HW1-LIF_Model-4

September 3, 2021

0.1 Leaky Integrate and Fire Model

Following the arguments of Gerstner et al (2014), we consider a simple resistor/capacitor (RC) circuit model of a neuron integrating current input. The law of current conservation implies

$$I(t) = I_R + I_C$$

input current $I(t)$ is split into resistive and capacitive current. Ohm's law implies $I_R = u_R/R$ where $u_R = u - u_{\text{rest}}$ is the voltage across the resistor. Capacitive current is $I_C = dq/dt = C du/dt$ where we used the definition of capacity $C = q/u$, so that

$$I(t) = \frac{u(t) - u_{\text{rest}}}{R} + C\frac{du}{dt}.$$

Rearranging and defining $\tau_m = RC$ (since R has units of ohms (Ω) and C has units of farads and ohms times farads equals seconds (the curious reader should look this up!), we have:

$$\tau_m \frac{du}{dt} = -\left[u(t) - u_{\text{rest}}\right] + RI(t),$$

where u is the membrane potential and τ_m is the membrane time constant. Clearly, as the above equation is linear, it will simply filter any input $I(t)$ with some lag. Therefore, to account for the spiking mechanism of natural neurons, we assume that there is a threshold voltage u_{th} at which a *spike* is initiated, followed by a reset of the voltage to the resting potential. This suggests the following conditional reset equation

$$\text{if } u(t) \geq u_{\text{th}} \text{ then } u(t^+) \mapsto u_{\text{rest}},$$

and any time t_j at which $u(t_j) \geq u_{\text{th}}$ is deemed a *spike time*, leading to a vector of spike times $(t_1, t_2, t_3, ...)$. You will study this model in detail on HW1.

Below we instantiate python code associated with the above differential equation (these are called lif_mod.py and lif_per.py in the python code folder). Note, we will want to use numerical methods and plotting, and as such we import *numpy* and *matplotlib*.

[3]:
```
import numpy as np
import matplotlib.pyplot as plt
```

Now we can refer to any numpy functions using np.* and any matplotlib python plotting functions using plt.*

To begin, let's use Euler's method to solve the above LIF model in the case of a constant current input $I(t) = \bar{I}$. This requires initializing model parameters and running a for-loop. All relevant model parameters are given (units are in comments):

[598]:

```
taum = 10     # membrane time constant (ms)
urest = 0    # resting potential (mV)
R = 1        # resistance (ohms)
I = 3      # input current (mA)
uth = 0.5      # spiking threshold (mV)
```

And simulation parameters are given (relevant units also in comments):

[599]:

```
T = 3      # total time to run
dt = 0.001    # time step
nt = int(np.round(T/dt)+1)      # number of entries in vector array
tvec = np.linspace(0,T,nt)      # time vector (ms)

u = np.zeros(nt)    # vector of voltage entries (mV)
st = 0              # initialize vector of spike times (ms)
```

If you're a MATLAB user, you will recognize common MATLAB numerical/vector functions in the above, preceded by *np.*. Now we run the for-loop for Euler's method with the timestep (dt) given above:

[600]:

