$w_1' = \eta v(v - v_\theta)u_1$ $\qquad$ $w_2' = \eta v(v - v_\theta)u_2$

$v = w_1 u_1 + w_2 u_2$

$\eta = 1,\ u_1 = 3,\ u_2 = 0$:

$$= \eta(w_1 u_1 + w_2 u_2)(w_1 u_1 + w_2 u_2 - 2)u_1$$

$$= \eta[w_1^2 u_1^2 + w_1 w_2 u_1 u_2 - 2 w_1 u_1 + w_2 u_2 w_1 u_1$$
$$+ w_2^2 u_2^2 - 2 w_2 u_2] u_1$$

$$= \eta w_1^2 u_1^3 - \eta w_1 w_2 u_1^2 u_2 - 2\eta w_1 u_1^2 + \eta w_2 w_1 u_1^2 u_2$$
$$+ \eta w_2^2 u_2^2 u_1 - 2\eta w_2 u_2 u_1$$

$$= 27 w_1^3 - 0 - 18 w_1 + 0 + 0 - 0$$

$w_1' = 9(3w_1^2 - 2w_1)$

$w_2' = 0$

$\eta = 2,\ u_1 = 0,\ u_2 = 1$:

$w_1' = 0$

$w_2' = 2w_2^2 - 4w_2$

When we solve for w_1' & w_2' for $\eta = 1$ and $\eta = 2$:

$\eta_1 = 1$:

$$w_1 = -\frac{2}{(-3+e^{18t})}, \quad w_2 = 0$$

$\eta = 2$:

$$w_2 = \frac{2}{1+e^{4t}}, \quad w_1 = 0$$

So w_1 evolves by changing faster towards 0 therefore the synapse is not strength.

b) $\eta = 2, \quad u_1 = 0 \quad , u_2 = 2.5$

$w_2' = 31.25 w_1 - 25 w_2 \quad , \quad w_1' = 0$

c) $V_\theta' = v^2 - V_\theta \qquad w_1' = \eta v (v - V_\theta) u_1$

$V_\theta' = (w_1 u_1 + w_2 u_2)^2 - V_\theta \qquad w_1' = \left[(w_1 u_1 + w_2 u_2)^2 - (w_1 u_1 + w_2 u_2) V_\theta \right] u_1$

$V_\theta = 0 \qquad 0 = w_1$

$(\bar{w}_1, \bar{V}_\theta) = (0,0) : \quad u_2 = 0 \quad , \eta = 1,$

$$J(\bar{w}_1, \bar{V}_\theta) = \begin{pmatrix} 2u_1^2(u_1 w_1 + w_2 u_2) - u_1^2 V_\theta & 2(w_1 u_1 + w_2 u_2) u_1 \\ 2u_1(w_1 u_1 + w_2 u_2) & -1 \end{pmatrix}$$

$2u_1^2 1$

$$J(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\det(\lambda I - J) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda+1 \end{pmatrix}$$

$$\Rightarrow \lambda^2 + \lambda = 0 \Rightarrow \lambda = 0, \ \lambda = -1$$

Now for $(\bar{w}_1, \bar{V}_\theta) = (\frac{1}{u}, 1)$

$$V_\theta' = (w_1 u_1 + w_2 u_2)^2 - V_\theta, \quad w_1' = \left[(w_1 u_1 + w_2 u_2)^2 - (w_1 u_1 + w_2 u_2) V_\theta\right] u_1$$

$$0 = (w_1 u)^2 - V_\theta$$

$$V_\theta = w_1^2 u_1^2$$
$$= V_\theta^2 - V_\theta$$
$$V_\theta (V_\theta - 1) = 0$$
$$V_\theta = 1$$

$$0 = w_1^2 u_1^2 - w_1 u_1 V_\theta$$
$$0 = w_1^2 u_1 - w_1 V_\theta$$
$$0 = w_1 u_1 - V_\theta$$
$$w_1 = \frac{V_\theta}{u_1} \Rightarrow w_1 = \frac{1}{u_1}$$

So, let $V_\theta = 1$:

$$J\left(\frac{1}{u_1}, 1\right) = \begin{pmatrix} 2u_1^2 & u_1 \\ 2u_1 & -1 \end{pmatrix}$$

$$\det(\lambda I - J) = \begin{pmatrix} \lambda - 2u_1^2 & -u_1 \\ -2u_1 & \lambda+1 \end{pmatrix}$$

$$\Rightarrow \lambda^2 + \lambda(1 - 2u_1^2) - 4u_1^2$$

$$\lambda = u_1^2 - \frac{1}{2} \pm \frac{\sqrt{(1-2u_1^2)^2 + 16u_1^2}}{2}$$

Note that the real part depends on u_1^2. So we can get a stable spiral when for $0.172 < u_1^2 - \frac{1}{2} < 1$ and an unstable spiral for $u_1^2 - \frac{1}{2} > 1$. also we get a stable node for $u_1^2 - \frac{1}{2} < 0.172$.