ME-467: Turbulence 15 November 2018
Tobias Schneider
Alessia Ferraro

Course Project: Finite Reynolds Number Effects in Turbulence

This 5-page document contains the instructions for the project of the course **ME-467: Turbulence**, which determines the final grade. You will be analyzing data from a modern wind tunnel experiment, and relating the findings to the predictions made by the Kolmogorov theory (hereafter called K41).

Material

To complete the assignment, you need the following files:

- `instructions.pdf`: This instruction sheet.
- `report.tex` (with files `titlepic.sty`, `fancyhdr.sty`, `figures/EPFL_LOG.pdf`):
  Please use this template to prepare your report. Set your name in the `MyName` variable, so that it is displayed on every page.
- `veldata1.txt`, `veldata2.txt`, `veldata3.txt` : The ascii datafiles (29.5MB, 3.3 million data points each) that contain all velocity data for the data analysis.

Rules / Honor Code

- **Individual report:** Each student has to write and submit her/his own report.
- **Collaborations:** We allow and encourage discussions and collaborations with your classmates. However, write the report individually, and at the end of the report cite all sources including a list of the people with whom you collaborated.
- **Resources:** You are free to use any sources that you find useful, including books, scientific papers and internet resources.
- **Citing:** Please cite all literature and other sources used in preparing the report.
- **Page limits:** The report should be only as long as is needed to convey the message clearly, and the template gives recommendations for the maximal length of several sections. Please respect the page limits.
- **Software:** For the data analysis, please use `Matlab` or `Python`.

Submission

The deadline for the project is **December 13, 2018, 3PM**. Please submit in class a signed printout of your report, plus electronically a `zip`-archive named `<Lastname>_<Firstname>.zip` including

- your report, as `.pdf` and `.tex` source code;
- the analysis scripts (readable ASCI source code with comments) you used for data analysis and production of figures;

- sources you have used to complete the assignment, if they are not available through the usual library channels.

Please use the Moodle upload function to submit these files. **The upload closes at 3PM sharp. Late submissions will not be accepted.**

Tasks

You will study turbulence at finite Reynolds numbers based on experimental data obtained in a modern wind tunnel using state-of-the-art instrumentation. Specifically, you will

- **Task 1:** Process and analyse experimental data using `Python` or `Matlab`.
- **Task 2:** Interpret the data in view of K41 turbulence theory and write a report presenting, discussing and interpreting the findings.

1 Data Analysis

The files `veldata1.txt`, `veldata2.txt` and `veldata3.txt` contain a time series of the downstream velocity from a wind tunnel experiment in the Warhaft Wind and Turbulence Tunnel at Cornell University[1], in units of m/s for different Reynolds numbers. The data were acquired in turbulent air at ambient pressure, using a hot wire of length 1 mm, positioned a few meters downstream of an array of randomly rotating paddles about 100 mm in size. The first three lines of each file report the number of data blocks acquired, the number of datapoints per block and the sampling frequency in Hz, as shown in the example box below.

`200`	number of data bloks
`16384`	number of datapoints per block
`6940`	sampling frequency
`5.168848`	
`5.212441`	downstream velocity data
`...`	

To investigate the influence of the Reynolds number, carry out all analysis steps for all provided datasets. Where appropriate, combine the data for different flow-speeds in a single plot to simplify comparisons.

1.1 Velocity Signal in the Spatial Domain

With x as the downstream coordinate, interpret the time series as a spatial measurement $U(x)$ via Taylor's frozen flow hypothesis. Quickly discuss to what extent this interpretation is or is not appropriate for the given datasets. Give the relevant velocity ratio and plot the velocity signal against x (Plot A). Is the velocity continuous between two data blocks? If not, keep this in mind when calculating correlations, increments and structure functions.

1.2 Correlation Length of the Velocity Signal

For short distances, the velocity signal is correlated, whereas for larger distances velocity values can be considered statistically independent. For the velocity fluctuations

$$v(x) \equiv U(x) - \langle U \rangle, \tag{1}$$

define the autocorrelation $C(l)$ as

[1] Yoon and Warhaft, J. Fluid Mech. **215**, 601–38 (1990), DOI: 10.1017/S0022112090002786

$$C(l) \equiv \frac{\langle v(x+l)v(x)\rangle}{\langle v^2(x)\rangle}. \tag{2}$$

The correlation length L_C is the length over which the fluctuations are correlated. Define L_C as the length at which $C(l)$ has dropped to $1/e$. Plot the autocorrelation against l (Plot B) and note the correlation length (which approximates the integral scale). [Extra: Describe your approach or approaches to calculating the correlation function, and briefly describe how the calculation can be performed without too much computational effort.]

1.3 Energy Spectrum of the Flow

Compute the spectral energy density $E(k)$. The velocity data is not available over an infinite spatial domain but only for a downstream distance L. The spectral energy density is then

$$\tilde{E}(k) \equiv \frac{1}{2}\left|\frac{1}{\sqrt{2\pi L}}\int_0^L v(x)e^{-ikx}\,\mathrm{d}x\right|^2 \tag{3}$$

for $k \in \mathbb{R}$. Calculate the energy spectrum for $k > 0$, which is $E(k) \equiv \tilde{E}(k) + \tilde{E}(-k)$. Use Parseval's theorem,

$$\frac{1}{2}\langle v^2\rangle = \int_0^\infty E(k)\mathrm{d}k, \tag{4}$$

to confirm that your normalization is correct.

