High-frequency Trading

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Abstract : *Statistical pair trading is a strategy used by many hedge funds, investment banks, and other investors and traders. Using two assets that have been historically traded at a narrow range of spread, when the spread widens one opens a position on the pair. That trading position includes shorting (selling) the asset with price gain and going long (buying) the asset with depreciated price. When the spread retreats to its mean or a threshold close to that, the trading position is closed and a profit is earned. In this project, we develop a pair trading strategy using the spread model which is an OU process.*

1 Introduction

The fundamental idea of pair trading is that knowing that a pair of financial instruments has historically moved together and kept a specific pattern for their spread, we could take advantage of any disturbance over this historic trend. The pair trading system is similar to the study of a steady state equilibrium in mechanical or electrical system that is expected to remain in the steady state in the absence of any perturbation or shock. When a shock or perturbation is introduced to the system, the property of the system would dampen out these forces and take the system back to its steady state equilibrium state. The basic understanding of pair trading strategy is to take advantage of a perturbation, when noise is introduced to the system, and take a trading position realizing that the noise will be removed from the system rather shortly.

The foundation of the idea of pair trading can be attributed to Schroder Salomon Smith Barney (SSSB) stating that the instruments that historically have the same trading patterns, will have so in the future as well. Historically [6] pair trading was introduced by Nunzio Tartaglia,working then for Morgan Stanley, in mid-1980's. Tartaglia believed that pair trading strategy works because of rather psychological reasons, ie. "...Human being don't like to trade against human nature, which wants to buy stocks after they go up not down.

Pair trading and in general statistical arbitrage investment shows that financial markets are not completely efficient. The idea of pair trading includes all asset classes and in general the financial instrument in a pair could be of different asset types. In this project the instruments are of a pair of the same asset type, more specifically of stocks.

2 Theory and Model

The continuous model [3] of the spread is a stochastic differential equation, OU (Ornstein-Uhlenbeck [5]) process is of the form;

$$dX(t) = (a - bX(t))dt + \sigma dW(t) \tag{1}$$

The equivalent discrete model for the spread is,

$$X_{k+1} - X_k = (a - bX_k)\tau + \sigma\sqrt{\tau}\epsilon_{k+1} \tag{2}$$

where

$$\mu_k = \frac{a}{b} + [\mu_{k-1} - \frac{a}{b}](1 - b\tau)^k \tag{3}$$

and

$$\sigma_k^2 = \frac{\sigma^2\tau}{1-(1-b\tau)^2}[1-(1-b\tau)^{2k}] + \sigma_{k-1}^2(1-b\tau)^{2k} \tag{4}$$

for $|1 - b\tau| < 1$.
Eq. (2) can be rewritten as;

$$X_{k+1} = A + BX_k + C\varepsilon_{k+1} \tag{5}$$

where

$$A = a\tau \geq 0, \;\; 0 \leq B = 1 - B\tau < 1, \;\text{ and }\;\; C = \sigma\sqrt{\tau}$$

There are three known methods to estimate the parameters in (2):

1. Method of Maximum Likelihood Estimation (MLE)
2. Method of Moments (MOM)
3. Least Squares Method (LSM)

In this project LSM method is used to estimate the model parameter in (2). This project's primary goals are to:

i Estimate the parameters of (2) based on 15 minute changes in the future data.

ii Enhance the Visual Basic (Excel) code which is given by Interactive Brokers in order to test the model by trading through the virtual account of IB.

3 Trading Strategies

Our trading Strategies are based on three different tasks of pair selection, establishing trading rules and risk management;

3.1 Selection of Stock Pairs

Selection of the best possible pairs of assets is fundamental to the overall strategy of pair trading scheme. These are the principles we consider in choosing the asset pairs:

1. We choose two assets that historically (at least for 5 years) moved together.
2. We choose pairs from the same sectors.
3. We test the pairs for mean reversion and stationarity.
4. We choose pairs of assets of ideally identical β ,and at most with $\Delta\beta \leq 0.1$. The β's are based on one year daily data.
5. Only stocks with high liquidity (of at least one million of average daily trading volume) are selected.

