a) $\frac{dv}{dt} = -v + wv$, $\frac{dw}{dt} = \gamma(1-v)v$, $\gamma > 0$

The v means the input in the neuro population and the w is the weight vector or weighted sum of inputs. As v increase our weighted sum will also increase, increasing the weights and inputs current.
For v decreasing the opposite will occur

b)

1) Find Nullclines:

$0 = -v + wv$, $0 = \gamma(1-v)v$

$v = wv$ $\quad$ $v = 1$

2) Find equilibrium: $v = wv = 0$

$1 = w$, $v = 1$

&

$w = 0$ $\quad$ $v = 0$

3) Find stability $\begin{pmatrix} F_v & F_w \\ G_v & G_w \end{pmatrix}$ eigen values

$$\begin{pmatrix} F_v & F_w \\ G_v & G_w \end{pmatrix} = \begin{pmatrix} w-1 & v \\ \gamma - 2v & 0 \end{pmatrix}$$

$$J(1,1) = \begin{pmatrix} 0 & 1 \\ r-2 & 0 \end{pmatrix}$$

$$|\lambda I - J| = \begin{pmatrix} \lambda & -1 \\ 2-r & \lambda \end{pmatrix} = 0$$

$$= \lambda^2 - (r-2) = \lambda^2 - r + 2$$

$$\lambda_1 = \lambda - i\sqrt{2-r} \quad , \quad \lambda_2 = \lambda + i\sqrt{2-r}$$

For a positive λ, the real part will be unstable however the imaginary is unstable and stable hence we will still have an unstable node.

For a negative λ, the real part will be stable and we will have similar imaginary parts from the previous statement, so we will have a saddle.

For $J(0,0)$:

$$J(0,0) = \begin{pmatrix} -1 & 0 \\ r & 0 \end{pmatrix}$$

$$\det(\lambda I - J) = \begin{pmatrix} \lambda+1 & 0 \\ -r & \lambda \end{pmatrix} = 0$$

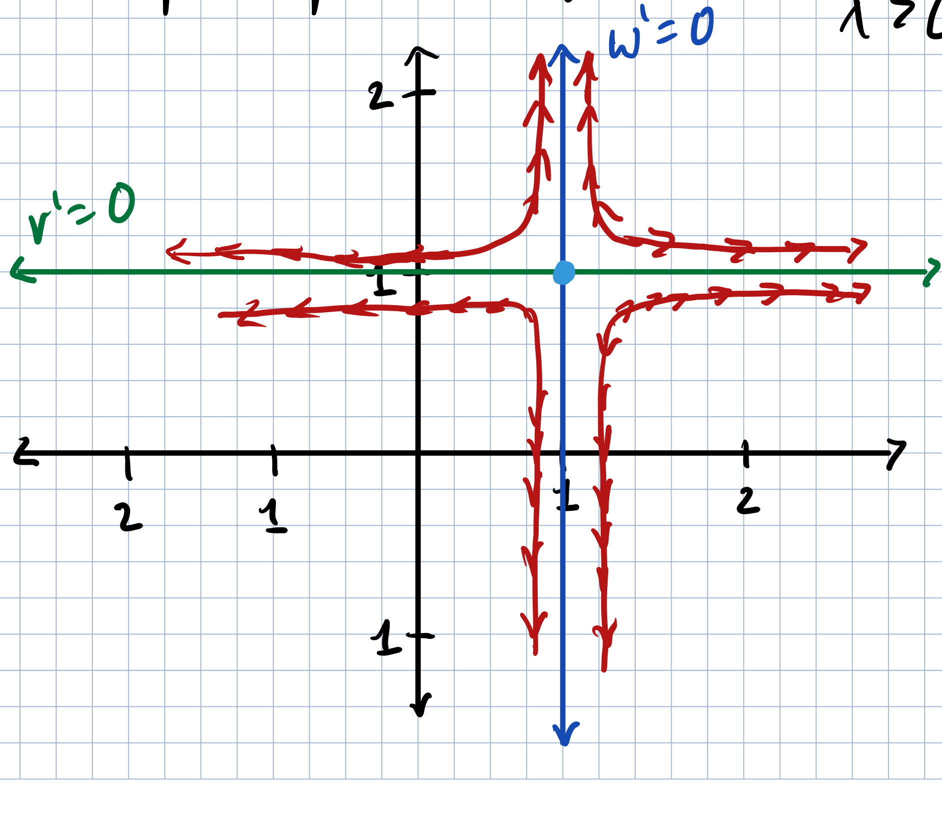
$$= \lambda^2 + \lambda = 0$$

$$\lambda = 0, -1$$

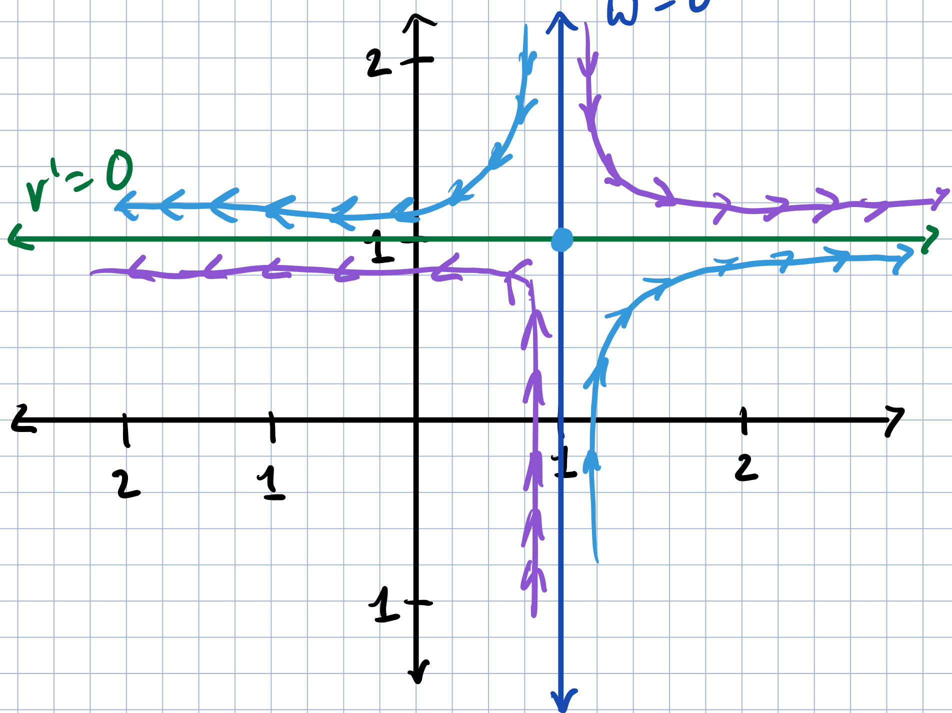
From $J(0,0)$ we have a stable saddle since the real part is $0, -1$.

c) Sketch phase plane : $J(1,1)$

$\lambda > 0$



$\lambda < 0$



$J(0,0)$: