Learning with differentiable and amortized optimization

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github.com/facebookresearch/presentations

Optimization is crucial technology

Optimization is a **modeling** and **decision-making** paradigm and **encodes reasoning operations**

Breakthroughs over the past century enabled by optimization advancements include:

1. **controlling systems** (robotic, autonomous, mechanical, and multi-agent)
2. **making operational decisions** based on future predictions
3. efficiently **transporting** or **matching** resources, information, and measures
   for **optimal transport** and **generative modeling**
4. **allocating** budgets and portfolios,
5. **designing** materials, molecules, and other structures,
6. **solving inverse problems**
   infers underlying hidden costs, incentives, geometries, terrains, and other structures
7. **parameter learning** of predictive and statistical models

Optimal control drives systems

Setting: deterministic, discrete-time system with a **continuous state-action space**

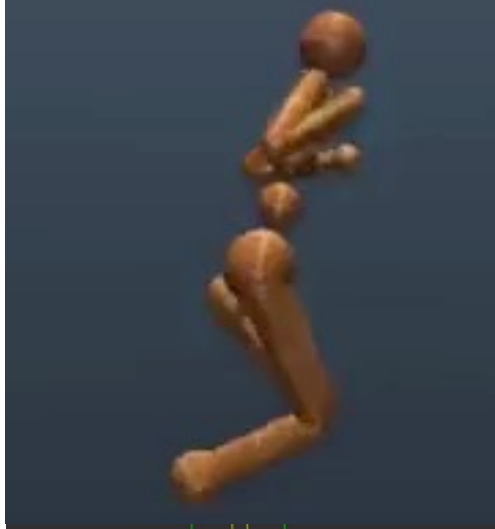
$$x^\star_{1:T}, u^\star_{1:T} \in \underset{x_{1:T}, u_{1:T}}{\operatorname{argmin}} \sum_t \overbrace{C_\theta(x_t, u_t)}^{\text{cost}} \text{ s.t. } \overbrace{x_1 = x_{\text{init}}}^{\text{initial state}} \quad \overbrace{x_{t+1} = f_\theta(x_t, u_t)}^{\text{dynamics}} \quad \overbrace{u_t \in \mathcal{U}}^{\text{constraints}}$$

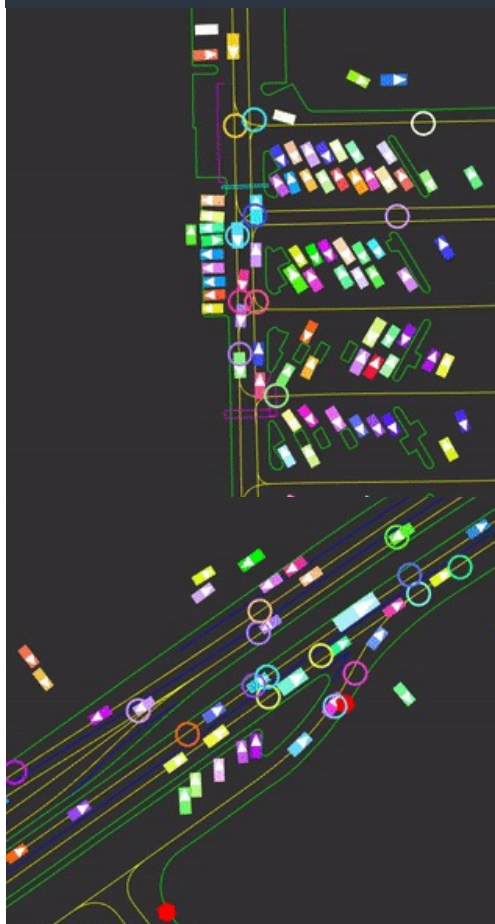
Full notation: $u^\star_{1:T}(x_{\text{init}}, f_\theta)$

Widely deployed over the past century in aviation, robotics, vehicles, HVAC
Often for a **Markov decision process** but doesn't have to be

The **real-world is non-convex**, so are our controllers
Convex in some cases, e.g., with quadratic cost/linear dynamics (LQR)

No learning necessary if we know the system — just pure optimization





On the model-based stochastic value gradient for continuous reinforcement. B. Amos et al., L4DC 2021
Nocturne: a driving benchmark for multi-agent learning. E. Vinitsky et al., NeurIPS Datasets and Benchmarks 2022

Optimal transport connects measures

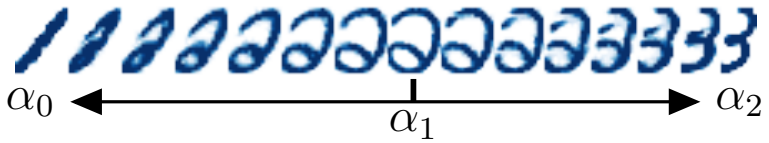
Setting: transport between measures α and β

Monge problem for Euclidean Wasserstein-2

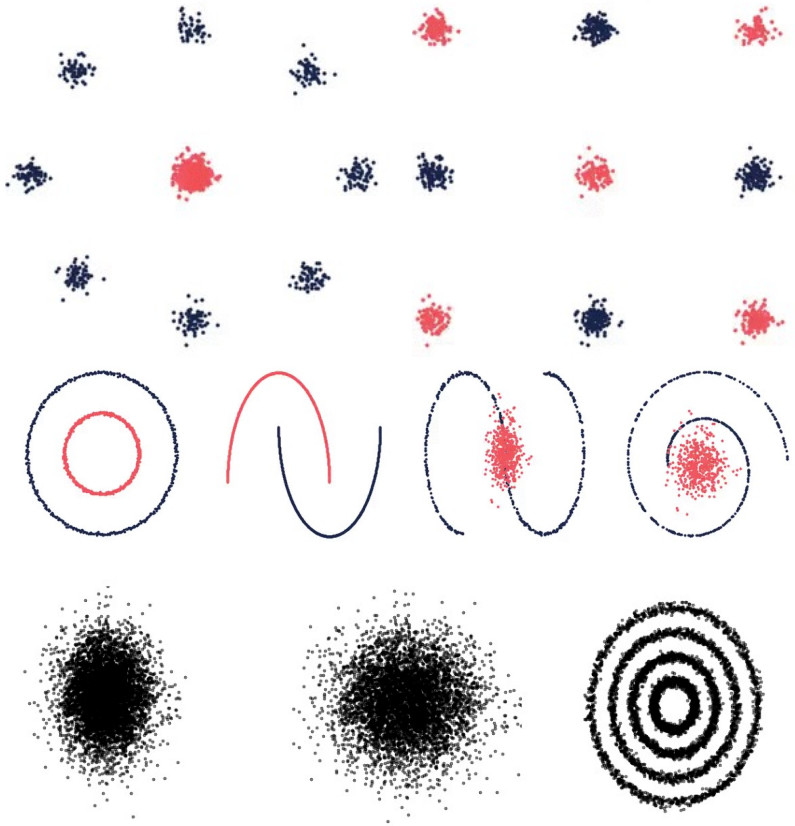
$$T^\star(\alpha,\beta) \in \underset{T\in\mathcal{C}(\alpha,\beta)}{\operatorname{argmin}} \mathbb{E}_{x\sim\alpha}\|x - T(x)\|_2^2$$

The transport map $T^\star$ **connects** α to β

No learning necessary, again just optimization



Meta Optimal Transport. Amos et al., 2022



On amortizing convex conjugates for optimal transport. Amos, 2022

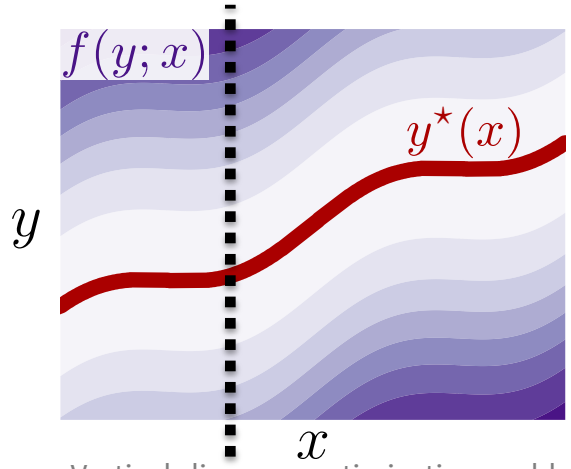
The magic of optimization

Find the **best way to interact** with a **representation of the world**

optimal solution — objective — context (or parameterization)

$$y^\star(x) \in \underset{y \in \mathcal{C}(x)}{\operatorname{argmin}} f(y; x)$$

optimization variable — constraints



Vertical slices are optimization problems

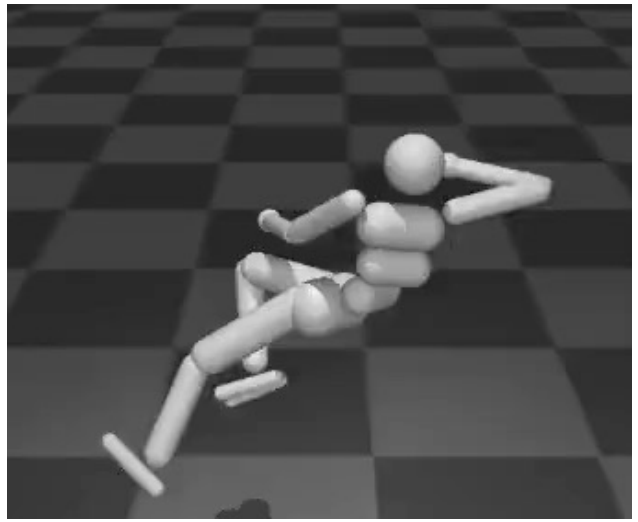
In control, x is the **system state**, y is the **action**, $f(y; x)$ is a **cost**, and $y^\star(x)$ is an **optimal action**

When optimization fails, machine learning helps

$$y^\star(x) \in \underset{y \in \mathcal{C}(x)}{\operatorname{argmin}} f(y; x)$$

Optimization starts failing us when:

1. Analytically **encoding every detail** into f and $\mathcal{C}$ is impossible
   - May be **unknown, mis-specified,** or **inaccurate**
   - Especially difficult with **high-dimensional data** (text, images, video)
2. **Solving** for $y^\star(x)$ is **intractable** and **difficult to compute**

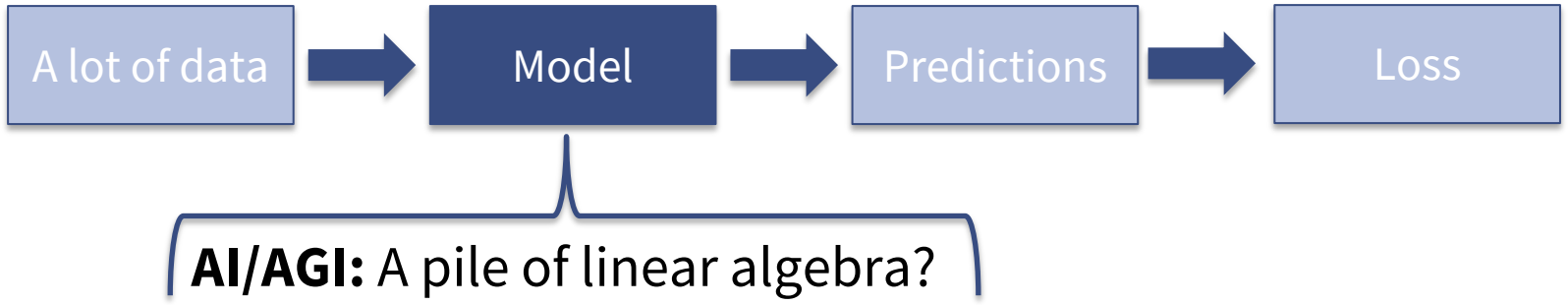


Machine learning systems are **part of the solution** and **excel** at:

1. **Modeling systems** we can extract a lot of data from
2. Extracting **meaningful latent representations**
3. Making **fast and accurate** predictions

When optimization fails, machine learning helps

When machine learning fails, optimization helps

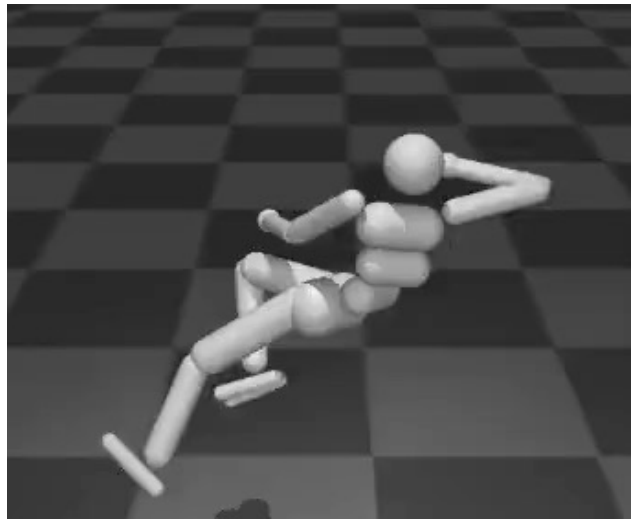


Is a **black-box of linear algebra** the **solution to AI/AGI**? *Maybe…*

Without enough data, learning systems struggle to:

1. **Encode hard constraints** and **rules** of the world crucial for safety
2. Perform **reasoning operations**
3. Isolate **interpretable** and **modular subcomponents**
4. **Generalize** to new settings beyond the training distribution

Optimization often **excels in these** and **can be brought into the model/loss**



Integrating optimization and machine learning

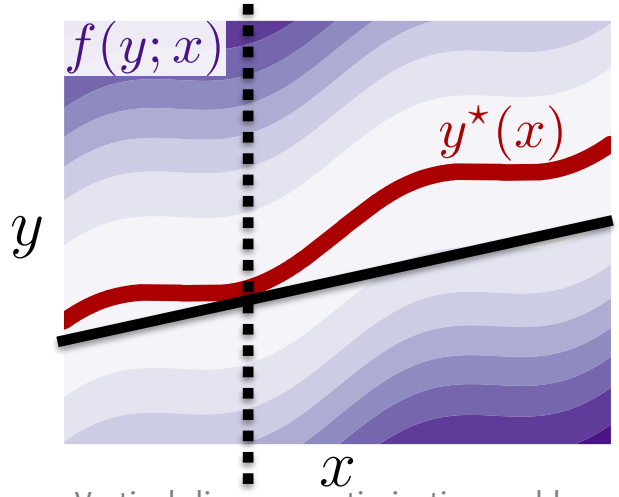
Key: view **optimization as a function** from the context x to the solution $y^\star(x) \in \underset{y \in \mathcal{C}(x)}{\operatorname{argmin}} f(y; x)$

Differentiable optimization — $\frac{\partial}{\partial x} y^\star(x)$

Optimization is just a function that can be *differentiated*

Tells how **perturbing the context changes the solution**

Enables **optimization as a layer** or loss for ML



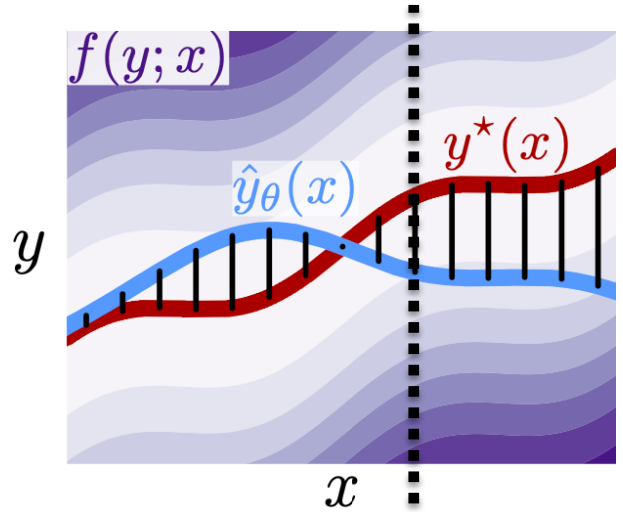
Vertical slices are optimization problems

Amortized optimization — $\hat{y}_\theta(x) \approx y^\star(x)$

Optimization is just a function that can be *approximated*

Approximate the solution map with $\hat{y}_\theta$

Power of optimization, fast **speed of an ML model**



Vertical slices are optimization problems

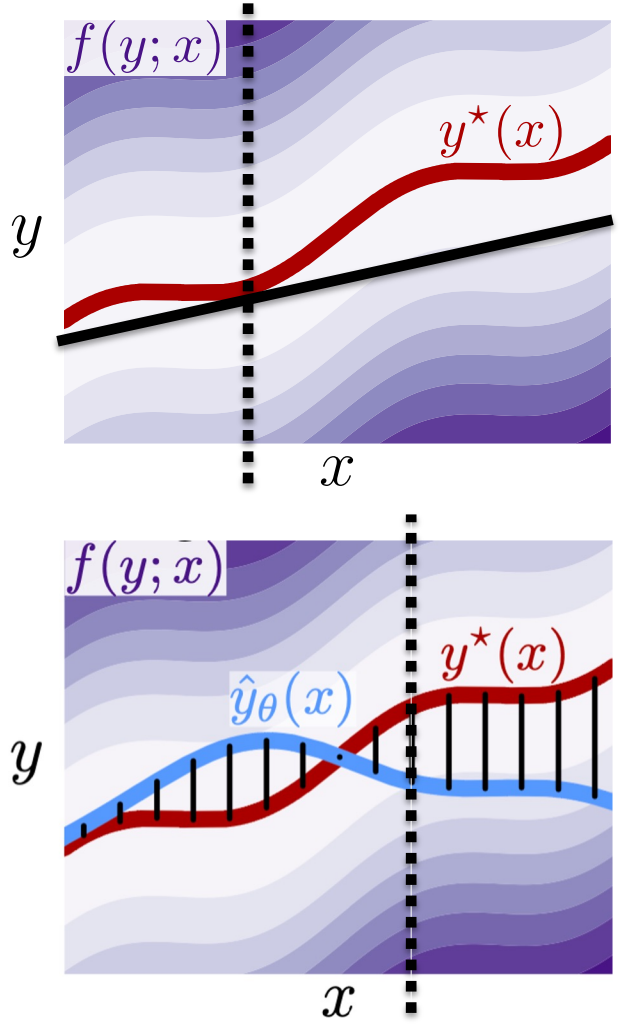
This talk

Differentiable optimization — $\frac{\partial}{\partial x} y^\star(x)$

Foundations
Differentiable convex optimization layers
Differentiable control
Optimization-based task losses

Amortized optimization — $\hat{y}_\theta(x) \approx y^\star(x)$

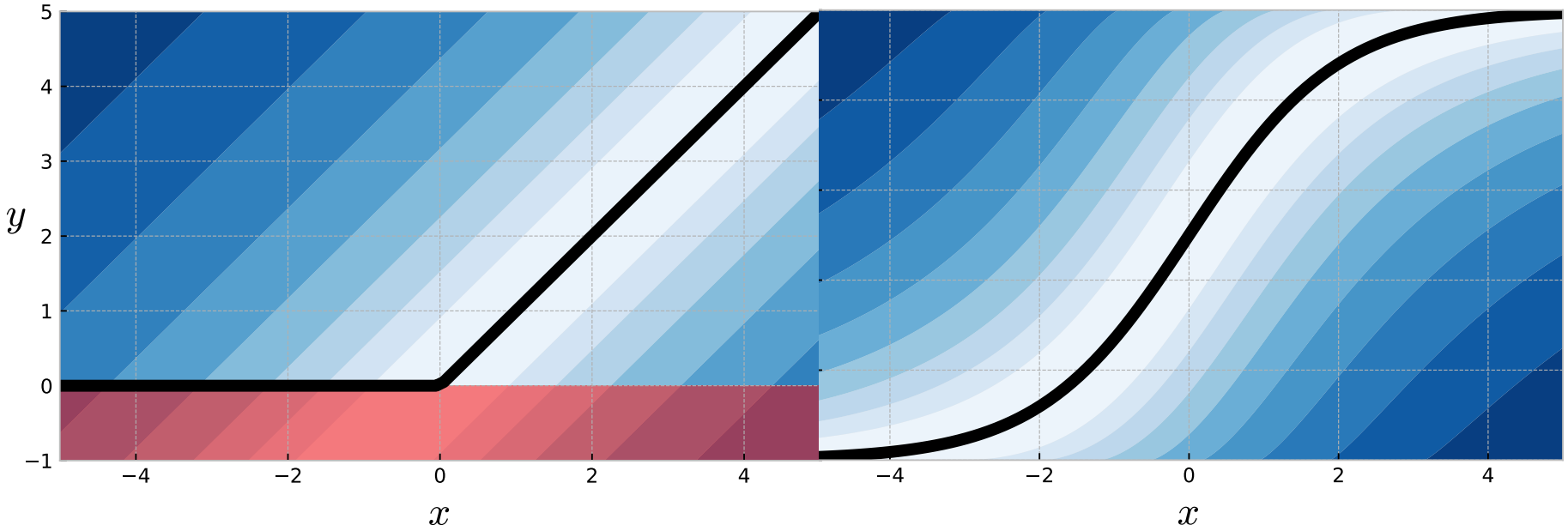
Foundations
RL as amortized optimization
Amortization via learning latent subspaces
Meta Optimal Transport
Amortizing convex conjugates



Convex optimization is expressive

The **argmin** of a **convex** optimization problem is **non-convex** and **expressive**
Standard non-linearities to be seen as **solutions** to convex optimization problems
We'll start simple for **intuition** and **motivation to generalize beyond these**

$$y^\star(x) = \underset{y}{\text{argmin}}\ f(y; x)\ \text{subject to}\ y \in C(x)$$



The ReLU is a convex optimization layer

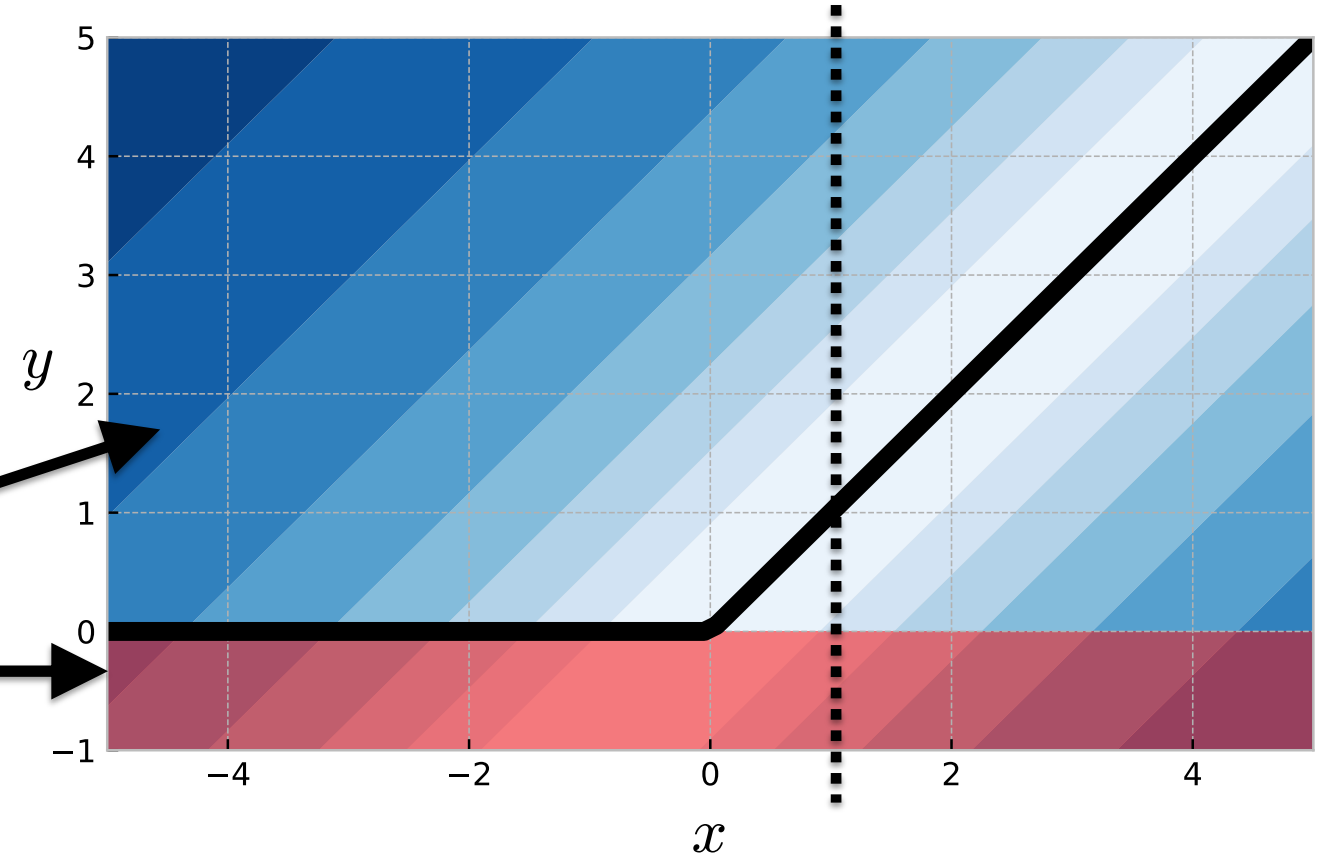
Section 2 of Differentiable optimization-based modeling for machine learning. Amos, PhD Thesis 2019

Proof: Comes from first-order optimality

$$\text{ReLU}(x) = \max\{0, x\}$$

$$\Updownarrow$$

$$\text{ReLU}(x) = \underset{y}{\text{argmin}} \quad \|y - x\|_2^2$$
$$\text{s.t.} \quad y \geq 0$$

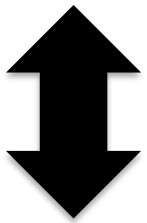


The sigmoid is a convex optimization layer

Section 2 of Differentiable optimization-based modeling for machine learning. Amos, PhD Thesis 2019