3.2 Trading Rules

Establishing trading rules that a trader or investor has to follow at all times is very critical to a successful trading strategy. This factor has perhaps a more pronounced impact in the case of pair trading where any loosening or negligence of the trading rules could lead to undesirable consequences. These are the trading rules in our portfolio of pair trading assets:

1. We open a position in a pair when their spread has hit a threshold of 2σ for the second time. This is to keep ourselves from trading on a pair when the likelihood and magnitude of mean reversion is not substantial and also to invest only when the spread is moving in the direction of mean reversion and not the other way. In doing so we open a long position on the asset that has decreased in price and a short position in the one with increased price. Each asset of the pair would get half of the investment designated for the pair.

2. In addition to the above criteria, we only want to trade in a pair when the rate of return we obtain is greater than the risk-free rate (3% in our case). This is because it is always better to invest in risk-free assets when a higher return cannot be obtained.

3. Now that we know which pairs should be considered, we use the Markowitz Model to determine portfolio allocation since we are making a choice between the 15 possible pairs and the risk-free asset. The rate of return and standard deviation of return is calculated for each pair. We then choose a target rate of return (in our case we arbitrarily choose it to be 2.5 times the return on the risk-free asset), and then choose portfolio weights to minimize the overall variance of our portfolio. Mathematically, the Markowitz problem attempts to solve the following equations:

$$\min_{\sigma_{ij}} \left(\frac{1}{2} \sum_{i,j=1}^{n} w_i . w_j . \sigma_{ij} \right)$$

   subject to

$$\sum_{i=1}^{n} w_i \bar{r_i} = \bar{r}$$

$$\sum_{i=1}^{n} = 1$$

4. We then perform the actual trades, going short on the stock that is expected to fall and going long on the stock that is expected to rise in order to close the deviating spread.

5. Finally we close the position and cash out once the spread reaches a lower threshold, in our case it is 0.5σ. The new funds are then used to repeat the process and reinvest in more pairs.

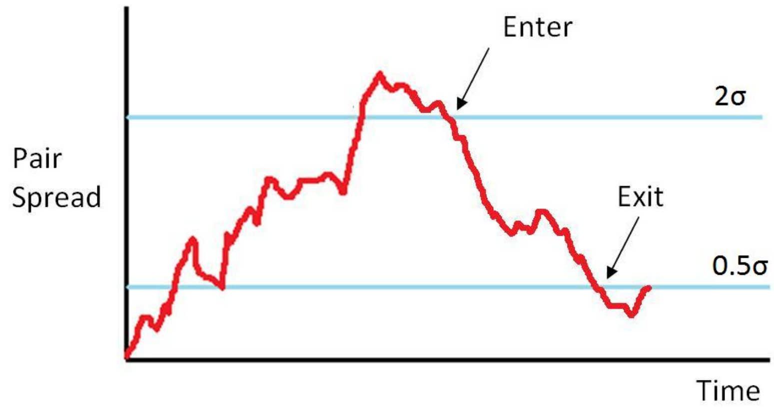


Figure 1: Trading positions are opened when the spread go below the upper threshold after having crossed it previously. Trading positions are closed when the spread drops below the lower threshold and profits are made.

6. At any given time, we use the unused fund in risk-free interest rate investment.
7. Transaction fee is assumed to be constant and applicable for both cases of selling and buying assets.
8. We update the whole trading strategy at any time point. (continuous strategy)

3.3 Risk Management

Risk management, or lack of it, has received a lot more publicity in the recent years, in the aftermath of 2008 crash. It seems that lack of proper risk management has contributed to the crash or at least to its amplitude. Thus, we are even more mindful of considering an effective risk management strategy in this project. There are different methods of dividing investment strategies into different type of investments. From a point of view of investment risks, we could envision two basic types of investments.

(1) Conventional investment, where an investor take positions by assuming / predicting the future price trend of a financial instrument and/or the market in general. The risk in this type of investment is the systematic market risk, and the return depends mainly on the accuracy of the prediction of the future prices.

