

## ● HW 2

### I. Two sample data with equal variance case and without outliers

#### A. Normal Distribution

We want to generate  $X_1, \dots, X_m$  from  $N(\mu_x, \sigma_x^2)$  and  $Y_1, \dots, Y_n$  from  $N(\mu_y, \sigma_y^2)$ , where  $\mu_x = 0$ ,  $\mu_y = \Delta$ ,  $\sigma_x^2 = \sigma_y^2 = 1$ .

1. For  $m = n = 5$  and  $\Delta = 0$ , generate one set of samples and perform
  - (1) Two sample t-test with the equal variance assumption
  - (2) Two sample t-test without the equal variance assumption
  - (3) Wilcoxon rank sum test
  - (4) Permutation test using t-statistics with the equal variance assumption (1000 replication)

(5) Permutation test using t-statistics without the equal variance assumption (1000 replication)

2. Repeat the whole process 100 times and compute the type one errors
3. For  $m = n = 5$ , and  $\Delta = 1$ , repeat the whole process 100 times and compute the power
4. For  $m = n = 5$  and  $\Delta = 2$ , repeat the whole process 100 times and compute the power
5. For  $m = n = 5$  and  $\Delta = 3$ , repeat the whole process 100 times and compute the power
6. Draw a power graph where the x-axis represents the values of  $\Delta$  and the y-axis does the power. Make your conclusion.

## B. Normal + Gamma Distribution

Generate  $X_1, \dots, X_m$  from  $N(\mu_x, \sigma_x^2)$  and  $Y_1, \dots, Y_n$  from  $\Gamma(\alpha_y, \beta_y)$ , where  $\mu_y = \alpha_y \beta_y$ ,  $\sigma_y^2 = \alpha_y \beta_y^2$ . Assume  $\mu_x = 2$ ,  $\sigma_x^2 = 1$ . If we set  $\beta_y = 1/\sqrt{\alpha_y}$ , then  $\sigma_y^2 = 1$ .

1. For  $m = n = 5$ , choose the value of  $\alpha_y$  so that  $\Delta = 0$ .  
Perform the type one error analysis.
2. For  $m = n = 5$ , choose the several values of  $\alpha_y$  in a systematic manner for  $\Delta > 0$ . Perform the type power analysis.
3. Make your conclusion.

## C. Gamma Distributions

Generate  $X_1, \dots, X_m$  from  $\Gamma(\alpha_x, \beta_x)$ , where  $\mu_x = \sqrt{\alpha_x}$ ,  $\sigma_x^2 = 1$  and  $Y_1, \dots, Y_n$  from  $\Gamma(\alpha_y, \beta_y)$ , where  $\mu_y = \sqrt{\alpha_y}$ ,  $\sigma_y^2 = 1$ .

1. For  $m = n = 5$ , assume that  $\alpha_x = \alpha_y = 4$  so that  $\Delta = 0$ .  
Perform the type one error analysis.
2. For  $m = n = 5$ , fix  $\alpha_x = 4$  and choose the several values of  $\alpha_y$  in a systematic manner for  $\Delta > 0$ . Perform the type power analysis.
3. Make your conclusion.

## II. Two sample data with equal variance case and **with outliers**

Among the generated  $X$ 's, choose the largest value and add 5(?) to it. Perform the same analysis.

**A'. Normal Distribution**

**B'. Normal + Gamma Distribution**

**C'. Gamma Distributions**

### III. Two sample data with different variances and without outliers

A''. Normal Distribution

B''. Normal + Gamma Distribution

C''. Gamma Distributions

### IV. Two sample data with different variances and with outliers

A'''. Normal Distribution

B'''. Normal + Gamma Distribution

C'''. Gamma Distributions