Design of Butterworth Crossover Network

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I. Introduction

The crossover network of filters are a parallel combination of filter systems designed such that the cutoff frequencies cross over each other to provide a uniform frequency response. Each of these filters separates the incoming signal from the audio amplifier to provide the power to the driver in their optimal frequencies. The 3 kinds of filters: low pass, band pass and high pass filter allows the incoming frequencies to be segregated into low, mid range and high frequencies. This then allows the drivers to be connected in their respective operating range frequencies. The overall design of this network occurs in 3 main steps: Low-Pass Filter Design , Band-Pass Filter Design and High-Pass Filter Design. For each of these designs, initially pass band edge, stop band edge and their attenuation information is used to determine the order of the required Butterworth filter polynomial which can be calculated as:

$$K = \left\lceil \frac{\log\left[\left(10^{\alpha_s/10}-1\right)/\left(10^{\alpha_p/10}-1\right)\right]}{2\log(\omega_s/\omega_p)} \right\rceil$$

where, α_s = min stopband attenuation
α_p = max passband attenuation
w_s = stopband edge
w_p = passband edge

Eq 1: Order of Filter

Similarly, 3 dB point or half power point is the point in the magnitude plot where the signal reaches the half of its original power of the input signal. The cutoff frequency is chosen at the half power point value. The cutoff frequency is determined using the following equations and their average value is taken as the cutoff frequency for the specific filter.

$$\omega_c = \frac{\omega_p}{\left(10^{\alpha_p/10}-1\right)^{1/2K}}, \quad \omega_c = \frac{\omega_s}{\left(10^{\alpha_s/10}-1\right)^{1/2K}}$$

where, K = Order of the Filter
w_c = cutoff frequency

Eq 2: Cutoff Frequency of Filter

After that, the ceiling value of the order of filter is used to match a corresponding Butterworth filter polynomials. This polynomial is the denominator of the normalized prototype filter. After this, the appropriate transformation terms are used to convert the prototype low pass filter to low pass, band pass and high pass filter.

II. Design of Low Pass Filter

For the low pass filter design, the pass band edge frequency was 100Hz, stop band edge frequency was 1000 Hz, maximum pass band attenuation was <= 1 dB and minimum stop band attenuation was>= 30 dB. The frequencies were multiplied by a factor 2*Pi and the angular frequencies were used in the design calculation of filters.

A. Determination of Filter Order

The provided parameters were used to evaluate the order of the filter using the Eq:1. By using the ceiling value for K , the order of the filter was calculated to be 2.

B. Determination of 3 dB Point

Under the provided parameters, the cutoff frequency of the filter was found to be 881.17 rad/s and 1118.05 rad/s. Hence, the cutoff frequency was taken as the average of the frequencies to be 999.6 rad/s.

C. Selection of Filter Family and Transfer Function

Using the evaluated filter order K =2, the Table 2.3 from textbook was used to get the normalized filter polynomial "$s^2 + 1.414214 * s + 1$". Hence, the butterworth prototype filter for the Low pass filter is ;

$$H(s) = 1/(s^2 + 1.414214s + 1)$$

Now, the frequencies are re-calibrated with the following transformation equation to get the transfer function for the low pass filter.

$$H_{\mathrm{lp}}(s) = H_{\mathrm{p}}(s)\big|_{s \to \frac{\omega_0}{\omega_1}s} = H_{\mathrm{p}}\left(\frac{\omega_0}{\omega_1}s\right)$$

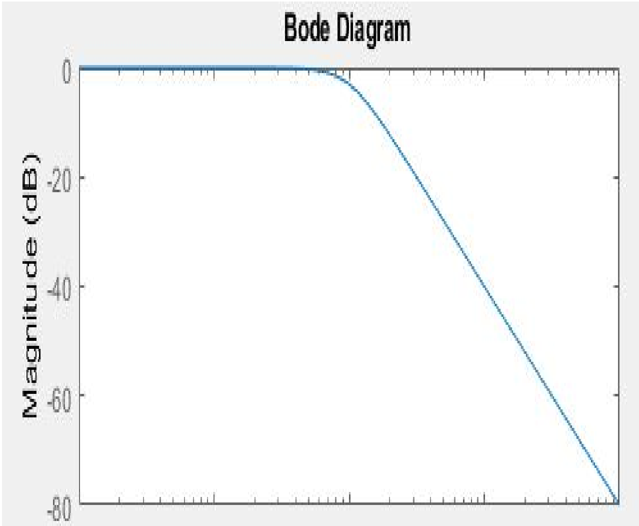
where, w_0 = prototype frequency = 1
w_1 = cutoff frequency of the filter

Eq 3: Prototype to Low pass transformation

Hence, using the calibration factor of s/4663.23, the transfer function of the low pass filter was evaluated to be

$$H(s) = 999200.16/(s^2 + 1413.65s + 999200.16)$$

D. Bode Magnitude Plot



III. Design of Band Pass Filter

For the band pass filter design, the pass band edge frequency was 100Hz and 3500 Hz, stop band edge frequency was 10 Hz and 35000 Hz, maximum pass band attenuation was ¡ = 1 dB and minimum stop band attenuation was ¿= 30 dB. The frequencies were multiplied by a factor 2*Pi and the angular frequencies were used in the design calculation of filters.

A. Determination of Filter Order

The min value of the stop band frequency of the prototype filter was evaluated. After that, the min stop band frequency was used to evaluate the Filter Order and cutoff frequency. By using the ceiling value for K , the order of the filter was calculated to be 2 and the cutoff frequency was 1.62 rad/s.

$$H_{\mathrm{bp}}(s) = H_{\mathrm{p}}\left(\omega_0 \frac{s^2+\omega_1\omega_2}{s\,(\omega_2-\omega_1)}\right)$$

where, w_0 = prototype frequency = 1
w_1 = cutoff frequency of high pass portion of filter
w_2 = cutoff frequency of low pass portion of filter

Eq 4: Prototype to Bandpass Filter Transformation

B. Determination of Prototype Filter

Then Eq: 5 was used to transform the poles to get the prototype Low pass filter from Band pass filter. $p_k = j\omega_c e^{\frac{j\pi}{2K}(2k-1)} \qquad k = 1, 2, 3, \ldots, 2K.$ Eq 6: Equation to evaluate pk Using the values for pk, the prototype filter transfer function obtained was,

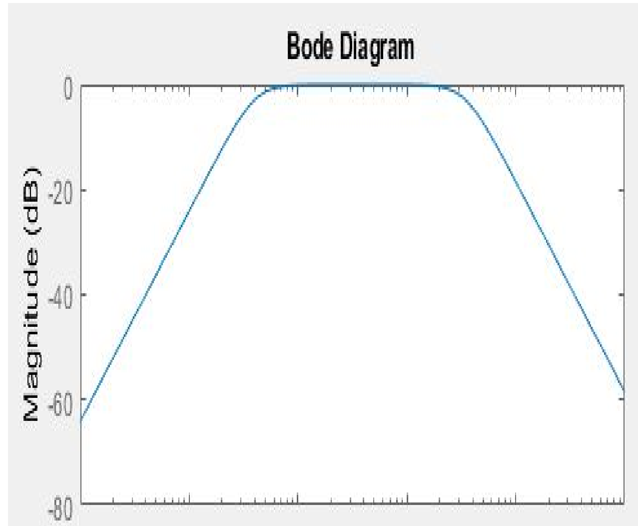
$$H(s) = 1/(s^2 + 2.29s + 2.61)$$

C. Evaluation of BP Transfer Function

Now the prototype filter was again calibrated and pole shifted to meet the band pass requirements of the filter and again Eq:6 is used to evaluate the actual band pass filter for the system. The transfer function evaluated was:

$$H(s) = 1.19*10^9 s^2/$$
$$(s^4+4.88*10^9 s^3+1.22*10^9 s^2+6.75*10^(11)s+1.91*10^(14))$$

D. Bode Magnitude Plot



IV. Design of High Pass Filter

For the low pass filter design, the pass band edge frequency was 3500 Hz, stop band edge frequency was 350 Hz, maximum pass band attenuation was ¡ = 1 dB and minimum stop band attenuation was ¿= 30 dB. The frequencies were multiplied by a factor 2*Pi and the angular frequencies were used in the design calculation of filters.

A. Determination of Filter Order

The provided parameters were used to evaluate the order of the filter using the Eq:1. By using the ceiling value for K , the order of the filter was calculated to be 2.

