Gauss sampling

1 Algorithm

I will explain outline of GS algorithm. GS algorithm is a bayesian extension of the EM algorithm. Suppose $\boldsymbol{X}$: observed variables and $\boldsymbol{Z}$: latent variables, we aim to estimate the parameters $\boldsymbol{\pi}$,$\boldsymbol{\mu}$,$\boldsymbol{\Lambda}$ of Finite Gaussian Mixture Models.

GS algorithm repeats to generate random samples from a probability distribution.

Sample $z_n \sim$

$$p\left(z_{nk}=1|x_n,\boldsymbol{\pi},\boldsymbol{\mu},\boldsymbol{\Lambda}\right)=\frac{\pi_k N\left(\boldsymbol{x}_n|\boldsymbol{\mu}_k,\boldsymbol{\Lambda}_k\right)}{\sum_{k'=1}^{K}\pi_{k'}N\left(\boldsymbol{x}_n|\boldsymbol{\mu}_{k'},\boldsymbol{\Lambda}_{k'}\right)}$$

Sample $\pi \sim$

$$p(\boldsymbol{\pi}|\boldsymbol{Z})=\mathrm{Dir}(\boldsymbol{\pi}|\boldsymbol{\alpha})$$

Sample $\mu \sim$

$$N\left(\boldsymbol{\mu}_k|\boldsymbol{m}_k,(\beta_k\boldsymbol{\Lambda}_k)^{-1}\right)$$

Sample $\Lambda \sim$

$$W\left(\boldsymbol{\Lambda}_k|\mathbf{W}_k,v_k\right)$$

Samples are iterated until convergence of probability distribution below threshold.

2 Result

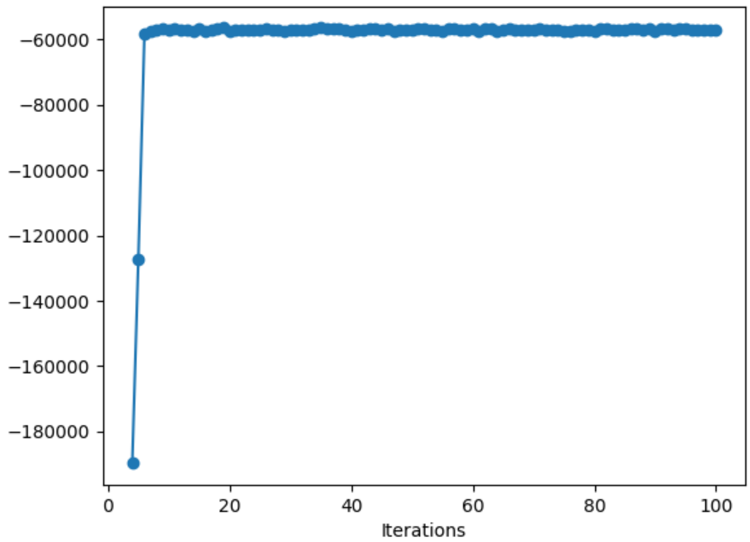
2.1 The value of the log likelihood

The loglikelihood functions are

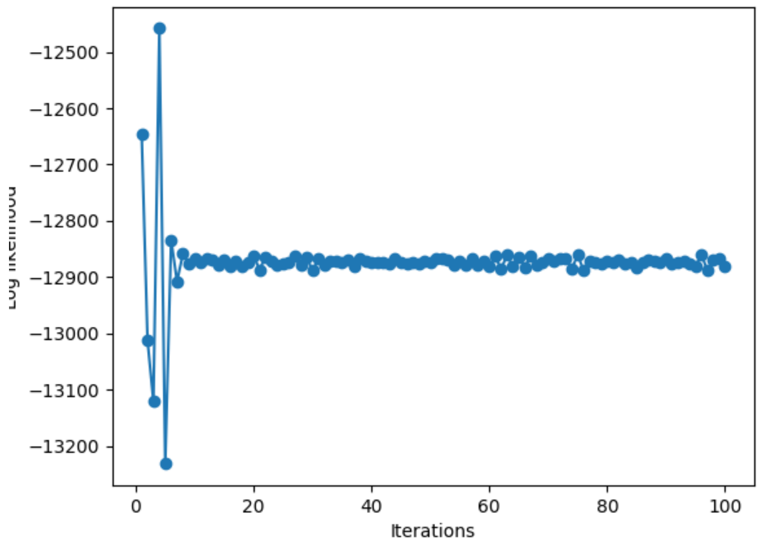
$$\log p(\boldsymbol{X}|\boldsymbol{Z},\boldsymbol{\mu},\boldsymbol{\Lambda})=\sum_{n=1}^{N}\sum_{k=1}^{K}z_{nk}\log N\left(x_n|\boldsymbol{\mu}_k,\boldsymbol{\Lambda}_k^{-1}\right) \tag{1}$$

$$\log p(\boldsymbol{Z}|\boldsymbol{\pi})=\sum_{n=1}^{N}\sum_{k=1}^{K}z_{nk}\log\pi_k \tag{2}$$

The value of the log likelihood(1) is



The value of the log likelihood(2) is



2.2 Classification

I classify each point into the class with the value 1 of z_{nk}. This result is shown below.