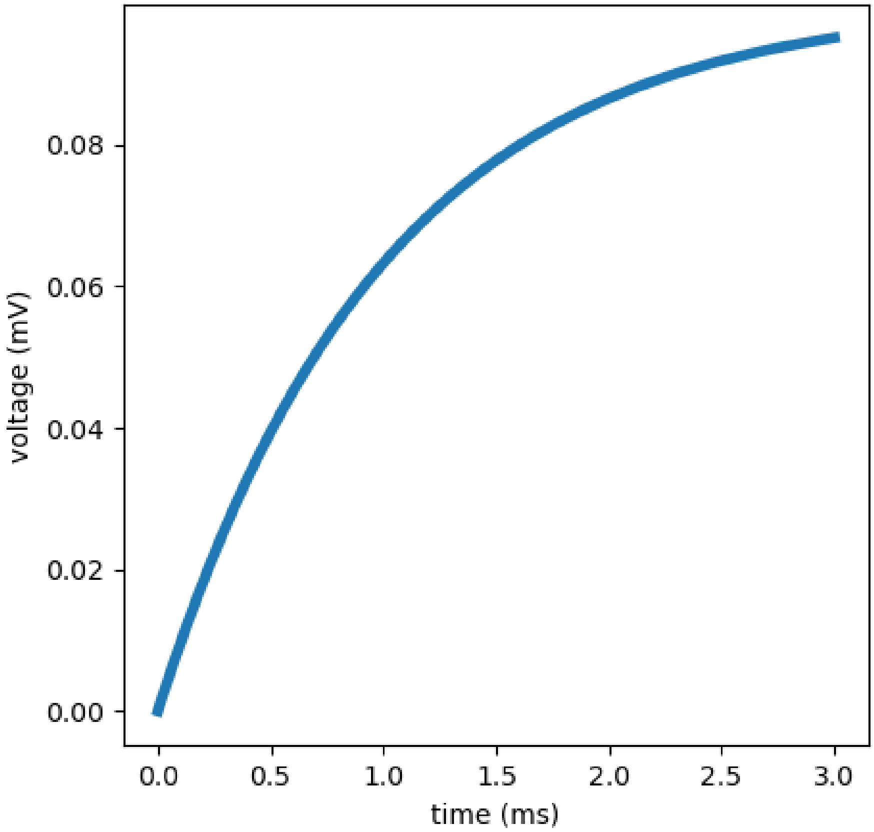
```
for j in np.arange(nt-1):
    u[j+1] = u[j]+dt*(R*I-u[j])/taum;
    if u[j+1]>uth:
        u[j+1]=urest;                        # reset the voltage to resting potential
        st = np.append(st,tvec[j+1])     # add on another spike time
```

So now, we have the vector u, which contains all output voltage values from the simulation. Note, we had to insert an *if* statement to instantiate the reset condition and within this reset condition, we updated the spike time vector (with the append function).

HW-1: 4a When $I < I_{crit}$:

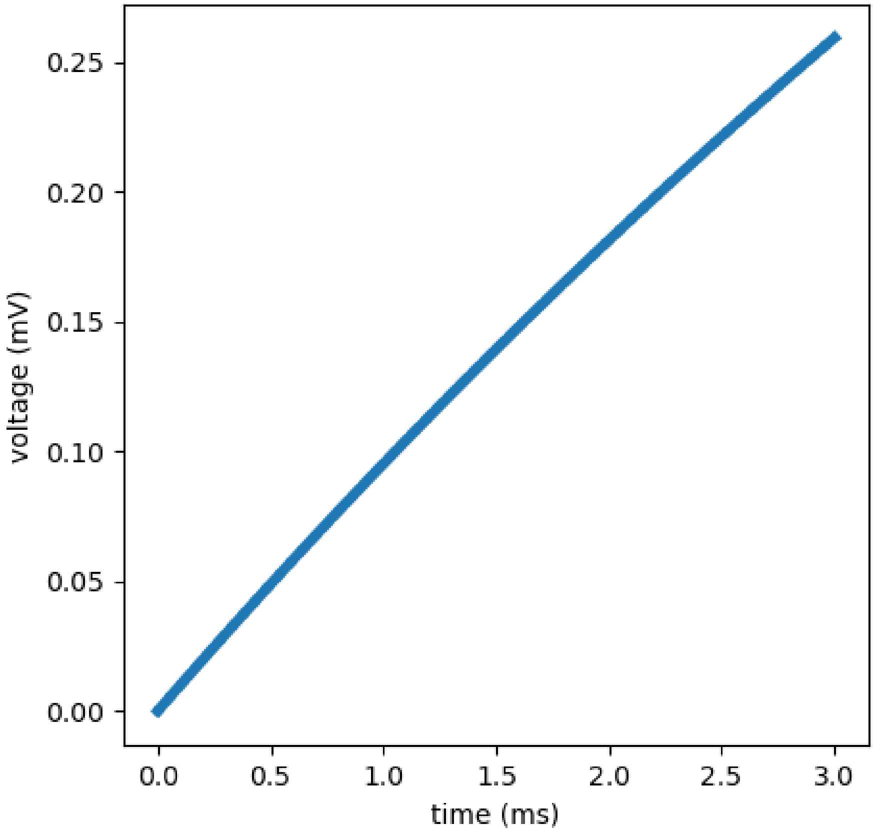
[41]:

```
fig = plt.figure(figsize=(5,5))  # initialize figure with a given area in inches
plt.plot(tvec,u,linewidth=4.0)  # plot command with vectors as arguments
plt.xlabel('time (ms)')         # label for x-axis
plt.ylabel('voltage (mV)')      # label for y-axis
plt.show()                      # needed in order to actually produce the plot␣
 ↪for you to see
```