B. Determination of 3 dB Point

Under the provided parameters, the cutoff frequency of the filter was found to be 1.56E4 rad/s.

C. Selection of Filter Family and Transfer Function

Using the evaluated filter order K =2, the Table 2.3 from textbook was used to get the normalized filter polynomial "$s^2 + 1.414214 * s + 1$". Hence, the butterworth prototype filter for the Low pass filter is ;

$$H(s) = 1/(s^2 + 1.414214s + 1)$$

Now, the frequencies are re-calibrated with the following transformation equation to get the transfer function for the high pass filter.

$$H_{\mathrm{hp}}(s) = H_{\mathrm{p}}\left(\frac{\omega_0\omega_1}{s}\right)$$

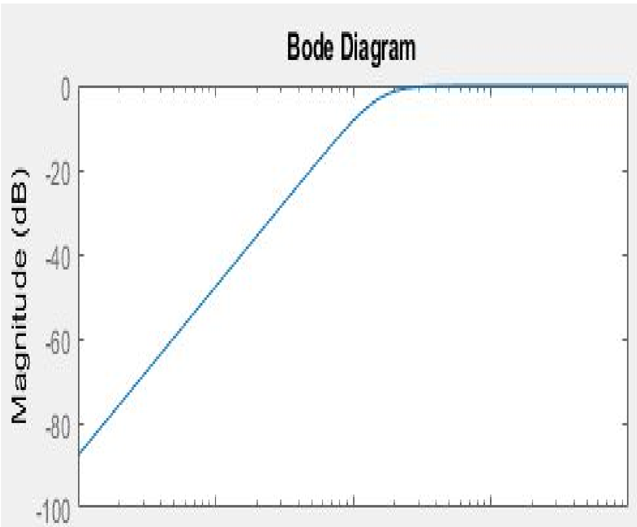
where, w_0 = prototype frequency = 1
w_1 = cutoff frequency of the filter

Eq 5: Prototype to High Pass Filter Transformation

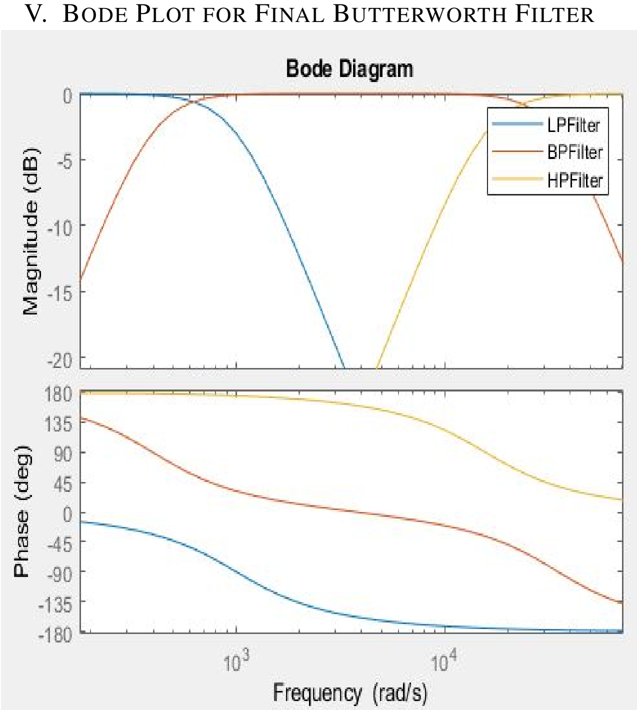
Hence, using the calibration factor of $'(1.56 * 10^4)/s'$, the transfer function of the high pass filter was evaluated to be

