3a) $$u_1' = -u_1 + H[I_1(t) + u_1]$$
$$u_2' = -u_2 + H[I_2(t) + u_2]$$
$$u_3' = -u_3 + H[I_3(t) + u_3]$$

Let $I_1(t) \equiv I_2(t) \equiv I_3(t) \equiv 0$ and $w = 0$:

$$u_1' = -u_1 + H[u_1]$$
$$u_2' = -u_2 + H[u_2]$$
$$u_3' = -u_3 + H[u_3]$$

Setting $u_1' = u_2' = u_3' = 0$:

$$u_1 = H[u_1]$$
$$u_2 = H[u_2]$$
$$u_3 = H[u_3]$$

which is $u_1 \equiv u_2 \equiv u_3 \equiv 1$.

3b) $u_1' = -u_1 + H[u_1 - wu_2 - wu_3]$

$u_2' = -u_2 + H[u_2 - wu_1 - wu_3]$

$u_3' = -u_2 + H[u_3 - wu_1 - wu_2]$

Let $u_1 = 1$:

$$u_1 = H[1 - w(0) - w(0)] = 1$$

$$u_2 = H[0 - w(1) - w(0)] = 0$$

$$u_3 = H[0 - w(1) - w(0)] = 0$$

So to find w_c:

$$1 - wu_2 - wu_3 \geq 0$$

$$wu_2 + wu_3 \geq 1$$

$$w(u_2 + u_3) \geq 1$$

$$w \geq \frac{1}{u_2 + u_3}$$

and so W_c:

$$W_c \geq \frac{1}{\sum_{a=1, a\neq j}^{N} u_a}$$

This is a better regime than $w=0$ b/c we have a precise winner depend on u_a and u_j.

3c)
$$u_1' = -u_1 + H[I_1(t) + u_1 - 2u_2 - 2u_3]$$
$$u_2' = -u_2 + H[I_2(t) + u_2 - 2u_1 - 2u_3]$$
$$u_3' = -u_2 + H[I_3(t) + u_3 - 2u_1 - 2u_2]$$

$$0 = -u_1 + H[I_1(t) + u_1 - 2u_2 - 2u_3]$$
$$0 = -u_2 + H[I_2(t) + u_2 - 2u_1 - 2u_3]$$
$$0 = -u_2 + H[I_3(t) + u_3 - 2u_1 - 2u_2]$$

$$u_1 = H[0 + 1 - 2(0) - 2(0)]$$
$$u_2 = H[3 + 0 - 2 - 2(0)]$$
$$u_3 = H[0 + 0 - 2 - 2(0)]$$

$t=0$:
$$u_1 = H[1] = 1 \qquad u_3 = H[0] = 0$$
$$u_2 = H[0] = 0$$

u_1 is winning

at $t=1$:

$u_1 = H[1] = 1$, $u_2 = H[3] = 1$

$u_3 = H[0] = 0$

u_1 and u_2 are equal

at $t=2$:

$u_1 = H[-1] = 0$, $u_2 = H[2] = 1$

$u_3 = H[0] = 0$

u_2 wins.

at $t=3$:

$u_1 = H[-2] = 0$ $\qquad$ u_2 wins as $t \to \infty$.

$u_2 = H[1] = 1$

$u_3 = H[0] = 0$

So for $u_1 = 0$ and $u_2 = u_3 = 0$, as $t \to \infty$ $u_2 \to 1$ and $u_1 \to 0$, $u_3 \to 0$ b/c of $I_2 = 3$. u_1 starts of winning but u_2 slowly begins to take over. As shown above, we can switch I_2 to 0 when $t=3$.

At $t=3$ it does not matter if there is input, u_2 will always win due to the fact that u_1 and u_3 are 0 for $t \geq 3$.