$$T''(v) + T'(v) = -1, \quad T_1(\theta) = 0$$

$$\& \quad \lim_{v \to \infty} T''(v) = 0$$

Let $T'' = y'$ and $T' = y$

$$y' + y = -1 \qquad \mu = e^t$$

$$e^t y = -\int e^t dt$$

$$e^t y = -e^t + C \Rightarrow y = Ce^{-t} - 1$$

So $y = T'$, so to obtain T, we integrate:

$$T(v) \to \int y\,dt = \int Ce^{-t} - 1\,dt$$

$$T(v) = -C_1 e^{-v} - v + C_2$$

$T_1(\theta) = 0$ and $\lim_{v \to \infty} T''(v) = 0$:

$$T(v) = C_2 - C_1 e^{-v} - v \quad ; T_1(\theta) = 0$$

$$\Rightarrow 0 = C_2 - C_1 e^{-\theta} - \theta$$

$$\Rightarrow C_2 = C_1 e^{-\theta} + \theta$$

$$\Rightarrow T(v) = C_1 \left(e^{-\theta} - e^{-v}\right) - v + \theta$$

Now $T''(v)$: $\lim_{v \to \infty} T''(v) = 0$

$$T(v) = C_2 - C_1 e^{-v} - v \Rightarrow T'(v) = C_1 e^{-v} - 1$$

$$\Rightarrow T''(v) = -C_1 e^{-v} \Rightarrow 0 = -C_1 \Rightarrow C_1 = 0$$

So we have:

$$T(v) = 0\left(e^{-\theta} - e^{-v}\right) - v + \theta$$

$$\Rightarrow T(v) = \Theta - v$$

Well Θ is our threshold and v is our initial input. As v gets closer to Θ we are getting closer to the threshold therefore $T_1(v)$ deceases to allow a spike to accur more easily.

b) $T_2''(v) + T_2'(v) = -2T_1(v)$, $T_2(\theta) = 0$ & $\lim_{v \to \infty} T_2'''(v) = 0$

Plug in $T_1(v)$:

$$T_2''(v) + T_2'(v) = -2(\Theta - v)$$

Let $y' = T_2''$ and $y = T_2'$

$$y' + y = -2(\theta - v) \quad , \mu = e^{v}$$

$$e^v y = \int (-2\theta + 2v) e^v$$

$g = v \quad g' = 1$
$f' = e^v \quad f = e^v$

$$e^v y = \int -2\theta e^v dv + \int 2v e^v dv$$

$$= -2\theta e^v + v e^v - \int e^v dv$$

$$= -2\theta e^v + 2[v e^v - e^v] + C$$

$$e^v y = -2\theta e^v + 2(v-1)e^v + C$$

$$y = -2\theta + 2(v-1) + Ce^{-v}$$

$$T_2 = \int y \, dv = \int -2\theta dv + 2\int v-1 \, dv + \int Ce^{-v} dv$$

$$T_2 = -2\theta v + v^2 - 2v - C_1 e^{-v} + C_2$$

$\lim_{v \to \infty} T_2'''(v) = 0$:

$$T_2' = -2\theta + 2(v-1) + C_1 e^{-v}$$

$$T_2'' = 2 - C_1 e^{-v}$$

$$T_2''' = C_1 e^{-v} \Rightarrow 0 = C_1$$

So,

$$T_2(v) = -2\theta v + v^2 - 2v + C_2$$

$T_2(\theta) = 0$:

$$0 = -2\theta^2 + \theta^2 - 2\theta + C_2$$

$$= -\theta^2 - 2\theta + C_2$$

$$C_2 = 2\theta + \theta^2$$

Therefore:

$$T_2(v) = \theta(\theta - 2v + 2) + v(v-2)$$

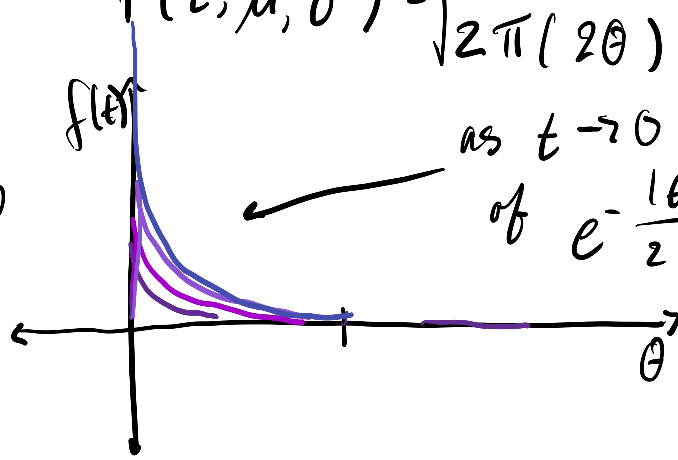
c) $v = 0$, $\mu(\theta) = T_1(v;\theta)$

$T_1(0,\theta) = \theta$ and $T_2(v;\theta) = \theta(\theta+2)$

$$\sigma^2(\theta) = T_2(v;\theta) - T_1(v,\theta)^2 = \theta(\theta-2) - \theta^2 = 2\theta$$

Plug in :

$$f(t;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi(2\theta)}} e^{-(t-\theta)^2/2(2\theta)}$$



when $t > 0$

as $t \to 0$ b/c of $e^{-\frac{(t-\theta)^2}{2(2\theta)}}$

The reason why is b/c thats when the neuron spikes.