$$H(s) = s^2/(s^2 + 2.21 * 10^4 s + 2.43 * 10^8)$$

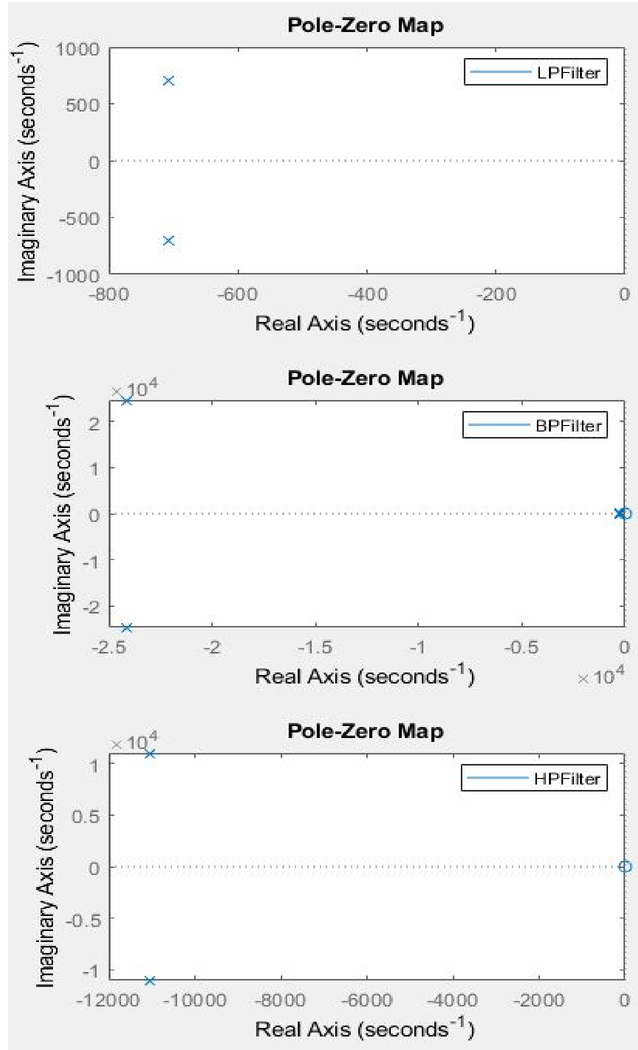
D. Bode Magnitude Plot



V. Bode Plot for Final Butterworth Filter



VI. Pole Map of Each Filters



VII. Conclusions

Hence, the butter worth filter for the system was thus obtained by individually designing a butterworth low pass, band pass and a high pass filter. The final system of filters have cross-over points as -0.6 dB and -0.8 dB from left to right.

References:

1.Essentials of Digital Signal Processing, Lathi

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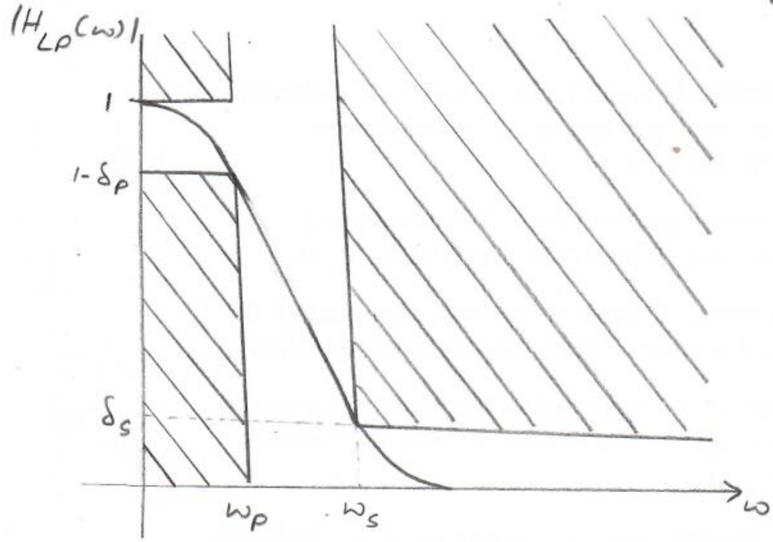
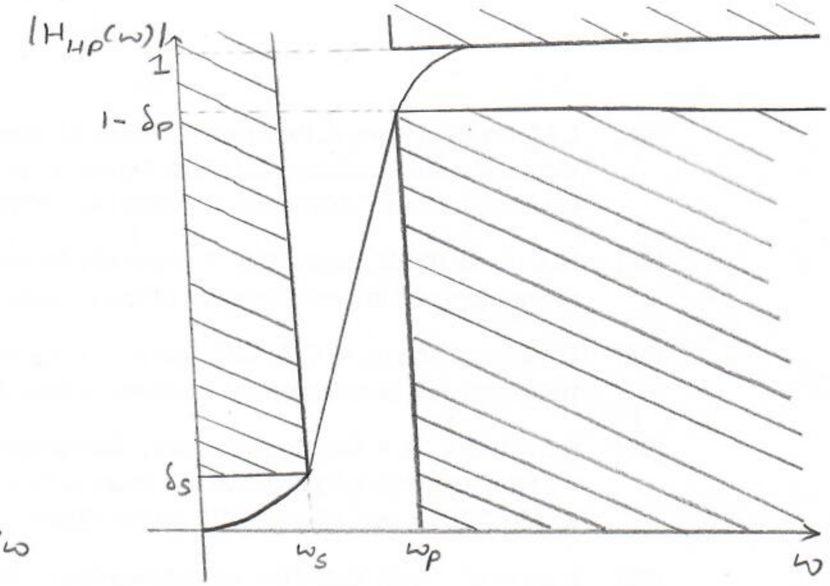
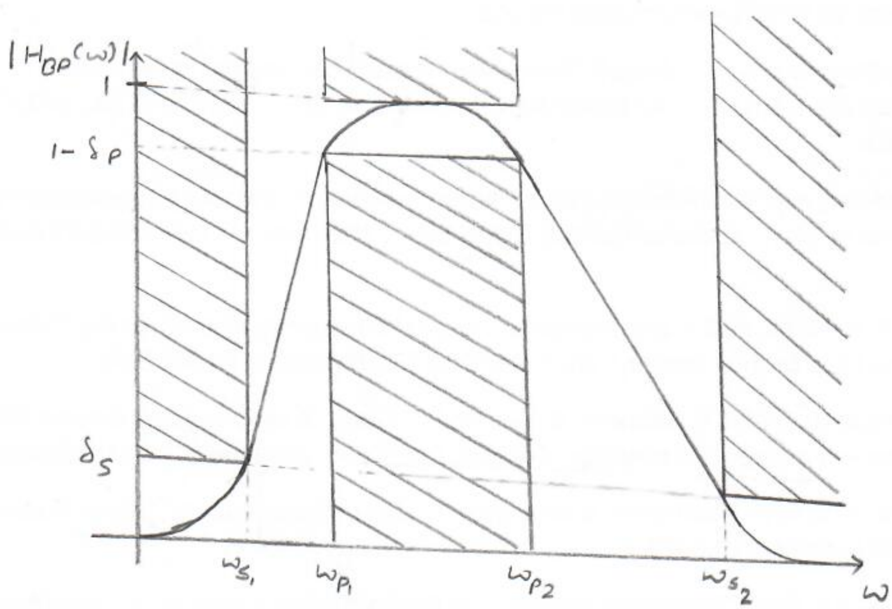






$$\alpha_s = -20\log_{10}(\delta_s)$$

$$\alpha_p = -20\log_{10}(1-\delta_p)$$

$$1\ db = -20\log_{10}(1-\delta_p)$$

$$1-\delta_p = 10^{-\frac{1}{20}}$$

$$\boxed{\delta_p = 0.11}$$

$$30dB = -20\log_{10}(\delta_s)$$

$$\delta_s = 10^{-\frac{30}{20}}$$

$$\boxed{\therefore \delta_s = 0.032}$$

For low pass Filter,

$f_s = 1000\ Hz$; $\omega_s = 2\pi f_s = 6285.7\ rads^{-1}$

$f_p = 1000\ Hz$; $\omega_p = 2\pi f_p = 628.57\ rads^{-1}$

Using 2.38,

$$K = \left[\frac{\log[(10^{\alpha_s/10}-1)/(10^{\alpha_p/10}-1)]}{2\log(\omega_s/\omega_p)}\right]$$

$$= \left[\frac{\log[(10^{30/10}-1)/(10^{1/10}-1)]}{2\log(6285.7/628.57)}\right] \approx 1.79$$

$$\boxed{\therefore K = 2}$$

2. Determine the 3dB point

$$\omega_c = \frac{\omega_p}{(10^{\alpha_p/10}-1)^{1/2K}} \qquad \omega_c = \frac{\omega_s}{(10^{\alpha_s/10}-1)^{\frac{1}{2K}}}$$

$$= \frac{628.57\,\text{rads}^{-1}}{(10^{1/10}-1)^{\frac{1}{2.2}}} \qquad = \frac{6285.7}{(10^{30/10}-1)^{\frac{1}{2.2}}}$$

$$\therefore \omega_c = 881.17\,\text{rads}^{-1} \qquad \therefore \omega_c = 1118.05\,\text{rads}^{-1}$$

$$881.17\,\text{rads}^{-1} \leq \omega_c \leq 8208.4\,\text{rads}^{-1}$$

Average : $\frac{(1118.05 + 881.17)\,\text{rads}^{-1}}{2} = \underline{\underline{999.6\ \text{rads}^{-1}}}$

3. Select a Filter Family & Find a Transfer Function

i) LP- to LP transform: $H_{lp}(\omega) = H_p(\omega)\Big|_{\omega \to \frac{\omega_0}{\omega_1}} = H_p\left(\frac{\omega_0}{\omega_1}\omega\right)$

In terms of $H(s)$: $\frac{\omega_0}{\omega_1}s \rightarrow \frac{s}{\omega_c} = \frac{s}{999.6\,\text{rads}^{-1}}$

ii) For $K=2$: Normalized Butterworth filter polynomial from Table 2 3 is,

$$D(s) = s^2 + 1.414214s + 1$$

Prototype filter:

$$H_p(s) = \frac{1}{s^2 + 1.414214s + 1}$$

For $\frac{s}{4663.23}$,

$$H_{LP}(s) = \frac{1}{\left(\frac{s}{999.6}\right)^2 + \left(1.414214 \times \frac{s}{999.6}\right) + 1}$$

$$= \frac{(999.6)^2}{s^2 + 1413.65s + (999.6)^2}$$

$$\boxed{\therefore H_{LP}(s) = \frac{999200.16}{s^2 + 1413.65s + 999200.16}}$$

Bandpass Filter:

3. Select a Filter Family & Find a Transfer Function

i) LP to BP transform: $H_{BP}(\omega) = H_{LP}(\omega)\Big|_{\omega \to \omega_P \frac{\omega^2 + \omega_{P_1}\omega_{P_2}}{\omega(\omega_{P_2} - \omega_{P_1})}}$

$\omega_P = 1$
(cutoff point at 1 rads^{-1})
(for prototype filter)

From Matlab (3db points),

$\omega_{P_1} \approx 446 \text{ rads}^{-1}$

$\omega_{P_2} \approx 34972 \text{ rads}^{-1}$

$$\omega_P \frac{\omega^2 + \omega_{P_1}\omega_{P_2}}{\omega(\omega_{P_2} - \omega_{P_1})} = 1 \cdot \frac{\omega^2 + 446 \cdot 34972}{\omega(34972 - 446)} = \frac{\omega^2 + 15597512}{34526\omega}$$

In terms of H(s),

$$s \longrightarrow \omega_P \frac{s^2 + \omega_{P_1}\omega_{P_2}}{s(\omega_{P_2} - \omega_{P_1})} = \frac{s^2 + 15597512}{34526s}$$

ii) For K=2: Normalized Butterworth filter polynomial from Table 2.3 is,

$$Q(s) = s^2 + 1.414214s + 1$$

Since prototype is all-LP (all poles) filter,

$$\frac{1}{s - P_k} \rightarrow \frac{1}{\frac{s^2 + 15597512}{34526s} - P_k} = \frac{34526s}{s^2 + 15597512 - 34526P_k s}$$

$$= \frac{(34526)s}{s^2 + (-34526P_k)s + 15597512}$$

From the Filter transformation eq^n:

$$as^2 + bs + c = s^2 - P_k(\omega_{P_2} - \omega_{P_1})s + \omega_{P_1}\omega_{P_2} = s^2 + (-34526P_k)s + 15597512$$

(Further evaluations in Matlab)

$$\boxed{H_{BP}(s) = \frac{1.19 \times 10^9 s^2}{s^4 + 4.88 \times 10^4 s^3 + 1.22 \times 10^9 s^2 + 6.75 \times 10^{11} s + 1.91 \times 10^{14}}}$$

High - Pass Filter:

1. From Matlab,

Order(K) = 2

Cutoff frequency (ω_c) = 1.56×10^4 rads^{-1}

2. Select a Filter Family & Find a Transfer Function

i) LP to HP transform:

$$H_{hp}(\omega) = H_p(\omega)\Big|_{\omega \to \frac{\omega_0\omega_1}{\omega}} = H_p\left(\frac{\omega_0\omega_1}{-\omega}\right)$$

In terms of H(s):

$$\frac{\omega_0\omega_1}{s} \rightarrow \frac{\omega_c}{s} = \frac{1.56 \times 10^4}{s}$$

ii) For K = 2: Normalized filter polynomial from Table 2.3 is;

$$D(s) = s^2 + 1.414214s + 1$$

Prototype filter:

$$H_p(s) = \frac{1}{s^2 + 1.414214s + 1}$$

For $\frac{1.56 \times 10^4}{s}$,

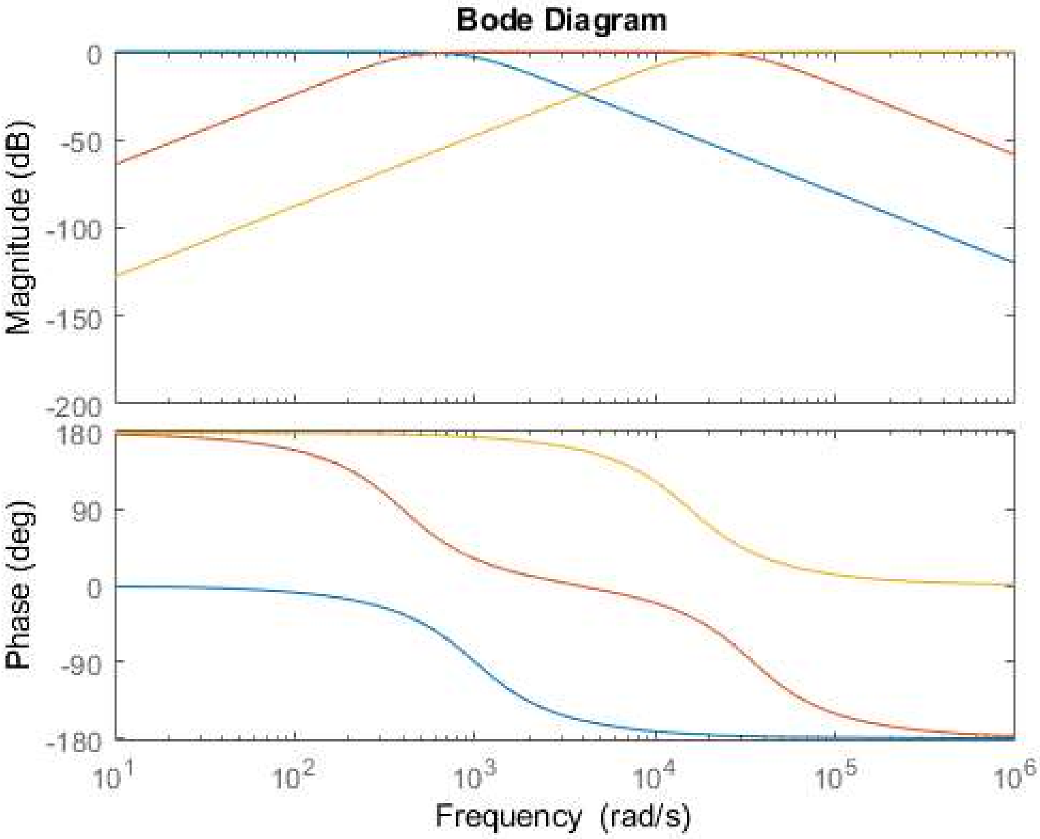
$$H_{HP}(s) = \frac{1}{\left(\frac{1.56\times10^4}{s}\right)^2 + 1.414214\left(\frac{1.56\times10^4}{s}\right) + 1}$$

$$= \frac{s^2}{2.43\times10^8 + 2.21\times10^4 s + s^2}$$

$$\boxed{\therefore H_{HP}(s) = \frac{s^2}{s^2 + 2.21\times10^4 s + 2.43\times10^8}}$$

```matlab
%Shishir Khanal
%EE5575
%Project-1
%Matlab code to evalaute filter Order
%Information Source: Lathi Page 124, Essentials of Digital Signal
%Processing
%-----------------------------------------------------------------

clc;clear;
%max passband attenuation
filter_ap = 1;
%min stopband attenuation
filter_as = 30;

%---------------------------------------------------------------
% define passband & cutoff frequencies
% LPFfp = 100;
% LPFfs = 1000;
LPFwp = 2*pi*100;
LPFws = 2*pi*1000;
LPF_K = ceil(abs(Order(LPFwp, LPFws, filter_ap, filter_as)));
LPFwc = cutoffrequency(filter_ap, filter_as,LPFwp, LPFws,LPF_K);
%Using the Normalized Transfer function to find the prototype filter
%transfer function
%Using the calibration parameter 's/4663.23' to calibrate the prototype
%filter transfer function to actual Low Pass Filter Transfer Function

%Transfer function for the Low Pass Filter
numLP = [999200.16];
denLP = [1 1413.65 999200.16];
LPFilter = tf(numLP,denLP);
bode(LPFilter)
%--------------------------------------------------------------------

hold all
%--------------------------------------------------------------------
% define passband & cutoff frequencies
% BPF1fp = 100;
% BPF1fs = 10;
BPF1wp = 2*pi*100;
BPF1ws = 2*pi*10;
% BPF2fp = 3500;
% BPF2fs = 35000;
BPF2wp = 2*pi*3500;
BPF2ws = 2*pi*35000;
wp = 1;
% stopband frequency for the prototype low pass filter
BPFwss = abs([wp*(BPF1ws^2-BPF1wp*BPF2wp)/(BPF1ws*(BPF2wp-BPF1wp)),...
wp*(BPF2ws^2-BPF1wp*BPF2wp)/(BPF2ws*(BPF2wp-BPF1wp))]);
BPFws = min(BPFwss);
%order of filter and cutoff frequency of Bandpass filter
BPF_K = ceil(abs(Order(wp, BPFws, filter_ap, filter_as)));
BPFwc = cutoffrequency(filter_ap, filter_as,wp, BPFws,BPF_K);
% Evaluation of Prototype Transfer function for Low pass filter
k = 1:BPF_K;
```

```
pk = (1j*BPFwc*exp(1j*pi/(2*BPF_K)*(2*k-1)));
A = poly(pk);

% calibration of prototype filter for the band pass filter
a = 1;
b = -pk*(BPF2wp-BPF1wp);
c = BPF2wp*BPF1wp;
pk = [(-b+sqrt(b.^2-4*a*c))./(2*a),(-b-sqrt(b.^2-4*a*c))./(2*a)];
B = (BPFwc^BPF_K)*((BPF2wp-BPF1wp)^BPF_K)*poly(zeros(BPF_K,1));
A = round(poly(pk), 1);
BPFilter = tf(B,A);
bode(BPFilter)
%----------------------------------------------

%----------------------------------------------
% define passband & cutoff frequencies
% HPFfp = 3500;
% HPFfc = 350;
HPFwp = 2*pi*3500;
HPFws = 2*pi*350;
HPF_K = ceil(abs(Order(HPFwp, HPFws, filter_ap, filter_as)));
HPFwc = cutoffrequency(filter_ap, filter_as, HPFwp, HPFws,HPF_K);
%Using the Normalized Transfer function to find the prototype filter
%transfer function
%Using the calibration parameter '(1.56E4)/s' to calibrate the prototype
%filter transfer function to actual Low Pass Filter Transfer Function

numHP = [1 0 0];
denHP = [1 2.21E4 2.43E8];
HPFilter = tf(numHP,denHP);
bode(HPFilter)
%----------------------------------------------

%----------------------------------------------
%function to evaluate order of filter 'K'
function [K] = Order(wp, ws,attpass, attstop)
    term1 = 10^(attstop/10)-1;
    term2 = 10^(attpass/10)-1;
    term3 = term1/term2;
    term4 = ws/wp;
    K = log(term3)/(2*log(term4));
end

%Function to evaluate cutoff frequency
function wc = cutoffrequency(ap, as, wp, ws,k)
wc1 = wp / (10^(ap/10) - 1)^(1/(2*k));
wc2 = ws / (10^(as/10) - 1)^(1/(2*k));
% fprintf("%d ",wc1)
% fprintf("%d ",wc2)
wc = (wc1+wc2)/2;
end
%-----------------------------------------------
```



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