Kalman Filter

Problem

Consider linear dynamic model:

$$\begin{aligned}
x_{t+1} &= Ax_t + w_t, \quad w_t \sim N(0, \Gamma) \\
y_t &= Cx_t + v_t, \quad v_t \sim N(0, \Sigma) \\
\theta &= \{A = 1, C = 1, \Gamma = 1, \Sigma = 49\} \\
t &= \{t \in \mathbb{Z} | 0 \le t \le T\}
\end{aligned}$$

Solve $\hat{\alpha}(x_t) = p(x_t | y_{1:t})$ by observation $y_{1:t} = (y_1, \ldots, y_t)$ and parameter θ. Since $\hat{\alpha}(x_t) = N(x_t; \mu_t, V_t)$, find μ_t, V_t.

Algorithm

μ_t, V_t is found by the iteration $(t = 1, 2, 3, \ldots)$:

$$\begin{aligned}
\hat{\mu_t} &= A\mu_{t-1} \\
P_{t-1} &= AV_{t-1}A^T + \Gamma \\
K_t &= P_{t-1}C^T \left(CP_{t-1}C^T + \Sigma\right)^{-1} \\
\mu_t &= \hat{\mu_t} + K_t \left(y_t - C\hat{\mu_t}\right) \\
V_t &= \left(I - K_tC\right) P_{t-1}.
\end{aligned}$$

The initial values are

$$\begin{aligned}
\mu_0 &= 0 \\
V_0 &= 1
\end{aligned}$$

When there are missing values in the observation series,

$$\begin{aligned}
P_{t-1} &= AV_{t-1}A^T + \Gamma \\
\mu_t &= A\mu_{t-1} \\
V_t &= AV_{t-1}A^T + \Gamma.
\end{aligned}$$

Result