Finals Review

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0.1

[1] Suppose wait times for a particular bank teller are exponentially distributed with rate parameter λ minutes. A random sample $n = 100$ customer wait times were selected, $\bar{x} = 1.2$ and $s^2 = 2.1$. Compute an approximate 90% CI (two-sided) for the population mean $\mu = \frac{1}{\lambda}$.

0.2

Suppose a company is considering putting a new type of coating on bearings that it produces. The true average wear life with the current coating is know to be 1000 hours. With μ denoting the true average life for the new coating, the company would not want to make any (costly) changes unless evidence strongly suggested that μ exceeds 1000. Write the appropriate hypothesis formulation.

0.3

A company A produces circuit boards, but 10% of them are defective. Company B claims that they produce fewer defective circuit boards. What are the null and alternative hypothesis?
Suppose that in the sample of $n = 200$, 7 circuit boards from Company B were defective. Calculate the test statistics, and decide, based on a lower tailed test with significance level $\alpha = 0.05$, whether we should reject the null hypothesis.

0.4

Suppose that 12 people in a sample of 95 are members of the Green Party. Calculate an approximate 90% confidence interval for the true proportion of Green Party members in the population. Interpret this interval.

0.5

Let $X_1, ..., X_n \overset{iid}{\sim} N(\mu, 1)$. Lets calculate the power for a size $\alpha = 0.05$ test of $H_0 : \mu = 0$ vs $H_1 : \mu > 0$, for the specific value of $0.08 \in H_1$. Let $n = 5$.

[1] Answers are in the lecture and HW

0.6

Let $X_1, ..., X_n \overset{iid}{\sim} Binomial(1, p)$. Let the prior distribution on p be a beta distribution. Find the posterior distribution of $p|x$, i.e, the posterior distribution of the parameter p given that we observed the data x.
The posterior is given by?
Estimator of p?

0.7

Suppose researchers would like to know the probability, p, that an individual in a given population has a genetic marker that predisposes them for disease D (this genetic marker is such that an individual has it or doesn't; assume that our test for the marker is completely accurate). Researchers collected data in the following way: they tested people for this genetic marker until they found a person who had it. They stopped data collection at that point. Here's the data: $x = (0, 0, 0, 0, 1)$. Suppose your prior beliefs are best represented by a beta(2,2) distribution. What is the posterior distribution for $p|x$?
95% credible interval? Let F^{-1} be the inverse cdf of the posterior distribution for $p|x$, i.e, the cdf of beta$(\alpha + 1, y + \beta)$. Then a 95% credible interval is given by?

0.8

A sample of 50 lenses used in eyeglasses yields a sample mean thickness of 3.05 mm and a sample standard deviation of 0.34 mm. The desired true average thickness of such lenses is 3.20 mm. Do the data suggest that the true average thickness of such lenses is something other than what is desired? Test using $\alpha = 0.05$.