(2) Statistical Arbitrage investment is the second type of investment. In this type investment there is no need of having a prediction of future assets prices. The basic idea in these type of investments/trades is that regardless of the

assets moving up or down, the investment/trade should make profits. These profits are obtained by assuming that some prices or price relationships follow an Ornstein-Uhlenbeck (mean-reverting) process. Thus the statistical arbitrage investor makes money whenever there is deviation from the mean. These deviations are caused by noise and investors / traders believe that the noise will be gone after a short time and prices return to their equilibrium (historic mean). The main risk is that the process may change its regime and thus no longer follows OU process. This kind of risk could be proven to be very costly for an investor/trader.

In this project, we have managed the risks using the following steps:

1. We try to avoid systematic market risk by making our pairs of assets with $\Delta\beta \leq 0.1$. Thus we have a very small β spread with very small systematic market risk.
2. We avoid the sector (industry) risk by choosing our pairs from the same sectors. The main risks we face is to have a change of a regime in their basic features of the pairs, i.e.,
   (a) The spread of the pairs is no longer following OU (mean-reversion) Process, and/or
   (b) The time for mean reversion process has drastically changed.

   To address the first risk (a), we enter a trade only after the spread has hit the trading threshold for the second time. To avoid the risk type (b), we make sure that we keep an open position no longer than its historical mean-reversion time period.
3. A stop-loss strategy for our overall portfolio is used.
4. We use diversification by using several pairs in our portfolio.
5. Since pair trading is very time sensitive, the risk of il-liquidity could lead to large losses. To avoid this risk, only sufficiently liquid stock of average daily trading volume of one million or more are considered.
6. Like all other strategies, we need to filter our model and algorithm through our own judgement. That is to review the ever fluid environment of any company or asset and make sure that there is a good financial and economic sense for the changes in the price of assets we are considering for the pair trading strategy.

4 Implementation

4.1 Backtesting

In this section, we will explain about the backtesting prototype that we built based on the trading strategy discussed in the previous section. We built this trading prototype fully in MATLAB, meaning that it will handle all of the data processing, regression, threshold analysis, portfolio optimization process, and the transaction process within the MATLAB itself. For this backtesting purpose, we decided to choose four stocks from the energy sector:

1. XOM: Exxon Mobil Corporation
2. CVX: Chevron Corporation
3. COP: Conoco Phillips
4. MRO: Marathon Oil Corporation

Out of these four assets, we formed four different pairs to trade on. The first pair is Exxon Mobil and Chevron (XOM-CVX) which is a well known pair that tends to move together historically. The second pair is Conoco Phillips and Marathon Oil (COP-MRO). The third pair is Exxon Mobil and Marathon Oil (XOM-MRO) and the last pair is Chevron and Conoco Phillips (CVX-COP). We decided to limit the number of pairs to these four for simplicity reason. We also assume that an annual risk-free rate of 3% is available for us to use.

The data that we use for backtesting is the historical daily data from September 2000 to March 2010 for all of those four assets. This means that the $\Delta\tau$ for our model is 1 day.

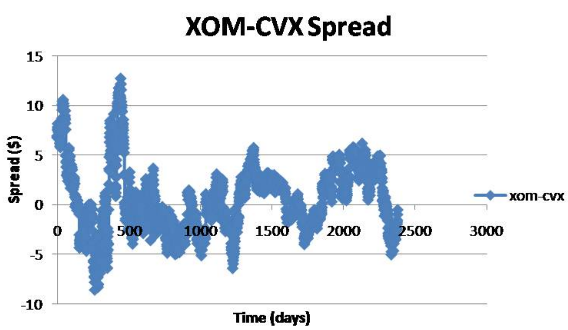


Figure 2: Historical Spread for XOM-CVX

If we look at Figure 2 above, we can see that the spread for XOM-CVX pair over the historical period is quite stable. There was no shift in the spread level of the two assets and the spread has a historical mean of around 0.5.

However, this is not true for all pairs that we have chosen. If we look at Figure 3 below, there were shifts in the spread level for the CVX-COP pair and this might hurt us during the actual trading process because when there were shifts in the spread level, our stop-loss strategy would kick in causing us to exit the position with loss. Nevertheless, we still did our pairs trading model using this

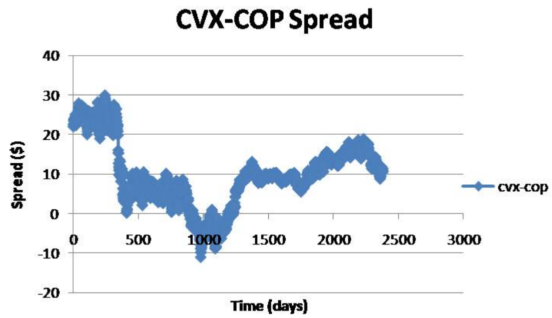


Figure 3: Historical Spread for CVX-COP

set of data and we tried to get a convincing backtesting results out of this.

