1.1) $n = 154$ $\qquad$ $z_{\alpha/2} = 2.576$

$$\hat{p} = \frac{76}{154} = 0.49$$

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\left(\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

$$(0.276,\ 0.710)$$

1.2) Yes, if you calculate the new interval for $\hat{p} = 0.65$, then we get:

$$(0.44,\ 0.85)$$

therefore their is evidence that the treatments are different b/c the first interval is lower then the second interval.

2.1) 1) pmf: $f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$

2) joint pmf:

$$f(x;\lambda) = \prod_{i=1}^{n} f(x_i,\lambda)$$

$$= \frac{\lambda^{x_1} e^{-\lambda}}{x_1!} \cdot \frac{\lambda^{x_2} e^{-\lambda}}{x_2!} \cdot \frac{\lambda^{x_n} e^{-\lambda}}{x_n!}$$

$$L(P) = \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-\lambda n}}{\prod_{i=1}^{n} x_i!}$$

3) $$l'(L(P)) = \log\left(\frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-\lambda n}}{\prod_{i=1}^{n} x_i!} \right)$$

$$= \sum_{i=1}^{n} x_i \log(\lambda) - n\lambda - \sum_{i=1}^{n} \log(x_i!)$$

4) max:

$$0 = \sum_{i=1}^{n} x_i \log(\lambda) - n\lambda - \sum_{i=1}^{n} \log(x_i!)$$

$$= \frac{\sum_{i=1}^{n} X_i}{\lambda} - n$$

$$MLE \to \hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

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2.2)

$$E(\hat{\lambda}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}(n\mu) = \mu$$

Yes $\hat{\lambda}$ is an unbiased estimator of λ. 4

2.3) $\theta = \alpha\hat{\lambda} + (1-\alpha)X_1$

$$E(\theta) = E\left(\alpha\hat{\lambda} + (1-\alpha)X_1\right)$$

$$= \alpha E(\hat{\lambda}) + E(X_1) - \alpha E(X_1)$$

We showed above $\sum E(X_i) = n\mu$ so $E(X_1) = \mu$

$$= \alpha\mu + \mu - \alpha\mu = \mu$$

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2.4) $\operatorname{Var}(\hat{\lambda}) < \operatorname{Var}(\alpha\hat{\lambda} + (1-\alpha)X_1)$

$$\operatorname{Var}\left(\frac{1}{n}\sum_1^n X_i\right) < \left(\frac{1}{3}\right)^2 \operatorname{Var}(\hat{\lambda}) + \operatorname{Var}(X_1) - \left(\frac{1}{3}\right)^2 \operatorname{Var}(X_1)$$

$$\Rightarrow \quad \left(\frac{1}{n}\right)^2 n\sigma^2$$

$$\Rightarrow \quad \frac{\sigma^2}{n} < \frac{1}{9}\frac{\sigma^2}{n} + \lambda - \frac{\lambda}{9}$$

$$\Rightarrow \quad \frac{\sigma^2}{n} < \lambda\left(\frac{8}{9}\right) + \frac{\sigma^2}{9n}$$

$$\Rightarrow \quad \frac{\sigma^2}{3} < \lambda\left(\frac{8}{9}\right) + \frac{\sigma^2}{27}$$

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2.5) Yes it is still true b/c

$$\frac{\sigma^2}{n} < \lambda\left(\frac{8}{9}\right) + \frac{\sigma^2}{9n}$$

we will always be adding a $\lambda\left(\frac{8}{9}\right)$ in our right side.

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