1) $f(x) = Log_{10}\left(\frac{x}{x-1}\right)$, $x = \{2, 3, \dots, 9\}$

i) $f(x) \geq 0, \forall x$

This is true because our base $a > 1$ and $x > 1$. If $x < 1$ then we would get a negative number, however, $2 \leq x \leq 9$ These are from rules of Logs.

2) $\sum_{x=2}^{9} f(x) = 1$

sum(log10((2:9)/(1:8)))

$$\sum_{x=2}^{9} \log_{10}\left(\frac{x}{x-1}\right) = 0.95 \neq 1$$

Hence not a PMF.

2.1) $f(x) = \frac{4}{3}x(2-x^2)$, $0 \leq x \leq 1$

i) $f(x) \geq 0, \forall x$

$$f'(x) = \frac{4}{3}\left[\frac{d}{dx}x \cdot (2-x^2) + x \cdot \frac{d}{dx}(2-x^2)\right]$$

$$= \frac{4}{3}\left[2-x^2 + x(0-2x)\right]$$

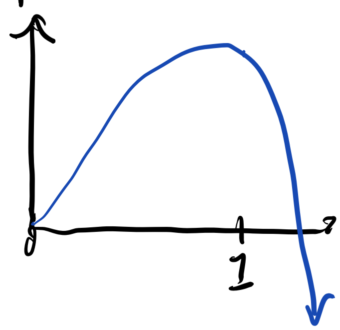
$$= \frac{4}{3}(2-3x^2)$$

$$f''(x) = \frac{4}{3}(2-3x^2)$$

$$= \frac{4}{3}(0-6x)$$

$$= -8x$$

Since we know its concave down and its from $0 \le x \le 1$, therefore $f(x) \ge 0$



2) $$\int_0^1 \frac{4}{3}x(2-x^2)\,dx = \frac{4}{3}\int_0^1 2x - x^3\,dx$$

$$= \frac{4}{3}\left[x^2 - \frac{x^4}{4}\right]_0^1$$

$$= \frac{4}{3}\left[1 - \frac{1}{4} - (0-0)\right]$$

$$= \frac{4}{3}\cdot\frac{3}{4} = 1$$

2.2) $P\left(\frac{1}{2} \le x \le 1\right)$

$$\int_{\frac{1}{2}}^{1} \frac{4}{3}x(2-x^2)\,dx = \frac{4}{3}\left[x^2 - \frac{x^4}{4}\right]\Bigg|_{1/2}^{1}$$

$$= \frac{4}{3}\left[1 - \frac{1}{4} - \left(\frac{1}{4} - \frac{\frac{1}{16}}{4}\right)\right]$$

$$= \frac{11}{16}$$

3.1) $P(B|C) = 0.5 \quad P(C) = \frac{1}{100} \quad P(C^c) = \frac{99}{100}$

$P(B|C^c) = 0.4$

$P(B|C) = 0.5$

3.2) $P(C|B) = \dfrac{P(C \cap B)}{P(B)}$ or $\dfrac{P(B|C)P(C)}{\text{[illegible]}}$

$P(R) \qquad\qquad P(R)$

$$P(R) = P(R|C)P(C) + P(R|C^c)P(C^c)$$
$$= 0.5\left(\frac{1}{100}\right) + 0.4\left(\frac{99}{100}\right)$$
$$= 0.401$$

$$P(C|R) = \frac{0.5\left(\frac{1}{100}\right)}{0.401} = 0.0124$$

3.3) The probability that a research claim is true given that a research finding says that it is, is 1%.

This is very alarming b/c that means the chances of them getting it wrong is 99%. You would expect it would be higher b/c they have a 50% of correctly reporting it.

However, $\frac{1}{100}$ claims are true so maybe I can see why.