From the historical data, we have around 2380 data points for each asset and we used a rolling window size of 50 days to do the initial regression analysis and get the parameters (A,B, and C) to predict all the μ and σ. From that, we then cycle through the data by getting the next day prices and do the threshold analysis, optimization, actual trading, and repeat. We also assume that our initial wealth is US$1 Million at the time we first enter trade. The result from our MATLAB backtesting prototype is shown in Figure 4.

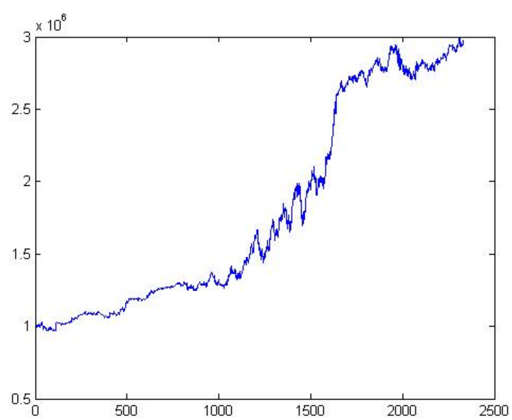


Figure 4: Backtesting Result with Optimization for weights on each pair

The result shows that after the 10 years period we ended up with US$2.9 Million which means that we have a total return on invested principal of 190%. This translates into a Compound Annual Growth Rate (CAGR) of 11.23% for our trading strategy. We believe that this result agrees with the paper by Gatev et al [6] that stated a typical good pair would generate around 11% annualized return. From this result, we can see that in theory our trading strategy would work, although we believe that we can still improve on this result.

Next, using our MATLAB trading model we wanted to see how big is the effect of implementing the portfolio optimization process against dividing the principal equally across the four pairs that we have in our portfolio. In order to check this, we built another model in which we removed the portfolio optimization part and left the other process to be the same. The result is shown in Figure 5.

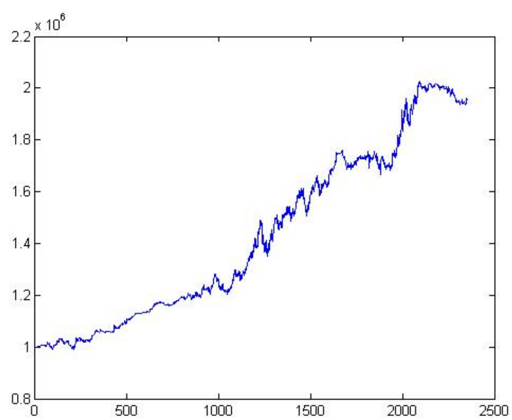


Figure 5: Backtesting Result Using Equal Weight Portfolio

Figure 5 shows that by removing the portfolio optimization process we only end up with US$1.9 Million (total return on invested principal of 90%), which means that we have a US$1 Million reduction compared to the result that we got using the portfolio optimization process. This translates to CAGR of only 6.63%, compared to 11.23% that we had before. From this result we can clearly see that the portfolio optimization process have a dramatic effect on the overall results and therefore form a crucial part of the whole trading strategy.

Finally, we also would like to show the effect of the rolling window sizes to the overall performance results. For the purpose of this analysis, we use the Bootstrapping method and find the optimal rolling window size that provides the least amount of fluctuations in the final return that we obtain. We use the same data set of 2380 data points and as given by the equation (5) for our model, we introduce random gaussian noise with $\mu = 0$ and $\sigma = 1$. We simulate this scenario multiple times and find the average return and return's standard deviation for different rolling window sizes. Figure 6 shows two different results that we got by using 30 days rolling window size for the regression analysis on the same data. The first picture shows that we ended up with US$4.2 Million after 10 years that translates to 320% total return. However, the next picture shows the opposite in which we only ended up with US$1.6 Million or 60% total return. During our analysis, we noted that we would get a very different result everytime we run the backtesting process for the 30 days rolling window size.

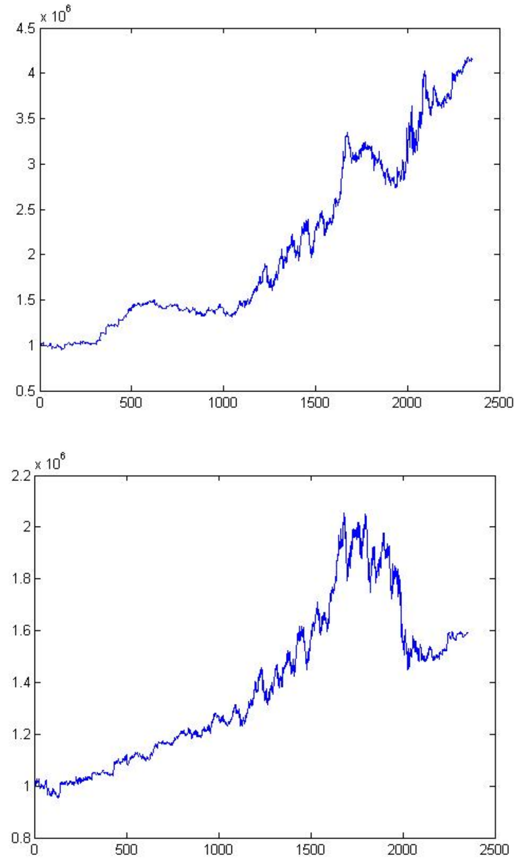


Figure 6: Backtesting Results Using 30-days Rolling Window Size

We gathered 200 data points out of the simulation results and we get that the mean was US$2.39 Million with standard deviation of US$1.02 Million (or roughly 60%). On the other hand, with 50 days rolling window size, we would get the mean to be around US$2.67 Million with standard deviation of US$135,000 (or roughly 8%). We can clearly see that by using a bigger rolling window sizes we would decrease the variation on the simulation results considerably and creating a more robust performance out of the trading strategy.

4.2 Algorithmic Trading Implementation

The implementation of our trading strategy in High Frequency Algorithmic Trading is spread across three domains, which are:

1. InteractiveBrokers Trading Platform
2. Visual Basic Trading Platform - which interfaces the user with the IB Trading Platform
3. MATLAB Platform - to implement the mathematical models for the trading strategy.

The complete architecture is illustrated in the Figure 7 (shown below).

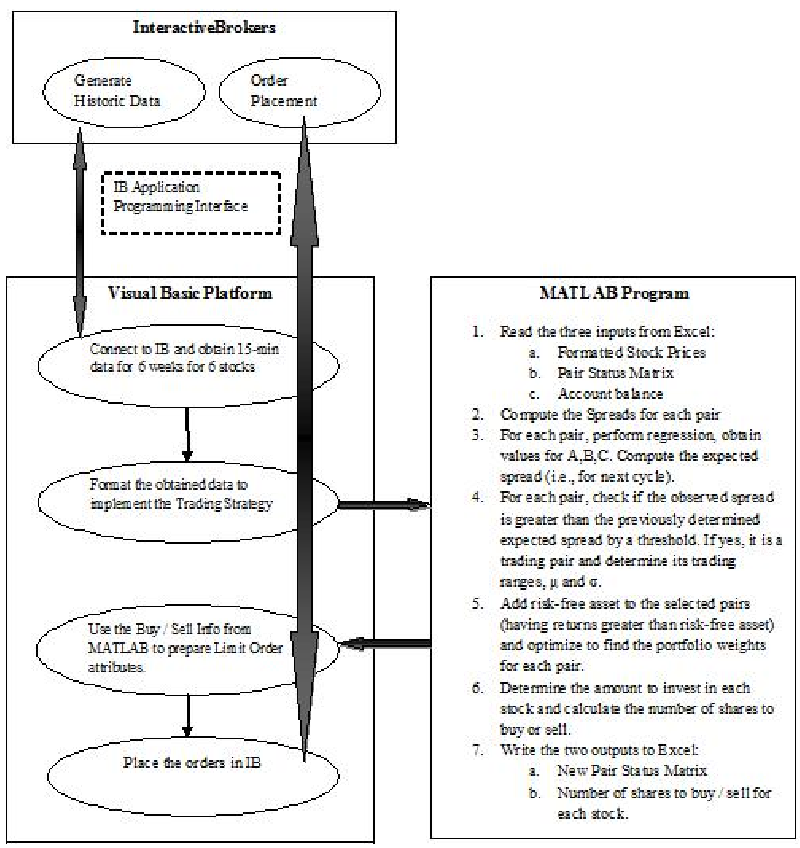


Figure 7: Algorithmic Trading Architecture

For the purpose of implementing our strategy, we consider all combination of following stocks to obtain the pairs.

1. CVX: Chevron Corporation
2. COP: ConocoPhilips
3. XOM: Exxon Mobil Corporation
4. MRO: Marathon Oil Corporation
5. HES: Hess Corp.
6. MUR: Murphy Oil Corporation

When considering two of these stocks at a time, we obtain 15 pairs that can be utilized for Pairs Trading. Since the pairs trading cycle repeats every 15 minutes in the Algorithmic trading implementation, we consider 15-minute interval data over six weeks for each of these stocks and compute the spreads for each Pair. Now, let us look at the function of each platform in implementing the Pairs Trading strategy.

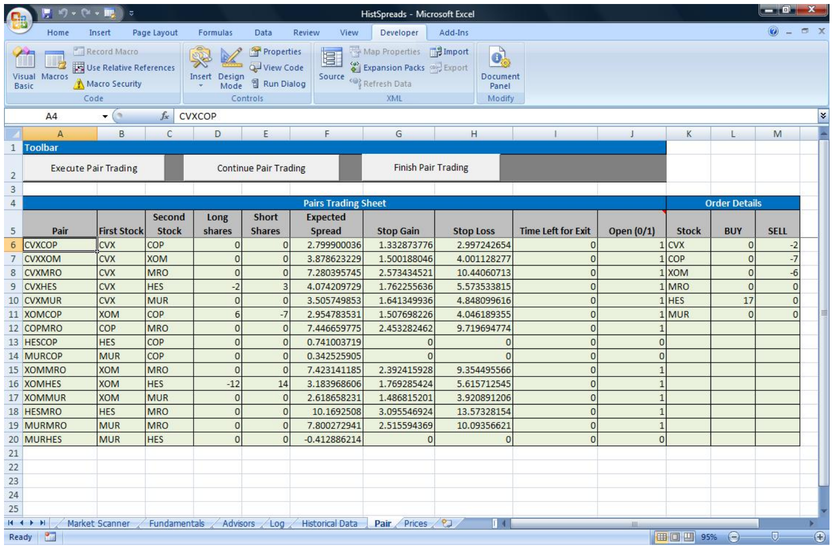
4.3 InteractiveBrokers Trading Platform

The InteractiveBrokers Trading Platform is the actual platform that interacts with the Stock Exchange. While the software for requesting data and placing orders in exchange is proprietary to InteractiveBrokers - and hence not available to us - these functions, however, can be triggered from platforms outside of InteractiveBrokers through its Application Programming Interface (API). So, in our implementation, the IB platform serves two purposes.

1. Whenever requested provides Visual Basic platform with stock price data as per the given specifications.
2. Whenever requested by the VB platform place BUY/SELL orders in the stock exhange and update VB on the status of the order, bank account, etc.

4.4 Visual Basic Trading Platform

The Visual Basic Trading Platform is the front end for the algorithmic trader and the trading actions placed here are executed in the stock exchange via the IB Trading Platform. The API from InteractiveBrokers allows traders to implement their trading strategy from multiple platforms. Since, we use Visual Basic for our implementation, we take advantage of the MS-Excel Spreadsheet named “TwsActiveX.xlsm” and extend it with a new Workseet named “Pair”. Figure 8, depicts the layout of the newly created “Pair” workseet.