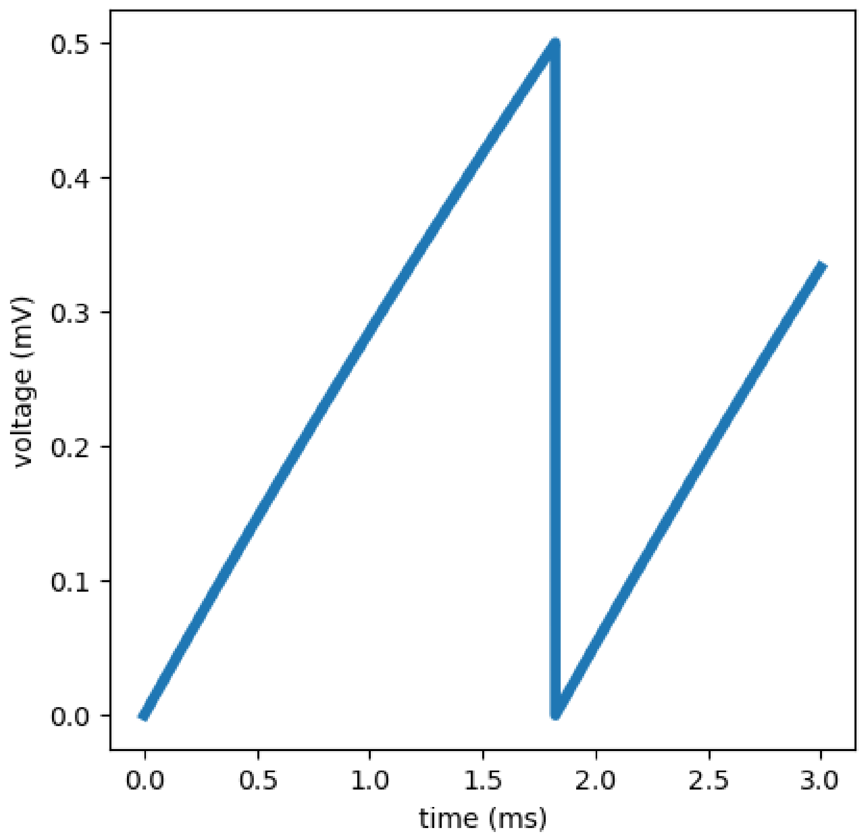
When $I = I_{crit}$:

[583]:
```
fig = plt.figure(figsize=(5,5))  # initialize figure with a given area in inches
plt.plot(tvec,u,linewidth=4.0)  # plot command with vectors as arguments
plt.xlabel('time (ms)')         # label for x-axis
plt.ylabel('voltage (mV)')      # label for y-axis
plt.show()                      # needed in order to actually produce the plot␣
 ↪for you to see
```



When $I > I_{crit}$:

[601]:
```
fig = plt.figure(figsize=(5,5))  # initialize figure with a given area in inches
plt.plot(tvec,u,linewidth=4.0)  # plot command with vectors as arguments
plt.xlabel('time (ms)')         # label for x-axis
plt.ylabel('voltage (mV)')      # label for y-axis
plt.show()
```



As I gets closer to I_{crit}, we get a linear line going towards the threshold.

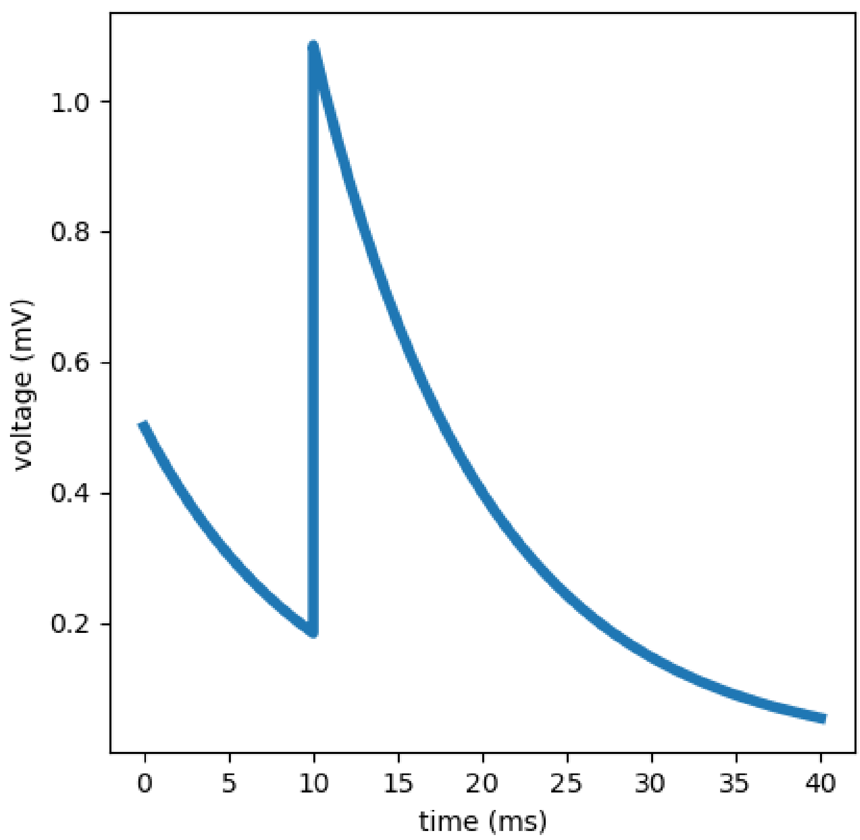
$4b$

$R = 1, \tau_M = 10, u_{rest} = 0, u_{th} = 1, u_{start} = 0.5, A = 1$

When $A > A_{min}$:

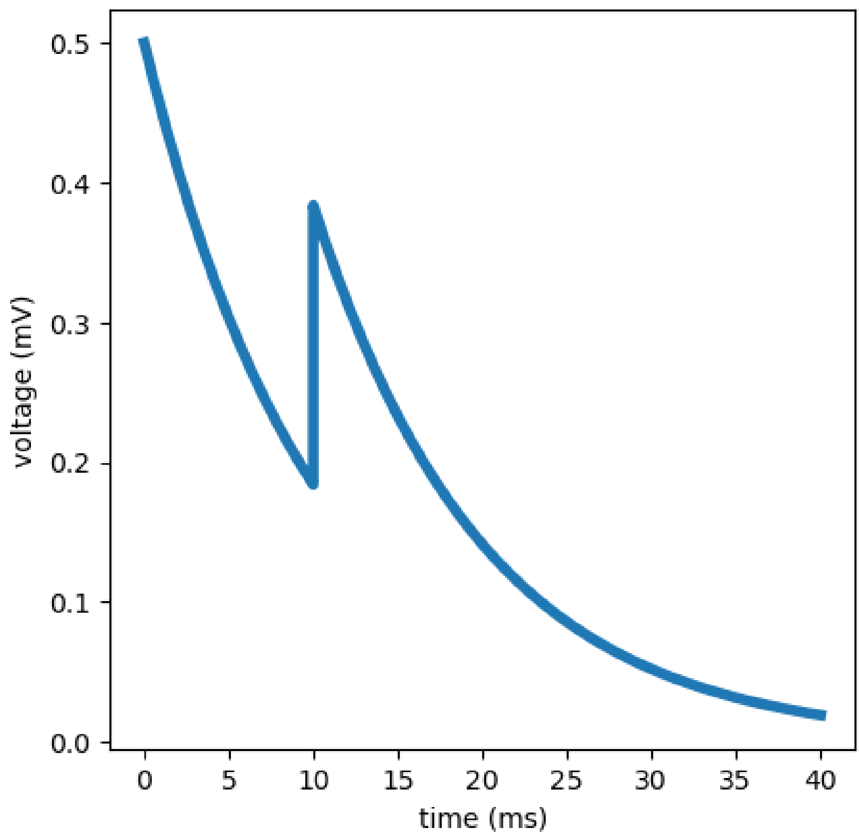
[20]:
```
taum = 10    # membrane time constant (ms)
urest = 0   # resting potential (mV)
R = 1       # resistance (ohms)
I = 2       # input current (mA)
uth = 1     # spiking threshold (mV)
A = 9   # current modulation amplitude (mA)

T = 40      # total time to run
dt = 0.001   # time step
nt = int(np.round(T/dt)+1)     # number of entries in vector array
tvec = np.linspace(0,T,nt)     # time vector (in ms)

u = [0.5 for i in range(nt)];   # vector of voltage entries
```

```python
st = 0                  # initialize vector to store spike times

tj = 10

for j in np.arange(nt-1):
    u[j+1] = u[j]+dt*(-u[j])/taum;

    if j*dt==tj:
        u[j+1]=u[j+1]+A*R/taum

#     if u[j+1]>uth:
#         u[j+1]=urest;                     # reset the voltage to resting␣
 ↪potential
#         st = np.append(st,tvec[j+1])    # add on another spike time

fig = plt.figure(figsize=(5,5))
plt.plot(tvec,u,linewidth=4.0)
plt.xlabel('time (ms)')
plt.ylabel('voltage (mV)')
plt.show()
```



When $A < A_{min}$:

[440]:
```
taum = 10    # membrane time constant (ms)
urest = 0   # resting potential (mV)
R = 1       # resistance (ohms)
I = 2       # input current (mA)
uth = 1     # spiking threshold (mV)
A = 2   # current modulation amplitude (mA)

T = 40      # total time to run
dt = 0.001   # time step
nt = int(np.round(T/dt)+1)     # number of entries in vector array
tvec = np.linspace(0,T,nt)     # time vector (in ms)

u = [0.5 for i in range(nt)];   # vector of voltage entries

st = 0              # initialize vector to store spike times

tj = 10

for j in np.arange(nt-1):
    u[j+1] = u[j]+dt*(-u[j])/taum;

    if j*dt==tj:
        u[j+1]=u[j+1]+A*R/taum

    if u[j+1]>uth:
        u[j+1]=urest;                   # reset the voltage to resting potential
        st = np.append(st,tvec[j+1])    # add on another spike time

fig = plt.figure(figsize=(5,5))
plt.plot(tvec,u,linewidth=4.0)
plt.xlabel('time (ms)')
plt.ylabel('voltage (mV)')
plt.show()
```



