2a) $P(K; v, t) = \frac{(vt)^K}{K!} e^{-vt}$

$v_1 = 3hz$ or $v_2 = 4hz$

$P(\overset{K}{10}; \overset{v}{3}, \overset{t}{3(s)})$ or $P(\overset{K}{10}; \overset{v}{4}, \overset{t}{3(s)})$

$= \frac{9^{10}}{10!} e^{-9}$	$= \frac{12^{10}}{10!} e^{-12}$
$= 0.118 = P(10/N_1)$	$= 0.104 = P(10/N_2)$

From the calculations done above we see that neuron 1 is more likely to emit 10 spikes in 3(s).

$$P(10) = P(10/N_1)\overset{0.5}{P(N_1)} + P(10|N_2)\overset{0.5}{P(N_2)} = 0.11$$

$$P(N_1|10) = \frac{P(10|N_1)\,P(N_1)}{P(10)} = \frac{(0.118)(0.5)}{0.11} = 0.53$$

$$P(N_2|10) = \frac{P(10|N_2)\,P(N_2)}{P(10)} = \frac{(0.104)(0.5)}{0.11} = 0.46$$

By above calculations if a neuron emits 10 spikes in 3s, it is more likely to come from neuron 1.

b) Bayes Rule :

$$P(v \mid k, t) = \frac{vt^k}{k!} e^{-vt} \frac{p_0(v)}{p(k,t)}$$

We can assume $p_0(v) = 1$, $k = 4$ $t = 3(s)$
$v_1 = 1hz$ and $v_2 = 2hz$,

$$P(k; v, t) = \frac{(vt)^k}{k!} e^{-vt}$$

$$P(4|v_1) = \frac{3^4}{4!} e^{-3} = 0.168$$

$$P(4|v_2) = \frac{6^4}{4!} e^{-6} = 0.13$$

$$p(4) = P(4|v_1) \underbrace{P(v_1)}_{0.5} + P(4|v_2) \underbrace{P(v_2)}_{0.5} = 0.1509$$

$$P(v \mid k,t) = \frac{vt^k}{k!} e^{-vt} \frac{p_0(v)}{p(k,t)}$$

$v_1 = 1$

$$P(v_1 \mid 4,3) = \frac{3^4}{4!} e^{-3} \cdot \cancel{\frac{1}{0.1509}} = .1680$$

$v_2 = 2$

$$P(v_2 \mid 4,3) = \frac{6^4}{4!} e^{-6} \cdot \cancel{\frac{1}{.1509}} = 0.1338$$

Therefore it is more probable to be neuron 1.

c) $$\frac{d}{dv} P(v \mid k,t) = \frac{vt^k}{k!} e^{-vt} \frac{p_0(v)}{p(k,t)}$$

$$\frac{d}{dv} \frac{vt^k}{k!} e^{-vt} = \frac{-t^4 v^3 (tv - k) e^{-tv}}{(k!)^k}$$

$$\Rightarrow \quad V = \frac{k}{t}$$