 5.68468983 21.76168775]
```

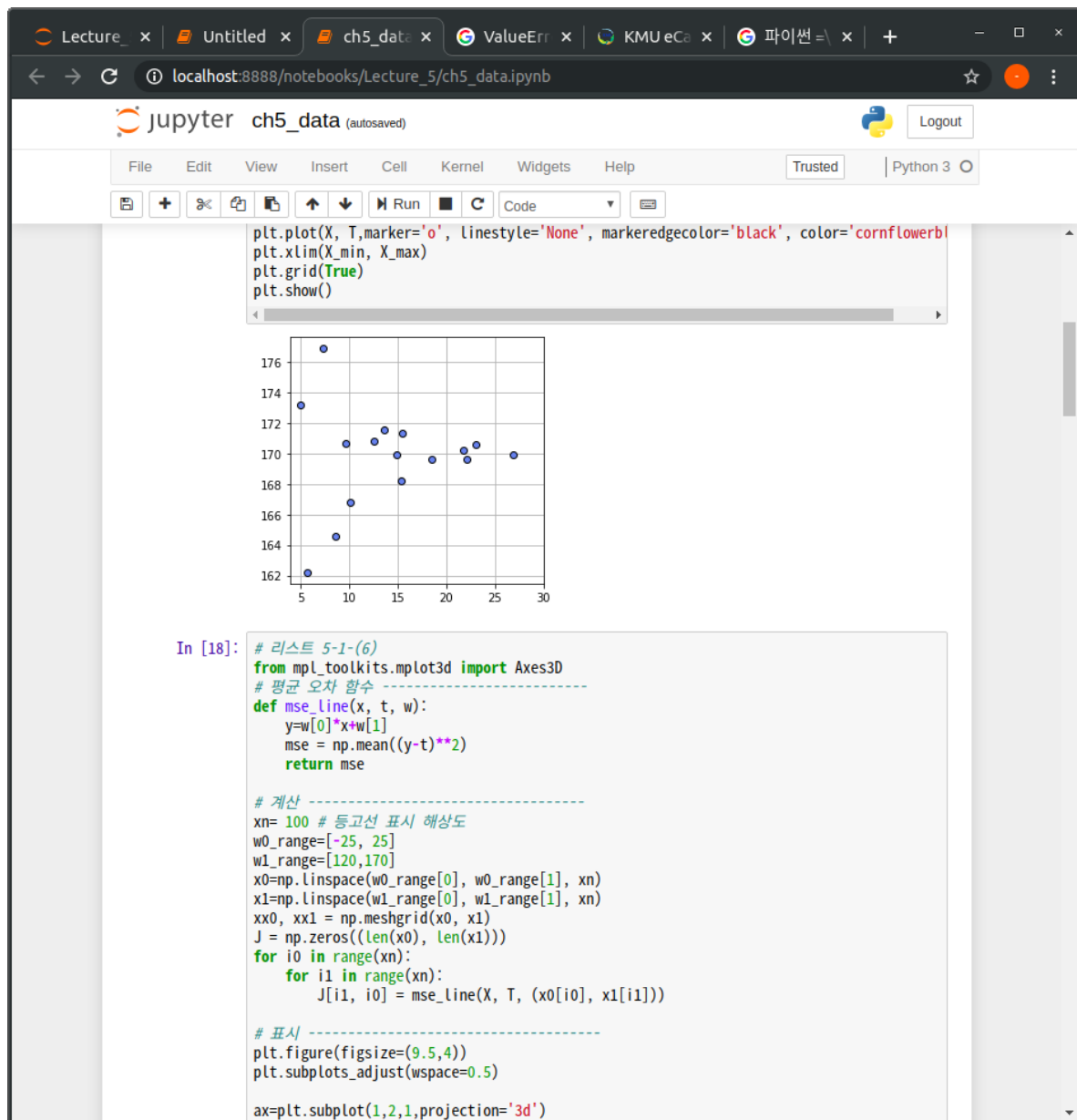
```
In [15]: # 리스트 5-1-(3)
print(np.round(X, 2))
```

```
[15.43 23.01 5. 12.56 8.67 7.31 9.66 13.64 14.92 18.47 15.48 22.13
10.11 26.95 5.68 21.76]
```

```
In [16]: # 리스트 5-1-(4)
print(np.round(T, 2))
```

```
[168.19 170.56 173.2 170.84 164.59 176.89 170.67 171.55 169.94 169.61
171.34 169.63 166.78 169.94 162.2 170.24]
```

```
In [17]: # 리스트 5-1-(5)
# 데이터 그래프 -----
plt.figure(figsize=(4,4))
plt.plot(X, T, marker='o', linestyle='None', markeredgecolor='black', color='cornflowerbl
plt.xlim(X_min, X_max)
plt.grid(True)
plt.show()
```



Lecture\_ x | Untitled x | ch5\_data x | ValueErr x | KMUEC x | 파이썬 x | +

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 Code

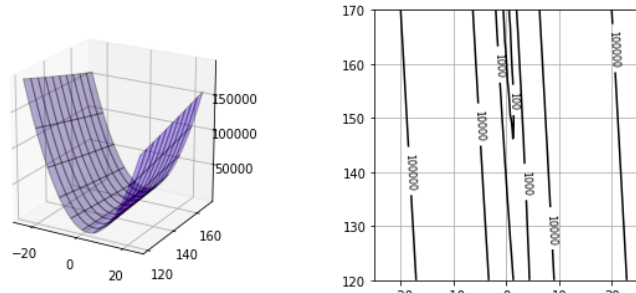
```

# 표시 -----
plt.figure(figsize=(9.5,4))
plt.subplots_adjust(wspace=0.5)

ax=plt.subplot(1,2,1,projection='3d')
ax.plot_surface(xx0,xx1,J,rstride=10, cstride=10, alpha=0.3, color='blue', edgecolor='b')
ax.set_xticks([-20,0,20])
ax.set_yticks([120,140,160])
ax.view_init(20,-60)

plt.subplot(1,2,2)
cont=plt.contour(xx0,xx1, J ,30, colors='black', levels=[100,1000,10000,100000])
cont.clabel(fmt='%1.0f', fontsize=8)
plt.grid(True)
plt.show()

```



In [19]:

