A Brief Introduction to Topological Games
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1   The Scottish Book

The Scottish Book is a book of problems, written from 1935 to 1941 in the city of Lwów, Poland. The namesake of the book comes from "Café Szkocka" (translated from Polish: Scottish Café) in the ciy of Lwów; from 1935 before the outbreak of World War II in 1939 and the invasion of Poland by Nazi Germany, it was a place where many of the mathematicians working at the time at the University of Lwów would come together to drink coffee, play chess, talk about politics and most importantly, discuss mathematical problems and ideas. **Stefan Banach** (1892 - 1945) is said to have had the idea to purchase a large notebook and to write down the problems discussed in the café in it. The notebook remained in the hands of the owner of the café; whenever mathematicians such as Banach, Ulam, Mazur (the three most prominent frequenters of the café), Kuratowski, Schauder, Kac, Borsuk, ... arrived at the café, they would request the book and it would be brought for them, alongside their coffees.

During the war some problems were added but in the end it remained with Banach until his unfortunate death in January of 1945 from lung cancer. His son Stephan Banach Jr., later a prominent physician, rediscovered the book and immediately sent it to **Hugo Dyonizy Steinhaus** (1887 - 1972), Banach's doctoral advisor. Steinhaus copied the book verbatim by hand and then sent a copy in 1956 to **Stanisław Ulam** (1909 - 1984) who translated it and printed three hundred copies of it to be distributed. The current version in circulation is one edited by Robert Mauldin and distributed by Birkhäuser Publications [3]. It includes forewords and lectures from Ulam, Kac, Erdös and others from a series of conferences related to the problems in the book.

The Scottish book has 193 problems; some were solved near-instantly by other mathematicians, some were solved years later, some opened active fields of research and some have been deemed "unsolvable". The problems are from many different fields of mathematics: from topology, functional analysis, measure theory and set theory, to graph theory, probability theory and group theory. It has inspired generations of mathematicians and has been a source of research problems for quite some time.

It is in this book, at Problem 43, written by Mazur and answered by Banach, where we find the first topological game ever discussed in the history of mathematics: the Banach-Mazur game.

2   Definitions, Notations and Example Games

First let us define what we mean exactly by a topological game (most definitions and results are taken from [1]):

Defintion 1. *Let X be a topological space. A **topological game** is a game played on X with the following properties:*

- *There are two players: the first one is usually labelled ALICE and the second is labelled BOB.*

- *There are no draws; one of ALICE or BOB wins the game.*

- *In each turn, players choose a "topological object" such as a point, an open set, a closed set, a compact set, an open cover of a set, etc. in X.*

- *The game has **transfinite** time; meaning that for each $n \in \mathbb{N}$, both players make a move in turn n. In other words, the game does not end in finite time, and the winning condition is usually stated in terms of the infinite sequence of moves made by both players.*

- *The game is played under perfect information; meaning that the sequence of moves played by both players until that turn are public information for both players.*

Defintion 2. *Let $G(X)$ be a topological game played on the topological space X. We say a player has a **winning strategy** a function φ exists from the set of all valid move sequences to the set of objects the player must choose in their move, in such a way that the sequence of moves decided by φ in each turn results in a win for the player. We write $PLAYER \uparrow G(X)$ if PLAYER has a winning strategy for $G(X)$ and $PLAYER \nmid G(X)$ if such a strategy does not exist.*

A more precise definition can be found in [1, p. 308].

Defintion 3. *For a topological game $G(X)$ we say $G(X)$ is **determined** if either ALICE $\uparrow G(X)$ or BOB $\uparrow G(X)$ (note that at most one of the players can have a winning strategy). We say $G(X)$ is **undetermined** if ALICE $\nmid G(X)$ and BOB $\nmid G(X)$.*

The surprising fact is that not necesarily every topological game is determined! This arises due to the fact that the game has transfinite time and therefore the existance of a winning strategy φ for a player usually will hinge on set-theoretical constructions, such as the Axiom of Choice. We will talk about this concept later on.