Pair	First Stock	Second Stock	Long shares	Short Shares	Expected Spread	Stop Gain	Stop Loss	Time Left for Exit	Open (0/1)	Stock	BUY	SELL
CVXCOP	CVX	COP	0	0	2.799900036	1.332873776	2.997242654	0	1	CVX	0	-2
CVXXOM	CVX	XOM	0	0	3.878623229	1.500188046	4.001128277	0	1	COP	0	-7
CVXMRO	CVX	MRO	0	0	7.280395745	2.573434521	10.44060713	0	1	XOM	0	-6
CVXHES	CVX	HES	-2	3	4.074209729	1.762255636	5.573533815	0	1	MRO	0	0
CVXMUR	CVX	MUR	0	0	3.505749853	1.641349936	4.848099616	0	1	HES	17	0
XOMCOP	XOM	COP	6	-7	2.954783531	1.507698226	4.046189355	0	1	MUR	0	0
COPMRO	COP	MRO	0	0	7.446659775	2.453282462	9.719694774	0	1			
HESCOP	HES	COP	0	0	0.741003719	0	0	0	0			
MURCOP	MUR	COP	0	0	0.342525905	0	0	0	0			
XOMMRO	XOM	MRO	0	0	7.423141185	2.392415928	9.354495566	0	1			
XOMHES	XOM	HES	-12	14	3.183968606	1.769285424	5.615712545	0	1			
XOMMUR	XOM	MUR	0	0	2.618658231	1.486815201	3.920891206	0	1			
HESMRO	HES	MRO	0	0	10.1692508	3.095546924	13.57328154	0	1			
MURMRO	MUR	MRO	0	0	7.800272941	2.515594369	10.09356621	0	1			
MURHES	MUR	HES	0	0	-0.412886214	0	0	0	0			

Figure 8: "Pair" worksheet (in VB platform) to implement Pair Trading

The layout of this worksheet consists of two main sections

1. The first ten columns on the left side of sheet contain information about each of the 15 pairs. The columns within this range serve as both input and output for the MATLAB program. Let us look at each column in detail

   (a) The first column ("A") contains the list of all possible pairs from the six stocks that are considered in our implementation

   (b) The second column ("B") contains the stock within that pair that is considered as 'long' when computing the spread and the third column ("C") contains the stock within that pair that is considered as 'short' when computing the spread. The stocks for these two columns are chosen based on the historic prices of these stocks. Generally, the stock that has had higher prices historically is chosen as the 'long' stock and the other is chosen as the 'short' stock. This arrangement is primarily to obtain positive values in spread.

   (c) The fourth ("D") and the fifth column ("E') contain number of stocks bought or sold from previous pair trades. This information is needed at the time of closing out already opened pairs as we would want to take the opposite position for the exact number of shares when closing our pair trade.

   (d) The sixth ("F") column contains the expected spread for each pair in the next trading cycle. It is this value that is compared with the observed spread to determine whether to enter into pair trade or not. The expected spread for next trading cycle is computed in every trading cycle and is stored in this column so that it can be used in the next trading cycle.

(e) The seventh ("G") column contains the lower threshold - which in our case is 0.5σ - at which the pair is unwound and profit is made.

(f) The eighth ("H") column contains the uppermost threshold - which in our case is 3σ - at which we unwind the pair with a loss. This threshold is a "Stop-Loss" position to manage the risk that the stocks considered in the pair are divergent and have stopped being a mean-reverting OU pair.

(g) The ninth ("I") column is also a risk management metric used to keep an upper limit on the number of trading cycles a pair can exist without being unwound. This figure is obtained from the historical time taken to revert back to mean.

(h) The tenth ("J") column is a 'Flag' that says whether the current pair is active or otherwise. This column can take any of the following values.

    i. 0 - Inactive Pair
    ii. 1 - Active/Open Pair
    iii. 2 - Candidate Pair ,i.e., a pair that has crossed 2σ already and we are waiting for it to go below 2σ to open a pair trading position. This is to incorporate our strategy of starting a pair trade only on the second time the spread hits the threshold 2σ.
    iv. 3 - A pair that has just been closed. This is for internal program logic to calculate the number of shares to buy/sell.
    v. 4 - A pair that has just been opened. This is for internal program logic to calculate the number of shares to buy/sell and also to calculate the expected return and expected standard deviation.