```

# 리스트 5-1-(7)
# 평균 제곱 오차의 기울기 -----
def dmse_line(x, t, w):
    y=w[0]*x+w[1]
    d_w0=2*np.mean((y-t)*x)
    d_w1=2*np.mean(y-t)
    return d_w0, d_w1

```

In [20]:

```

# 리스트 5-1-(8)
d_w = dmse_line(X, T, [10, 165])
print(np.round(d_w,1))

```

[4841. 279.]

Lecture\_ x | Untitled x | ch5\_data x | ValueErr x | KMU eCa x | 파이썬 x | + - □ x

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In [20]:

```
# 리스트 5-1-(8)
d_w = dmse_line(X, T, [10, 165])
print(np.round(d_w,1))
```

[4841. 279.]

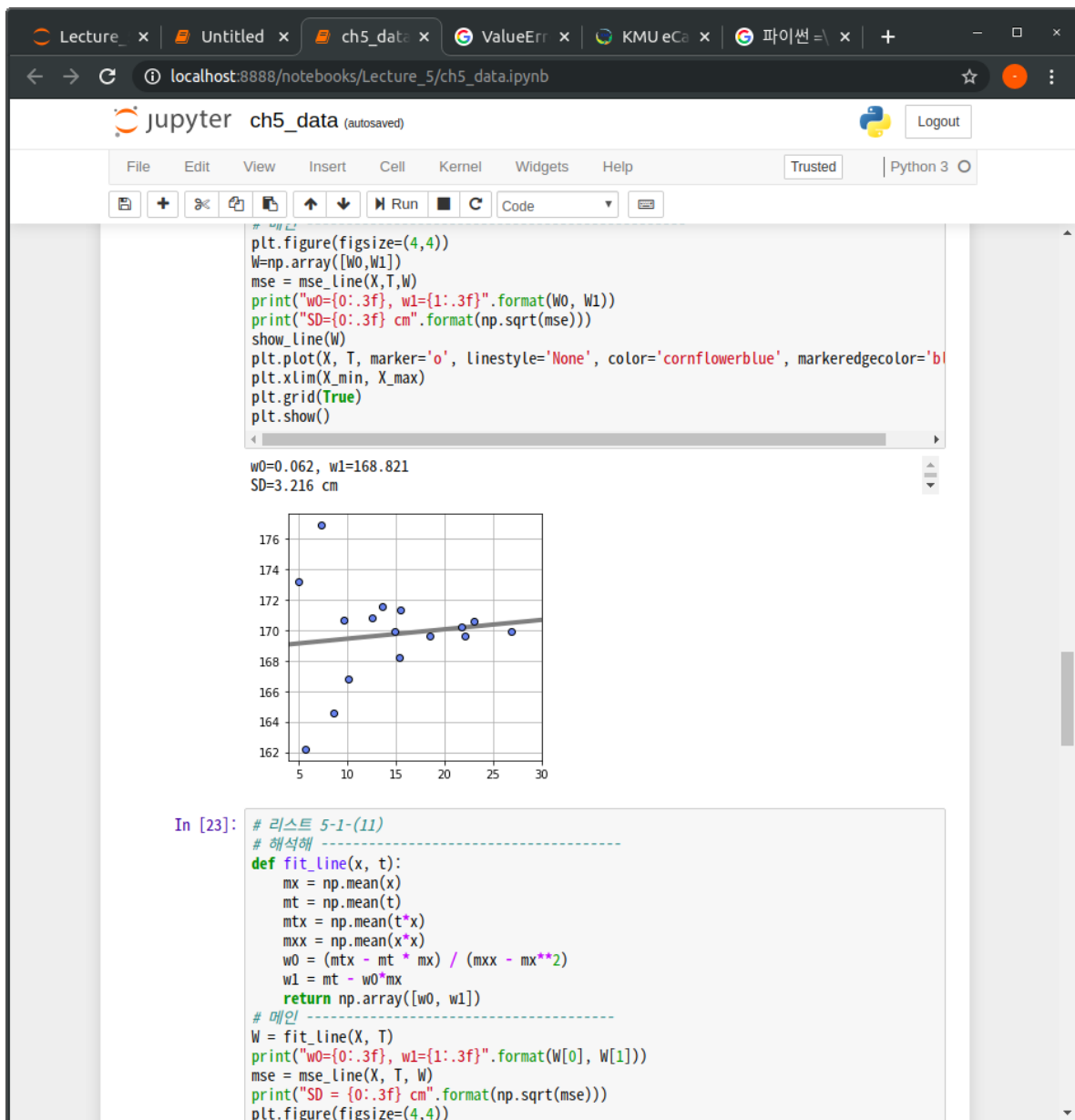
In [21]:

```
# 리스트 5-1-(9)
# 경사 하강법 -----
def fit_line_num(x,t):
    w_init = [10.0, 165.0]
    alpha = 0.001
    i_max = 100000
    eps = 0.1
    w_i = np.zeros([i_max, 2])
    w_i[0, :] = w_init
    for i in range(1, i_max):
        dmse = dmse_line(x, t, w_i[i-1])
        w_i[i, 0] = w_i[i-1, 0] - alpha * dmse[0]
        w_i[i, 1] = w_i[i-1, 1] - alpha * dmse[1]
        if max(np.absolute(dmse)) < eps:
            break
    w0 = w_i[i, 0]
    w1 = w_i[i, 1]
    wi = w_i[:, :]
    return w0, w1, dmse, w_i

# 메인 -----
plt.figure(figsize=(4,4))
xn=100 # 등고선 해상도
w0_range = [-25, 25]
w1_range = [120, 170]
x0 = np.linspace(w0_range[0], w0_range[1], xn)
x1 = np.linspace(w1_range[0], w1_range[1], xn)
xx0, xx1 = np.meshgrid(x0, x1)
J = np.zeros((len(x0), len(x1)))
for i0 in range(xn):
    for i1 in range(xn):
        J[i1, i0] = mse_line(X, T, (x0[i0], x1[i1]))
cont = plt.contour(xx0, xx1, J, 30, colors='black', levels=(100,1000,10000,100000))
cont.clabel(fmt='%0.1f', fontsize=8)
plt.grid(True)

# 경사 하강법 호출
W0, W1, dMSE, W_history = fit_line_num(X, T)
# 결과보기
```





Lecture\_ x | Untitled x | ch5\_data x | ValueError x | KMU eC x | 파이썬 = x | + - □ ×

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Run Code