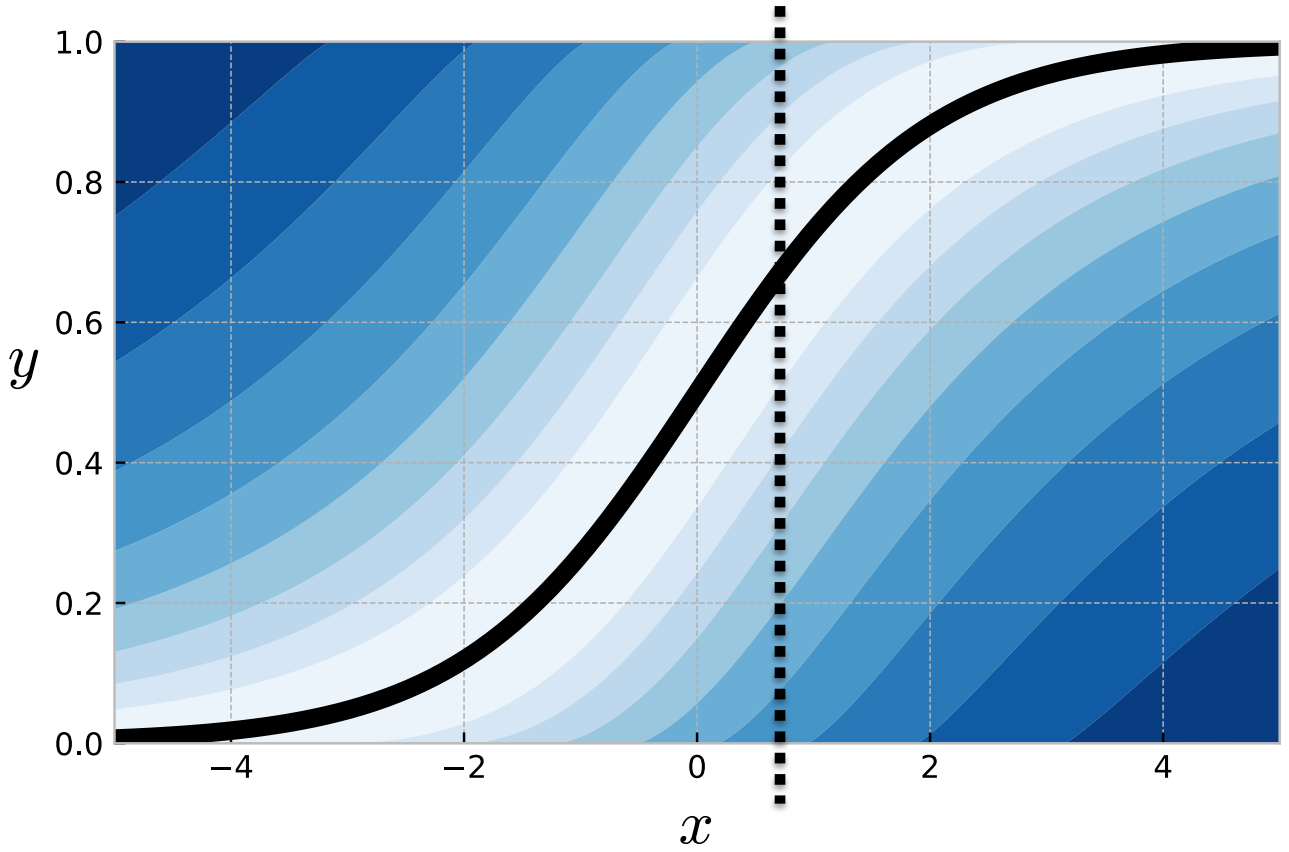
Proof: Comes from first-order optimality

$$\sigma(x) = \frac{1}{1 + \exp\{-x\}}$$



$$\sigma(x) = \underset{y}{\text{argmin}} \; -y^\top x - H_b(y)$$

$$\text{s.t.} \;\; 0 \leq y \leq 1$$



$H_b(y) \coloneqq -\sum_i (y_i \log y_i + (1 - y_i) \log(1 - y_i))$ is the binary cross-entropy function

The softargmax is a convex optimization layer

Section 2 of Differentiable optimization-based modeling for machine learning. Amos, PhD Thesis 2019

Proof: Comes from first-order optimality

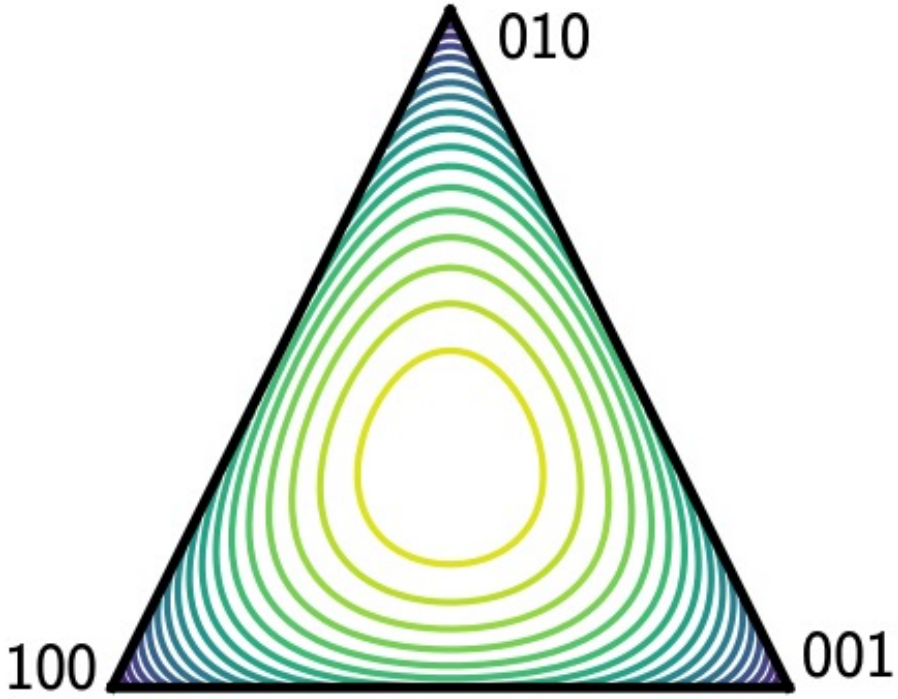
$$\pi_\Delta(x) = \frac{\exp x}{\Sigma_i \exp x_i}$$

$$\pi_\Delta(x) = \underset{y}{\operatorname{argmin}} \; -y^\top x - H(y)$$

$$\text{s.t. } 0 \leq y \leq 1$$

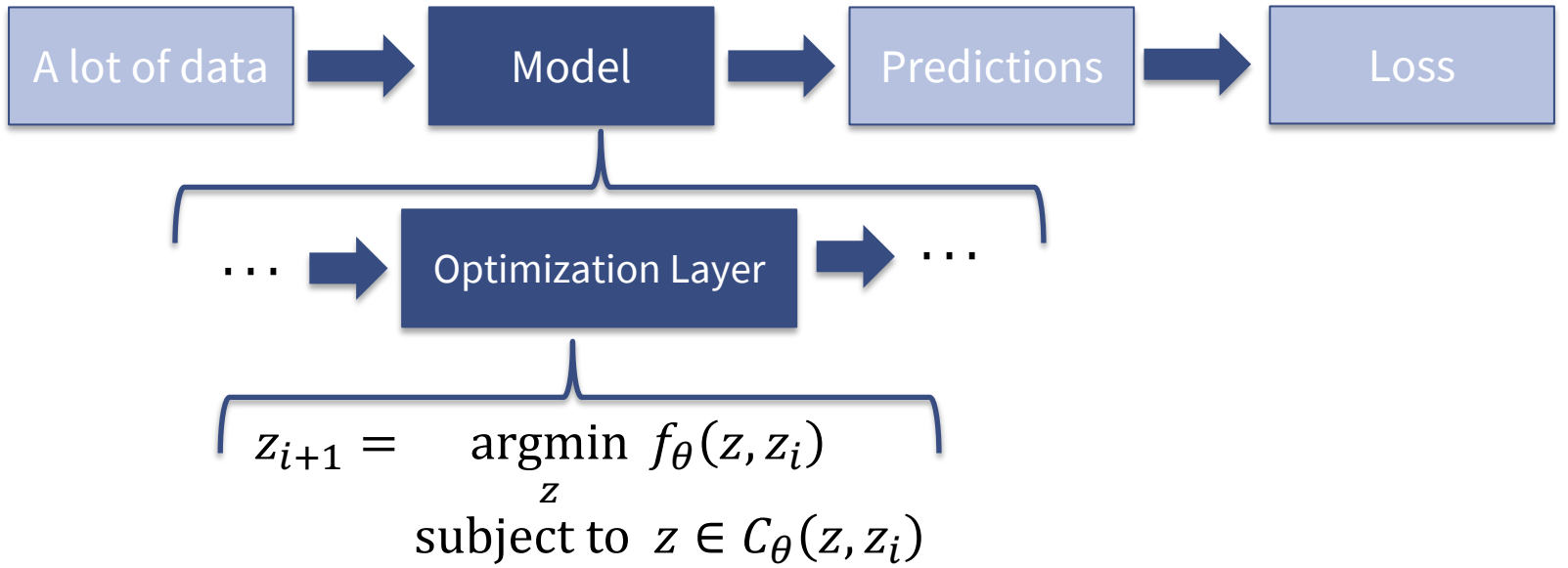
$$1^\top y = 1$$

$H(y) \coloneqq -\sum_i y_i \log y_i$ is the entropy function



Contours of the entropy $H(y)$ over the simplex

How can we generalize this?



Derivatives and backpropagation

For learning, we **differentiate** or backpropagate through these layers — **differentiable optimization**

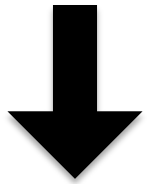
Easy if the optimization problem has an **explicit, closed-form solution** (often standard differentiation)

Otherwise, need to use **implicit differentiation,** which is also used for sensitivity analysis

Differentiable convex quadratic programs

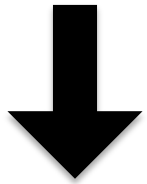
OptNet: Differentiable Optimization as a Layer in Neural Networks. Amos and Kolter, ICML 2017.

$$x^\star = \underset{x}{\text{argmin}} \ \frac{1}{2} x^\top Q x + p^\top x$$
$$\text{subject to} \ \ Ax = b \quad Gx \leq h$$



KKT Optimality

Find $z^\star$ s.t. $\mathcal{R}(z^\star, \theta) = 0$ where $z^\star = [x^\star, \dots]$ and $\theta = \{Q, p, A, b, G, h\}$



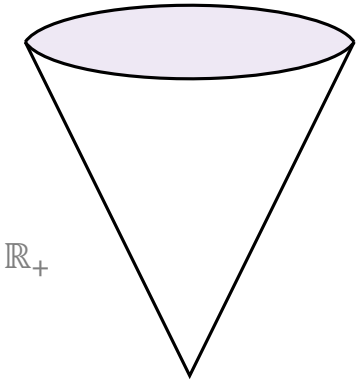
Implicitly differentiating $\mathcal{R}$ gives $D_\theta(z^\star) = -\left(D_z \mathcal{R}(z^\star)\right)^{-1} D_\theta \mathcal{R}(z^\star)$

Differentiable convex conic programs

Section 7 of Differentiable optimization-based modeling for machine learning. Amos, PhD Thesis 2019
Differentiating through a cone program. Agrawal et al., 2019

$$x^\star = \underset{x}{\text{argmin}}\ c^\top x \quad \text{subject to}\ \ b - Ax \in \mathcal{K}$$

- **Zero:** $\{0\}$
- **Free:** $\mathbb{R}^n$
- **Non-negative:** $\mathbb{R}^n_+$
- **Second-order** (Lorentz): $\{(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n \,|\, \|x\|_2 \leq t\}$
- **Semidefinite:** $\mathbb{S}^n_+$
- **Exponential:** $\{(x, y, z) \in \mathbb{R}^3 \,|\, ye^{x/y} \leq z, y > 0\} \cup \mathbb{R}_- \times \{0\} \times \mathbb{R}_+$
- **Cartesian Products:** $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_p$



Conic Optimality

Find $z^\star$ s.t. $\mathcal{R}(z^\star, \theta) = 0$ where $z^\star = [x^\star, \dots]$ and $\theta = \{A, b, c\}$

Implicitly differentiating $\mathcal{R}$ gives $D_\theta(z^\star) = -\left(D_z \mathcal{R}(z^\star)\right)^{-1} D_\theta \mathcal{R}(z^\star)$

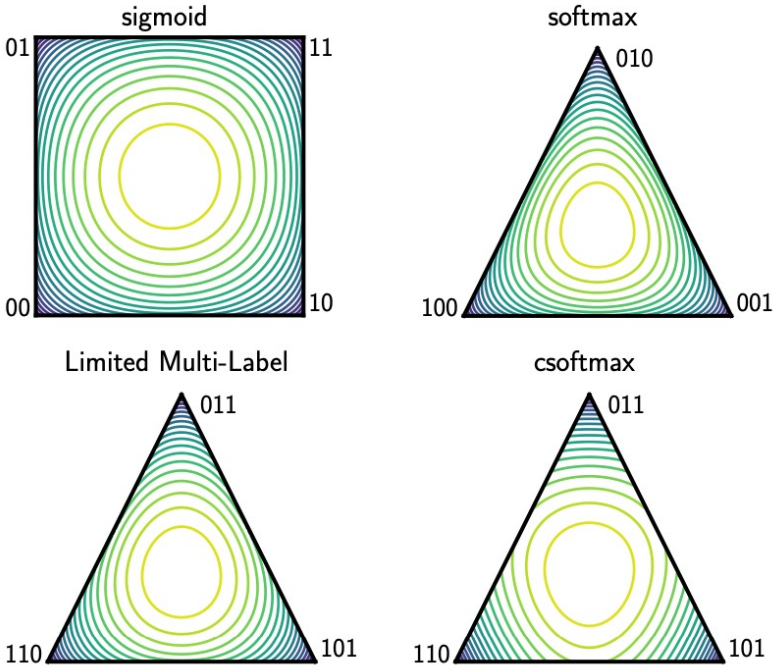
From the softmax to soft and differentiable top-k

Limited multi-label projection layer. Amos et al., 2019.

