$$0 = -u_j + g(u_{j-1} - 2u_j + u_{j+1})$$

$$g u_{j+1} = u_j - g u_{j-1} + 2g u_j$$

$$\Rightarrow \quad u_{j+1} = \left(\frac{1}{g} + 2\right) u_j - u_{j-1}$$

$$\begin{bmatrix} u_{j+1} \\ u_j \\ u_{j-1} \end{bmatrix} = \begin{bmatrix} \frac{1}{g}+2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_j \\ u_{j-1} \\ u_{j-2} \end{bmatrix}$$

$$(A - \lambda I)$$

$$\begin{bmatrix} \frac{1}{g}+2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{g}+2-\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{bmatrix}$$ take det(A)

$$\Downarrow$$

$$0\begin{pmatrix} 1 & -\lambda \\ 0 & 1 \end{pmatrix} - 0\begin{pmatrix} \frac{1}{g}+2-\lambda & -1 \\ 0 & 1 \end{pmatrix} + (-\lambda)\begin{pmatrix} \frac{1}{g}+2-\lambda & -1 \\ 1 & -\lambda \end{pmatrix}$$

$$\Rightarrow 0 = \frac{-\lambda}{g} - 2\lambda + \lambda^2 + 1$$

Now we use quadratic equation to find λ:

$$\lambda^2 - \lambda\left(2 + \frac{1}{g}\right) + 1 = 0$$

$$\lambda = \frac{2 + 1/g \pm \sqrt{\frac{4}{g} + \frac{1}{g^2}}}{2}$$

We obtain,

$$\lambda_1 = \frac{2 + 1/g + \sqrt{\frac{4}{g} + \frac{1}{g^2}}}{2}$$

this goes to explodes therefore we dont need it

B

$$\lambda_2 = \frac{2 + 1/g - \sqrt{\frac{4}{g} + \frac{1}{g^2}}}{2}$$

this stays between 0 and 1

A

Therefore,

$$u_1 = A\lambda_2 \quad \text{and} \quad u_2 = A\lambda_2^2$$

$$0 = I - A\lambda_2 + g(A\lambda_2^2 - A\lambda_2)$$

$$= I - \lambda A + gA\lambda_2^2 - gA\lambda_2$$

$$= I + (g\lambda_2^2 - g\lambda_2 - \lambda_1)A$$

$$A = -\frac{I}{g\lambda_2^2 - g\lambda_2 - \lambda_2}$$

Now for $R_{input}(g) = \frac{u_1}{I}$,

$$R_{input}(g) = -\frac{\lambda}{g\lambda_2 - g - 1}$$

$$\boxed{R_{input}(g) = -\frac{2 + 1/g - \sqrt{\frac{4}{g} + \frac{1}{g^2}}}{2 + 1/g - \sqrt{\frac{4}{g} + \frac{1}{g^2}} - 2g - 2}}$$

As more and more compartments get added we get g's that become insignificant to neuron. We reach a limit of ion channels.