```
# 데이터 -----
W = fit_line(X, T)
print("w0={0:.3f}, w1={1:.3f}".format(W[0], W[1]))
mse = mse_line(X, T, W)
print("SD = {0:.3f} cm".format(np.sqrt(mse)))
plt.figure(figsize=(4,4))
show_line(W)
plt.plot(X, T, marker='o', linestyle='None', color='cornflowerblue', markeredgecolor='b')
plt.xlim(X_min, X_max)
plt.grid(True)
plt.show()
```

w0=0.044, w1=169.125  
SD = 3.214 cm

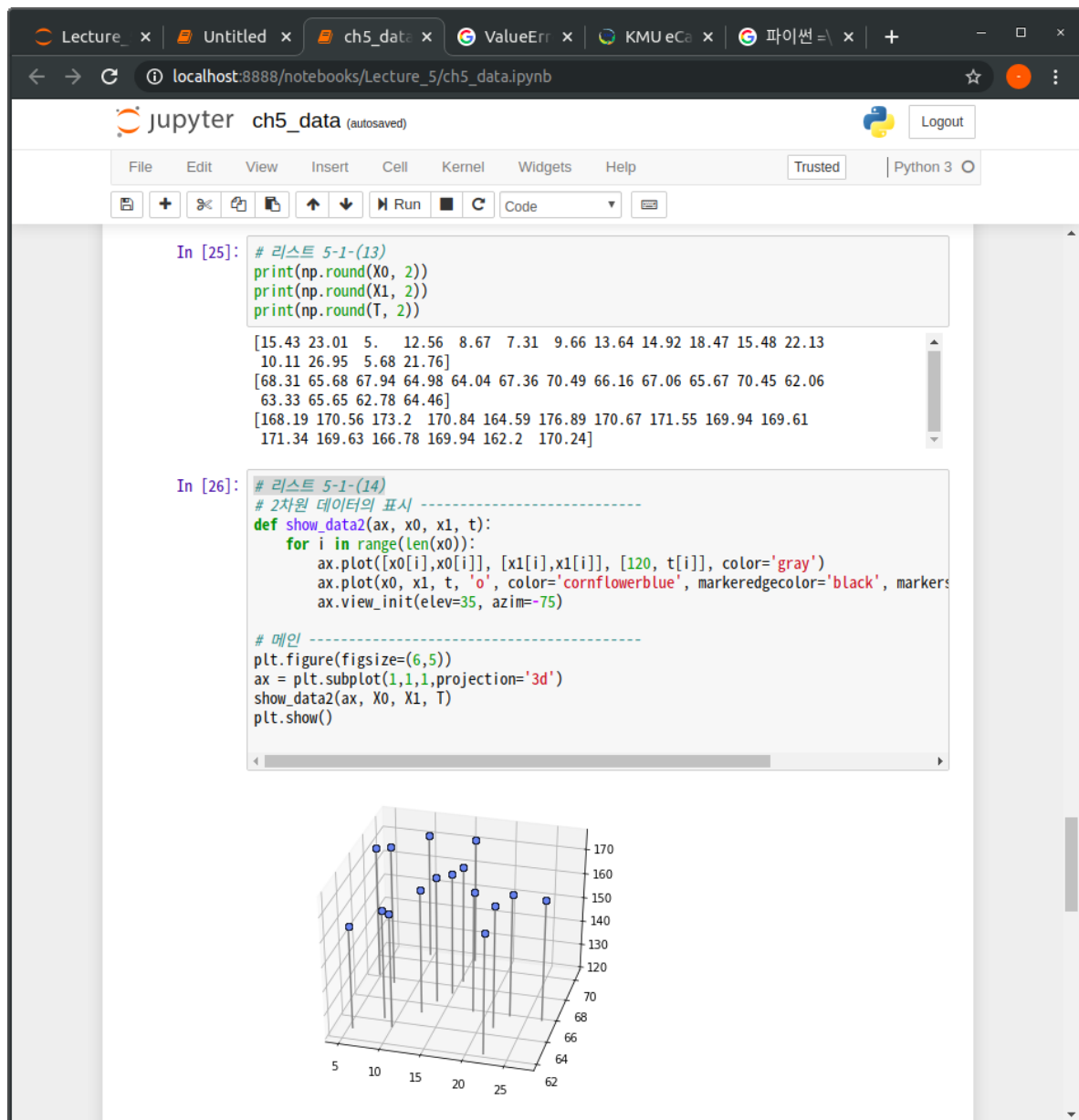
In [24]:

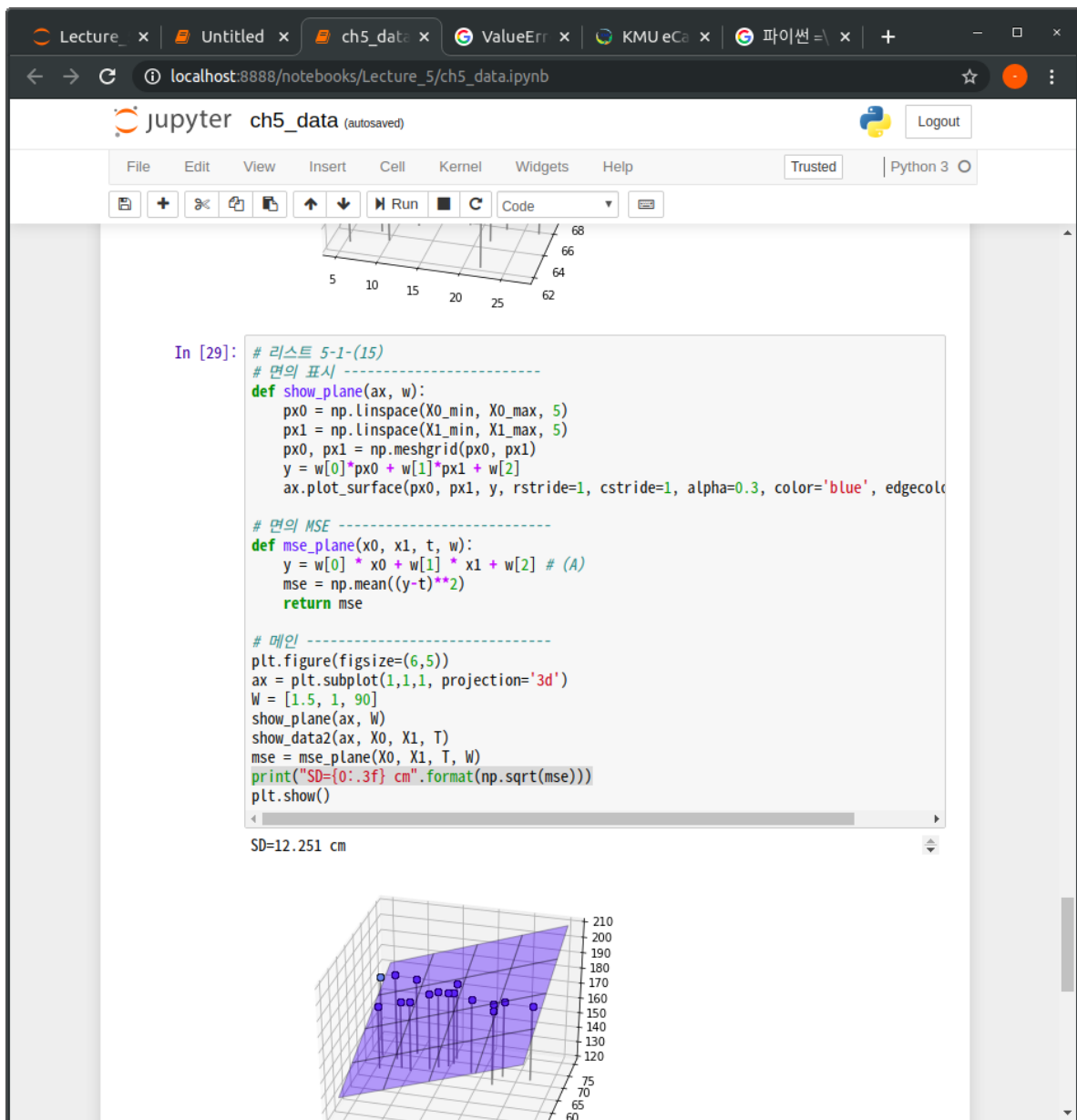
```
# 리스트 5-1-(12)
# 2차원 데이터 생성 -----
X0= X
X0_min = 5
X0_max =30
np.random.seed(seed=1) # 난수를 고정
X1 = 23*(T/100)**2 + 2 * np.random.randn(X_n)
X1_min =40
X1_max = 75
```

