a)

$$\frac{dx}{dt} = I + x^2 = f(x)$$

So let:

$$0 = I + x^2$$

$$x^2 = -I \Rightarrow x = \pm\sqrt{-I}$$

So we have,

$$f'(x) = 2\sqrt{-I} \qquad f'(x) = 2(-\sqrt{-I})$$

For $x = -\sqrt{-I}$, let $I = -4$, $f'(x) = \pm 2$.

for $f'(x) < 0$:

$x' = I + x^2$

Let $x = -1$ and $x = 3$

$x' = -4 + 1 = -3$ (left)

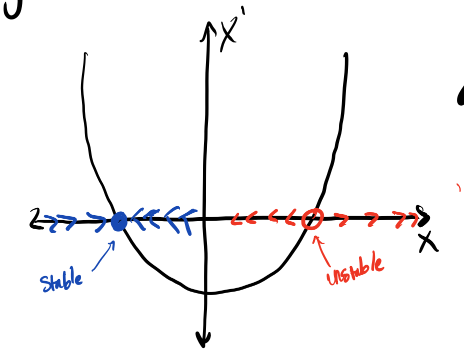
$x' = -4 + 9 = 5$ (right)

for $f'(x) > 0$:

Let $x = 1$ and $x = 3$

$x' = -4 + 1 = -3$

$x' = -4 + 9 = 5$



When $I \to 0$, we get something like this:

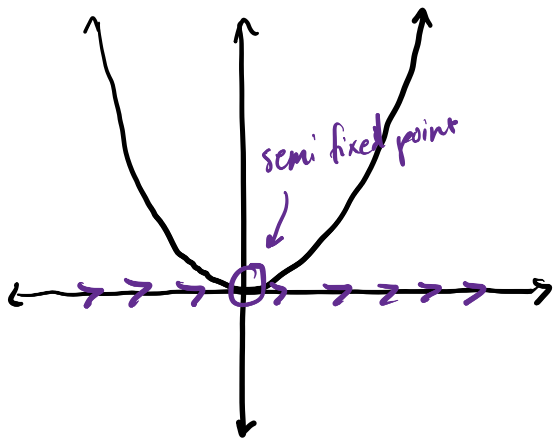
$$f'(x) = 2(\pm\sqrt{-0}) = 0$$

at $x = -2$

$x' = 4$

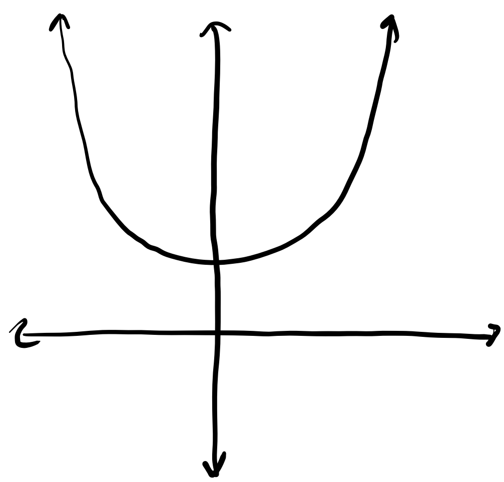
at $x = 2$

$x' = 4$



When $I > 0$ we get:

No fixed pts at all.



For $I < 0$, this represents a repetitive spiking, for $I \to 0$, a single spike, and for $I > 0$ no spiking.

b)

$$\frac{dx}{dt} = I + x^2$$

$$\int \frac{dx}{I+x^2} = \int dt \Rightarrow \frac{\arctan\left(\frac{x}{\sqrt{I}}\right)}{\sqrt{I}} = t + C$$

$$\Rightarrow \arctan\left(\frac{x}{\sqrt{I}}\right) = \sqrt{I}t + C$$

$$\frac{x}{\sqrt{I}} = \tan\sqrt{I}t + \tan(c)$$

$$x = \sqrt{I}\tan\left(\sqrt{I}t + c\right) \qquad x(0) = 0$$

$$\boxed{x = \sqrt{I}\tan\left(\sqrt{I}t\right)}$$

The way T changes with I is by $I > 0$ we have a positive graph but when $I < 0$ we have a negative graph. Positive meaning toward a spike and negative back to rest

c) $\frac{dx}{dt} = 1 + |x|, \quad x(0) = -1$

$$\int \frac{dx}{1+|x|} = dt$$

For $\int \frac{dx}{1+|x|}$ we look at $|x| \begin{cases} x \mid x \geq 0 \\ -x \mid x < 0 \end{cases}$

1) $x \geq 0$:

$$\int \frac{dx}{1+x} = dt \Rightarrow \ln|1+x| = t + C$$

$$1+x = e^{t+C}$$

$$x = Ce^{t} - 1$$

$x(0) = -1$,

$$-1 = C - 1$$

$$C = 0$$

$$x = 0$$

2) $x < 0$:

$$\int \frac{dx}{1-x} = \int dt$$

$$-\ln|1-x| = t + C$$

$$\ln|1-x| = C - t$$

$$1 - x = e^{C-t}$$

$$x = Ce^{-t} + 1$$

$x(0) = -1$ $\quad -1 = C + 1$

$$C = -2$$

$$\boxed{x = -2e^{-t} + 1}$$

So for a spike to occur $-2e^{-t}$ has to be greater than 1.