Plot the energy spectrum $E(k)$ against k (Plot C) [*Extra: Use appropriate smoothing to minimize the noise. Show the result in a separate plot*].

1.4 The dissipation rate and Reynolds number

The Taylor Reynolds number is

$$Re_\lambda = \frac{\sqrt{\langle v^2\rangle}\lambda}{\nu} \tag{5}$$

with the Taylor scale $\lambda = \sqrt{15\nu\langle v^2\rangle/\epsilon}$ depending on the energy dissipation rate ϵ and the kinetic energy. The energy dissipation can be estimated from the velocity fluctuations at the integral scale where energy is injected into the cascade, $\epsilon = \frac{1}{2}\langle v^2\rangle^{\frac{3}{2}}/L_c$. On the basis of this analysis, estimate ϵ and the Taylor Reynolds number of the turbulence. How much does the Reynolds number vary between the datasets? In plot C, mark the correlation length, $L_c/2\pi$ and the Taylor scale $\lambda/2\pi$.

1.5 Velocity Increments

Consider now the longitudinal velocity increment

$$\delta v_\parallel(x,l) \equiv v(x+l) - v(x). \tag{6}$$

For $l \in \{0.5\,\mathrm{mm}, 1\,\mathrm{cm}, 10\,\mathrm{cm}, 10\,\mathrm{m}\}$, plot $\delta v_\parallel$ against x for suitable x- and $\delta v_\parallel$-ranges (Plot D). Where do these l-values lie relative to the relevant turbulent length scales you determined from the energy spectrum? Describe possible trends, qualitative and quantitative differences in the signal between small and large length scales.

1.6 Statistics of Velocity Increments

Plot the probability density function (PDF) of $\delta v_\parallel$ for $l \in \{0.5\,\mathrm{mm}, 1\,\mathrm{cm}, 10\,\mathrm{cm}, 10\,\mathrm{m}\}$ (Plot E) and compare it to a Gaussian distribution. How does the distribution change with l? Discuss in what way the l-dependence of the PDF supports or contradicts the assumption of self-similarity, on which K41 is based. Comment on the different Reynolds number data.

1.7 Structure functions

Calculate longitudinal structure functions $S_n(l) = \langle \delta v_{\|}^n(l) \rangle$ for $n = 2, 3, 4$ (with $\langle ... \rangle$ the spatial average over x) and plot them as a function of l for suitable l-ranges (Plot F). Discuss the observations:

- K41 predicts a scaling of S_2 as a function of l. How is the scaling of S_2 related to scaling of the spectral energy density $E(k)$? Compare ranges over which a clear scaling is observed for $E(k)$ and $S_2(l)$.
- According to K41, the third order structure function follows the four-fifth law. Discuss to which extent your data supports the K41 prediction.
- Can you estimate the energy dissipation rate ϵ from $S_2(l)$ and $S_3(l)$? Discuss how the estimates compare to the estimates based on integral scale quantities in section 1.4. (Note: According to K. Sreenivasan, the prefactor of the second order structure function is $C_2 \approx 2.1$ with $S_p(l) = C_p \epsilon^{p/3} l^{p/3}$)

1.8 Flatness of Velocity Increments

The flatness of the velocity increment signal $\delta v_{\|}(x, l)$ is defined as

$$f(l) \equiv \frac{\langle \delta v_{\|}^4 \rangle}{\langle \delta v_{\|}^2 \rangle^2} \,. \tag{7}$$

Plot the flatness as a function of l (Plot G). Relate the behavior in flatness to the shape of the PDFs of $\delta v_{\|}$. Explain the convergence behaviour of $f(l)$ for large l.

2 Interpretation

Use the template `report.tex` to prepare your report. Fill in the following sections:

2.1 Introduction

Review the significance of the $Re \to \infty$ limit assumed in the derivation of K41 theory. Discuss which hypotheses are not strictly valid if the Reynolds number is finite. Specifically, do you expect H2 (self-similar scaling) to be valid on all scales, only in the inertial range or not on any scale? How would predictions of K41 theory be modified if the flow is not self-similar. (Limit: 1 page)

2.2 Data Analysis

Present and discuss the data analysis from **Task 1**.

2.3 Discussion

In light of the statistical analysis you have performed on the velocity data from an actual turbulent flow at three different flow conditions: Discuss to which extend the data supports K41 theory. Are there discrepancies between observations and K41 predictions? Which of them might be explained by the finite Reynolds number and could disappear at higher values of Re. (Limit: 1 page)

Hints / Suggestions

The following ideas might be helpful:

- When developing your analysis code, use only part of the full data (to speed up the computation).
- Label the axes of your plots in the right physical units. Deliberately choose axis ranges and a linear/logarithmic scaling of the axes to support your analysis.
- When discussing data compared to theoretical predictions, add the predictions to the plot.
- When discussing the dependence on flow speeds, you may want present data for the three datasets in a single plot.