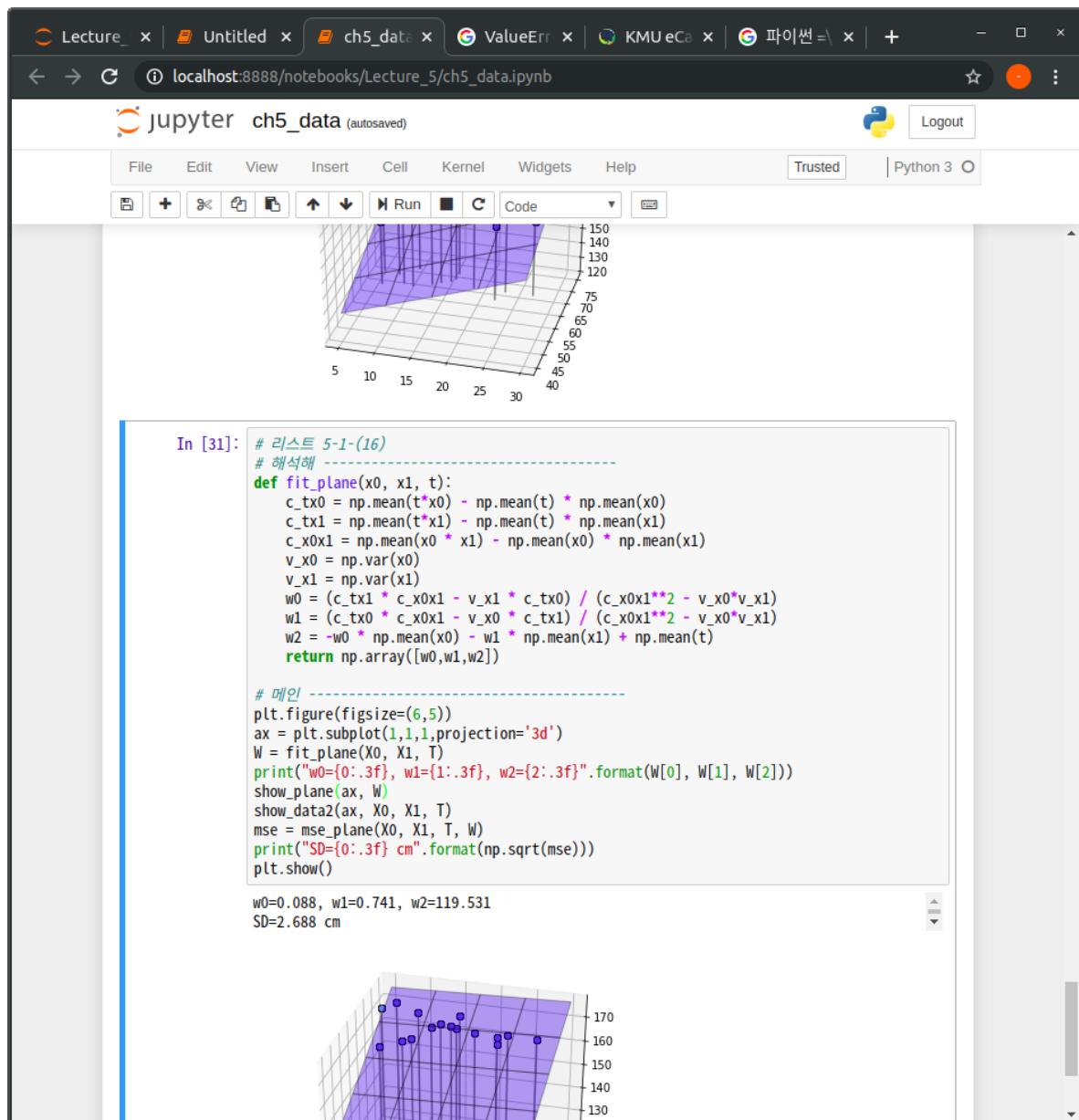
In [25]:

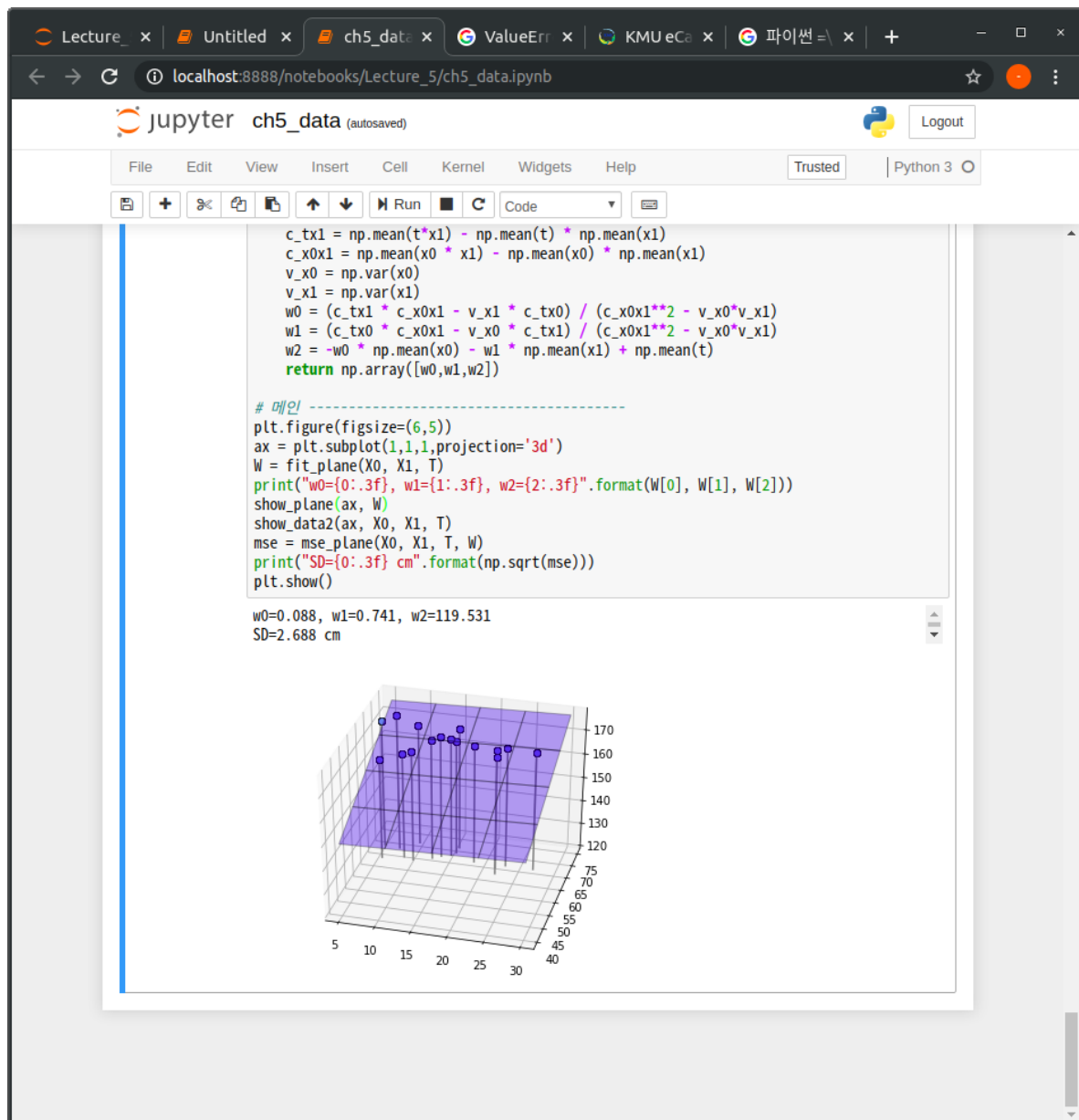
```
# 리스트 5-1-(13)
print(np.round(X0, 2))
print(np.round(X1, 2))
print(np.round(T, 2))
```

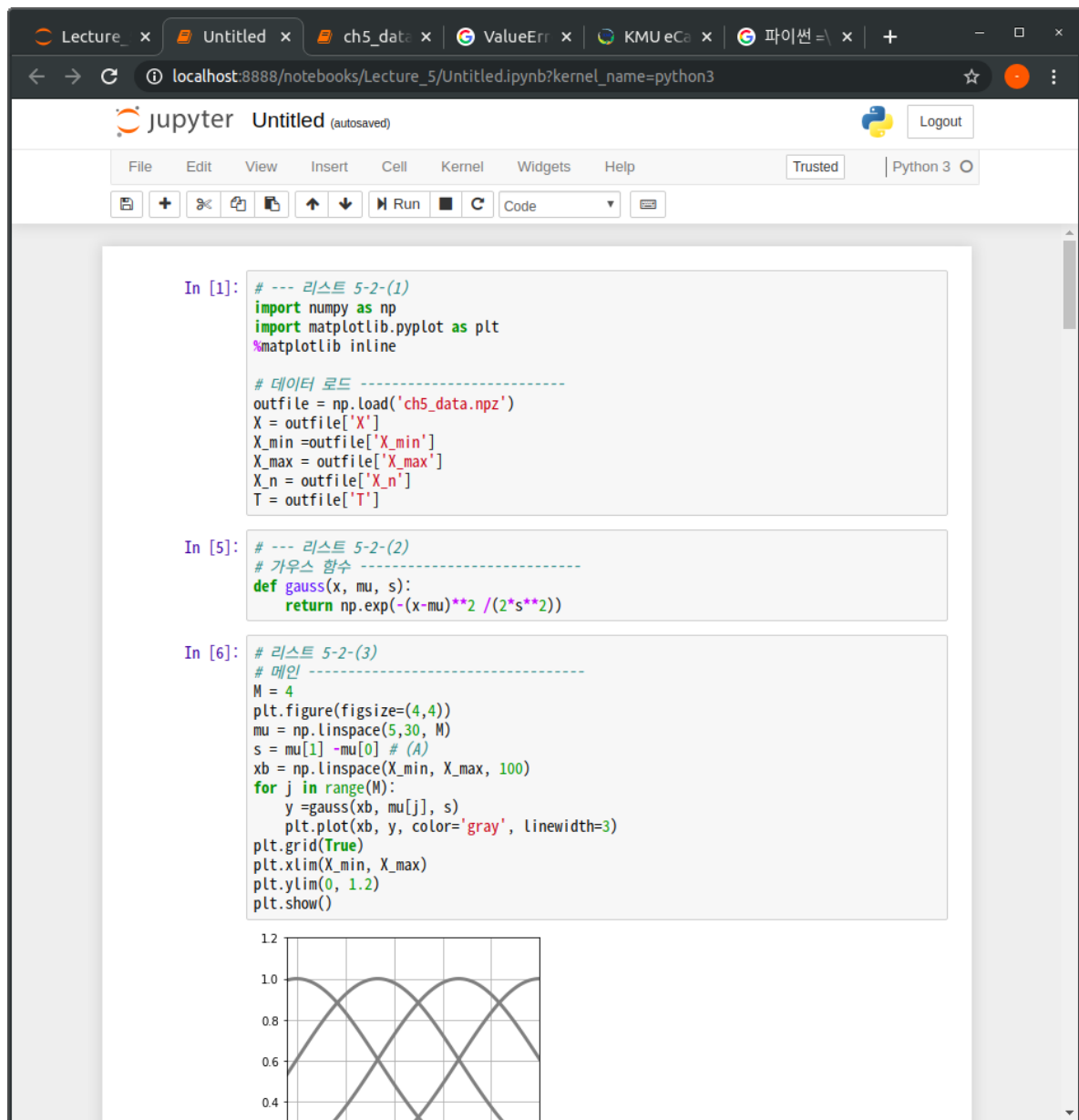












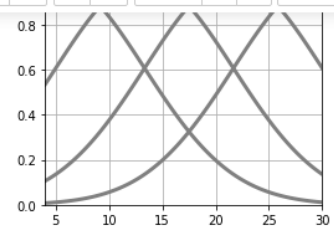
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Run Code



