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MCE 6642
Project: Design of Luenberger observer for Proof Mass Actuator System
Problem Source: Linear State-Space Control Systems; CE-5

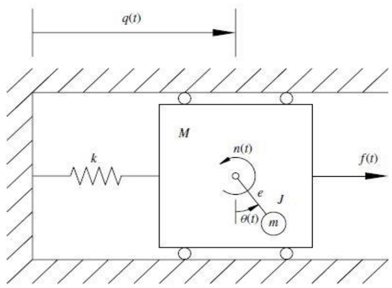

FIGURE 1.17 Diagram for Continuing Exercise 5 (top view).

CE1.5a A nonlinear proof-mass actuator system is shown in Figure 1.17. This system has been proposed as a nonlinear controls benchmark problem (Bupp et al., 1998). However, in this book, the system will be linearized about a nominal trajectory, and the linearization then will be used in all ensuing chapters as Continuing Exercise 5.
This is a vibration-suppression system wherein the control goal is to reject an unknown, unwanted disturbance force $f(t)$ by using the control torque $n(t)$ to drive the unbalanced rotating pendulum (proof mass) to counter these disturbances. The block of mass M is connected to the wall via a spring with spring constant k and is constrained to translate as shown; $q(t)$ is the block displacement. The rotating pendulum has a point mass m at the tip, and the pendulum has mass moment of inertia J. The pendulum length is e and the pendulum angle $\theta(t)$ is measured as shown. Assume that the system is operating in the horizontal plane, so gravity need not be considered.

TABLE 2.7 Numerical Parameters for CE5 System

Parameter	Value	Units	Name
M	1.3608	kg	cart mass
k	186.3	N/m	spring stiffness constant
m	0.096	kg	pendulum-end point mass
J	0.0002175	kg-m^2	pendulum mass moment of inertia
e	0.0592	m	pendulum length

CE8.5 CE4.5 results should indicate that the original system is not observable. Therefore, add a second output: Make $\theta(t)$ an output in addition to $q(t)$; check observability again and proceed. For the control law designed in CE7.5b, design an observer-based compensator for this modified system with two outputs. Use observer error eigenvalues that are your desired state feedback eigenvalues scaled by 10. Evaluate your results: Plot and compare the simulated open-loop, closed-loop with state feedback, and closed-loop with observer output responses for the same case (given initial conditions) as in CE2.5. Introduce an initial observer error; otherwise, the closed-loop with state feedback and closed-loop with observer responses will be identical.

Solution for CE1.5:

$$(M+m)\ddot{q}(t) + kq(t) + me(\ddot{\theta}(t)\cos\theta(t) - \dot{\theta}^2(t)\sin\theta(t)) = 0$$
$$(J+me^2)\ddot{\theta}(t) + me\ddot{q}(t)\cos\theta(t) = n(t)$$

A valid state-space realization for this system is given below:

$$x_1(t) = q(t) \qquad x_3(t) = \theta(t)$$
$$x_2(t) = \dot{q}(t) = \dot{x}_1(t) \qquad x_4(t) = \dot{\theta}(t) = \dot{x}_3(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k(J+me^2)}{d(\tilde{\theta})} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{kme\cos(\tilde{\theta})}{d(\tilde{\theta})} & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \dfrac{-me\cos(\tilde{\theta})}{d(\tilde{\theta})} \\ 0 \\ \dfrac{M+m}{d(\tilde{\theta})} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \qquad D = 0$$

where $d(\tilde{\theta}) = (M+m)(J+me^2) - (me\cos(\tilde{\theta}))^2$. Note that this linearized state-space realization depends on the zero-torque