APPM 5500 Stats Math

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1) Prove or disprove: $P(A^c) = 1 - P(A)$
2) For any two sets E_1 and E_2. Show that:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

3) Let $X = \#$ of heads in 3 tosses of a fair coin. What is the underlying probabilistic process? What is the sample space? What are the possible values of X? What is the probability that X is equal to 1: $P(X = 1)$?

4) Suppose that your music app contains 100 songs, 10 of which are by the Beatles. Using the shuffle feature, what is the probability that the first Beatles song heard is the fifth song played? (With out replacement)

5) What is the probability of a flush in poker(five cards)?

6) What is the probability of exactly three queens in poker (five cards)?

7) Plot the proportion of times the 'coin' landed on heads on the ith flip, for $i = 1...500$?

8) A specific smartphone part is assembled at a plant that uses two different assemly lines, A and B. Line A uses older equipment than B, so it is somewhat slower and less reliable. Suppose that on a given day, line A has assembled 8 parts, whereas B has produced 10. From the 8 parts from A, 2 were defective and 6 as not defective. From the 10 parts from B, 1 was defective and 9 not defective. The manager chose a part that turned out to be defective - i.e. the event $D = \{defective\}$ has occurred. What is the chance that is was made by the line A?

9) You roll a 6-sided die twice and hope for two fours. What is the probability you roll two fours?

10) If your first roll is a four,what is the probability that your second roll as a four?

11) In a school of 1200 students, 250 are seniors, 150 students take math, and 40 students are seniors and are also taking math. One student is selected at random. Let S represent the a senior is chosen and M represent that a student taking math is chosen. If the randomly chosen student is a senior, then what is the probability that they are taking math?

12) Let A_1, A_2, A_3, and B be events where A_1, A_2, A_3, are all disjoint and $A_1, A_2, A_3 = \Omega$. Show,

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

13) An individual has 3 different email accounts. Most of the messages, in fact 70%, come into account 1, whereas 20% come into account 2 and the remaining 10% into account 3. Of the messages into account 1, only 1% are spam, whereas the corresponding percentages for accounts 2 and 3, are 2% and 5% respectively. What is the probability that a randomly selected message is spam? Say she randomly selected a message and it was indeed spam. What is the probability that it came from account 1?

14) An aircraft emergency locator transmitter (ELT) is a device designed to transmit a signal in the case of a crash. The Altigauge Manufacturing Company meakes 80% of the ELT's, the Bryant Company makes 15% of them, and the Chartair Company makes the other 5%. The ELT's made by Altigauge have 4% rate of defects, the Bryant ELT's have a 6% rate of defects, and the Chartair ELTs have a 9% rate of defects. A randomly selected ELT is tested and is found to be defective. What is the probability that it was made by the Altigauge Manufacturing Company? Which company do you suspect has the lowest market share? Why?

15) Let $P(B) > 0$, prove or disprove: $P(\Omega|B) = 1$

16) Let $P(B) > 0$, prove or disprove: For any $E, P(E|B) \geq 0$

17) Let $P(B) > 0$ and E_1 and E_2 be disjoints events. Prove or disprove:

$$P(E_1 \cup E_2) = P(E_1|B) + P(E_2|B)$$

18) A fair coin is tossed 4 times. A "successful toss" is defined to be the coin landing on heads. Let $X = \#$ of success/heads in 4 tosses. What is $P(X - 2)$? What is $P(X = 3)$?

19) What is the probability that you sell 1000 cookies when your rate is 5 per hour?

20) Let X denote the number of mosquitoes captured in a trap during a given time period. Suppose that X has a Poisson distribution with $\lambda = 4.5$. What is the probability that the trap contains 5 mosquitoes? Code this in R? Probability that the trap contains at least 5 mosquitoes? Code in R.

21) A factory makes parts for a medical device company. 6% of those parts are defective. For each one of the problems below:

0.1 Define an appropriate random variable for the experiment

0.2 Give the values that the random variable can take on.

0.3 Find the probability that the random variabel equals 2

0.4 State any assumptions you need to make

1) Out of 10 parts, x are defective.
2) Upon observing an assembly lien, x non-defective parts are observed before finding a defective part.
3) The number of defective parts made per day, where the rate of defective parts per day is 10.

22) Suppose we are given the following PMF:

$$f(x)\begin{cases} 0.5 & x = 0 \\ 0.167 & x = 1 \\ 0.333 & x = 2 \\ 0 & otherwise \end{cases}$$

Calculate $F(0), F(1), F(2)$
What is F(1.5)? F(20.5)?
Is $P(X < 1) = P(X \leq 1)$?

23) Consider the reference line connecting the valve stem on a tire to the center point. Let X be the angle measured clockwise to the location of an imperfection. The PDF for X is? Prove the PDF for X? How do we calculate $P(90 \leq X \leq 180)$? What is the probability that the angle of occurrence is within 90 of the reference line?

24) Find P(X ¿ x) for the Exponential Distribution?

25) Let $f(x) = e^{-x}$ for$x \geq 0$.

Show that $f(x)$ is a pdf. The median of a continuous random variable with pdf $f(x)$ is defined as the value m such that $P(X \leq m) = 0.5$, where

$P(X \leq m) = \int_{-\infty}^{m} f(x)dx/$ Show that the median of a random variable X with pdf $f(x)$ is given by $\log(2)$.