```

In [7]: # 리스트 5-2-(4)
# 선형 기저 함수 모델 -----
def gauss_func(w, x):
    m = len(w) - 1
    mu = np.linspace(5, 30, m)
    s = mu[1] - mu[0]
    y = np.zeros_like(x) # x와 같은 크기로 요소가 0의 행렬 y를 작성
    for j in range(m):
        y = y + w[j] * gauss(x, mu[j], s)
    y = y + w[m]
    return y

```

```

In [77]: # 리스트 5-2-(5)
# 선형 기저 함수 모델 MSE -----
def mse_gauss_func(x, t, w):
    y = gauss_func(w, x)
    mse = np.mean((y - t)**2)
    return mse

```

```

In [78]: # 리스트 5-2-(6)
# 선형 기저 함수 모델 정확한 해 -----
def fit_gauss_func(x, t, m):
    mu = np.linspace(5, 30, m)
    s = mu[1] - mu[0]
    n = x.shape[0]
    psi = np.ones((n, m+1))
    for j in range(m):
        psi[:, j] = gauss(x, mu[j], s)
    psi_T = np.transpose(psi)

    b = np.linalg.inv(psi_T.dot(psi))
    c = b.dot(psi_T)
    w = c.dot(t)
    return w

```



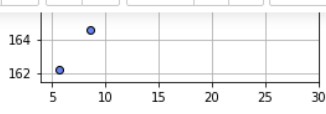
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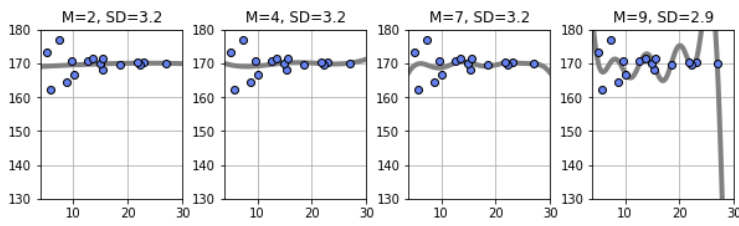
Code



```

In [80]: # 리스트 5-2-(8)
plt.figure(figsize=(10,2.5))
plt.subplots_adjust(wspace=0.3)
M = [2, 4, 7, 9]
for i in range(len(M)):
    plt.subplot(1, len(M), i+1)
    W = fit_gauss_func(X, T, M[i])
    show_gauss_func(W)
    plt.plot(X, T, marker='o', linestyle='None', color='cornflowerblue', markeredgcolor='black')
    plt.xlim(X_min, X_max)
    plt.grid(True)
    plt.ylim(130, 180)
    mse = mse_gauss_func(X, T, W)
    plt.title("M={0:d}, SD={1:.1f}".format(M[i], np.sqrt(mse)))

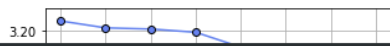
```



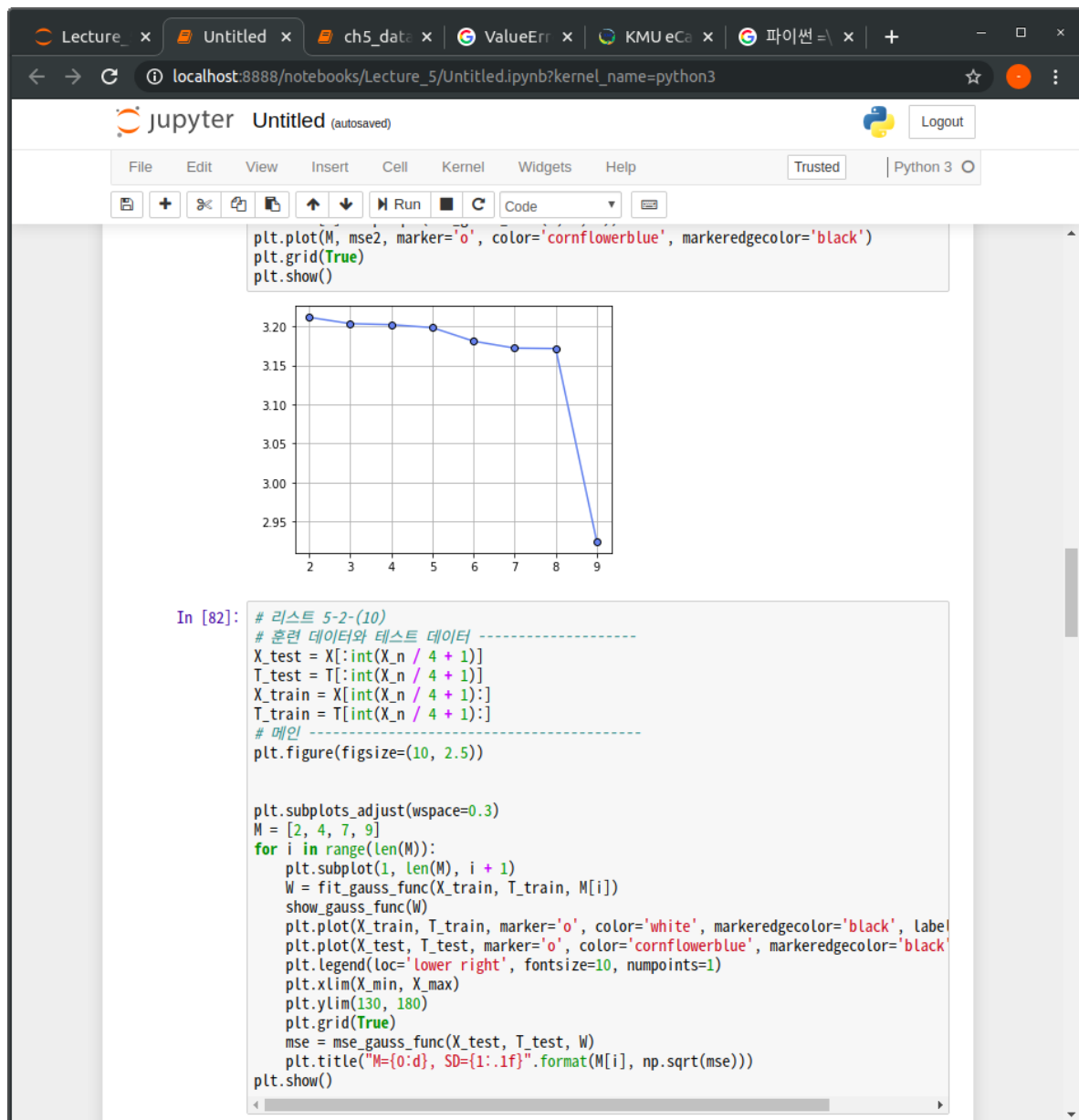
```

In [81]: # 리스트 5-2-(9)
plt.figure(figsize=(5, 4))
M = range(2, 10)
mse2 = np.zeros(len(M))
for i in range(len(M)):
    W = fit_gauss_func(X, T, M[i])
    mse2[i] = np.sqrt(mse_gauss_func(X, T, W))
plt.plot(M, mse2, marker='o', color='cornflowerblue', markeredgcolor='black')
plt.grid(True)
plt.show()

```







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Run
 Code

```

plt.grid(True)
mse = mse_gauss_func(X_test, T_test, W)
plt.title("M={0:d}, SD={1:.1f}".format(M[i], np.sqrt(mse)))
plt.show()

```

In [83]:

```

# 리스트 5-2-(11)
plt.figure(figsize=(5, 4))
M = range(2, 10)
mse_train = np.zeros(len(M))
mse_test = np.zeros(len(M))
for i in range(len(M)):
    W = fit_gauss_func(X_train, T_train, M[i])
    mse_train[i] = np.sqrt(mse_gauss_func(X_train, T_train, W))
    mse_test[i] = np.sqrt(mse_gauss_func(X_test, T_test, W))
plt.plot(M, mse_train, marker='o', markerfacecolor='white', markeredgcolor='black', color='black')
plt.plot(M, mse_test, marker='o', markerfacecolor='white', markeredgcolor='black', color='blue')
plt.legend(loc='upper left', fontsize=10)
plt.ylim(0, 12)

```

Out[83]: (0.0, 12.0)

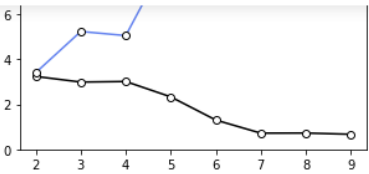
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Run



```

In [87]: # 리스트 5-2-(12)
# K겹 교차 검증 -----
def kfold_gauss_func(x, t, m, k):
    n = x.shape[0]
    mse_train = np.zeros(k)
    mse_test = np.zeros(k)
    for i in range(0, k):
        x_train = x[np.fmod(range(n), k) != i] # (A)
        t_train = t[np.fmod(range(n), k) != i] # (A)
        x_test = x[np.fmod(range(n), k) == i] # (A)
        t_test = t[np.fmod(range(n), k) == i] # (A)
        wm = fit_gauss_func(x_train, t_train, m)

        mse_train[i] = mse_gauss_func(x_train, t_train, wm)
        mse_test[i] = mse_gauss_func(x_test, t_test, wm)
    return mse_train, mse_test
  
```

```

In [88]: # 리스트 5-2-(13)
np.fmod(range(10), 5)
  
```

```

Out[88]: array([0, 1, 2, 3, 4, 0, 1, 2, 3, 4])
  
```

```

In [89]: # 리스트 5-2-(14)
M = 4
K = 4
kfold_gauss_func(X, T, M, K)
  
```

```

Out[89]: (array([ 9.25633247,  4.87774416,  5.57408757, 13.14858355]),
 array([ 23.55403547, 1308.00873405, 120.19314282,  2.56056657]))
  
```

```

In [108]: # 리스트 5-2-(15)
M = range(2, 8)
K = 16
Cv_Gauss_train = np.zeros((K, len(M)))
Cv_Gauss_test = np.zeros((K, len(M)))
for i in range(0, len(M)):
  
```

Lecture\_ x | Untitled x | ch5\_data x | ValueErr x | KMUEC x | 파이썬 x | +

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in [108]:

```

# 리스트 5-2-(15)
M = range(2, 8)
K = 16
Cv_Gauss_train = np.zeros((K, len(M)))
Cv_Gauss_test = np.zeros((K, len(M)))
for i in range(0, len(M)):
    Cv_Gauss_train[:, i], Cv_Gauss_test[:, i] = kfold_gauss_func(X, T, M[i], K)
    mean_Gauss_train = np.sqrt(np.mean(Cv_Gauss_train, axis = 0))
    mean_Gauss_test = np.sqrt(np.mean(Cv_Gauss_test, axis = 0))

plt.figure(figsize=(4, 3))
plt.plot(M, mean_Gauss_train, marker='o', color='k', markerfacecolor='w', label='training')
plt.plot(M, mean_Gauss_test, marker='o', color='cornflowerblue', markeredgecolor='black')
plt.legend(loc='upper left', fontsize=10)
plt.ylim(0, 20)
plt.grid(True)
plt.show()

```

M	training (mean)	test (mean)
2	~3.0	~4.0
3	~3.0	~4.5
4	~3.0	~5.0
5	~3.0	~6.0
6	~3.0	~7.0
7	~3.0	~9.5

in [109]:

