a) $w_{n+1} = w_n + \gamma v_n (u_n - w_n v_n)$

$v_n = u_n^T w_n \quad C = \langle u_n u_n^T \rangle$

So, $w_{n+1} - w_n = \gamma v_n (u_n - w_n v_n)$

$\langle w_{n+1} - w_n \rangle = 0$

$\langle \gamma v_n (u_n - w_n v_n) \rangle = 0$

$\langle \gamma v_n u_n - \gamma w_n v_n^2 \rangle = 0$

$\langle \gamma v_n u_n \rangle = \langle \gamma w_n v_n^2 \rangle$

$\langle u_n^T w_n u_n \rangle = \langle w_n v_n^2 \rangle$

$C \bar{w}_n = \mu \bar{w}$

So since μ is the eigenvalue then $\bar{w}$ is the eigenvector b/c we multiply $\mu \cdot \bar{w}$ which gives us our $C\bar{w}$.

b) $C \bar{w}_n = \mu \bar{w} \Rightarrow \bar{w}^T C \bar{w}_n = \mu \bar{w} \bar{w}^T$

$\mu = \langle v_n^2 \rangle = (w^T u_n)(w^T u_n)^T = w^T \langle u_n u_n^T \rangle w$

$= w^T C w$

$\bar{w}^T C \bar{w}_n = \mu \bar{w} \bar{w}^T$

$\mu = \mu \|w\|^2$

c) $w_{n+1} - w_n = \Delta \bar{w}_n$

$$\Rightarrow C\bar{\omega}_n - (\bar{\omega}_n^T C \bar{\omega}_n)\bar{\omega}_n$$

$$\Rightarrow C(\bar{\omega} + \varepsilon_n) - (\bar{\omega}_n^T C(\bar{\omega} - \varepsilon_n))(\bar{\omega} + \varepsilon_n)$$

$$\Rightarrow C(\bar{\omega} + \varepsilon_n) - ((\omega^T + \varepsilon_n^T) C(\bar{\omega} - \varepsilon_n))(\bar{\omega} + \varepsilon_n)$$

$$C\bar{\omega} + C\varepsilon_n - [(\omega^T C - \varepsilon_n^T C)(\bar{\omega} - \varepsilon_n)(\bar{\omega} + \varepsilon_n)$$

$$-[\omega^T C\bar{\omega} - \omega^T C\varepsilon_n - \varepsilon_n^T C\bar{\omega} + \varepsilon_n^T C\varepsilon_n]$$

$$-[\mu - \omega^T C\varepsilon_n - \varepsilon_n^T C\bar{\omega} + \varepsilon_n^T C\varepsilon_n](\bar{\omega} - \varepsilon_n)$$

$$\mu\bar{\omega} - \omega^T C\bar{\omega}\varepsilon_n - \varepsilon_n^T C\bar{\omega}^2 - \varepsilon_n^T C\varepsilon_n\bar{\omega}$$

$$-\mu\varepsilon_n + \omega^T C\varepsilon_n^2 + \varepsilon_n^T C\bar{\omega}\varepsilon_n + \varepsilon_n^T C\varepsilon_n^2$$

$$\mu\bar{\omega} - \mu\varepsilon_n - \varepsilon_n^T C\bar{\omega}^2 - \varepsilon_n^T C\bar{\omega}^2 - \varepsilon_n^T C\varepsilon_n\bar{\omega} - \mu\varepsilon_n - \cancel{\omega^T C\varepsilon_n^2}$$

$$+ \varepsilon_n^T C\bar{\omega}\varepsilon_n + \cancel{\varepsilon_n^T C\varepsilon_n^2}$$

$$\mu\bar{\omega} - \mu\varepsilon_n - 2\varepsilon_n^T C\bar{\omega}^2 - \cancel{\varepsilon_n^T C\varepsilon_n\bar{\omega}} - \mu\varepsilon_n + \cancel{\varepsilon_n^T C\bar{\omega}\varepsilon_n}$$

$$\mu\bar{\omega} - 2\mu\varepsilon_n \qquad -2\mu(\varepsilon_n^T\bar{\omega})\bar{\omega}$$

$$\Rightarrow \Delta\bar{w}_n = (C - \mu I)\varepsilon_n - 2\mu(\varepsilon_n^T\bar{\omega})\bar{\omega}$$

d) $e_j^T \Delta \bar{\omega}_n = \left[(C - \mu I) \varepsilon_n - 2\mu (\varepsilon_n^T \bar{\omega}) \bar{\omega} \right] e_j^T$

$= C e_j^T \varepsilon_n - \mu I \varepsilon_n e_j^T - 2\mu \varepsilon_n^T \bar{\omega}^2 e_j^T$

$= \lambda_j \varepsilon_n + \mu \varepsilon_n e_j^2 - 2\mu \varepsilon_n e_j^T$

$= \lambda_j e_j^T e_n - \mu e_j^T e_n$

$= (\lambda_j - \mu) e_j^T e_n$