2. The last three columns contain the output of the MATLAB program which is the information on the number of shares to BUY/SELL in each stock. This information is used to place orders in the IB Trading platform.

Within a trading cycle, the sequence of steps involved in the Visual Basic platform are:

1. Establish a connection with the IB Trading platform

2. Once "Execute Pair Trading" is trigerred, request and obtain stock prices from IB.

3. When "Continue Pair Trading" is trigerred, format the obtained prices for MATLAB.

4. When "Finish Pair Trading" is trigerred - after the execution of the MATLAB program, prepare the Order attributes to be executed in the IB trading platform.

5. Place the Orders on the exchange through the IB trading platform.

4.5 MATLAB Platform

The following algorithm was implemented in MATLAB to execute our trading strategy.

1. Read the three inputs that are obtained from Excel Spreadsheet:
   (a) The Formatted Stock Prices
   (b) The Pair Status Matrix (in the "Pair" worksheet)
   (c) The Account Balance (needed for optimization)
2. For each pair, calculate the spreads from the Stock Prices
3. Perform AutoRegression on each of these pairs and obtain the values for the co-efficients A, B and C given in the equation.
$$X_{k+1} = A + BX_k + C\varepsilon_{k+1}$$
4. From the obtained A, B, C values calculate the Expected Spread for the next cycle. This value would be compared with the observed spread in the next cycle to determine whether a pair trade should be entered into.
5. Also compute the mean (μ) and standard deviation (σ) for the sample.
6. Perform steps (6) through (8) for each pair. If the pair is already open, check if it has crossed the uppermost threshold (3σ) or the lower threshold (0.5σ) or if it is open for more than the threshold number of cycles. If yes, close the pair trade.
7. If the pair is not open, check if it has crossed the upper threshold (2σ) and if it is not a candidate pair, make it a candidate pair so that a pair trade can be opened the next time it falls below this threshold.
8. In all other cases, just continue without any action.
9. For pairs that have been opened in this trading cycle, calculate its expected return and expected standard deviation. If the expected return is greater than the risk-free rate of return, that is when we decide to trade on this pair.
10. For all pairs that were selected in the previous step, along with the risk-free asset, perform optimization using the Markowitz model. The Markowitz problem is defined as below:
$$\min_{\sigma_{ij}} \left(\frac{1}{2} \sum_{i,j=1}^{n} w_i.w_j.\sigma_{ij} \right)$$

subject to

$$\sum_{i=1}^{n} w_i \bar{r_i} = \bar{r}$$

$$\sum_{i=1}^{n} = 1$$

11. Based on the portfolio weights arrived at from the above optimization problem, determine the amount to be invested in each pair. In each pair, the investment is split equally between the stock in which we go 'long' and the stock in which we go 'short'.

12. Divide these amounts by the respective stock prices to obtain the number shares that are to bought and sold.

13. Write the following outputs into the Excel spreadsheet:

    (a) The New Pair Status Matrix

    (b) The number of shares to be bought/sold for each stock,i.e., the BUY / SELL Matrix.

5 Conclusions

Based on the backtesting of the model we implemented, we obtain a Compunded Annual Growth Rate (CAGR) of 11.23% each year over a period of 10 years. When compared with the returns from S&P index and also considering that the Pair Trading strategy is in vogue for a few years now, we find the returns to be excellent. The high returns in our trading strategy is partly attributable to the optimization of our portfolio by considering multiple pairs.

6 Future Work

We would like to develop our trading model further by undertaking the following steps:

- Though we optimize the amount to be invested in each pair, within the pair we split the investment equally between the stocks. We would like to optimize the investment within the pair too, as we believe that this would significantly augment our returns.

- In this trading strategy we considered assets from the same asset class for finding pairs. In future, we would like to work on pairs that might exist across asset classes.

- In the backtesting, we considered the time period (τ) as 1 day and in the algorithmic implementation, we kept τ as 15 minutes. We plan to analyze our trading strategy for different time periods and find the best conceivable time period (τ).
- We would also like to analyze the sensitivity of our trading strategy to different kinds / levels of transaction fee; as in reality the transaction fees can be a flat fee or a percentage fee. And the percentage fee also various according to the type of investors. So, analysis on transaction fees would help us identify how well our strategy suits with different investors.

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