```

# 리스트 5-2-(16)
M = 3
plt.figure(figsize=(4,4))
W = fit_gauss_func(X, T, M)
show_gauss_func(W)
plt.plot(X, T, marker='o', linestyle='None', color='cornflowerblue', markeredgecolor='b')
plt.xlim([X_min, X_max])
plt.grid(True)
mse = mse_gauss_func(X, T, W)
print("SD={0:.2f} cm".format(np.sqrt(mse)))
plt.show()

```

SD=3.20 cm

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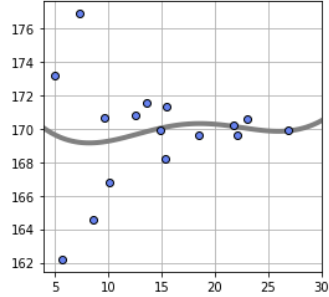
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```

mse = mse_gauss_func(x, t, w)
print("SD={0:.2f} cm".format(np.sqrt(mse)))
plt.show()

```

SD=3.20 cm



```

In [111]: # 리스트 5-2-(17)
# 모델 A -----
def model_A(x, w):
    y = w[0] - w[1] * np.exp(-w[2] * x)
    return y

# 모델 A 표시 -----
def show_model_A(w):
    xb = np.linspace(X_min, X_max, 100)
    y = model_A(xb, w)
    plt.plot(xb, y, c=[.5, .5, .5], lw = 4)

# 모델 A의 MSE -----
def mse_model_A(w, x, t):
    y = model_A(x, w)
    mse = np.mean((y - t)**2)
    return mse

```

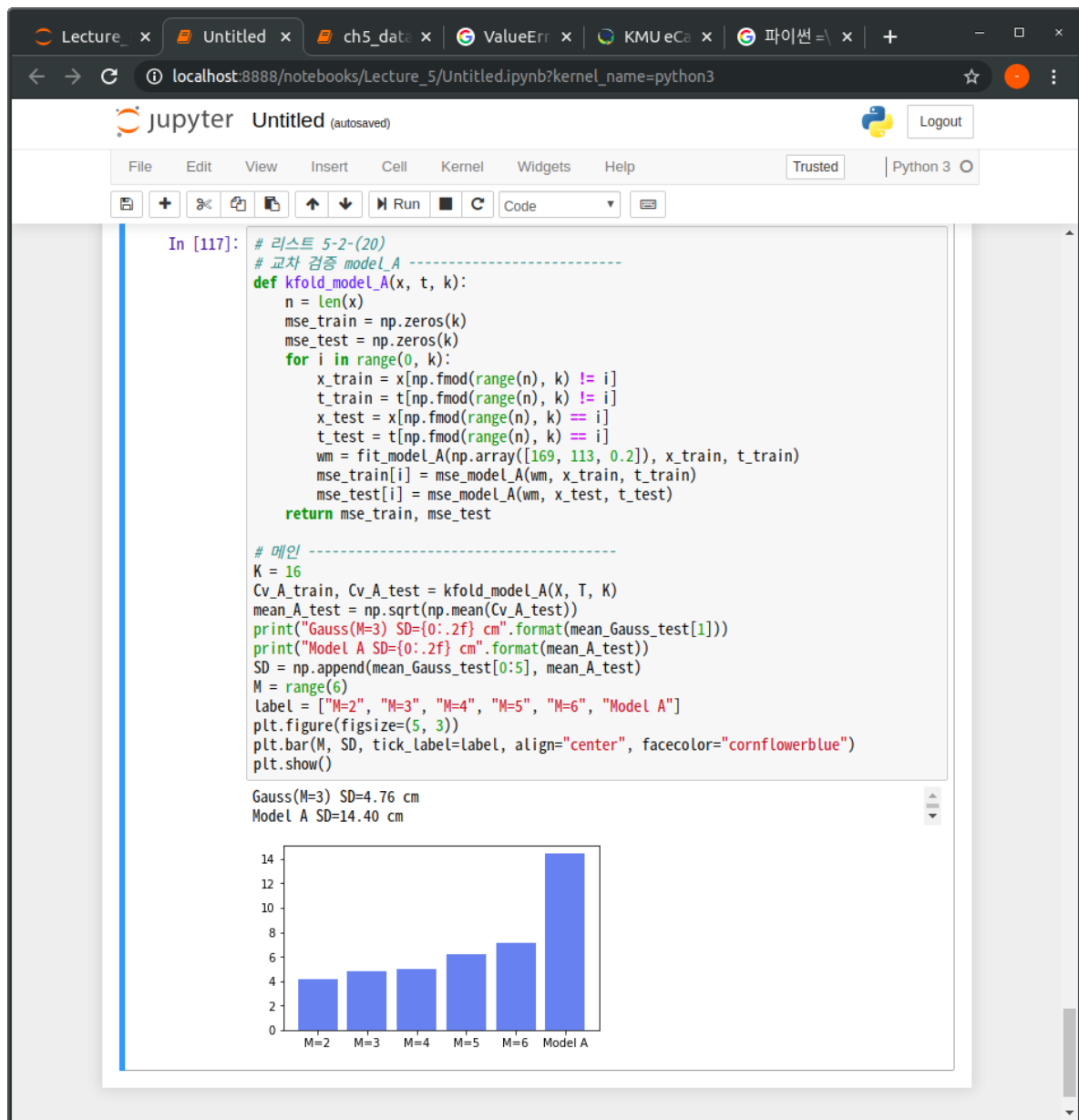
```

In [112]: # 리스트 5-2-(18)
from scipy.optimize import minimize

# 모델 A의 매개 변수 최적화 -----
def fit_model_A(w_init, x, t):
    res1 = minimize(mse_model_A, w_init, args=(x, t), method="powell")

```





### 3. 간단한 소감

그래프를 그려보면 데이터 시각화를 재밌게 공부할 수 있었다.