Now we introduce a few simple games, with which we shall introduce a few key concepts:

Defintion 4. *Let X be a topological space. The **point-open game** on X, which we shall denote as $PO(X)$, is played as such: in the n-th turn, ALICE chooses a point $x_n \in X$ and then BOB chooses a non-empty open set $U_n \subseteq X$ such that $x_n \in U_n$. The sequence $\langle x_1, U_1, x_2, U_2, ... \rangle$ is reached. ALICE wins if $X \subseteq \bigcup_{n=1}^{\infty} U_n$ and BOB wins if otherwise.*

Example 5. If X is countable, then no matter what the topology on X is, we have that ALICE $\uparrow PO(X)$; a winning strategy is simply to order the elements of X as $\{x_1, x_2, ...\}$ and for ALICE to choose the point x_n in the n-th turn.

Example 6. If $X = \mathbb{R}$, then $BOB \uparrow PO(X)$ since in the n-th turn, he can choose an open interval of length $\frac{1}{2^n}$ around x_n; the resulting union of open intervals cannot cover all of $\mathbb{R}$.

Defintion 7. *Let X be a topological space. The **Rothberger game** on X, which we shall denote as $ROTH(X)$, is played as such: in the n-th turn, ALICE chooses an open cover $\mathcal{U}_n$ of X and then BOB chooses an open set $U_n \in \mathcal{U}_n$. The sequence $\langle \mathcal{U}_1, U_1, \mathcal{U}_2, U_2, ... \rangle$ is reached. BOB wins if $X \subseteq \bigcup_{n=1}^{\infty} U_n$ and ALICE wins if otherwise.*

The Rothberger game has motivated the following property of topological spaces:

Defintion 8. *Let X be a topological space. We call X a* **Rothberger space** *if for every sequence $\{\mathcal{V}_n\}_{n=1}^{\infty}$ of open covers of X, we can choose $U_n \in \mathcal{V}_n$ for each $n \in \mathbb{N}$ such that $\{U_1, U_2, ...\}$ is itself an open cover of X.*

Remark 9. It can easily be seen that:

- If X is countable, then BOB $\uparrow ROTH(X)$.

- ALICE $\uparrow ROTH(\mathbb{R})$.

- If ALICE $\not\uparrow ROTH(X)$ then X is a Rothberger space.

The first two properties discussed in 9 motivate the following duality in topological games:

Defintion 10. *Let G and G' be two topological games. We say G and G' are* **dual** *to each other if the following occurs:*

- *ALICE $\uparrow G \Leftrightarrow$ BOB $\uparrow G'$*

- *BOB $\uparrow G \Leftrightarrow$ ALICE $\uparrow G'$*

Proposition 11. *The topological games $PO(X)$ and $ROTH(X)$ are dual.*

Proof. First let ALICE $\uparrow PO(X)$ and let their winning strategy be σ. We construct a winning strategy for BOB in $ROTH(X)$. How? If $\mathcal{V}_1$ is ALICE's first move in $ROTH(X)$, define the point $x_1 = \sigma(\langle \rangle)$ and let BOB play an open set $U_1 \in \mathcal{V}_1$ such that $x_1 \in U_1$. Next if ALICE plays $\mathcal{V}_2$, find $x_2 = \sigma(\langle U_1 \rangle)$ and then let BOB play $U_2 \in \mathcal{V}_2$ such that $x_2 \in U_2$. Continue this till we can form a strategy for BOB in $ROTH(X)$. Since σ is winning for ALICE in $PO(X)$, therefore $\bigcup_{n=1}^{\infty} U_n = X$. So these open sets also satisfy BOB's winning condition in $ROTH(X)$ and therefore BOB $\uparrow ROTH(X)$.

Now suppose ρ is a winning strategy for BOB in $ROTH(X)$; we want to find a winning strategy for ALICE in $PO(X)$. We claim that at the n-th turn, for every sequence $\langle \mathcal{V}_1, ..., \mathcal{V}_n \rangle$ of open covers of X, we can find a point $x \in X$ such that for every open set $U \ni x$ we can find an open cover $\mathcal{U}$ such that $U = \langle \mathcal{V}_1, ..., \mathcal{V}_n, \mathcal{U} \rangle$. In essence, we claim that for every sequence of open covers that BOB chooses in $PO(X)$, ALICE can make choices in choosing the open covers such that BOB's choices give her a winning strategy in $PO(X)$. We leave this simple claim to be proven (using proof by contradiction) by the reader. Proving ALICE $\uparrow ROTH(X) \Leftrightarrow$ BOB $\uparrow PO(X)$ is similar. ∎