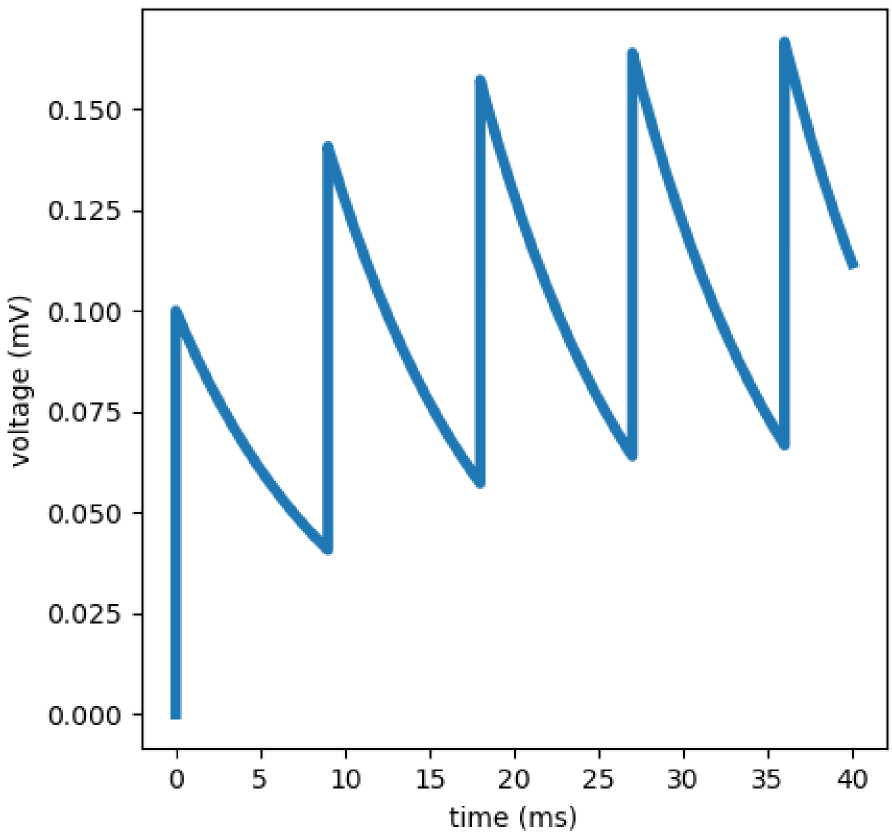
$4c$

$R = 1, \tau_M = 10, u_{rest} = 0, u_{th} = 1, u_{start} = 0, A = 1$

When $T < T_m$:

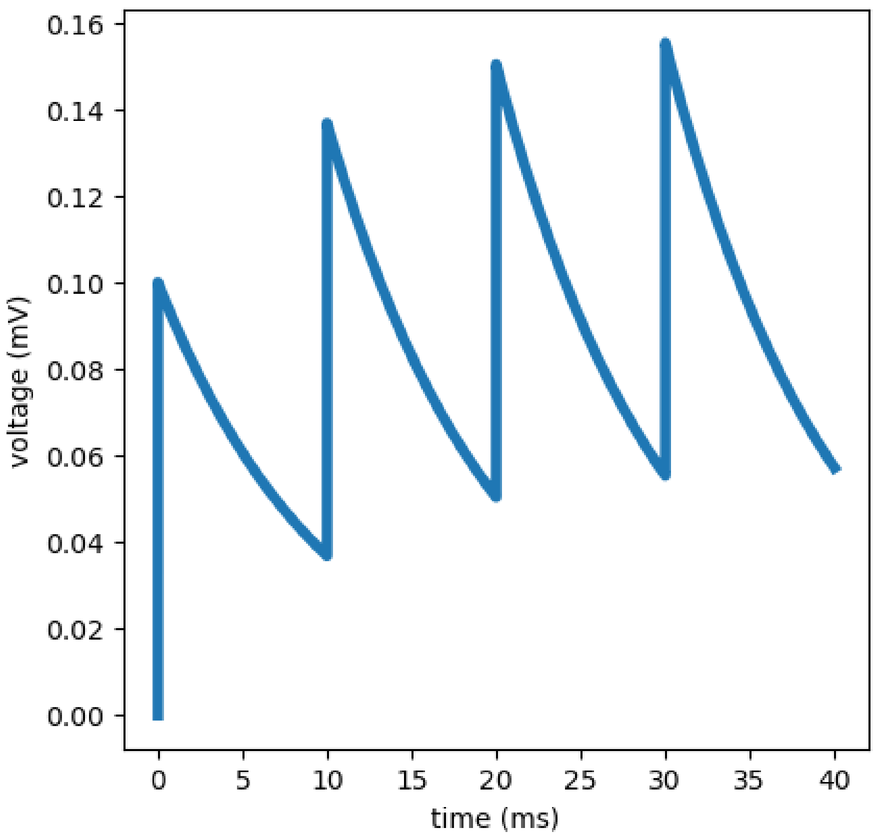
[16]:
```
taum = 10     # membrane time constant (ms)
urest = 0    # resting potential (mV)
R = 1        # resistance (ohms)
I = 2        # input current (mA)
uth = 1      # spiking threshold (mV)
A = 1    # current modulation amplitude (mA)

T = 40       # total time to run
dt = 0.001    # time step
nt = int(np.round(T/dt)+1)      # number of entries in vector array
tvec = np.linspace(0,T,nt)      # time vector (in ms)

u = np.zeros(nt);    # vector of voltage entries

st = 0              # initialize vector to store spike times
```

```
tj = 9          # T time units later

for j in np.arange(nt-1):                    #
    u[j+1] = u[j]+dt*(-u[j])/taum   # update the voltage

    if u[j+1]>uth:
        u[j+1]=urest                         # reset the spike voltage
        st = np.append(st,tvec[j+1])

    if j*dt%tj==0:
        u[j+1] = u[j]+dt*(-u[j])/taum +R*A/taum

fig = plt.figure(figsize=(5,5))
plt.plot(tvec,u,linewidth=4.0)
plt.xlabel('time (ms)')
plt.ylabel('voltage (mV)')
plt.show()
```



When $T > T_m$:

[13]:
```python
taum = 10     # membrane time constant (ms)
urest = 0    # resting potential (mV)
R = 1        # resistance (ohms)
I = 2        # input current (mA)
uth = 1      # spiking threshold (mV)
A = 1    # current modulation amplitude (mA)

T = 40       # total time to run
dt = 0.001    # time step
nt = int(np.round(T/dt)+1)      # number of entries in vector array
tvec = np.linspace(0,T,nt)      # time vector (in ms)

u = np.zeros(nt);    # vector of voltage entries

st = 0               # initialize vector to store spike times

tj = 10


for j in np.arange(nt-1):                 #
    u[j+1] = u[j]+dt*(-u[j])/taum     # update the voltage

    if u[j+1]>uth:
        u[j+1]=urest                    # reset the spike voltage
        st = np.append(st,tvec[j+1])

    if j*dt%tj==0:
        u[j+1] = u[j]+dt*(R*A-u[j])/taum +R*A/taum


# for j in np.arange(nt-1):
#     u[j+1] = u[j]+dt*(-u[j])/taum;

#     if j*dt<tj:
#         u[j+1]=u[j+1]+A*R/taum

#     if u[j+1]>uth:
#         u[j+1]=urest;                    # reset the voltage to resting␣
↪potential
#         st = np.append(st,tvec[j+1])    # add on another spike time

fig = plt.figure(figsize=(5,5))
plt.plot(tvec,u,linewidth=4.0)
plt.xlabel('time (ms)')
plt.ylabel('voltage (mV)')
plt.show()
```


0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
voltage (mV)
0
5
10
15
20
25
30
35
40
time (ms)

[ ]: