Digital Signal Processing(EE 5575)
Final Project
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Submitted to: Prof. Farshad Dailami

Design and Comparison of Infinite Impulse Response(IIR) and Finite Impulse Response(FIR) Low Pass Filter

Introduction:

IIR and FIR filters are used to process digital signals. The FIR filters are efficient to implement whereas FIR filters are simple to implement. The objective of the project was to convert the real low pass filter from project 1 to a digital low pass IIR and Filter and study their characteristics. The filters specifications are:

Stopband frequency(f_s') = 1000 Hz
Passband frequency(f_p) = 100 Hz
Stopband attenuation(α_s) >= 30 db
Passband attenuation(α_s) <= 1 db

$$\text{Filter Transfer Function} = \frac{394784.2}{s^2 + 888.5766s + 394784.2}$$

Rendered Passband frequency(f_p') = 160 Hz

I. IIR Filter Design:

The IIR low pass filter conversion from a Continuous Time(CT) low pass filter transfer function can be achieved by taking the bilinear transfer function of the CT transfer function. The bilinear transfer function causes a shift in the poles and zeros in the complex plot and hence frequency prewarping is required that shifts the poles and zeros from 0 to 2pi to 0 to sampling frequency(f_s) /2.

A. Sampling frequency specification

The bandwidth can be passed through the filter by using the passband frequency to evaluate the sampling rate. However, for the potential purposes of passing the signal through the reconstruction filter,it is a good idea to take a higher sampling than sufficient frequency value. Let's consider the stopband frequency to evaluate the sampling frequency.
Stopband frequency(f_s') = 1000 Hz
Nyquist sampling frequency(f_s) = 2*f_s' = 2000Hz
Sufficient Nyquist sampling (f_{suf}) = 2*f_p' = 320 Hz
Hence, the frequency is oversampled by 1680 Hz. Because of the mirroring effect around each interval of sampling frequency this value provides a margin of (1680/2 =) 840 Hz for sampling.

B. Bilinear Transform & z-domain transfer function

The bilinear transform was performed on a CT signal by substituting s→ $\frac{z-1}{z+1}$ and in matlab using the 'bilinear' keyword. Following transfer functions were evaluated:

$$H(z)_{Hand} = \frac{0.9978z^2+1.9956z+0.9978}{z^2+1.995z+0.9955} \text{(App.1)}$$

$$H(z)_{Matlab} = \frac{0.01979+0.03958z+0.01979}{z^2-1.565z+0.6437}$$

Comparison of these transfer functions show similarity in the order of transfer function but discrepancy in the location of poles of the transfer functions.

II. IIR Filter Evaluation

A. Pole-Zero Plot & Frequency Response

The poles of the IIR filter all lie within the unit circle. Hence the filter is stable. In the frequency response, exponentially decaying phase response was obtained.

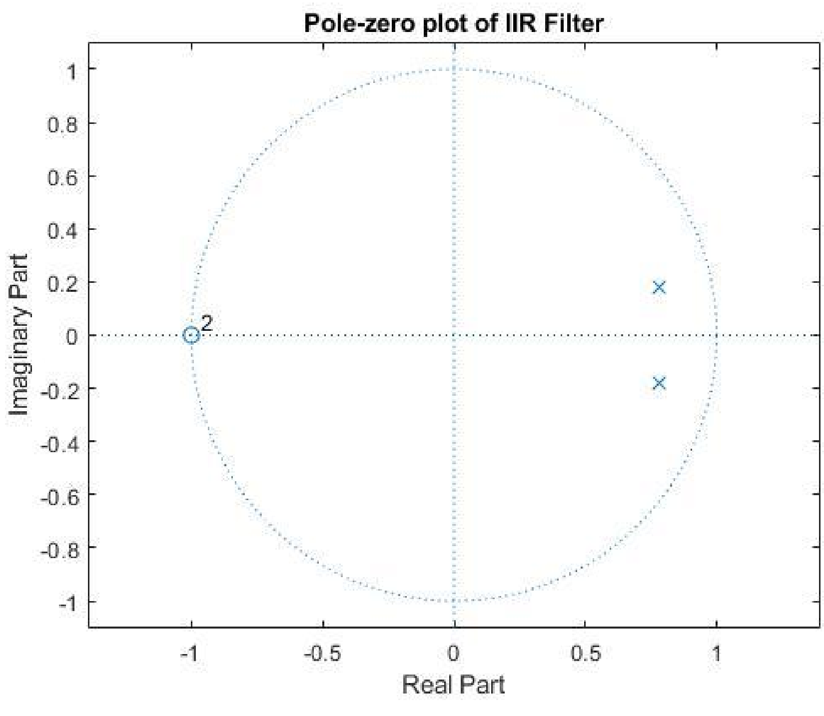


Fig 1: Pole-Zero Plot of IIR Filter

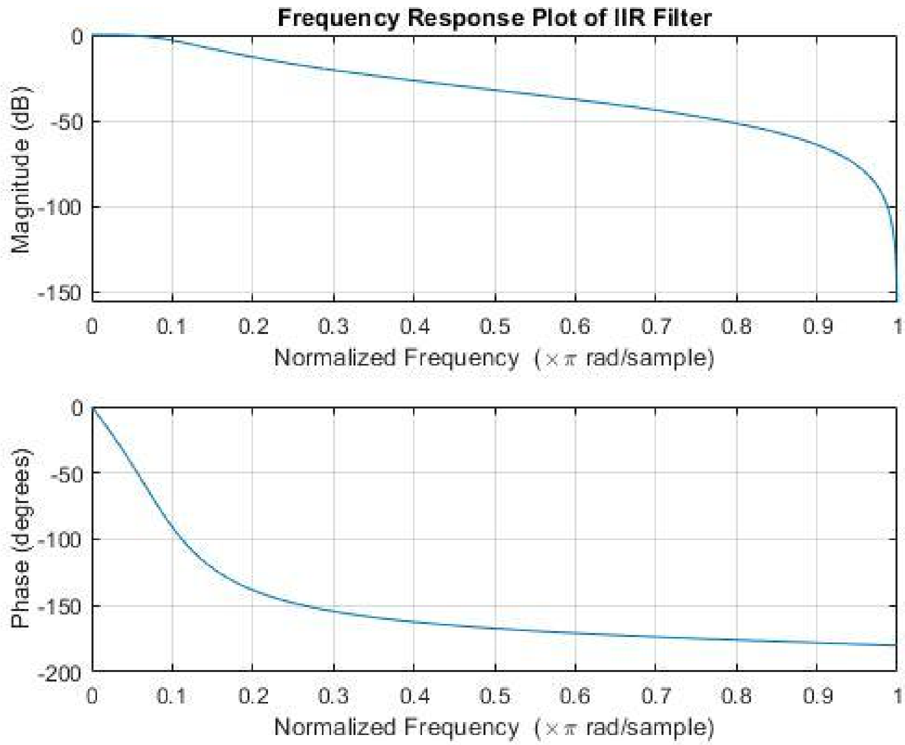


Fig 2: Frequency Response Plot of IIR Filter

B. Impulse Response

Impulse response plot show unsymmetric behavior about the largest amplitude point in the response curve.

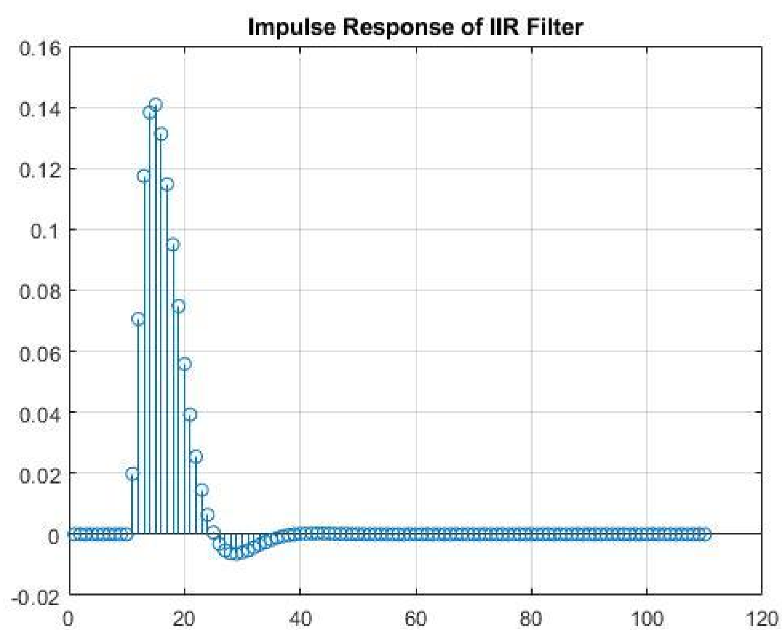


Fig 3: Impulse Response Plot of IIR Filter

III. **FIR Filter Design:**

A. Using Rectangular Window:

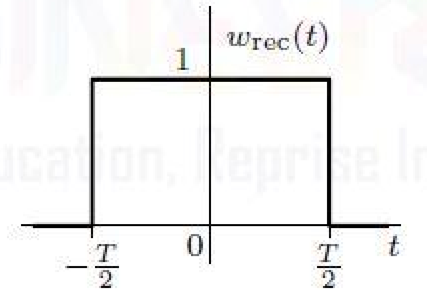


Fig 4: Rectangular window

Rectangular windows have no attenuation of the input signal but sharp cutoff edges. Also, there is a seeping effect as the cutoff is not absolute.

B. Using Hamming Window:

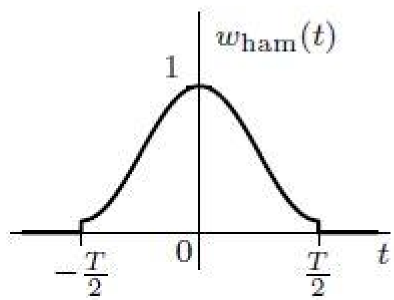


Fig 4: Hamming window

Hamming Window starts with no attenuation but there is a sharp attenuation after some steps. The seeping effect is not seen with this window.

IV. **FIR Filter Evaluation:**

A. Using Rectangular Window:

1. Frequency Response & Pole-Zero Plot

Magnitude plot shows the main lobe decaying around 160 Hz. The pointy decaying lobe points correspond to the zeroes in the pole-zero plots. Also, all the poles lie on the origin on the pole-zero plot which is the characteristic of FIR filters.

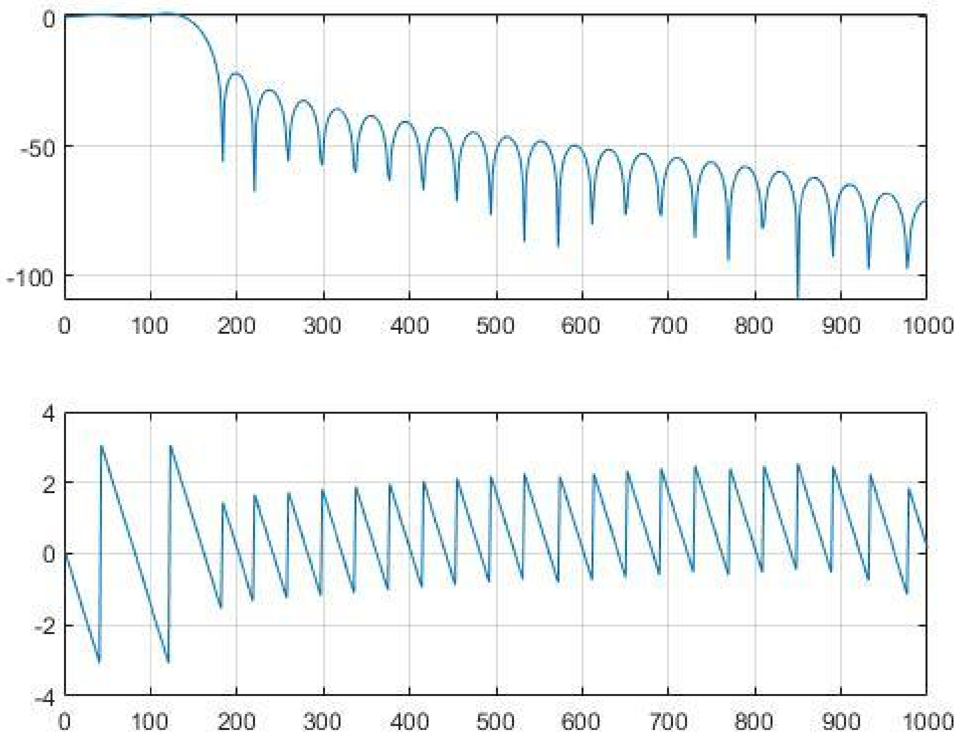


Fig 5: Frequency Response of FIR filter using Rectangular window

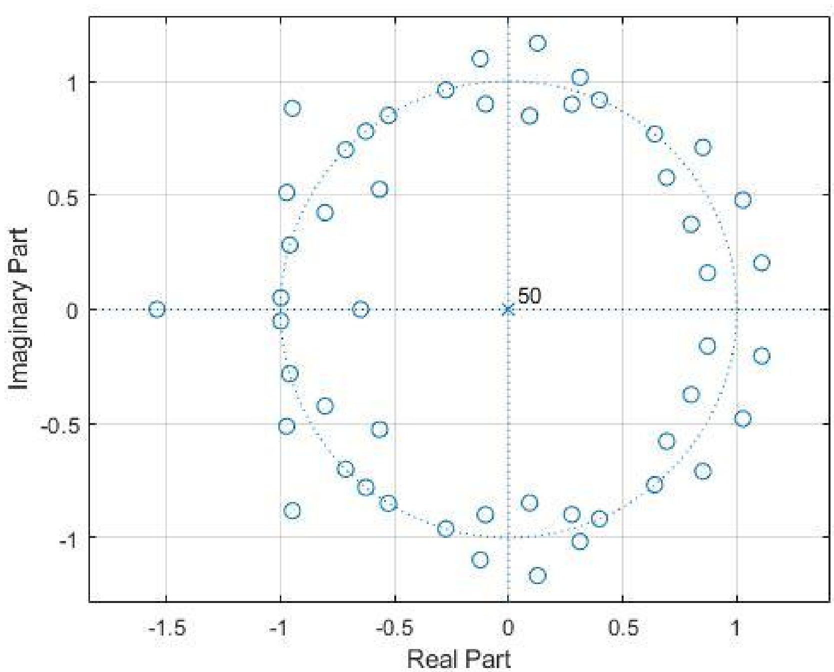


Fig 6: Pole Zero plot of the FIR Filter using Rectangular window

2. **Impulse Response**

A symmetric plot of the impulse response was obtained.

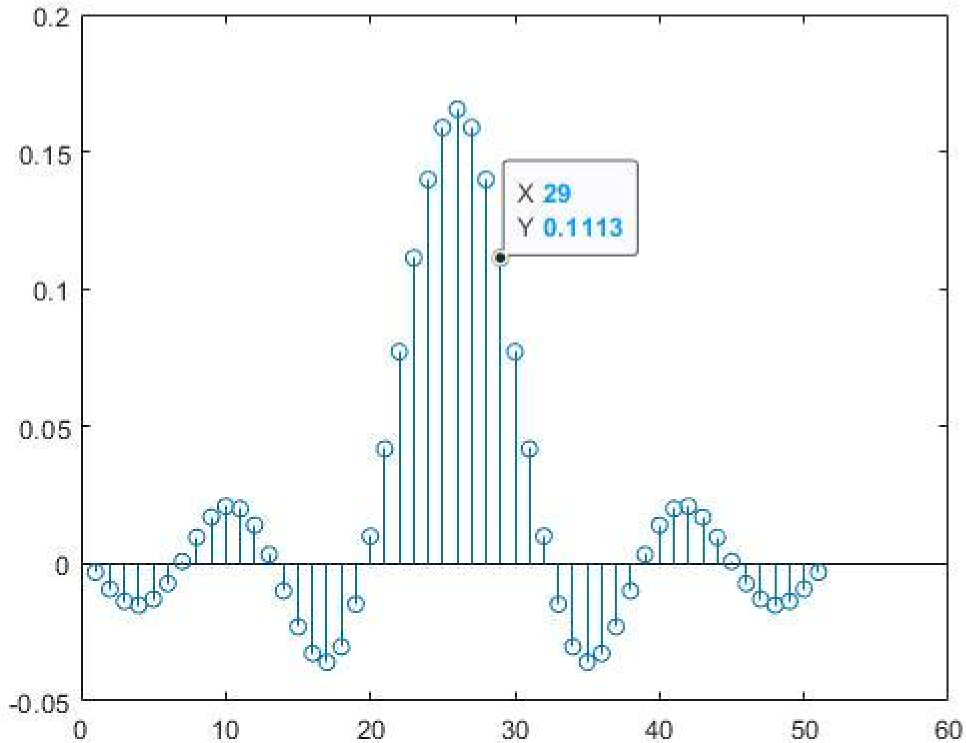


Fig 7: Impulse Response of FIR filter designed using Rectangular Window

3. **Coefficients of Filter**

Following coefficients were obtained for the FIR filter designed using rectangular window:

-0.0030 -0.0092 -0.0137 -0.0151 -0.0129 -0.0072 0.0009 0.0096 0.0170 0.0209

B. **Using Hamming Window**

1. **Frequency Response & Pole-Zero Plot**

The main lobe in the magnitude plot showed a shower change in the slope than that using the Rectangular window. This is because the hamming window has a slower attenuation than the rectangular window. Also, the cutoff frequency is a little bit off because of the frequency prewarping error in the code. However, it is really close.

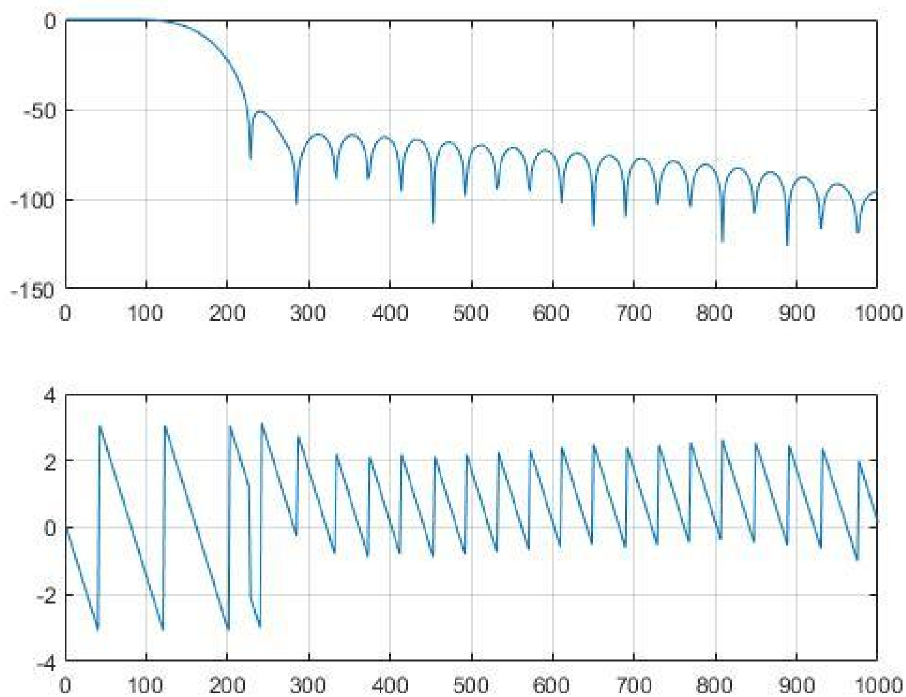


Fig 8: Frequency response of FIR filter designed using Hamming Window

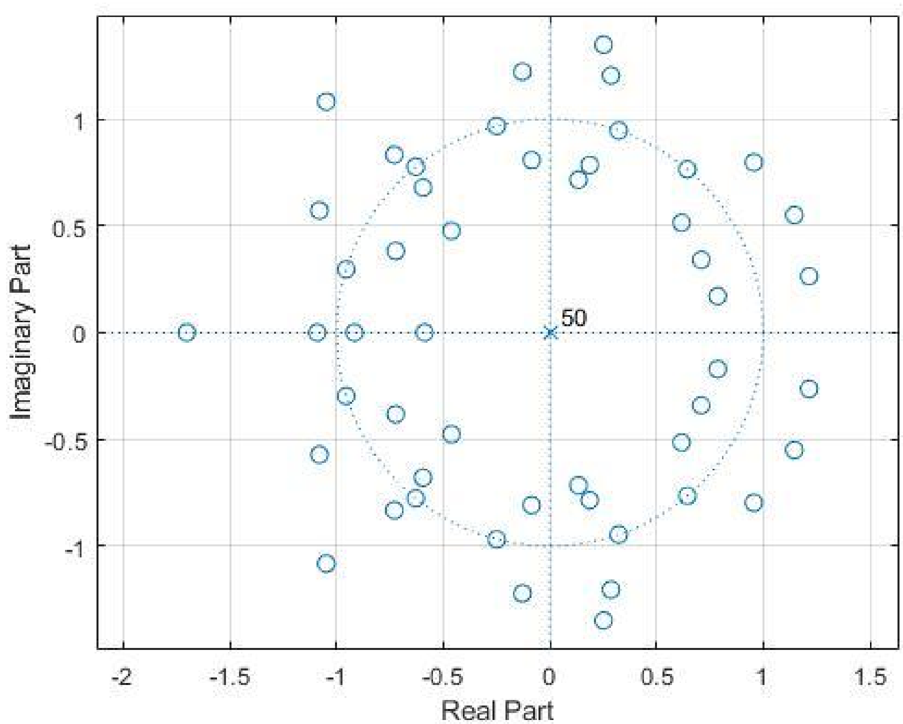


Fig 9: Pole-zero plot of FIR filter designed using Hamming Window

2. Impulse Response

A symmetric impulse response was obtained. The main lobes are already attenuated in the impulse response for the hamming windowed FIR filters as compared to the rectangular windowed FIR filters. This is because of the difference in the nature of the rectangular and hamming window. Also, the magnitudes of the secondary ripples were found smaller than that in the rectangular window. This was attributed to the minimal seeping from main lobe to the side lobe during the windowing.

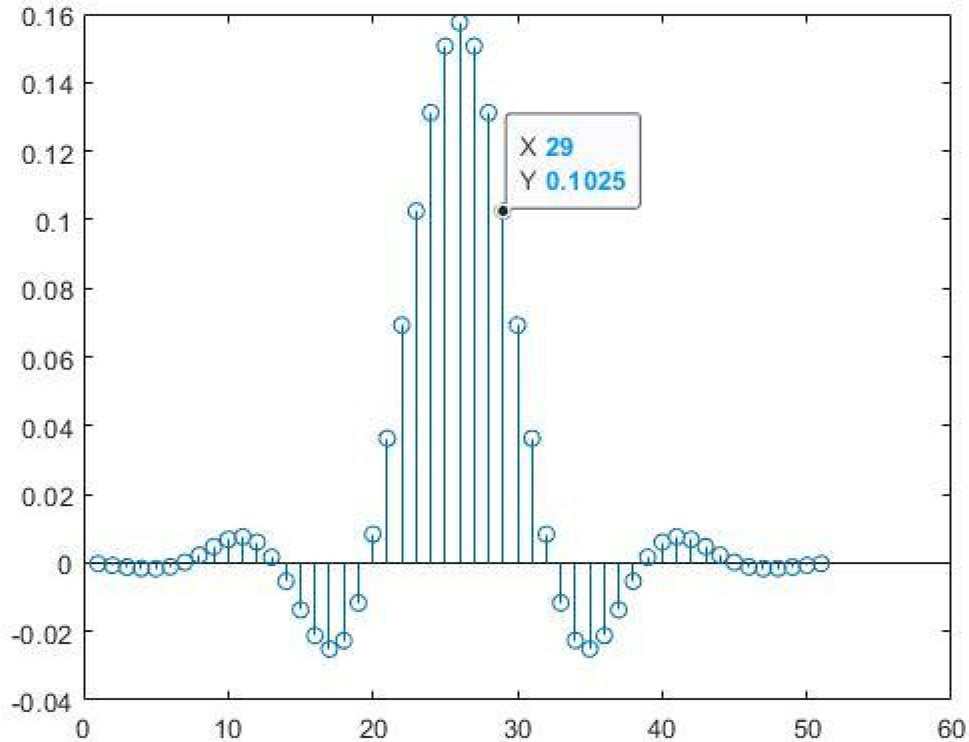


Fig 10: Impulse Response of FIR filter designed using Hamming Window

V. Comparison of FIR and IIR Filters:

A. Digital filter response

Input Signal Response Plots showed that the IIR Filters have lower delay as compared to 50 count FIR filters. However, 50 count FIR filters could provide cleaner signal than the IIR Filters. Also, during the experiment(plot not provided) it was found that increasing the count provided a sharper attenuation but increased the delay of the FIR Filter.

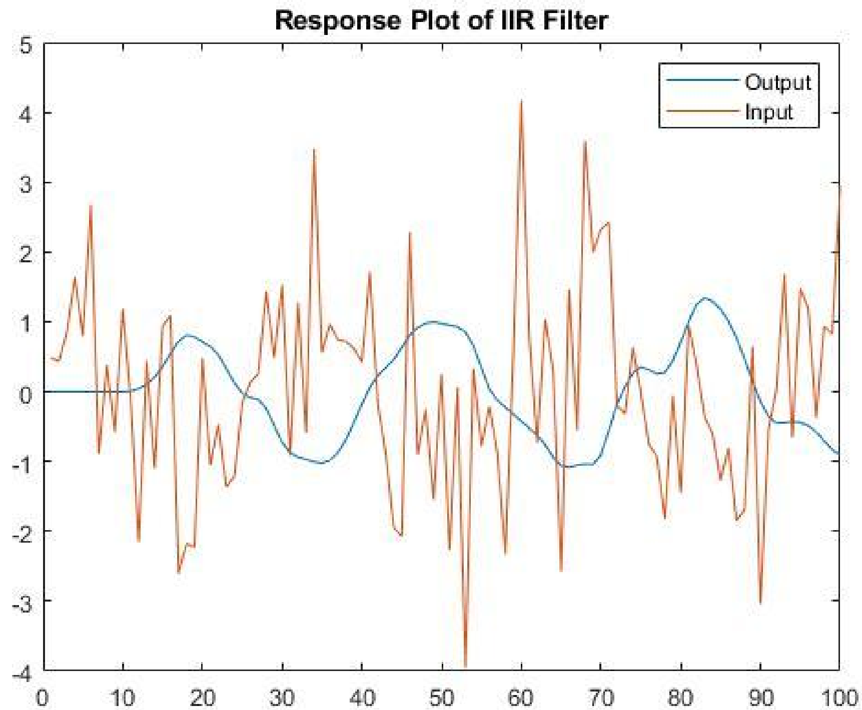


Fig 11: Response of IIR Filter to Input Signal

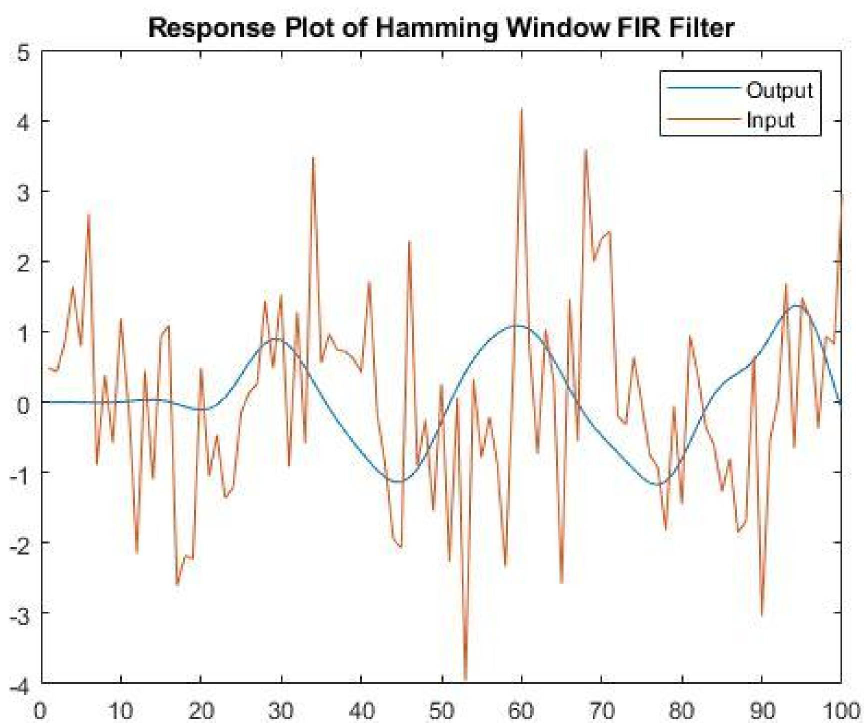


Fig 12: Response of Hamming Window FIR Filter to Input Signal

B. Spectra Evaluation of Input signal and Response Signal of IIR & FIR

The spectrum plots for the IIR and FIR filters showed that all three of the filter designs were able to properly attenuate the signal. Also, the Hamming window FIR Filter showed sharper attenuation than the rectangular window FIR filter.

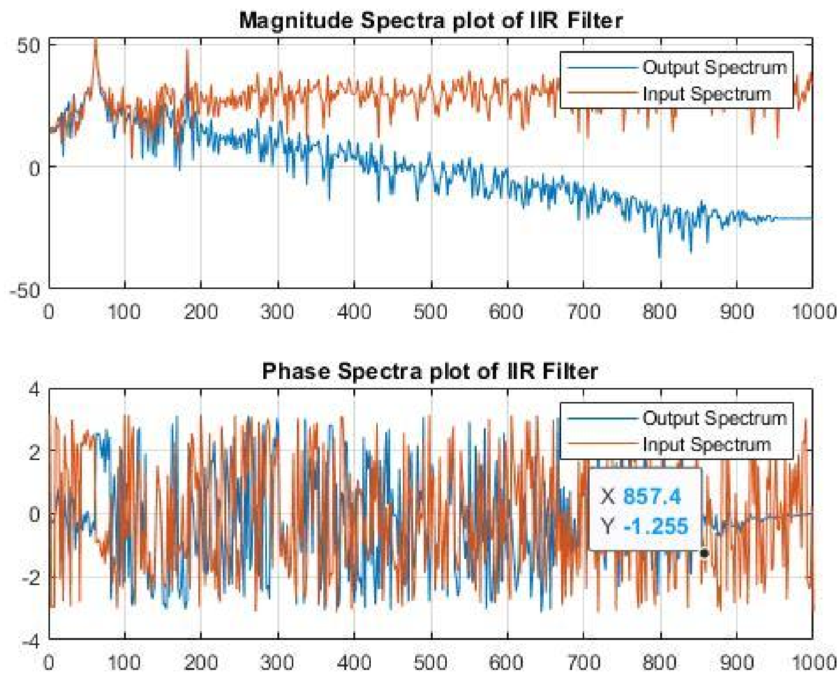


Fig 12: Spectrum Plot of Input Signal and IIR Filter Output Signal

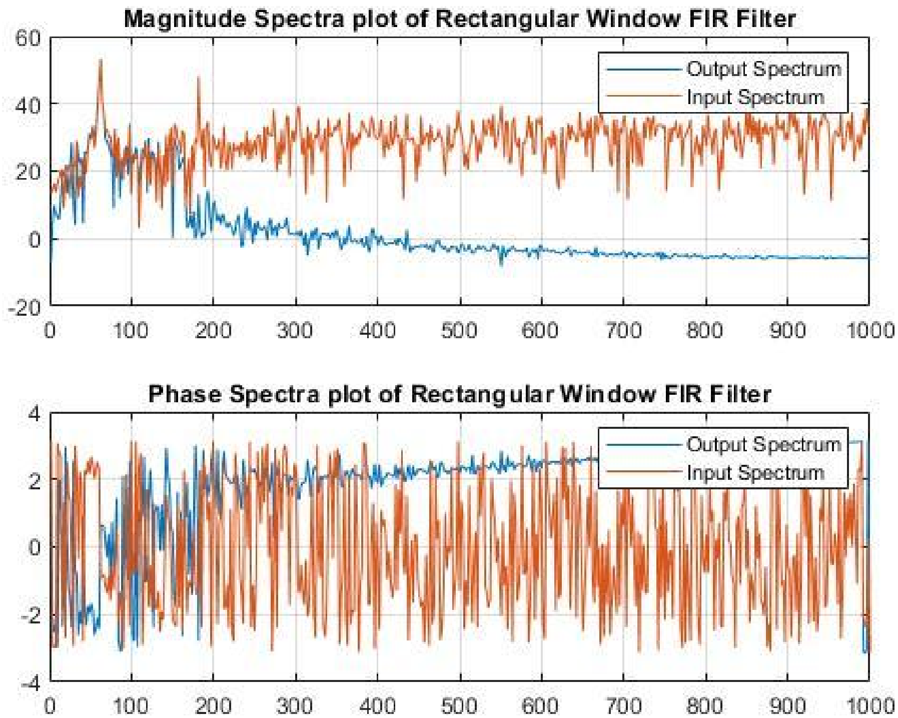


Fig 13: Spectrum Plot of Input Signal and Rectangular Window FIR Filter Output Signal

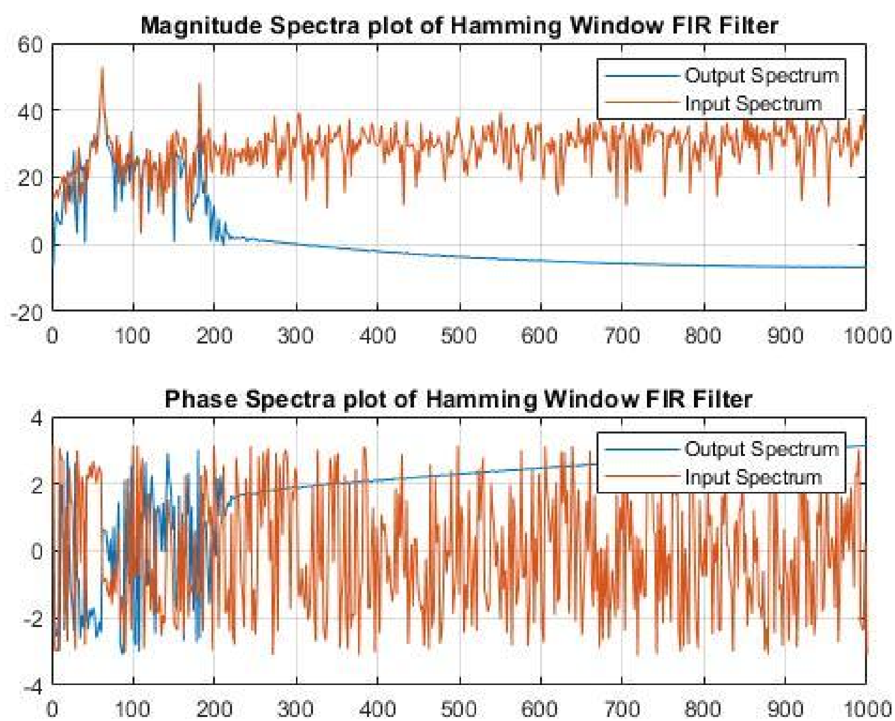


Fig 14: Spectrum Plot of Input Signal and Hamming Window FIR Filter Output Signal

Appendix 1: Hand Calculation for the Bilinear Transform

$$H_{LPF}(s) = \frac{394784.2}{s^2 + 888.7566\, s + 394784.2}$$

$$H(z) = H_{LPF}(s)\Big|_{\frac{z-1}{z+1}} = \frac{394784.2}{\left(\frac{z-1}{z+1}\right)^2 + 888.7566\left(\frac{z-1}{z+1}\right) + 394784.2}$$

$$= \frac{394784.2\,(z+1)^2}{(z-1)^2 + 888.7566(z-1)(z+1) + 394784.2(z+1)^2}$$

$$= \frac{394784.2\,(z+1)^2}{z^2 - 2z + 1 + 888.7566\,z^2 - 888.7566 + 394784.2(z^2+2z+1)}$$

$$= \frac{394784.2\,(z+1)^2}{395673.9566\,z^2 + 789566.4\,z + 393896.4434}$$

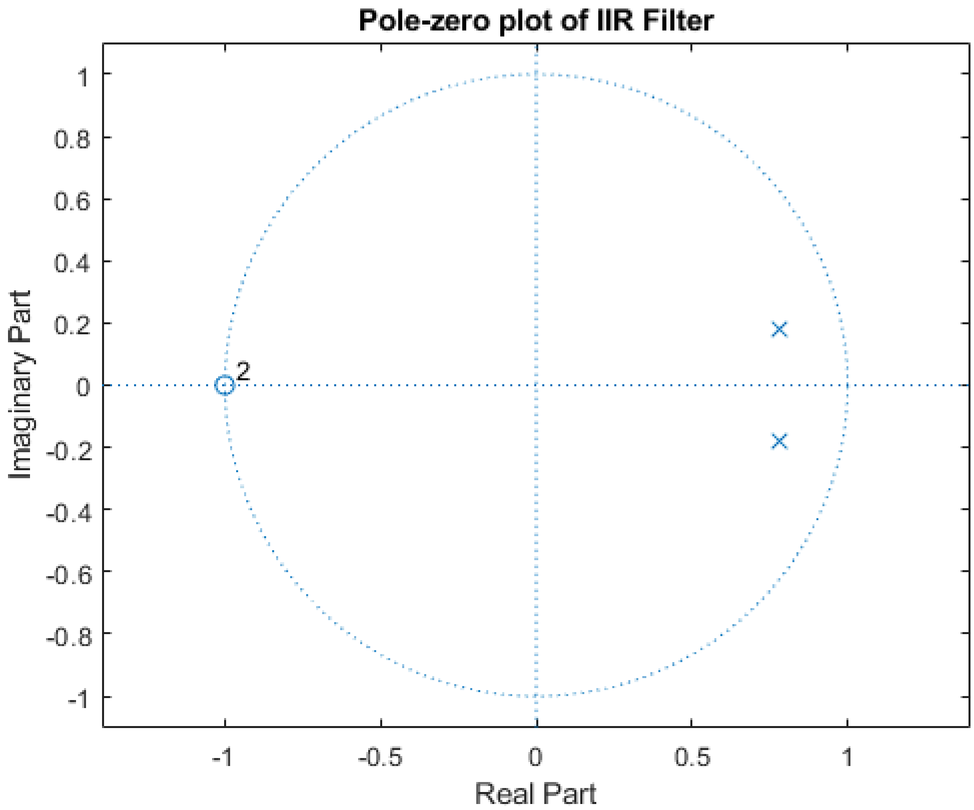
$$= \frac{0.9978(z+1)^2}{z^2 + 1.995z + 0.9955}$$

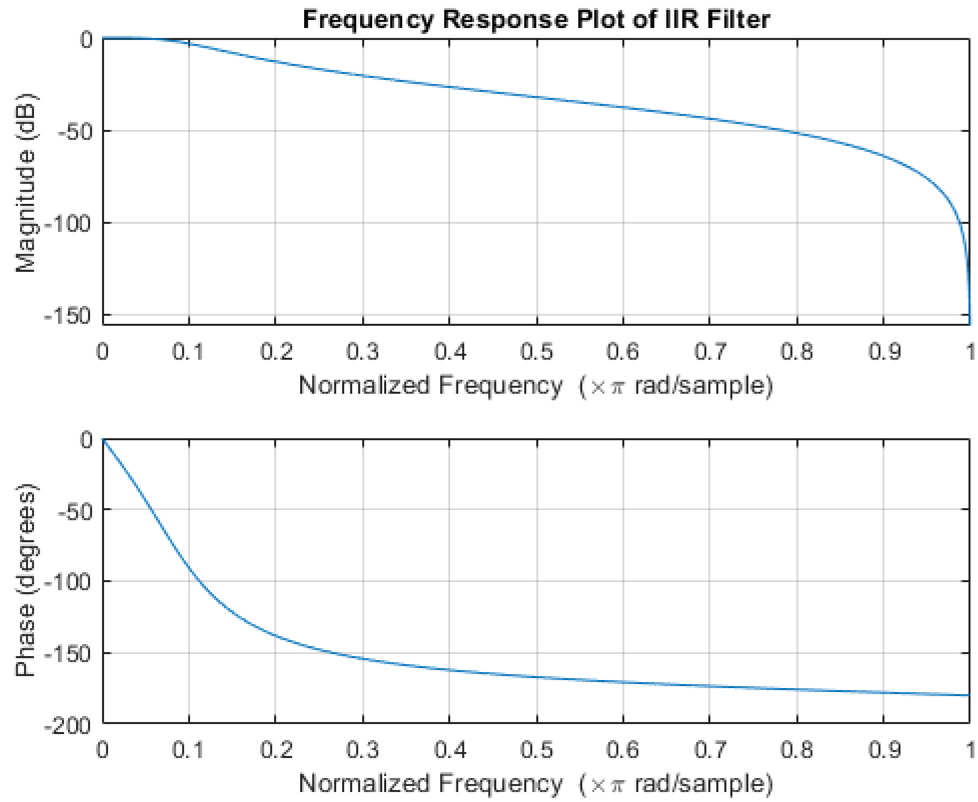
$$\boxed{\therefore H(z) = \frac{0.9978\,z^2 + 1.9956z + 0.9978}{z^2 + 1.995z + 0.9955}}$$

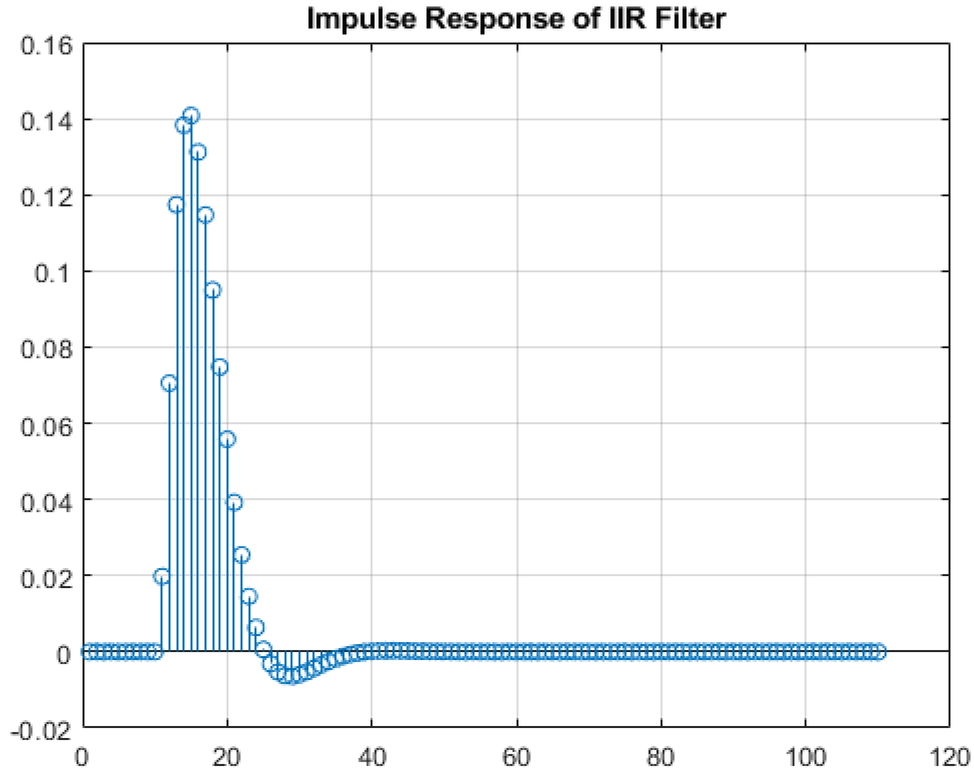
```
%Shishir Khanal
%Final Project
%EE5575
% Design of Low Pass IIR Filter
%-------------------------------------
%Part-1: IIR Filter
clc;clear; close all;
frequency_pass = 100;
frequency_stop = 1000;
alpha_pass = 1;
alpha_stop = 30;

num =[394784.2];
den = [1 888.5766 394784.2];
h_lpf = tf(num,den)
%1
frequency_sampling = 2*frequency_stop;
%3
[num_z,den_z] = bilinear(num,den,frequency_sampling);
H_z = tf(num_z,den_z,frequency_sampling)
%4
figure(1)
zplane(num_z,den_z)
title('Pole-zero plot of IIR Filter')
%5
figure(2)
freqz(num_z,den_z,frequency_sampling)
[w,p] = freqz(num_z,den_z,frequency_sampling);
title('Frequency Response Plot of IIR Filter');
%6
figure(3)
imp = [zeros(10,1);1;zeros(99,1)];
response_impulse = filter(num_z,den_z,imp);
stem(response_impulse);(grid);
title('Impulse Response of IIR Filter')
%----------------------------------------------------------------

%----------------------------------------------------------------
%Part 3: Apply Filters
%1.
load('proj_signal.mat');
y = conv(x,response_impulse);
figure(4)
plot(y(1:100))
hold on
plot(x(1:100))
title('Response Plot of IIR Filter');legend('Output','Input');
hold off
%2.
figure(5)
m = [1:1024];
f = m.*frequency_sampling/length(m);
```

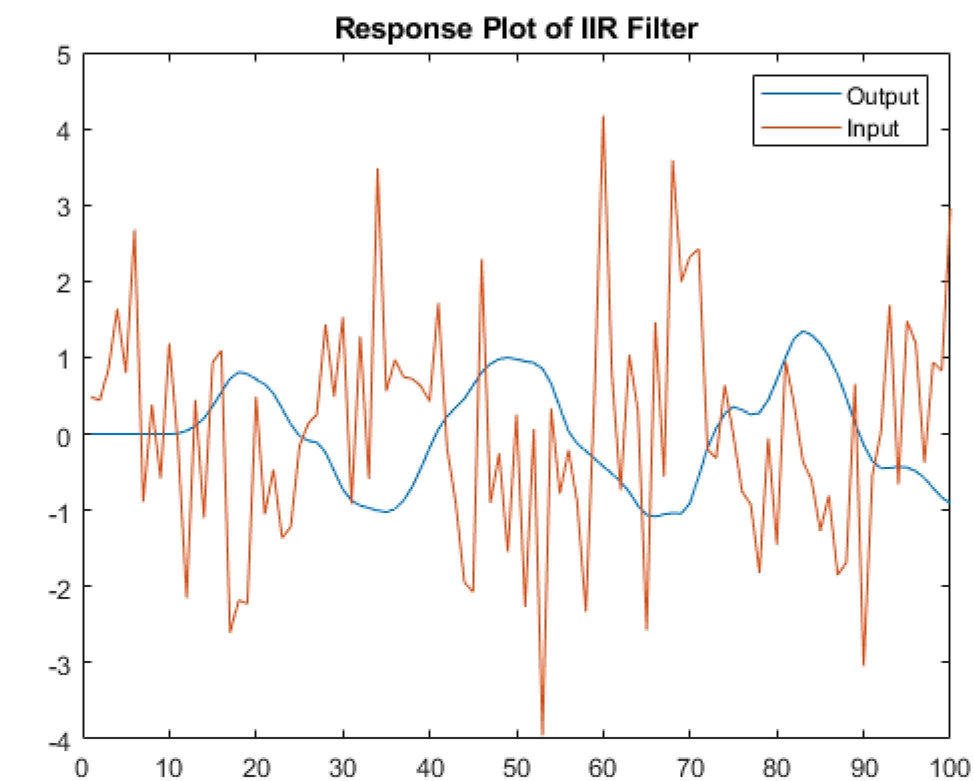
```
fft_x = fft(x,1024);
fft_y = fft(y,1024);
half_f = f(1:length(f)/2);
subplot(2,1,1);
plot(half_f,20*log10(abs(fft_y(1:length(f)/2))));
hold on
plot(half_f,20*log10(abs(fft_x(1:length(f)/2))));
grid;hold off;
title('Magnitude Spectra plot of IIR Filter')

subplot(2,1,2);
plot(half_f,angle(fft_y(1:length(f)/2)));
hold on
plot(half_f,angle(fft_x(1:length(f)/2)));
grid; hold off;
title('Phase Spectra plot of IIR Filter')

%---------------------------------------------------------------

%---------------------------------------------------------------
```

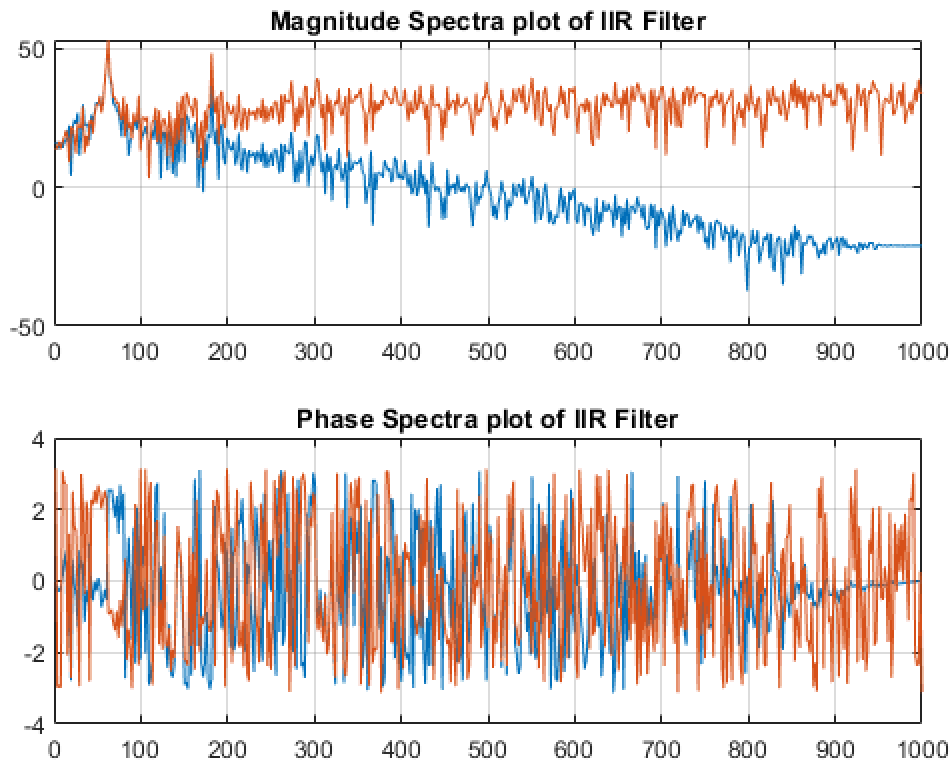
```
h_lpf =

          3.948e05
  ------------------------
  s^2 + 888.6 s + 3.948e05

Continuous-time transfer function.


H_z =

  0.01979 z^2 + 0.03958 z + 0.01979
  ---------------------------------
       z^2 - 1.565 z + 0.6437

Sample time: 2000 seconds
Discrete-time transfer function.
```


Pole-zero plot of IIR Filter
Imaginary Part
Real Part
2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1


Frequency Response Plot of IIR Filter
Magnitude (dB)
0
-50
-100
-150
Normalized Frequency (×π rad/sample)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Phase (degrees)
0
-50
-100
-150
-200
Normalized Frequency (×π rad/sample)


Impulse Response of IIR Filter
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
20
40
60
80
100
120


Response Plot of IIR Filter
Output
Input
5
4
3
2
1
0
-1
-2
-3
-4
0
10
20
30
40
50
60
70
80
90
100



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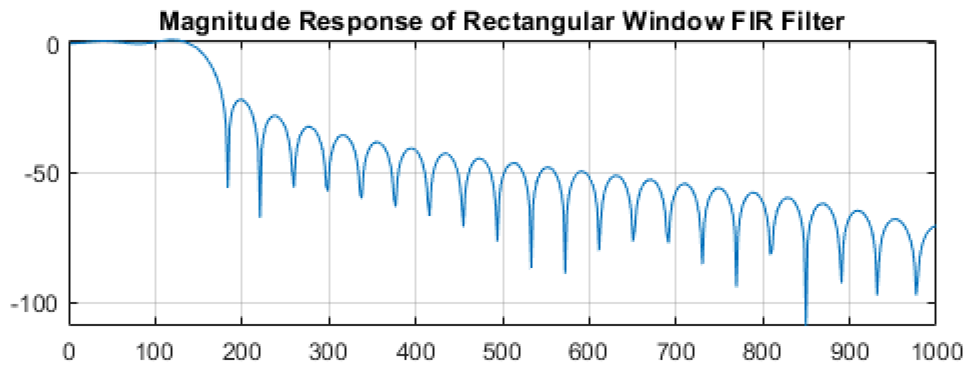
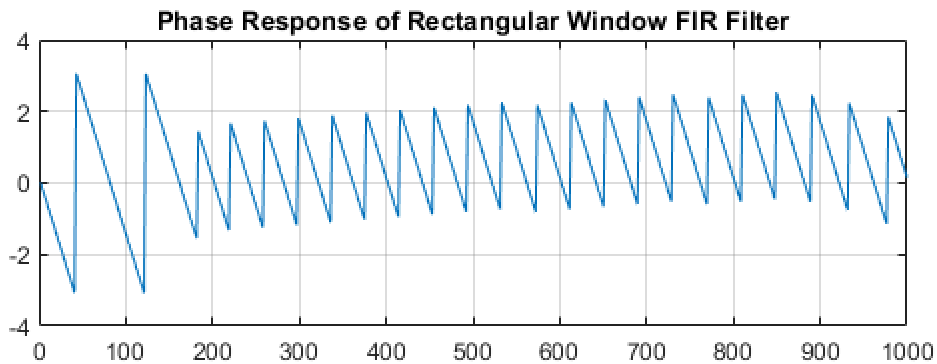
```
%Shishir Khanal
%Final Project
%EE5575
% Design of Low Pass FIR Filter
%-------------------------------------

clc;clear; close all;
frequency_pass = 100;
frequency_stop = 1000;
alpha_pass = 1;
alpha_stop = 30;
frequency_cut = 100;
wc = frequency_cut*2*pi;
frequency_sampling = 2*frequency_stop;
ws = frequency_sampling*2*pi;
%---------------------------------------
%1
n = 50;
Wn = (wc/ws)*pi;
H_n_rect = fir1(n,Wn,'low',rectwin(n+1));
%H_n_rect = fir1(50,0.2513*.5,'low',rectwin(n+1));
%a.
figure(1)
m = [1:1024];
f = m.*frequency_sampling/length(m);
H_f_rec = fft(H_n_rect,1024);
half_f = f(1:length(f)/2);
subplot(2,1,1);
plot(half_f,20*log10(abs(H_f_rec(1:length(f)/2))));
title('Magnitude Response of Rectangular Window FIR Filter');
grid;
subplot(2,1,2);
plot(half_f,angle(H_f_rec(1:length(f)/2)));
grid;
title('Phase Response of Rectangular Window FIR Filter');

%b.
figure(2)
zplane(abs(H_n_rect));grid;
title('Pole-Zero Plot of Rectangular Window FIR Filter');
%c
figure(3)
stem(H_n_rect)
title('Impulse Response of Rectangular Window FIR Filter');

%d.
filter_coeff_Rect = H_n_rect(1:10)
%-----------------------------------------

%-----------------------------------------
%2.
H_n_Hamming = fir1(n,Wn,'low');
```

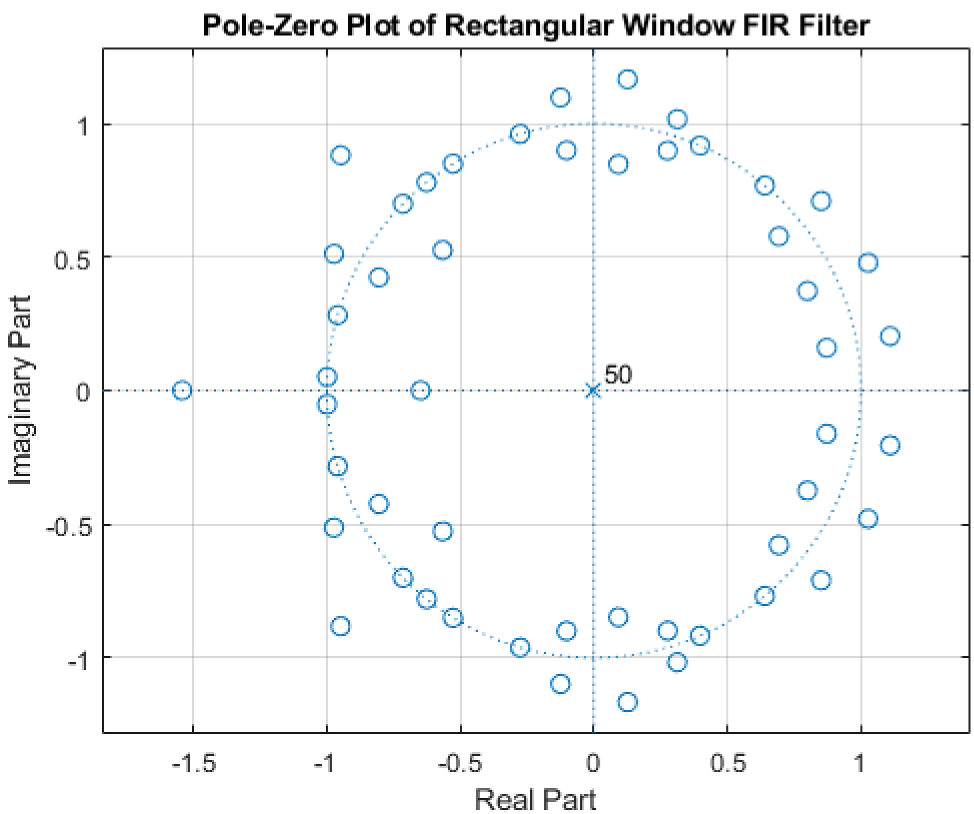
```
%a.
figure(4)
H_f_ham = fft(H_n_Hamming,1024);
half_f = f(1:length(f)/2);
subplot(2,1,1);
plot(half_f,20*log10(abs(H_f_ham(1:length(f)/2))));
title('Magnitude Response of Hamming Window FIR Filter');

grid;
subplot(2,1,2);
plot(half_f,angle(H_f_ham(1:length(f)/2)));
title('Phase Response of Hamming Window FIR Filter');

grid;
%b.
figure(5)
zplane(abs(H_n_Hamming));grid;
title('Pole-Zero Plot of Hamming Window FIR Filter');

%c
figure(6)
stem(H_n_Hamming)
title('Impulse Response of Hamming Window FIR Filter');
%-----------------------------------------

%-----------------------------------------------------------------
%Part 3: Apply Filters
%1.
load('proj_signal.mat');
y = conv(x,H_n_Hamming);
figure(7)
plot(y(1:100))
hold on
plot(x(1:100))
title('Response Plot of Hamming Window FIR
 Filter');legend('Output','Input');
hold off

%2.
figure(8)
m = 1:1024;
f = m.*frequency_sampling/length(m);
fft_x = fft(x,1024);
fft_y = fft(y,1024);
half_f = f(1:length(f)/2);
subplot(2,1,1);
plot(half_f,20*log10(abs(fft_y(1:length(f)/2))));
hold on
plot(half_f,20*log10(abs(fft_x(1:length(f)/2))));
grid;hold off;legend('Output Spectrum','Input Spectrum')
title('Magnitude Spectra plot of Hamming Window FIR Filter')

subplot(2,1,2);
plot(half_f,angle(fft_y(1:length(f)/2)));
```

