a) $P_F = \nu \Delta t$ is the probability of firing

We want to know the prob. of 0 firing in Δt.
So,

$$P(0 \text{ spiking during } \Delta t) = 1 - \nu \Delta t$$

b) Probabality neuron goes $N\Delta t$ without firing?
we know,

$$P(0 \text{ spiking during } \Delta t) = 1 - \nu \Delta t$$

Lets say we have 3 time steps where we dont fire
So,

$$(1-\nu\Delta t)(1-\nu\Delta t)(1-\nu\Delta t)$$

therefore

$$(1-\nu\Delta t)^3$$

So if we don fire for $N\Delta t$:

$$P(0 \text{ spiking in } N\Delta t) = (1-\nu\Delta t)^N$$

c) Probability it fires next exactly at $N\Delta t$, when it fired at $t=0$?

$$P(\text{first } N\Delta t \,/\, \hat{t}\,) = \nu\Delta t[1-\nu\Delta t]^{N-1}$$

d) $\lim_{\Delta t \to 0} (1-v\Delta t)^N$, we know $t = N\Delta t$

Notice by taking the inner limit $(1-v\Delta t)$ this get close to 1, (0.999) but as $N \to \infty$, the

$$\lim_{\Delta t \to 0} (1-v\Delta t)^{\lim_{N\to\infty} N} = 0$$

Also note that,

$\lim_{\Delta t \to 0} (1-v\Delta t)^N$ so $N = \frac{t}{\Delta t}$

$\Rightarrow \lim_{\Delta t \to 0} (1-v\Delta t)^{\frac{t}{\Delta t}}$ $\qquad \lim (1-v\Delta t)^{N\to\infty}$

Using taylor series,

$$e^{-tv} - \frac{1}{2}\Delta t\left(tv^2 - e^{-tv}\right) + \frac{1}{24} t v^3 \Delta t^2 e^{-tv}(3tv-8) - \ldots$$

Gives us,

$$\lim_{\Delta t \to 0} (1-v\Delta t)^{\frac{t}{\Delta t}} = e^{-vt}$$

and as $t \to \infty$, $e^{vt} = 0$

$$e^{-v(0)} = 1 = S_0(t:v)$$

e) $\text{PDF} = P_0(t;v) = -\dfrac{dS_0}{dt}$

We know $S_0(t:v) = e^{-vt}$ so,

$$\frac{-dS}{dt} = e^{-vt} \Rightarrow \frac{d}{dt}S = -e^{-vt}$$

$$\Rightarrow -e^{-vt}\frac{d}{dt} = ve^{-vt}$$

We can also show by taking the limit, and $t = N\Delta t$.

$$\lim_{\Delta t \to 0} \frac{P(N, \Delta t)}{\Delta t} = \frac{v\cancel{\Delta t}(1-v\Delta t)^{\frac{t}{\Delta t}}}{\cancel{\Delta t}}$$

$$\Rightarrow \lim_{\Delta t \to 0} v(1-v\Delta t)^{\frac{t}{\Delta t}} = ve^{-vt}$$

Hence,

$$P_0(t;v) = ve^{-vt}$$

f) $\rho_F^1 = 4\ hz \quad \rho_F^2 = 2\ hz$

$$P(n \text{ spikes during } \Delta t) = \frac{(\nu\Delta t)^n e^{-\nu\Delta t}}{n!}$$

So the prob. of no spikes would occur in neuron 1, $t = 2s$

$$P(0 \text{ spikes during } 2(s)) = \frac{(4\cdot 2)^0}{0!} e^{-4(2)}$$

$$= e^{-8} = .0003$$

The prob. of no spikes would occur in neuron 2, $t = 2s$

$$P(0 \text{ spikes in } 2(s)) = \frac{(2\cdot 2)^0}{0!} e^{-2(2)}$$

$$= e^{-4} = 0.018$$

It is more likely to be neuron 2.