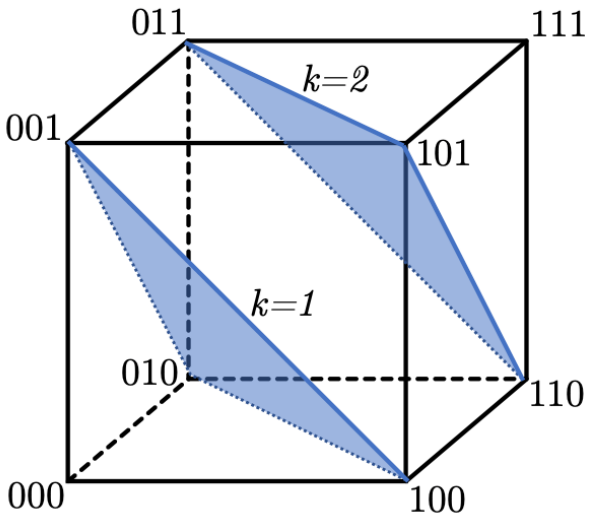
$$y^\star = \underset{y}{\text{argmin}} \; -y^\top x - H(y) \quad \text{subject to} \quad 0 \le y \le 1, \; 1^\top y = \boxed{1}$$

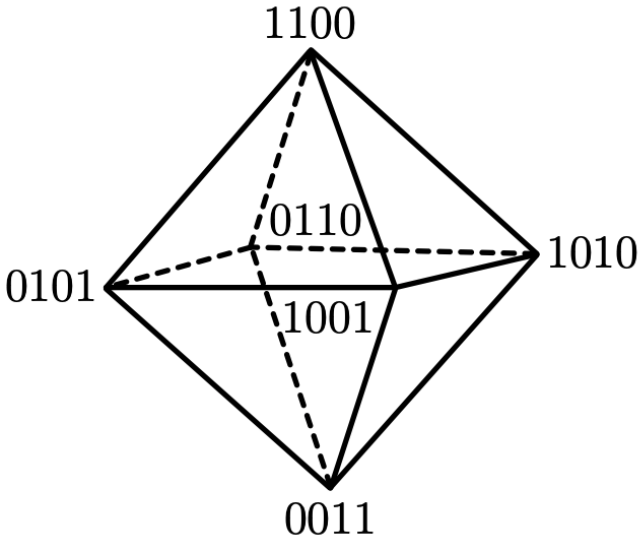
$$y^\star = \underset{y}{\text{argmin}} \; -y^\top x - H_b(y) \quad \text{subject to} \quad 0 \le y \le 1, \; 1^\top y = \boxed{k}$$

$H_b(y) \coloneqq -\sum_i (y_i \log y_i + (1-y_i)\log(1-y_i))$ is the binary cross-entropy function



Contours of the entropy penalties





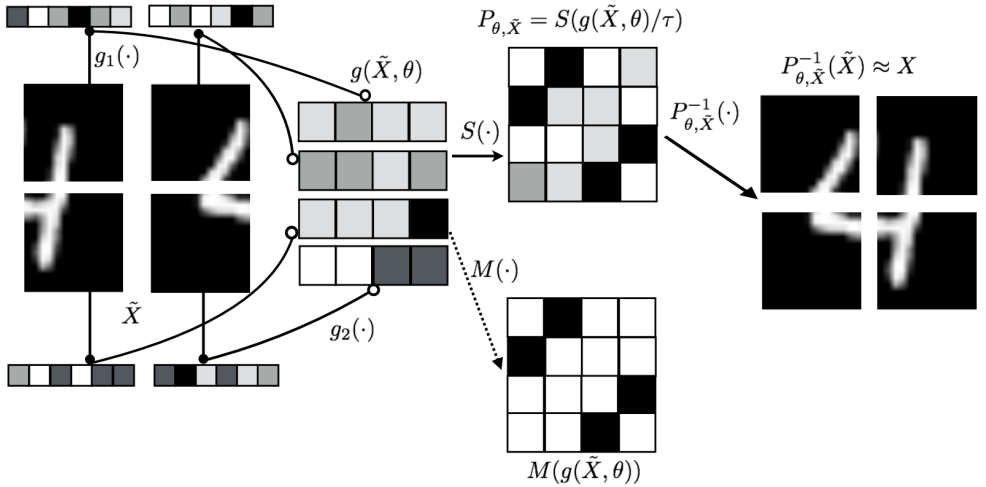
Differentiable permutations, sorting and SVMs

Differentiable permutations and sorting (Gumbel-Sinkhorn)
Projection onto the **Birkhoff polytope** $\mathcal{B}_N$**:**

$$S\left(\frac{X}{\tau}\right) = \underset{P \in \mathcal{B}_N}{\operatorname{argmax}} \ \langle P, X \rangle_F + \tau H(P)$$

$$\mathcal{B}_N = \left\{X : X \geq 0, \Sigma_i X_{ij} = \Sigma_j X_{ij} = 1\right\}$$

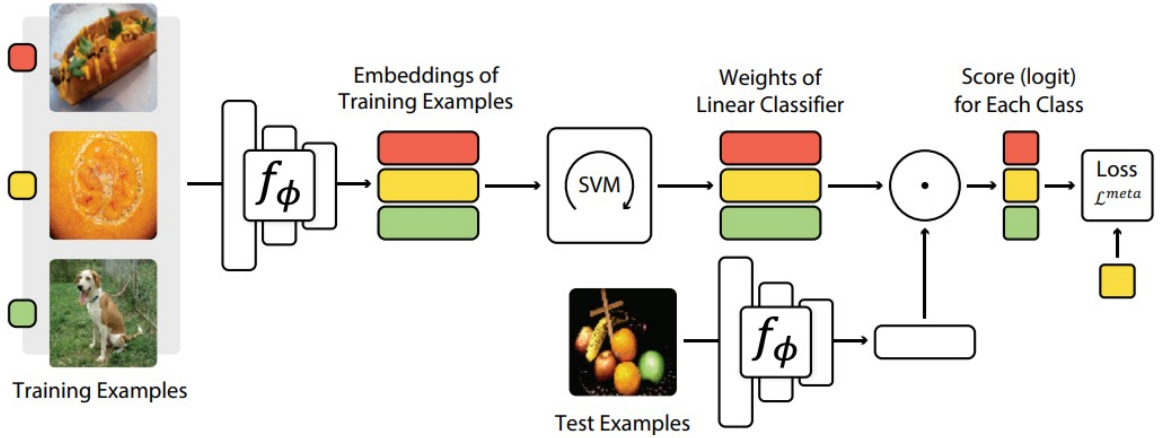
Learning latent permutations with Gumbel-Sinkhorn networks. Mena et al., ICLR 2018.



Differentiable SVMs (MetaOptNet)
Differentiate the decision boundary w.r.t. the dataset

$$w^\star(\mathcal{D}) = \underset{w}{\operatorname{argmin}} \ \|w\|^2 + C \sum_i \max\{0, 1 - y_i f(x_i)\}$$

Meta-learning with differentiable convex optimization. Lee et al., CVPR 2019.



Sensitivity and perturbation analysis

Differentiable convex optimization layers. Agrawal*, Amos*, et al., NeurIPS 2019.

Adjoint derivatives for optimization problems have been studied for decades
We have focused on uses for learning, but also widely used for **sensitivity analysis**

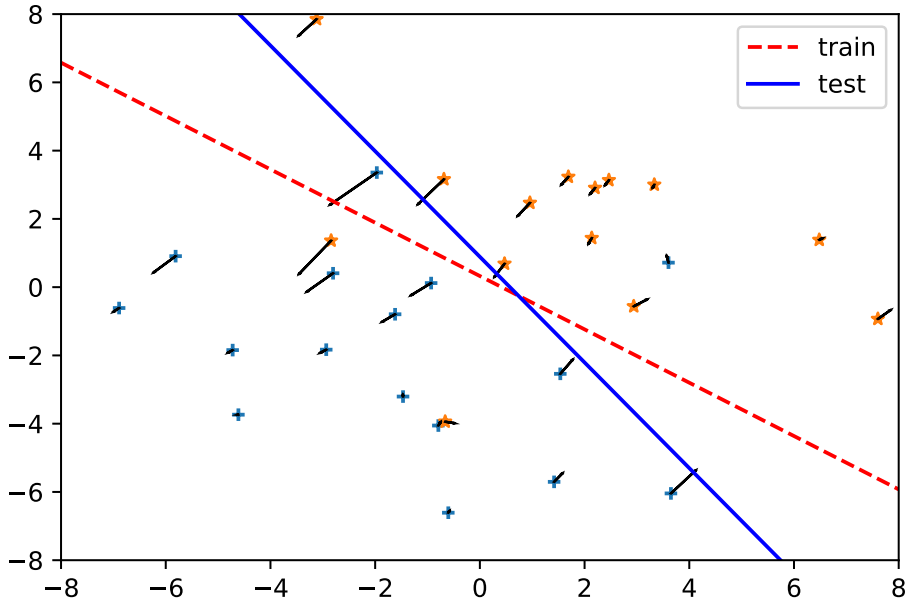
Logistic regression example
Find optimal decision boundary:

$$\theta^\star(\mathcal{D}) \in \operatorname*{argmax}_\theta \sum_i \log p_\theta(y_i \mid x_i)$$

$\theta^\star(\mathcal{D})$ is **just a function** of the data

Derivatives $\frac{\partial \theta^\star}{\partial x_i}$ provide **sensitivity** to the data points

- *Show how much the data impacts the decision boundary*



Differentiable control

Differentiable MPC for end-to-end planning and control. Amos et al., NeurIPS 2018. *The differentiable cross-entropy method.* Amos and Yarats, ICML 2020.

We can often measure the **downstream performance** induced by the controller
Idea: optimize (i.e., tune/learn) the parameters for a **downstream performance metric**
by **differentiating through the control optimization problem**

θ controller **parameters**
(design space of cost and model)

solve control optimization problem

$$x^\star_{1:T}, u^\star_{1:T} \in \underset{x_{1:T}, u_{1:T}}{\text{argmin}} \sum_t \overbrace{C_\theta(x_t, u_t)}^{\text{cost}} \text{ s.t. } \overbrace{x_1 = x_{\text{init}}}^{\text{initial state}} \quad \overbrace{x_{t+1} = f_\theta(x_t, u_t)}^{\text{dynamics}} \quad \overbrace{u_t \in \mathcal{U}}^{\text{constraints}}$$

Full notation: $u^\star_{1:T}(x_{\text{init}}, \theta)$

$\pi_\theta(x) \overset{\text{def}}{=} u^\star_1(x, \theta)$ **execute** control policy on the system

$\mathcal{J}(\pi_\theta)$ **measure downstream performance** of the controller

differentiable control

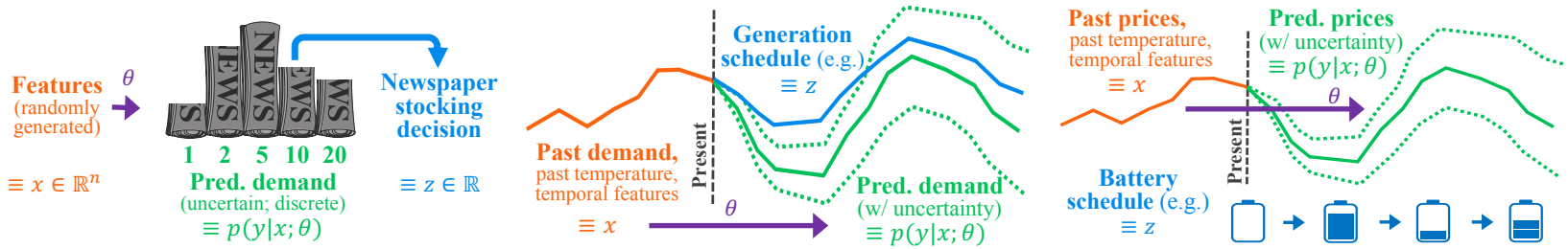
$\nabla_\theta \mathcal{J}(\pi_\theta)$

or $\mathrm{D}_\theta(\pi_\theta(x))$

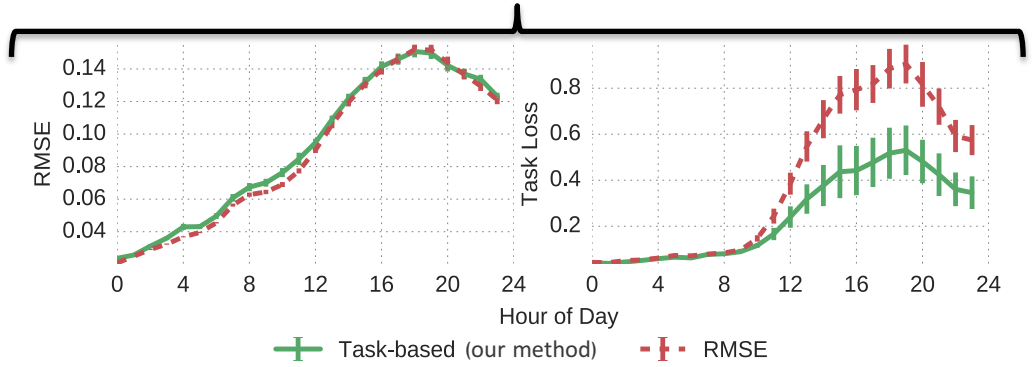
Optimization as a loss for downstream tasks

Task-based end-to-end model learning in stochastic optimization. Donti, Amos, and Kolter, NeurIPS 2017.

