a) $v_e' = w_{ee} v_e - w_{ei} v_i$, $v_i' = v_e - v_i$

$0 = w_{ee} v_e - w_{ei} v_i$ $\qquad$ $0 = v_e - v_i$

$v_i = v_e$

$0 = w_{ee} v_i - w_{ei} v_i$

$w_{ee} = w_{ei}$

$$\underline{J(v_E, v_I)} = \begin{pmatrix} w_{ee} & -w_{ei} \\ 1 & -1 \end{pmatrix}$$

$$\det(\lambda I - J) = \begin{pmatrix} \lambda - w_{ee} & w_{ei} \\ -1 & \lambda + 1 \end{pmatrix} = 0$$

$$\Rightarrow \lambda^2 + \lambda - \lambda w_{ee} - w_{ee} + w_{ei} = 0$$

$$\Rightarrow \lambda^2 + \lambda(1 - w_{ee}) - w_{ee} + w_{ei} = 0$$

$$\lambda = \frac{-(1-w_{ee})}{2} \pm \sqrt{\frac{(1-w_{ee})^2 - 4(-w_{ee} + w_{ei})}{2}}$$

So for the state to be stable $0 < w_{ee} < 1$, b/c after $(1-w_{ee})$ will be '+' and hence unstable.

$W_{ei} > W_{ee}$ for the imaginary part to be a stable spiral.

b)

$$V_1' = -V_1 + \frac{V_2^2}{K+V_2^2} \qquad V_2' = -V_2 + \frac{V_1^2}{K+V_1^2}$$

$$0 = -V_1 + \frac{V_2^2}{K+V_2^2} \qquad 0 = -V_2 + \frac{V_1^2}{K+V_1^2}$$

$$V_1 = \frac{V_2^2}{K+V_2^2} \qquad V_2 = \frac{V_1^2}{K+V_1^2}$$

$$V_2 = \frac{\left(\frac{V_2^2}{K+V_2^2}\right)^2}{K+\left(\frac{V_2^2}{K+V_2^2}\right)^2} = \frac{\frac{V_2^4}{(K+V_2^2)^2}}{K+\frac{V_2^4}{(K+V_2^2)^2}}$$

$$V_2 = \frac{V_2^4}{(K+V_2^2)^2+\frac{V_2^4}{(K+V_2^2)^2}} = (K+V_2^2)^2 V_2 = \frac{V_2^4}{1+V_2^4}$$

$$(K+V_2^2)^2 = \frac{V_2^3}{1+V_2^4}$$

$$\Rightarrow \underline{k} = v_1 - v_2^2 \quad , \quad \underline{k} = \frac{1}{2}\left(-v_2 - v_2^2 - v_2\sqrt{-3+2v_2+v_2^2}\right)$$

$$\underline{k} = \frac{1}{2}\left(-v_2 - v_2^2 + v_2\sqrt{-3+2v_2+v_2^2}\right)$$

So,

$$\underline{k} < v_1 - v_2^2 = k_c \quad , \quad \underline{k} < \frac{1}{2}\left(-v_2 - v_2^2 - v_2\sqrt{-3+2v_2+v_2^2}\right)$$

and $\underline{k} < \frac{1}{2}\left(-v_2 - v_2^2 + v_2\sqrt{-3+2v_2+v_2^2}\right)$

$$J(v_1, v_2) = \begin{pmatrix} -1 & \dfrac{2kv_2}{(k+v_2^2)^2} \\ \dfrac{2kv_1}{(k+v_1^2)^2} & -1 \end{pmatrix}$$

$$\det(\lambda I - J) = \begin{pmatrix} \lambda+1 & -\dfrac{2kv}{(k+v^2)^2} \\ -\dfrac{2kv}{(k+v^2)^2} & \lambda+1 \end{pmatrix}$$

$$0 = \lambda^2 + 2\lambda + 1 - \frac{4k^2v^2}{(k+v^2)^4}$$

$$\lambda = \frac{-1 + \sqrt{4 - 4\left(1 - \frac{4k^2r^2}{(k+r^2)^4}\right)}}{2}$$

So for our first case $k = r - r^2$:

$$r^2 - r - \frac{3}{16} = 0 \Rightarrow r = \frac{1}{4}(2 \pm \sqrt{7})$$

we are not Stable since our real part is positive.

For our second case $k = \frac{1}{2}\left(-r_2 - r_2^2 - r_2\sqrt{-3 + 2r_2 + r_2^2}\right)$

$$r = \frac{3}{76}\left(-2 \pm i\sqrt{15}\right)$$

we are stable since our real part is '−'. Stable spiral for imaginary part is '−'.

Last case, $k = \frac{1}{2}\left(-r_2 - r_2^2 + r_2\sqrt{-3 + 2r_2 + r_2^2}\right)$

r is not possible there fore not stable.

Yes, I think this network can contain memory of initial conditions b/c we have unstable spirals that will repeat the excitatory/inhibitory