

$$E(p|X) = \frac{n\bar{x} + \alpha}{n + \alpha + \beta}$$

if $\alpha + \beta \rightarrow 0$ then $\frac{n\bar{x} + \alpha}{n}$

and sample mean or prior mean is

$$\hat{p} = \bar{x} \neq \frac{n\bar{x} + \alpha}{n}$$