A model's predictions may be used in a **downstream optimization problem** (e.g., scheduling)
Challenge: models are inaccurate and **not aware of how errors impact the downstream problem**
Idea: use the **downstream optimizer as a loss** for the model using differentiable optimization



(a) Inventory stock problem (b) Load forecasting/generator scheduling (c) Price forecasting/battery arbitrage



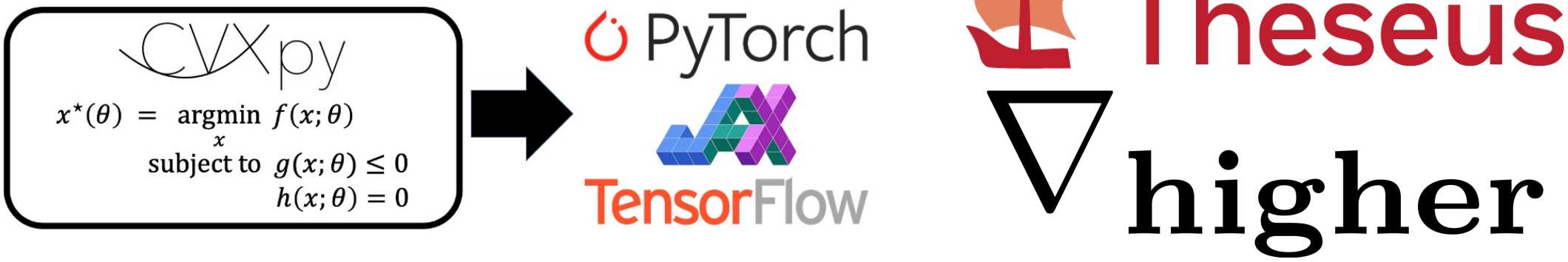
$$\underset{z \in \mathbb{R}^{24}}{\text{minimize}} \ \mathbf{E}_y \left[\sum_{i=1}^{24} \gamma_s [y_i - z_i]_+ + \gamma_e [z_i - y_i]_+ + \frac{1}{2} \|z_i - y_i\|^2 \right] \quad \text{subject to} \ |z_i - z_{i+1}| \leq c_{\text{ramp}}.$$

Implementing differentiable optimization

Manually: unroll or implicitly differentiate the optimality conditions (such as the KKT system)
My original differentiable QP implementation was **~1k lines of code** to make efficient

Use a software package to prototype such as:

- `cvxpylayers`: differentiable convex optimization in CVXPY [Agrawal*, Amos*, et al., NeurIPS 2019]
  Implements a differentiable QP in ~10 lines of code
- `Theseus`: differentiable non-linear least squares with a focus on robotics [Pineda et al., NeurIPS 2022]
- `higher`: differentiable unrolled optimizers [Grefenstette et al., 2019]



Code example: OptNet QP

Before: 1k lines of code → **Now:** ~10 lines of code

$$z_{i+1} = \underset{z}{\text{argmin}} \ \frac{1}{2} z^\top Q(z_i) z + q(z_i)^\top z$$
$$\text{subject to } A(z_i) z = b(z_i)$$
$$G(z_i) z \leq h(z_i)$$

Parameters/Submodules : Q, q, A, b, G, h

```
Q = cp.Parameter((n, n), PSD=True)
p = cp.Parameter(n)
A = cp.Parameter((m, n))
b = cp.Parameter(m)
G = cp.Parameter((p, n))
h = cp.Parameter(p)
x = cp.Variable(n)
obj = cp.Minimize(0.5*cp.quad_form(x, Q) + p.T * x)
cons = [A*x == b, G*x <= h]
prob = cp.Problem(obj, cons)
layer = CvxpyLayer(prob, params=[Q, p, A, b, G, h], out=[x])
```

```
import cvxpy as cp
from cvxpyth import CvxpyLayer
```

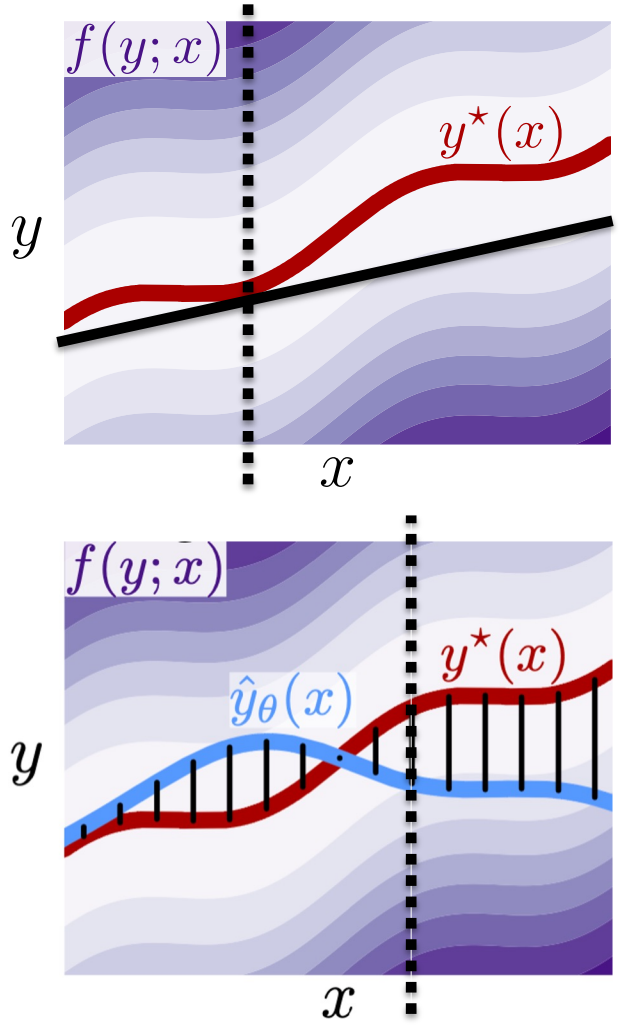
This talk

Differentiable optimization — $\frac{\partial}{\partial x} y^\star(x)$

Foundations
Differentiable convex optimization layers
Differentiable control
Optimization-based task losses

Amortized optimization — $\hat{y}_\theta(x) \approx y^\star(x)$

Foundations
RL as amortized optimization
Amortization via learning latent subspaces
Meta Optimal Transport
Amortizing convex conjugates



Difficulty: optimization is computationally expensive

Sometimes solving just **once** may be difficult
Exasperated when **repeatedly solving** during deployment

Insight: optimal solutions share structure

Optimization problems **share structure** and don't live in isolation

Solution: amortized optimization

Use **machine learning** to uncover the shared structure
Create **learning-augmented** versions of classical optimization solvers
Far surpasses average or worst-case convergence rates

Also referred to as **learning to optimize**

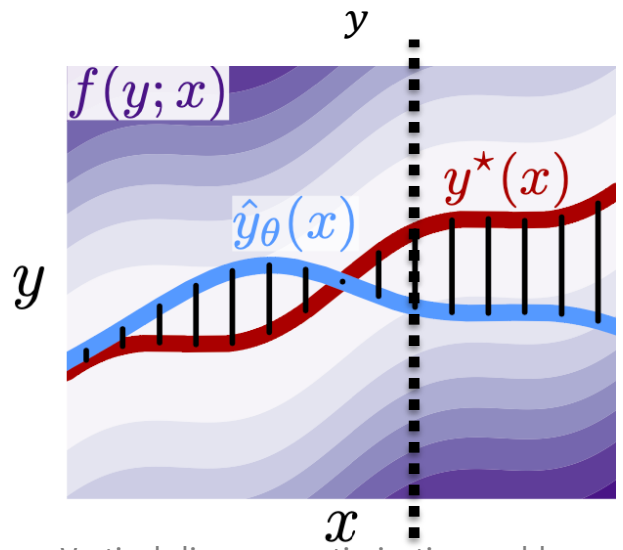
Amortized optimization

Two key components:

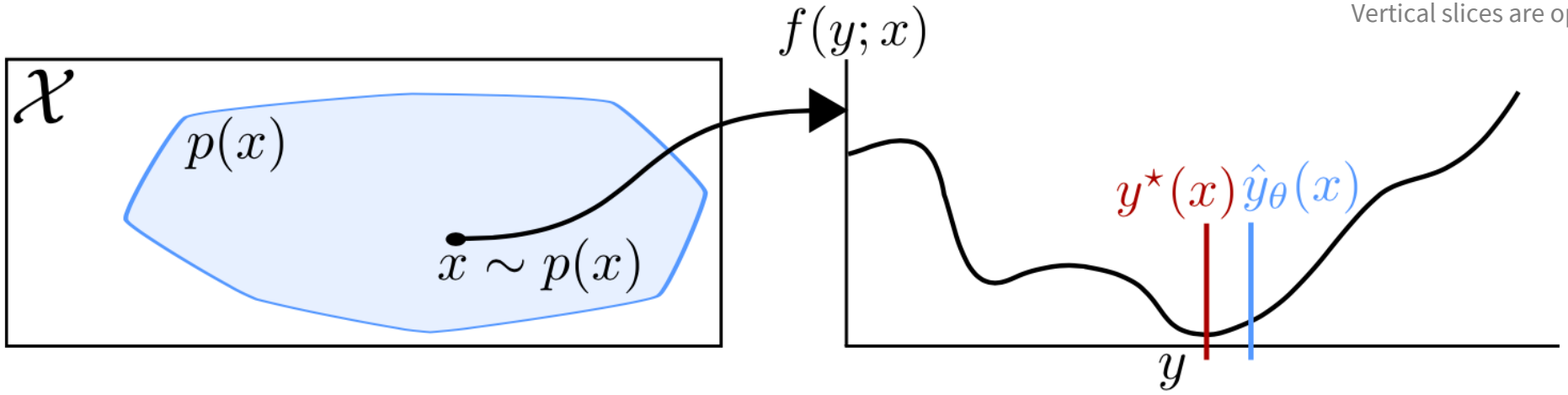
1. **Amortization model** $\hat{y}_\theta(x)$ tries to approximate $y^\star(x)$
   Fully-amortized: A neural network mapping from x to the solution
   Semi-amortized: Allowed to query the objective f

2. **Loss** $\mathcal{L}$ measures how well $\hat{y}$ fits $y^\star$ and optimized with $\min_\theta \mathcal{L}(\hat{y}_\theta)$
   Regression: $\mathcal{L}_{\text{reg}}(\hat{y}_\theta) \coloneqq \mathbb{E}_{p(x)} \|\hat{y}_\theta(x) - y^\star(x)\|_2^2$
   Objective-based: $\mathcal{L}_{\text{obj}}(\hat{y}_\theta) \coloneqq \mathbb{E}_{p(x)} f(\hat{y}_\theta(x); x)$

$y^\star(x) \in \operatorname{argmin}_y f(y; x)$



Vertical slices are optimization problems



RL policy learning is amortized optimization

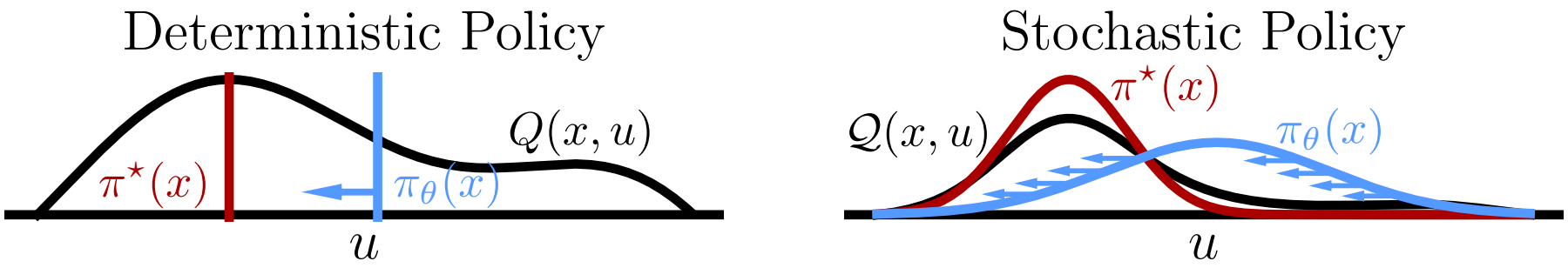
Tutorial on amortized optimization for learning to optimize over continuous domains. Amos, Foundations and Trends in Machine Learning (to appear)
On the model-based stochastic value gradient for continuous reinforcement learning. Amos et al., L4DC 2021.