3 The Banach-Mazur Game and Baire Spaces

Now we return to the first example of a topological game: the Banach-Mazur game:

Defintion 12. *Let X be a topological space. The* **Banach-Mazur game** *on X, denoted as $BM(X)$, is played as such: ALICE starts by choosing a non-empty open set $A_1 \subseteq X$. Then BOB chooses a non-empty open set $B_1 \subseteq X$. Then ALICE chooses another non-empty open set $A_2 \subseteq B_1$ and so on. In other words, a descending chain of open sets of X such as $A_1 \supseteq B_1 \supseteq A_2 \supseteq B_2 \supseteq ...$ is formed. If $\bigcap_{n=0}^{\infty} A_n \neq \emptyset$ then BOB wins; otherwise, ALICE wins.*

This game, originally assuming X to be a subspace of $\mathbb{R}$, was Problem 43 in the Scottish Book as mentioned before; Mazur came up with the idea of the game and Banach solved it. We now relate this game to an important topological property that frequently appears in topology, analysis and set theory:

Defintion 13. *Let X be a topological space. We say $S \subseteq X$ is*

- **rare** *or **nowhere-dense** if $\text{int}_X(\text{cl}_X(S)) = \emptyset$.*

- **meagre** *or of the **first category** if it is the countable union of rare subsets of X.*

- **nonmeagre** *or of the **second category** if S is not meagre.*

*If X is a nonmeagre subset of itself, we say X is a **Baire space**.*

Theorem 14. *(Baire Category Theorem)*
a) Every complete metric space is a Baire space.
b) Every locally-compact Hausdorff topological space is a Baire space.

Theorem 14.a is a powerful tool in functional analysis; three of the most well-known and fundamental theorems in functional analysis (the uniform boundedness principle, the open mapping theorem and the closed graph theorem) can be proved by using it. Theorem 14.b shows that topological manifolds, and many other well-known topological spaces, are Baire spaces. For a proof of both versions of Theorem 14, please refer to [7, Corollary 25.4].

Exercise 15. If X is a Baire space and $Y \subseteq X$ is open, show that the subspace topology on Y is Baire. Give an example of a Baire space X and a closed subspace $Z \subset X$ where the topology induced on Z is not Baire.

Remark 16. Even though Baire spaces are of great importance in different parts of mathematics, they do not necessarily behave well under products: Oxtoby (see [5]) constructed a Baire space Z such that, by assuming the continuum hypothesis, $Z \times Z$ is not Baire. Fleissner and Kunen (see [2]) also describe different situations in which the product of Baire spaces will be or cannot be Baire.

Here we shall use the Banach-Mazur Game to describe a condition in which Baire spaces are closed under product:

Proposition 17. *A topological space X is a Baire space if and only if ALICE $\nuparrow BM(X)$.*

Proof. First assume that X is not Baire. Therefore we can find a sequence $\{D_n\}_{n=1}^{\infty}$ of open dense subsets of X and another non-empty open set $U \subseteq X$ such that $U \cap \bigcap_{n=1}^{\infty} D_n = \emptyset$. We devise a winning strategy for ALICE as follows: in the first turn, ALICE should play $A_1 := V \cap D_1$. If BOB plays B_n in his n-th turn, ALICE should play $A_{n+1} := B_n \cap D_{n+1}$ in her $(n+1)$-th turn. Note that this is a legitimate strategy for ALICE since every D_n is open and dense, and therefore A_n will also be non-empty and open. To see that this strategy is winning, it suffices to note that:

$$\bigcap_{n=1}^{\infty} A_n = V \cap \bigcap_{n=1}^{\infty} (B_n \cap D_n) \subseteq V \cap \bigcap_{n=1}^{\infty} D_n = \emptyset$$

For a proof of the converse (including a "decision tree" approach similar to usual game-theoretic proofs, which is constructed using Zorn's Lemma) see [1, Thm 3.2, p. 320]. ∎

Using this characterization, we see that BOB having a winning strategy in a Banach-Mazur game, is a stronger condition than the Baire property. In fact, we can prove the following:

Theorem 18. *Let X and Y be two topological spaces, such that $BOB \uparrow BM(X)$ and Y is Baire. Then, $X \times Y$ is also Baire.*

Proof. Assume that $X \times Y$ is not Baire; therefore ALICE has a winning strategy σ for $BM(X \times Y)$. Also, let ρ be a winning strategy for BOB in $BM(X)$. We construct a winning strategy τ for ALICE in $BM(Y)$, which by Proposition 17 is a contradiction to Y being a Baire space.

Let $A_1 \times B_1$ be the first move for ALICE by σ. Let ALICE's first move by τ be B_1, and let V_1 be BOB's first move by ρ after ALICE plays A_1 in $BM(X)$. Now in $BM(Y)$ if BOB chooses W_1 as his move, we let $A_2 \times B_2 = \sigma(\langle V_1 \times W_1 \rangle)$. Again, let $\tau(\langle W_1 \rangle) = B_2$, and name $\rho(\langle A_1, A_2 \rangle)$ to be W_2. We continue to define τ for each n and each choice W_n that BOB makes.

Now since σ is a winning strategy for ALICE in $BM(X \times Y)$, we know that $\bigcap_{n=1}^{\infty} A_n \times B_n = \emptyset$. Similarly since ρ is a winning strategy for BOB in $BM(X)$, therefore we know that $\bigcap_{n=1}^{\infty} A_n \neq \emptyset$. Therefore, necessarily we should have $\bigcap_{n=1}^{\infty} B_n = \emptyset$. Therefore by the rules of the Banach-Mazur game, ALICE wins if they choose $\{B_n\}_{n=1}^{\infty}$ (the open sets given by the strategy τ); this means τ is a winning strategy for ALICE in $BM(Y)$ which means Y cannot be a Baire space. This contradiction tells us that $X \times Y$ has to be Baire. ■

Exercise 19. The Baire Category Theorem 14 shows that complete metric spaces are Baire. Prove a stronger result: show that if X is a complete metric space, then BOB $\uparrow BM(X)$.

However, this also opens up a surprising point where one can talk about the determinacy of the Banach-Mazur game:

Corollary 20. *The proposition "A topological space X exists such that $BM(X)$ is undetermined" is consistent with ZFC!*

Proof. If $BM(X)$ is determined for every topological space X, then Proposition 17 says that X is a Baire space if and only if ALICE $\not\uparrow BM(X)$, if and only if BOB $\uparrow BM(X)$. Now, Theorem 18 tells us that the product of every two Baire spaces should be Baire. And yet, by use of the Axiom of Choice, Fleissner and Kunen (as mentioned in Remark 16) give us examples of Baire spaces whose product is not a Baire space, which is a contradiction. [2]

Therefore, a topological space X exists such that $BM(X)$ is undetermined! ■

4 The D-Space Problem and the Menger Game

The D-space problem is a longstanding problem about a covering-type concept in which little has been done in the past few decades (see [6] for a more general exposition).

Defintion 21. *Let X be a T_1 topological space. We define an **open neighbourhood assignment** (or **o.n.a.** for short) as a function $N : X \to 2^X$ where to each $x \in X$ we assign an open neighbourhood of x named $N(x)$. We call X a **D-space** if for every o.n.a. such as N, we can find a closed and discrete set $D \subseteq X$ such that:*

$$\bigcup_{y \in D} N(y) = X$$

In other words, we are looking for a space such that we can always find a "small enough" set of points whose assigned neighbourhoods can cover the space. For example, every compact T_1 space is a D-space. Every second countable space and every metrizable space can also readily seen to be D-spaces. But other than that, especially when X is not Lindelöf, not many ideas for finding classes of D-spaces have been found.

Question 22. (D-Space Problem) Let X be a topological space which is T_1 and regular (equivalently, is T_3) and Lindelöf. Is X necessarily a D-space?

Here, similar to what we did when studying Baire spaces, we shall use modification of the Rothberger game topological game to describe a large class of topological spaces which are D-spaces:

Defintion 23. *Let X be a topological space. We define the **Menger game**, denoted as $M(X)$, as follows: in their n-th turn, ALICE chooses an open cover $\mathcal{V}_n$ for X, and after that BOB chooses a finite subset $F_n \subseteq \mathcal{V}_n$. A sequence $\langle \mathcal{V}_1, F_1, \mathcal{V}_2, F_2, ... \rangle$ is reached. BOB wins if $\bigcup_{n=1}^{\infty} F_n$ is an open cover for X; otherwise, ALICE wins.*

*We call a topological space **Menger**, if for every sequence $\{\mathcal{V}_n\}_{n=1}^{\infty}$ of open covers of X, we can choose finite $F_n \subseteq \mathcal{V}_n$ for each $n \in \mathbb{N}$ such that $\bigcup_{n=1}^{\infty} F_n$ is itself an open cover of X.*

Remark 24. We readily see that X is a Menger space if and only if ALICE $\not\uparrow M(X)$.

Proposition 25. *Let X be a σ-**compact** space, meaning it is a countable union of compact spaces. We have $BOB \uparrow M(X)$.*

Proof. Let $X = \bigcup_{n=1}^{\infty} K_n$ where each K_n is compact. In the n-th turn, BOB can choose a finite subset F_n of $\mathcal{V}_n$ which covers K_n (since K_n is compact); this leads to a winning strategy for BOB. ∎

Remark 26. Menger conjectured that the Menger game, unlike the Banach-Mazur game, is determined; essentially asking the question "is every Menger space σ-compact?" (making use of Remark 24 and Proposition 25). This was answered in the negative by Fremlin and Miller (see [4]); therefore, we can conclude that a topological space X exists such that $M(X)$ is undetermined.

We conclude this section with a partial solution to the D-space problem, related to the Menger game:

Theorem 27. *Every T_1 Menger space is a D-space.*

Proof. Let X be a Menger space, and let $N : X \to 2^X$ be an o.n.a.; we shall define a strategy for ALICE from this o.n.a., and since this strategy cannot be winning (by Remark 24) we can construct the closed and discrete subset D which shall prove the D-property.

First, ALICE will play the open cover $\mathcal{V}_1 := \{N(x) | x \in X\}$. From that, BOB will choose a finite number of open sets from that cover; essentially, he chooses a finite set D_1 and plays $\{N(x) | x \in D_1\}$ from the cover. Set $U_1 := \bigcup_{x \in D_1} N(x)$. Now we essentially "remove" U_1 from the o.n.a.: ALICE plays the open cover $\mathcal{V}_2 := \{N(x) \cup U_1 | x \in X - U_1\}$. From this, again, BOB chooses a finite set $D_2 \subseteq X - U_1$ and plays the open sets $\{U_1 \cup N(x) | x \in D_2\}$. We now set $U_2 := \bigcup_{x \in D_2} N(x)$. A sequence $\langle \mathcal{V}_1, D_1, \mathcal{V}_2, D_2, ... \rangle$ is constructed.

Note that D_n is finite and therefore discrete and closed for each n. Another fact is that BOB can choose these sets in a way so that $D_{k+1} \cap \bigcup_{j \leq k} \bigcup_{x \in D_j} N(x) = \emptyset$; meaning that he does not chooses neighbourhoods from the points he has already covered. Also, since ALICE cannot have a winning strategy, in the end we should have $\bigcup_{k=1}^{\infty} \bigcup_{x \in D_k} N(x) = X$ in order for BOB to win. Using these properties, we show that $D := \bigcup_{n=1}^{\infty}$ is closed and discrete; this is the D we need in order to prove X is a D-space and therefore the proof will be completed. D is obviously still discrete so only closed-ness needs to be proven.

Let $x \in X$. Since $x \in \bigcup_{d \in D} N(d)$, we can find a natural number k and a $d \in D$ such that $x \in N(d)$ and $d \in D_k$. Here we can see that if $y \in D \cap N(d)$, then we necessarily have $y \in D_j$ for some $j \leq k$. This means that $N(d) \cap D$ is finite and therefore, since X is T_1, we conclude that D cannot have a limit point not in it; therefore, D is closed. ∎

5 The Axiom of Determinacy

We can see that topological games give a good framework in order to understand complex concepts in topology. The explanation and intuition behind this fact essentially boils down to mathematical logic; specifically, to higher order logics. In zeroth-order and first order logic, we have either no quantifiers ($\forall$ and $\exists$) or a finite number in each sentence (respectively). In second-order logic, we can have an infinite quantifier. This tells us that the sentences in second-order languages have much more complexity behind them and are therefore much harder to work with.

In a topological game, having a winning strategy means a certain sequence of quantifiers is true. For example, if we have a topological game $G(X)$ and we let $s = \langle A_1, B_1, A_2, B_2, ... \rangle$ be the sequence of moves made by ALICE and BOB, and if $P(s)$ gives us the winning condition of the game, we see that:

$$ALICE \uparrow G(X) \Leftrightarrow (\exists A_1 \forall B_1 \exists A_2 \forall B_2 \exists A_3...)P(s) = ALICE$$

The same goes for BOB $\uparrow G(X)$, ALICE $\Uparrow G(X)$ and BOOB $\Uparrow G(X)$: in each case, an infinite sequence of alternating quantifiers is constructed; which is almost always a second-order sentence in the language describing the topological space X (there are exceptions where the sentence can be interpreted in first-order logic, but they are rare and of basically no use).

This is where the ingenuity of topological games comes in. By framing these concepts in a familiar language (the language of games and players and strategies and such) we get a precise and yet more intuitive point-of-view in order to work on these kinds of results and propositions. The reason why most of the winning strategies in these games characterise intricate topological properties is because of this second-order logic hiding behind the guise of a game. If everything is defined axiomatically and correctly - and as many have tried successfully to bring precision to definitions of winning strategies in topological games - these topological game become a powerful and enlightening method of discovery in topology.

Another point of these games is determinacy. As we have seen, topological games can be undetermined. This phenomenon seems strange and is a departure from our intuition about game theory, because in finite-time games with perfect information and no draws, a winning strategy always exists for some player. Also, most examples of topological games being undetermined relate to seemingly absurd and almost pathological topological spaces which seem too counter-intuitive to be true. In order to remedy this situation, logicians asked a question: can we define a non-trivial yet simple topological game whose determinacy is consistent with ZF set-theory? One such attempt was the following game:

Defintion 28. *Let $X = \mathbb{N}^{\mathbb{N}}$ be the set of all sequences of natural numbers, and let $A \subseteq X$ be a fixed subset of X. The **Gale-Stewart game** on A is denoted by $GS(A)$ and is played as follows: in each turn, ALICE and BOB each choose a natural number; let them be a_n and b_n respectively in their n-th turn. A sequence $s = \langle a_1, b_1, a_2, b_2, a_3, ... \rangle$ is constructed by the players. ALICE wins if $s \in A$ and BOB wins otherwise.*

Defintion 29. *The **Axiom of Determinacy**, usually abbreviated as **AD**, is the following statement:*

For every $A \subseteq \mathbb{N}^{\mathbb{N}}$, the game $GS(A)$ is determined.

Remark 30. One of the more interesting alternatives to the Axiom of Choice, to be used alongside the Zermelo-Fraenkel axioms, is AD. AD is inconsistent with ZFC: one can construct a subset A of $\mathbb{N}^{\mathbb{N}}$ where the Gale-Stewart game is undetermined (by a clever equivalence relation and a strategy-stealing argument). However, ditching the Axiom of Choice for AD is not without its merits. Indeed, in ZF+AD one can prove the following:

- Every subset of $\mathbb{R}$ is Lebesgue-measurable.

- For every subspace $X \subseteq \mathbb{R}$, $BM(X)$ is determined; therefore, every subset of $\mathbb{R}$ has the Baire property.

- A Hamel basis for $\mathbb{R}$ over $\mathbb{Q}$ does not exist.

In essence, just as determinacy of the Banach-Mazur game or Menger game would have given us a way to rule out certain topological spaces with pathological properties (for example a non-Baire product of Baire spaces), the determinacy of the Gale-Stewart game will rule out the existance of Vitali's non-measurable set. However even though ZFC is inconsistent with AD, AD is still consistent with DC (the Axiom of Dependant Choice) and with ZF is even able to prove CC (the Axiom of Countable Choice). It does force certain odd results in cardinal arithmetic, but in the end, it is still considered as one of the best possible replacements for the Axiom of Choice in alternative set-theories.

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