Particle filter

1 Problem

The white disk-shaped object with the diameter of about 6[pixel] is moving around in the binary movie data 30[pixel] × 40[pixel] × 1[bit] × 400[frames] containing noise. An object tracking system is constructed by estimating the trajectory of the object (two-dimensional position coordinates in each frame).

2 Algorithm

The particle filter does not examine all combinations of regions, but rather predicts the position according to the probability distribution, select several candidate positions for the object, and examines only them.

The candidates for each object are called particles. Sampling, reweighting, object position estimation, and resampling are repeated. Let $x\left(t\right)$ be the coordinates of the particle center at time t and $y\left(t\right)$ be the number of white pixels in the observed image of the particle. The total number of pixels in a particle is n, the generation ratio of white pixels in the particle is a, and the generation ratio of white pixels in the background is b.

In sampling, the particle position in the current frame is updated based on the particle position in the previous frame.

Consider two ways to update:

$$\begin{aligned} x\left(t+1\right) &= x\left(t\right) + \epsilon\left(t\right), \quad \epsilon\left(t\right) \sim N(0, \sigma^2) \\ x\left(t+1\right) &= x\left(t\right) + c(x\left(t\right) - x\left(t-1\right)) + \epsilon\left(t\right), \quad \epsilon\left(t\right) \sim N(0, \sigma^2), \end{aligned}$$

where σ, c are given appropriately and the particles are arranged uniformly in the initial state.

In order to estimate the object position from multiple particles, each particle is given the weight $w(t)$. In reweighting, observe the particle position and update the weights with

$$w\left(t\right) = w\left(t-1\right) \times p(y\left(t\right)|x\left(t\right), a).$$

$p(y\left(t\right)|x\left(t\right), a)$ is

$$y|x, a \sim Bi\left(n, a\right),$$

or

$$y|x,a \sim Bi\left(n,a\right)$$
$$y'|x,a \sim Bi\left(n',b\right)$$
$$p\left(y\left(t\right)|x\left(t\right),a\right) = p\left(y\left(t\right)|x\left(t\right),a\right) \times p\left(y'\left(t\right)|x\left(t\right),a\right),$$

where y' is the number of white pixels around the particle (outside the frame) and n' is the total number of pixels around the particle (outside the frame). Give a and b appropriately. In the initial state, all weights have the same value.

After the reweighting, the object position in each frame can be estimated by weighted averaging each particle position using the particle weight. In resampling, particles with small weights are deleted and particles with large weights are divided to update the particles. In updating, the particle with the lower d weights among the weights of each particle is replaced with the particle with the highest weight. Give d appropriately.

The parameters are set as $\sigma = 1$, $c = 1$, $a = \frac{3}{4}$, $d = 5$. The number of particles is 48, and the particle (frame) is the square of 7 [pixel] per side.