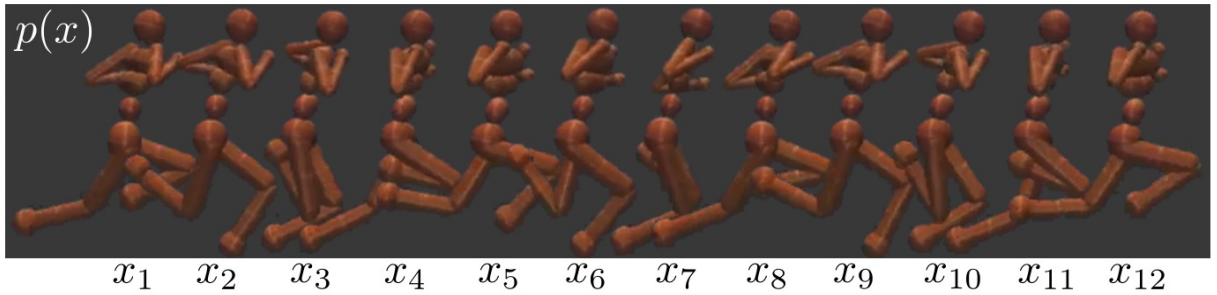
Setup: controlling a **continuous MDP** with a **model-free policy** $\pi_\theta(x)$

Review: Learning a policy with a **value gradient** amortizes over the Q-value:

$$\underset{\theta}{\text{argmax}}\, \mathbb{E}_{p(x)}\, Q(x, \pi_\theta(x))$$

The **amortization perspective** easily enables **expanding beyond this fully-amortized setting**



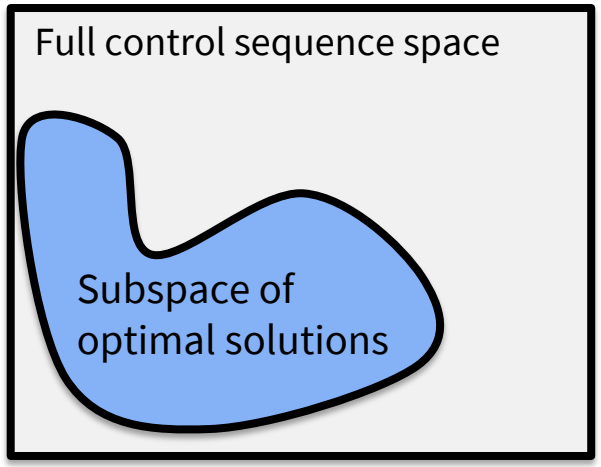


Amortization via learning latent subspaces

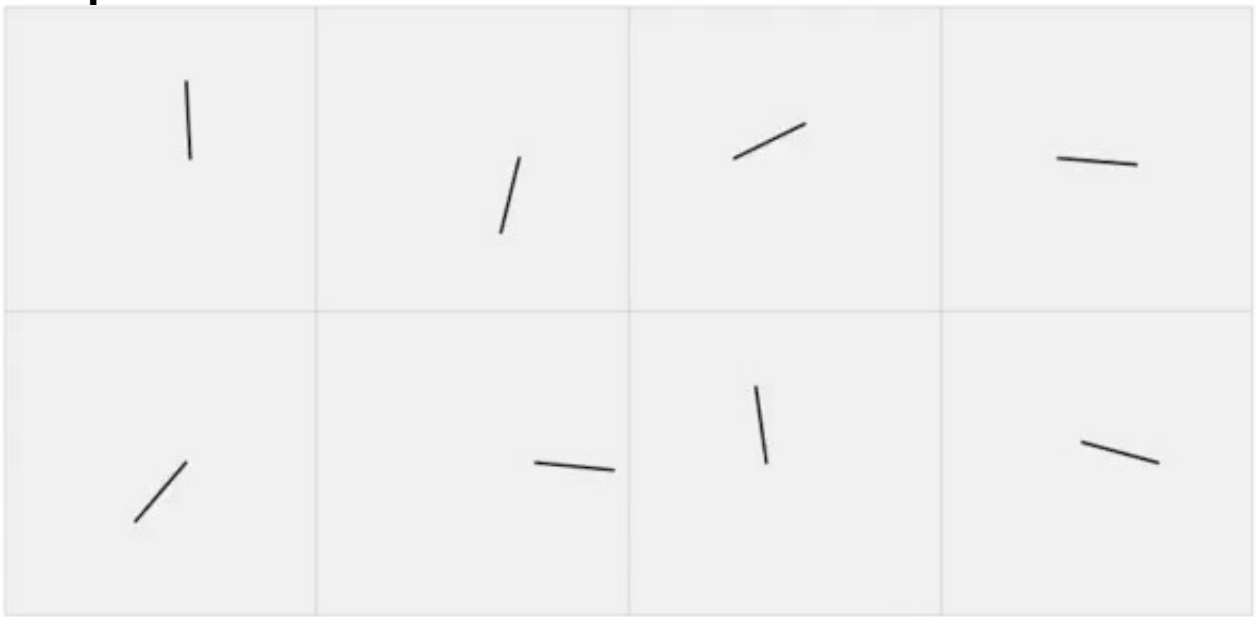
The differentiable cross-entropy method. Amos and Yarats, ICML 2020.

Amortize the problem by **learning a latent subspace** of optimal solutions
Only search over optimal solutions rather than the entire space

$$x^\star_{1:T}, u^\star_{1:T} \in \underset{x_{1:T}, u_{1:T}}{\operatorname{argmin}} \sum_t \underbrace{C_\theta(x_t, u_t)}_{\text{cost}} \text{ s.t. } \underbrace{x_1 = x_{\text{init}}}_{\text{initial state}} \quad \underbrace{x_{t+1} = f_\theta(x_t, u_t)}_{\text{dynamics}} \quad \underbrace{u_t \in \mathcal{U}}_{\text{constraints}}$$



Cartpole videos

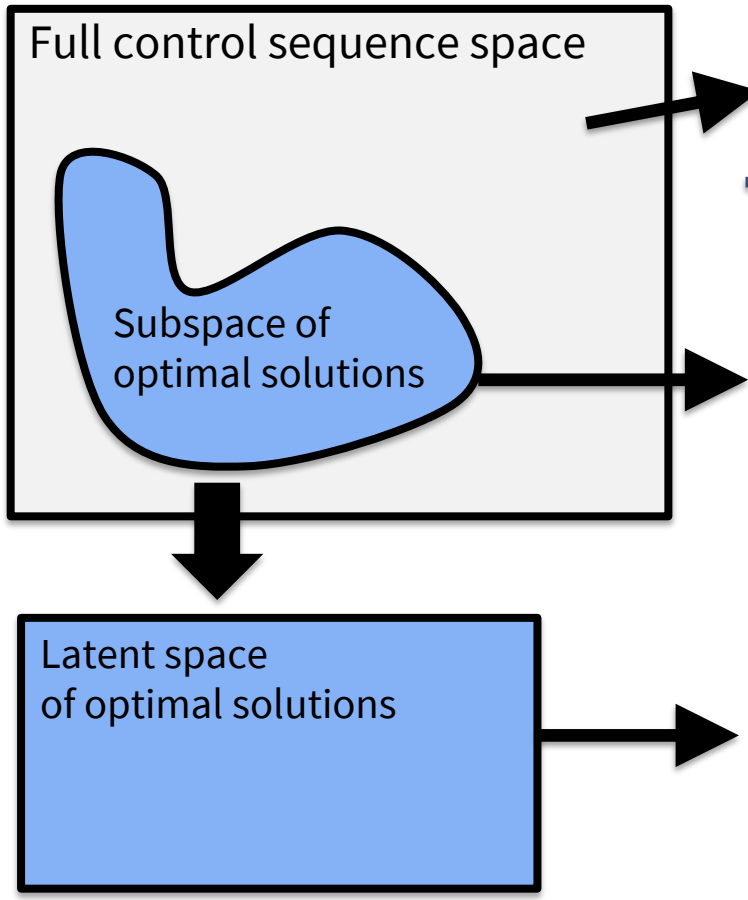


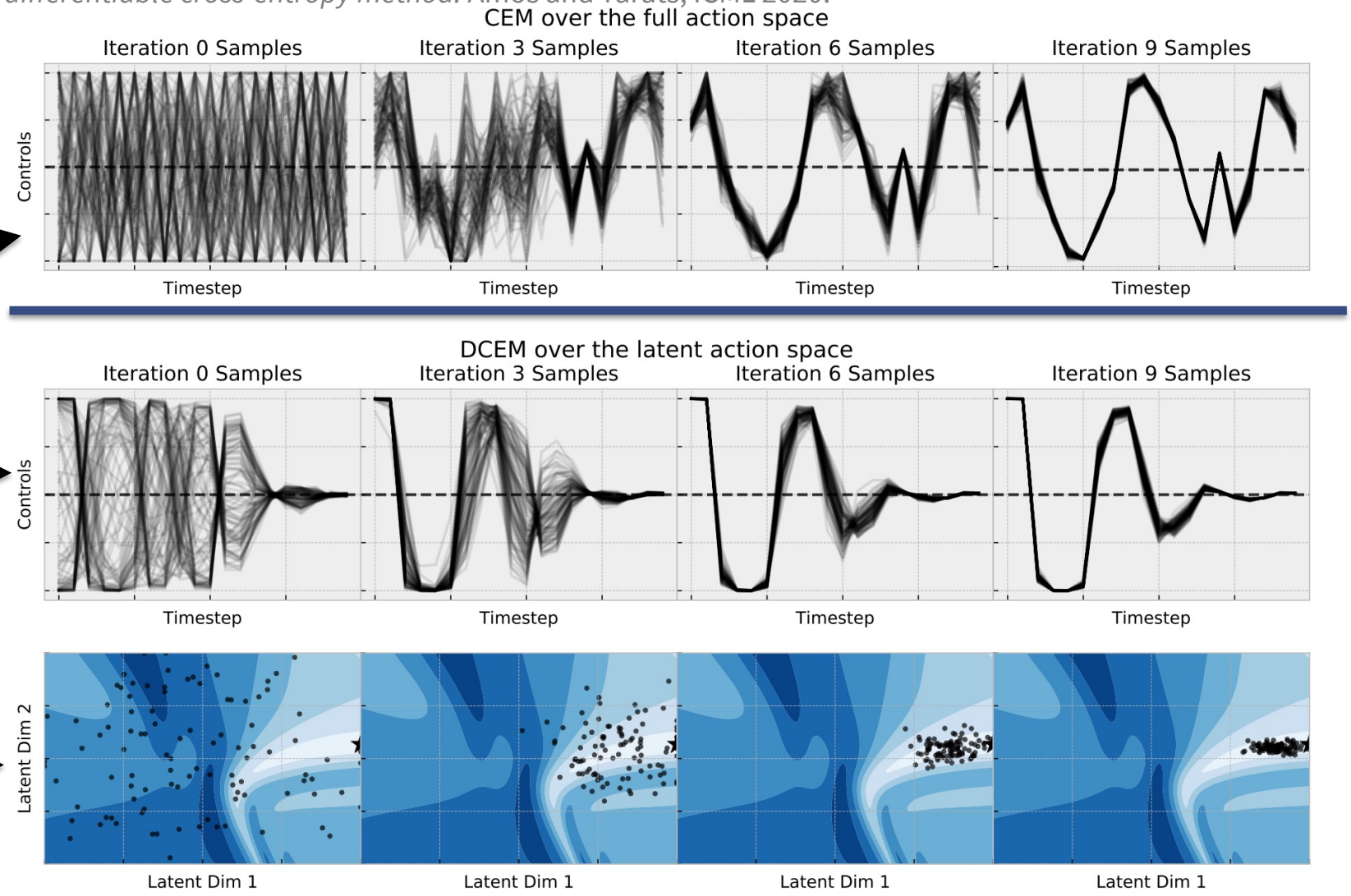
Optimal controls over time — force on the cartpole

Amortization via learning latent subspaces

The differentiable cross-entropy method. Amos and Yarats, ICML 2020.

