1)

$$T = \frac{\bar{X} - Y - \delta_0}{\sqrt{S_p^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}} \sim t_{n_x + n_y - 2}$$

$$S_p^2 = \frac{(n_x - 1) S_x^2 + (n_y - 1) S_y^2}{n_x + n_y - 2}$$

$H_0 : \mu_x - \mu_y = 0 = \delta_0 \quad H_1 : \mu_x - \mu_y \neq 0$

$n_x = 30 \quad n_y = 30 \quad \delta = 0 \quad \bar{x} = 1.76 \quad \bar{y} = 1.536$

$\alpha = 0.01 \quad \sqrt{S_x^2} = 0.385 \quad \sqrt{S_y^2} = 0.301$

So, $T = \dfrac{1.76 - 1.536}{\sqrt{S_p^2\left(\frac{1}{30} + \frac{1}{30}\right)}} = 2.510 < Z_{\alpha/2} = 2.57$

$$S_p^2 = \frac{(30-1)(0.14) + (30-1)(0.09)}{30 + 30 - 2}$$

$2.510 < 2.57$, we have enough evidence against the null hypothesis. That means we have evidence that the design modification has an effect on the refrigerators' avg. energy consumption.

2) $n = 15$ $\sigma^2 = 9$ $\alpha = 0.01$ $Z_\alpha = -2.32$

$H_0: \mu = 100$ $H_1: \mu < 100$ $Z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$$\gamma(98) = P(\text{reject } H_0;\ H_1: \mu = 98)$$

$$= P\left(\frac{\bar{X} - 100}{\frac{3}{\sqrt{15}}} < -2.32;\ \mu = 98 \right)$$

$$= P\left(\bar{X} < -2.32(3/\sqrt{15}) + 100;\ \mu = 98 \right)$$

$$= P\left(\bar{X} < -1.80;\ \bar{X} \sim N\left(98, \frac{9}{15}\right)\right)$$

$$= \text{pnorm}\left(-1.80,\ 98,\ \frac{3}{\sqrt{15}}\right) \approx 0.60$$

Without doing any calculations if $\mu = 96 \in H_1$, the probability it will reject H_0 when H_0 is false will increase.

3)

prior: $\pi(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} r^{\alpha-1} e^{-\beta r}$

likelihood : $f(\underline{x}|\lambda) \prod_{i=1}^{n} = \frac{e^{-r} r^{x}}{x!} = \frac{e^{-nr} r^{n\bar{x}}}{\prod_{i=1}^{n} x!}$

posterior : $\pi(\lambda|x) \propto f(\underline{x}|\lambda)\,\pi(\lambda)$

$$\Rightarrow \frac{e^{-nr} r^{n\bar{x}}}{\prod_{i=1}^{n} x!} \cdot r^{\alpha-1} \cdot e^{-\beta r}$$

$$= \frac{e^{-nr-\beta r} \cdot r^{\alpha-1+n\bar{x}}}{\prod_{i=1}^{n} x!} = \frac{e^{-r(n+\beta)}\, r^{\alpha-1+n\bar{x}}}{\prod_{i=1}^{n} x_i!}$$

$$= \lambda|x \sim \text{Beta}\,(n\bar{x}+\alpha, -nr-\beta r)$$