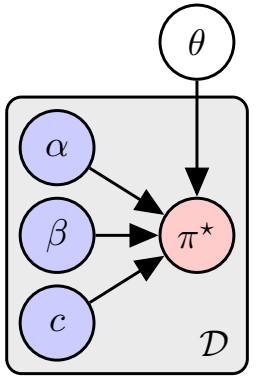
$$u^\star = \underset{u \in [0,1]^N}{\text{argmin}} \; f(u)$$





Meta Optimal Transport

[Amos et al., 2022]

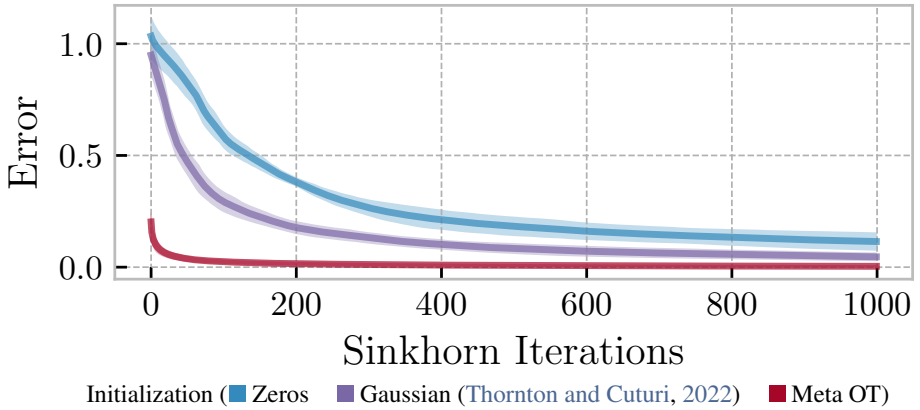


Goal: optimally transport mass between measures α to β

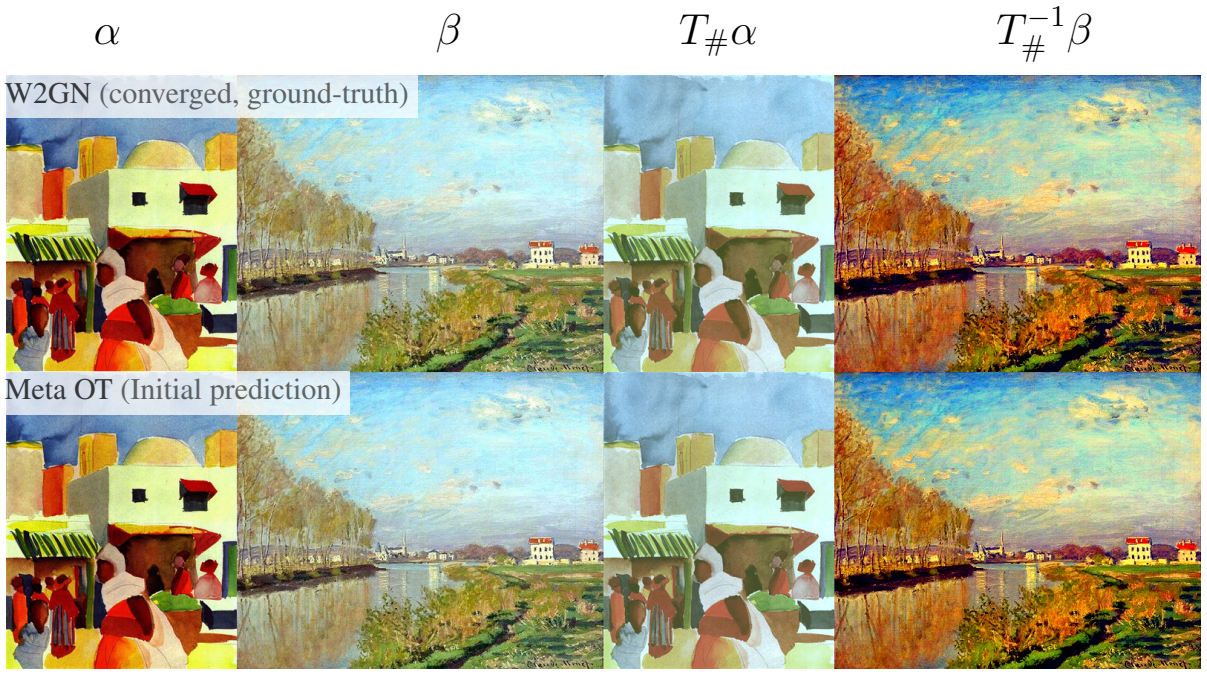
$$\pi^\star(\alpha, \beta, c) \in \underset{\pi \in \mathcal{U}(\alpha,\beta)}{\operatorname{argmin}} \int_{\mathcal{X}\times\mathcal{Y}} c(x,y)\mathrm{d}\pi(x,y)$$

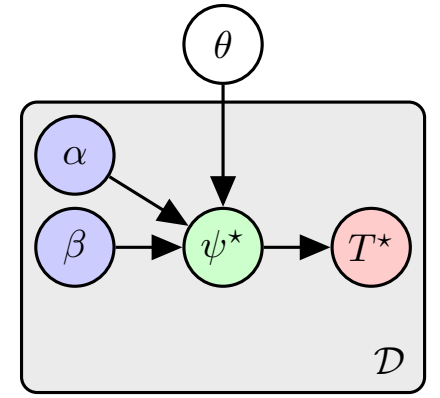
Use **amortization** when **repeatedly coupling measures**
e.g., between pairs of images or physical transport

Often amortize over **unconstrained dual potentials**

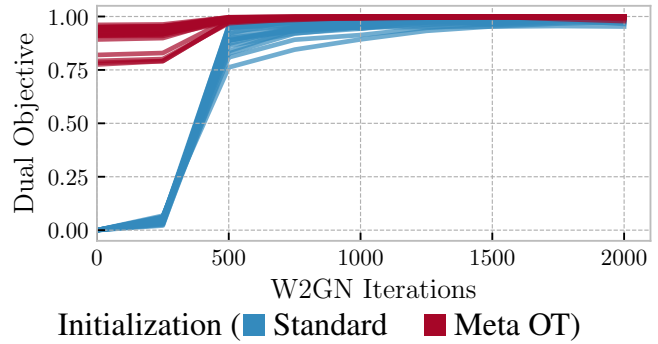


Meta OT for continuous OT with Meta ICNNs

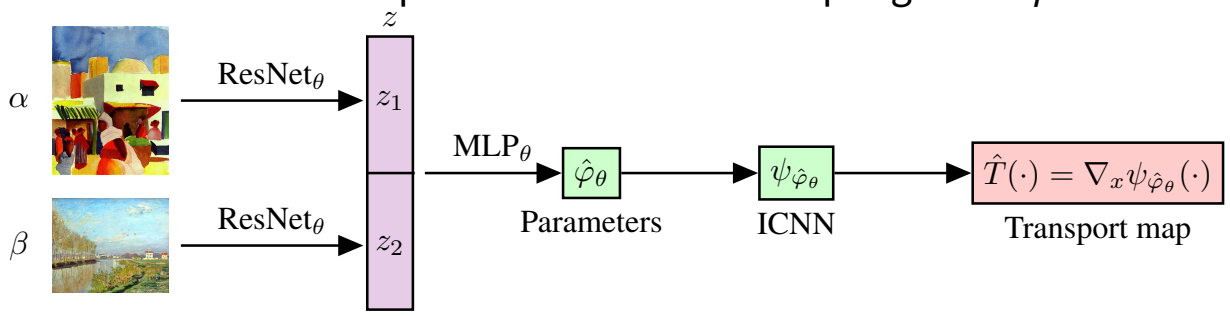




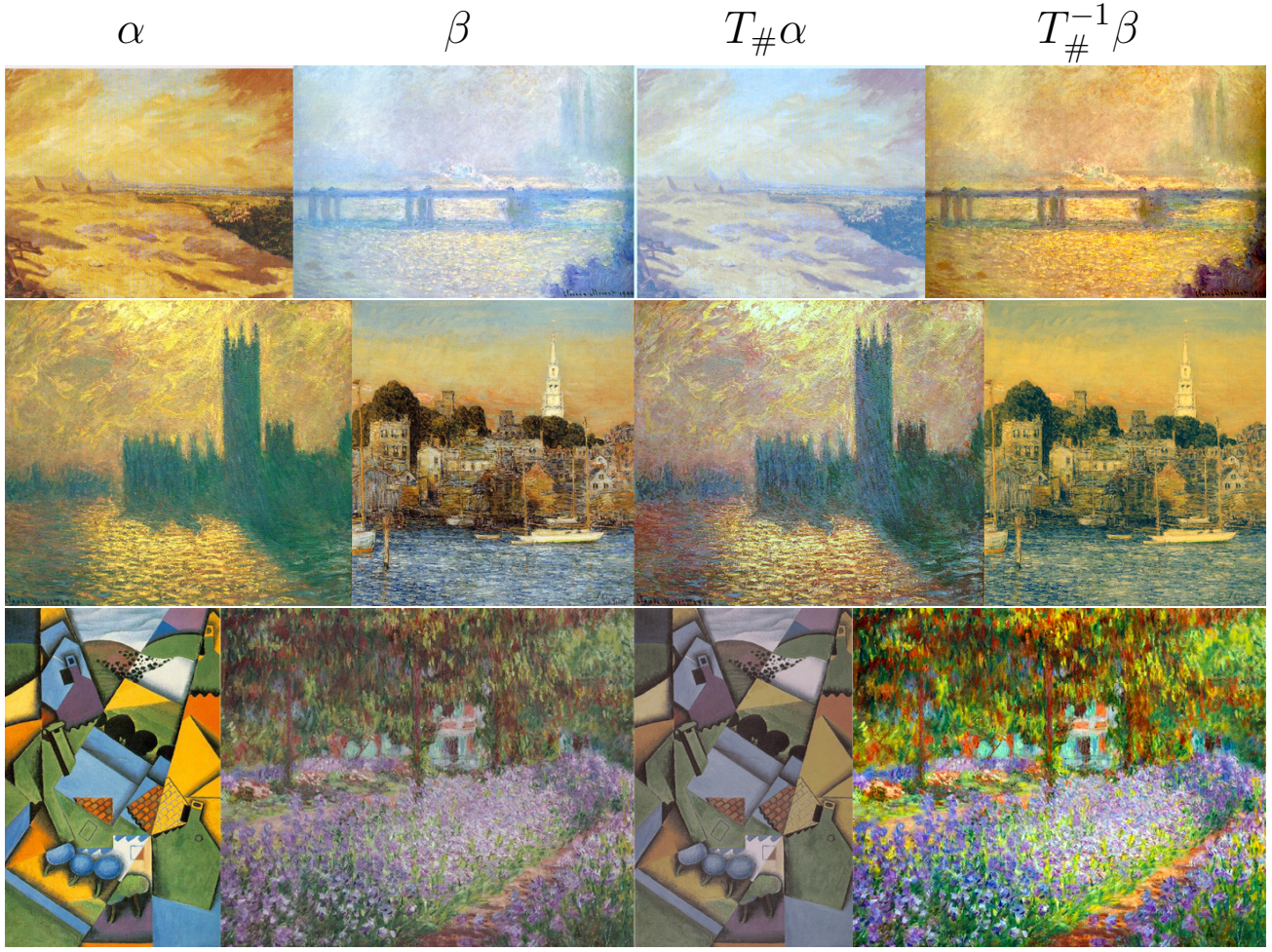
Continuous (Wasserstein-2)

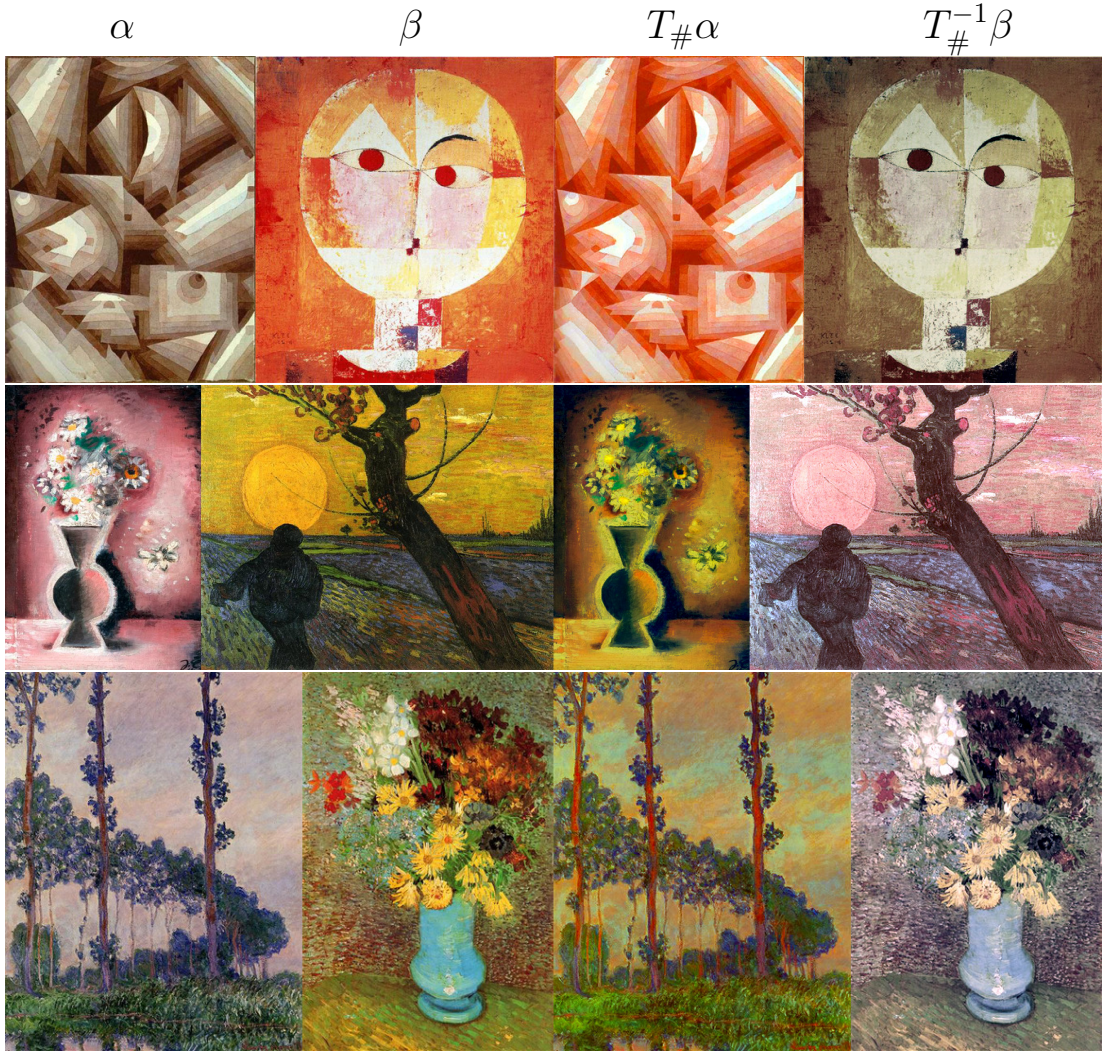


Meta ICNN: Predict parameters of ICNN coupling α and β



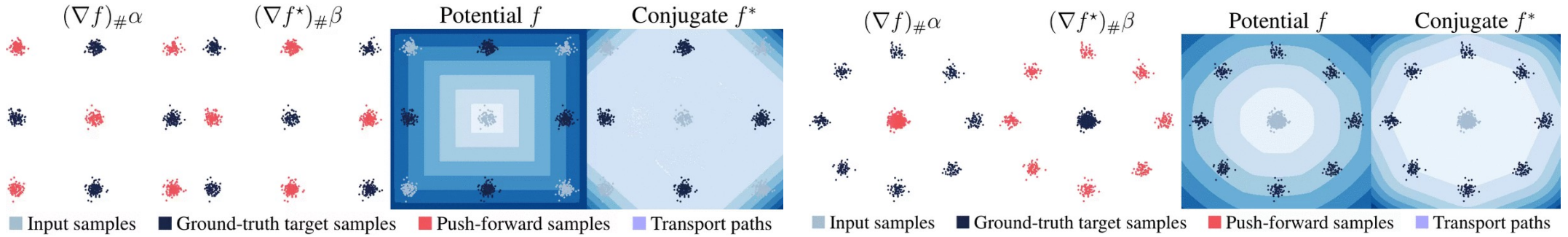
More Meta OT color transfer predictions





On amortizing convex conjugates for OT

[Amos, 2022]



Setting: transport between measures α and β

Monge problem for Euclidean Wasserstein-2

$$T^\star(\alpha,\beta) \in \underset{T\in\mathcal{C}(\alpha,\beta)}{\operatorname{argmin}} \mathbb{E}_{x\sim\alpha}\|x - T(x)\|_2^2$$

$f^\star(y) \coloneqq -\inf_x f(x) - x^\top y$ is the convex conjugate

Kantorovich dual

$$\hat{f} \in \underset{f\in\mathcal{L}^1(\alpha)}{\operatorname{argmax}} -\mathbb{E}_{x\sim\alpha}[f(x)] - \boxed{\mathbb{E}_{y\sim\beta}[f^\star(y)]}$$

$T^\star = \nabla\hat{f}$ from Brenier's theorem

Challenge: The **convex conjugate can be computationally expensive** to numerically estimate
Idea: Amortize and fine-tune the conjugate to rapidly estimate it

Improves Wasserstein-2 benchmark results by **~2-4x**

Closing thoughts

Optimization expresses **non-trivial reasoning operations**

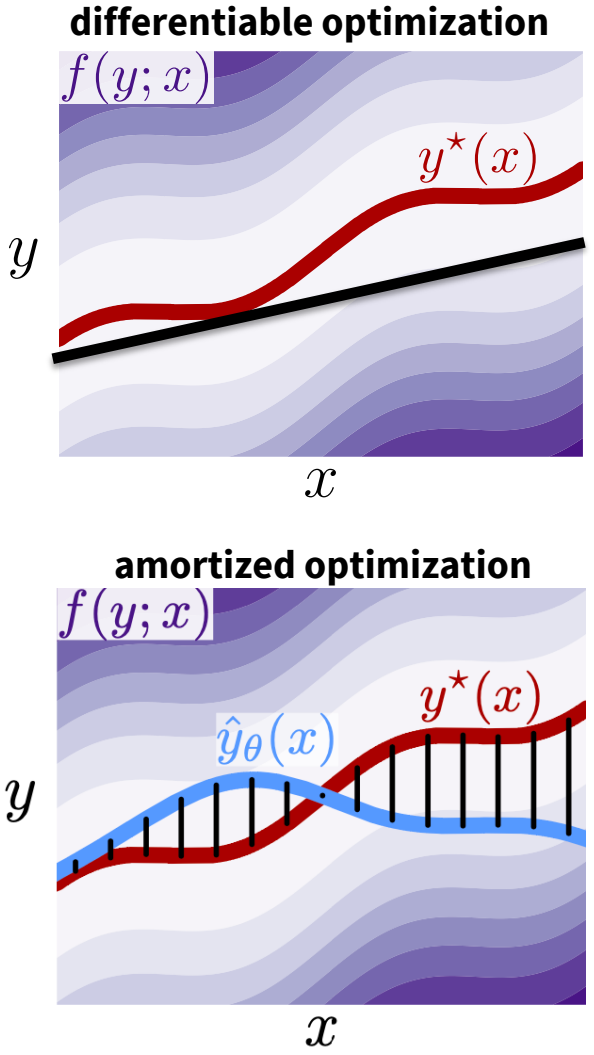
- Integrates nicely with machine learning systems by **seeing it as a function**

Differentiable optimization is a powerful primitive within larger systems

- Theoretical and engineering foundations are here
- Can be propagated through and learned, just like any layer
- Provides a perspective to analyze existing models and layers

Amortized optimization enables fast learning-augmented solvers

- Foundations are likewise here
- Provides a perspective on existing methods in RL, VI, OT



Future directions

Goal: build intelligent systems that **understand and interact** with the world
Advancing **optimization** and **machine learning foundations** is crucial

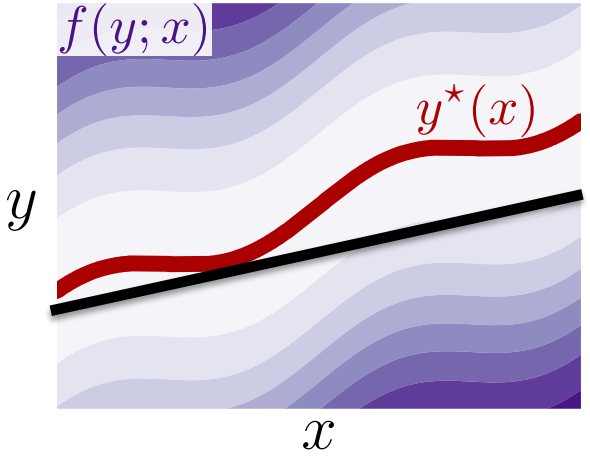
Differentiable optimization — $\frac{\partial}{\partial x} y^\star(x)$

- *Differentiable control: learning control-aware representations*
- *Numerics: "differentiating" ill-behaved functions*
- *Generalizing to non-convex or non-Euclidean settings*

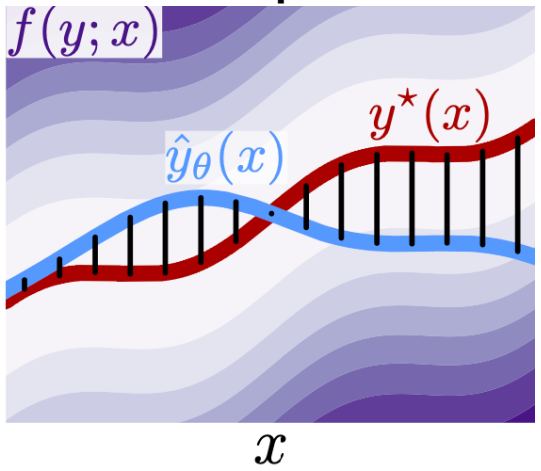
Amortized optimization — $\hat{y}_\theta(x) \approx y^\star(x)$

- *Numerics: amortizing with distributions and Markov kernels*
- *Building general learning-augmented optimizers*
- *Amortization models that use differentiable optimization*

differentiable optimization



amortized optimization

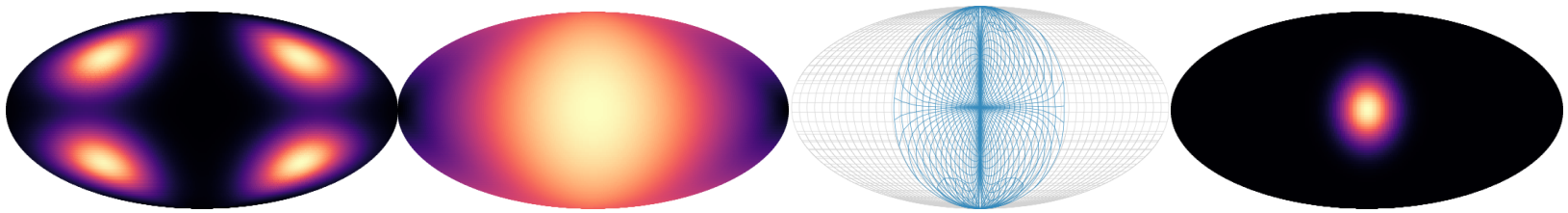


Learning foundations beyond optimization

I want to **build intelligent systems** that **understand and interact with the world**
Advancing **optimization** and **machine learning foundations** is crucial

Next frontier includes understanding, mapping between, and optimizing over:

- **Modeling** and **controlling non-trivial physical systems** (e.g. fluid dynamics, open-world robots)
- **High-dimensional** image, video, and text spaces
- **Discrete structures** arising from decisions (e.g. the knapsack) and other natural structures
- **Riemannian geometries** expressing 3D objects and many other mathematical spaces
- **Optimal transport** connecting spaces and geometries



Riemannian convex potential maps. Cohen*, Amos*, and Lipman, ICML 2021.

Learning with differentiable and amortized optimization

Brandon Amos • Meta AI (FAIR)

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[ICML 2017] *Differentiable QPs: OptNet*
[NeurIPS 2017] *Differentiable Task-based Model Learning*
[NeurIPS 2018] *Differentiable MPC for End-to-end Planning and Control*
[NeurIPS 2019] *Differentiable Convex Optimization Layers*
[Ph.D. Thesis 2019] *Differentiable Optimization-Based Modeling for ML*
[arXiv 2019] *Differentiable Top-k and Multi-Label Projection*
[arXiv 2019] *Generalized Inner Loop Meta-Learning*
[ICML 2020] *Differentiable Cross-Entropy Method*
[ICML 2021] *Differentiable Combinatorial Optimization: CombOptNet*
[L4DC 2021] *On the model-based stochastic value gradient*
[NeurIPS 2022] *Theseus: Differentiable Nonlinear Optimization*
[arXiv 2022] *Meta Optimal Transport*
[arXiv 2022] *On amortizing convex conjugates for optimal transport*
[Foundations and Trends in ML, to appear] *Tutorial on amortized optimization*

Collaborators: Akshay Agrawal, Alexander Rives, Andrew Gordon Wilson, Anselm Paulus, Arnaud Fickinger, Artem Molchanov, Austin Wang, Byron Boots, Caroline Chen, Daniel DeTone, Denis Yarats, Douwe Kiela, Edward Grefenstette, Franziska Meier, Georg Martius, Giulia Luise, Hengyuan Hu, Ievgen Redko, Ivan Jimenez, Jacob Sacks, Jason Liu, Jing Dong, Joseph Ortiz, Joshua Meier, Kyunghyun Cho,,Luis Pineda, Maurizio Monge, Michal Rolínek, Mustafa Mukadam, Nathan Lambert, Noam Brown, Omry Yadan, Paloma Sodhi, Phu Mon Htut, Priya Donti, Ricky Chen, Robert Verkuil, Roberto Calandra, Samuel Cohen, Samuel Stanton, Shane Barratt, Shobha Venkataraman, Soumith Chintala, Stephen Boyd, Steven Diamond, Stuart Anderson, Stuart Russel, Taosha Fan, Tom Sercu, Vít Musil, Yann LeCun, Zeming Lin, Zico Kolter