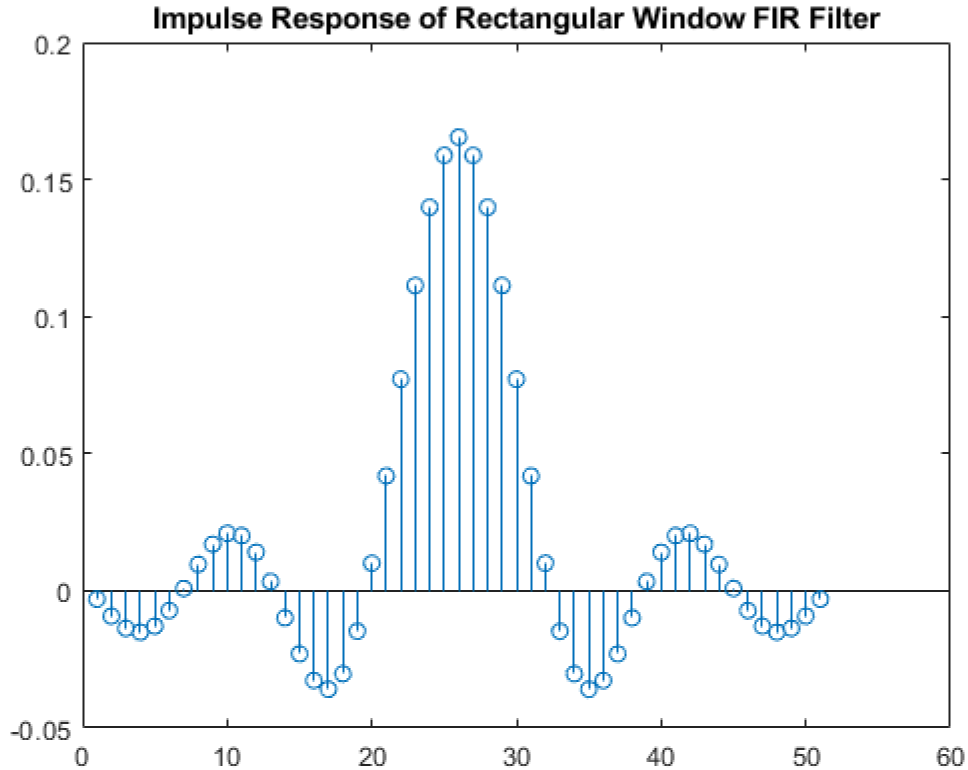
```
hold on
plot(half_f,angle(fft_x(1:length(f)/2)));
grid; hold off;legend('Output Spectrum','Input Spectrum');
title('Phase Spectra plot of Hamming Window FIR Filter')

figure(9)
y_rec = conv(x,H_n_rect);
fft_yrec = fft(y_rec,1024);
subplot(2,1,1);
plot(half_f,20*log10(abs(fft_yrec(1:length(f)/2))));
hold on
plot(half_f,20*log10(abs(fft_x(1:length(f)/2))));
grid;hold off;legend('Output Spectrum','Input Spectrum')
title('Magnitude Spectra plot of Rectangular Window FIR Filter')

subplot(2,1,2);
plot(half_f,angle(fft_yrec(1:length(f)/2)));
hold on
plot(half_f,angle(fft_x(1:length(f)/2)));
grid; hold off;legend('Output Spectrum','Input Spectrum');
title('Phase Spectra plot of Rectangular Window FIR Filter')
%------------------------------------------------------------

%------------------------------------------------------------
```

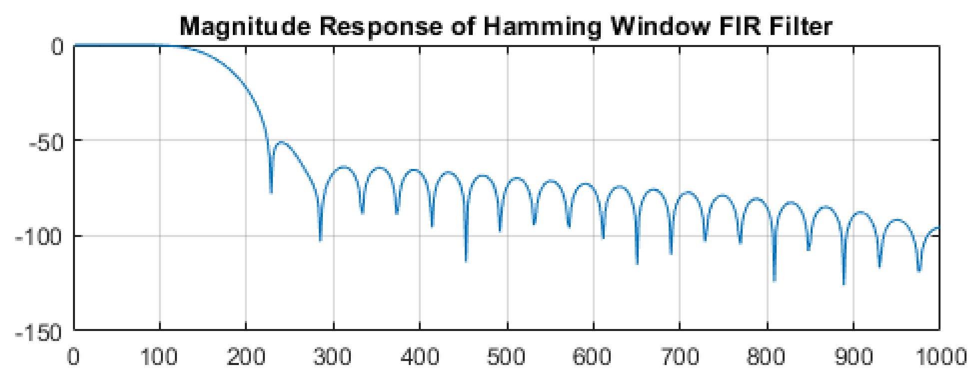
```
filter_coeff_Rect =

  Columns 1 through 7

   -0.0030   -0.0092   -0.0137   -0.0151   -0.0129   -0.0072    0.0009

  Columns 8 through 10

    0.0096    0.0170    0.0209
```

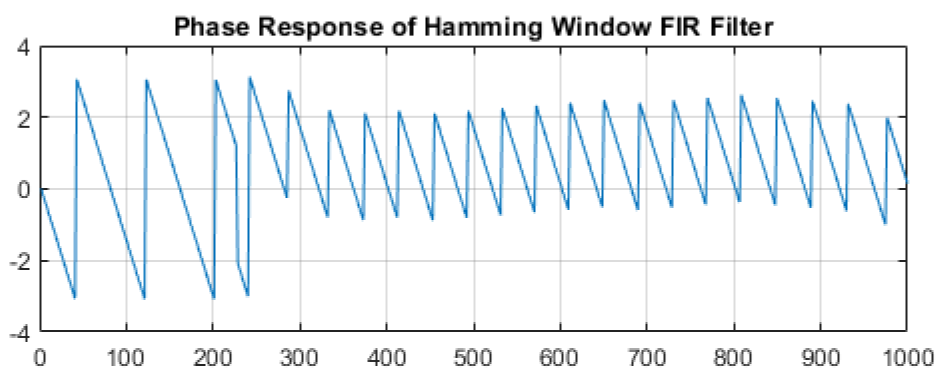

Magnitude Response of Rectangular Window FIR Filter
0
-50
-100
0
100
200
300
400
500
600
700
800
900
1000

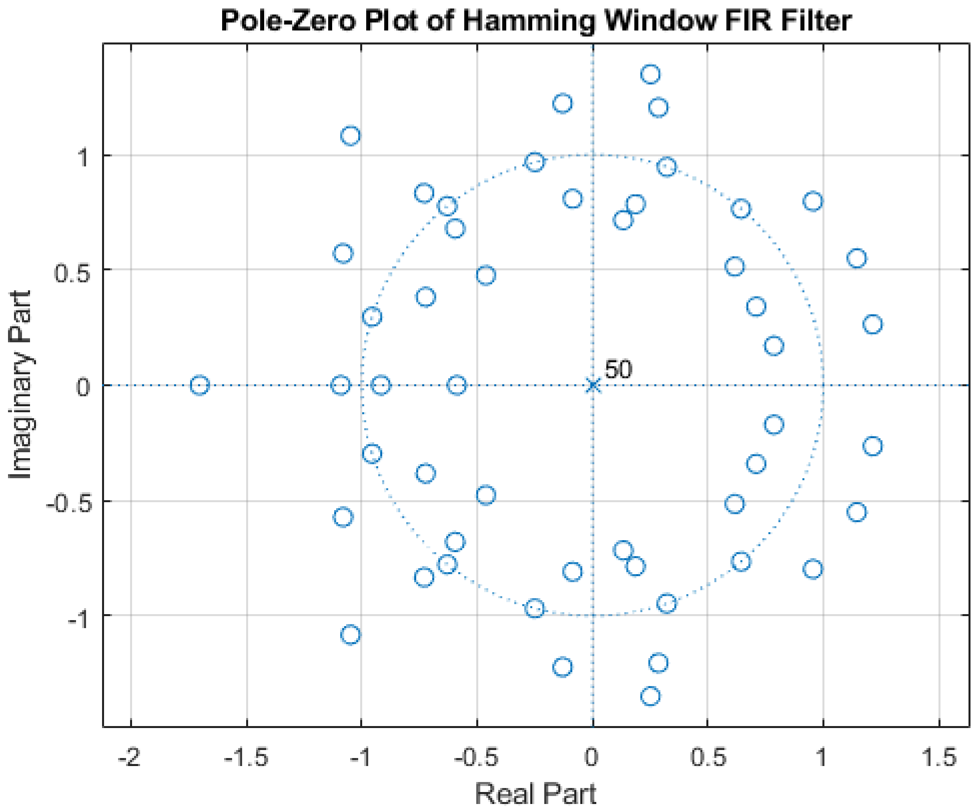

Phase Response of Rectangular Window FIR Filter
4
2
0
-2
-4
0
100
200
300
400
500
600
700
800
900
1000

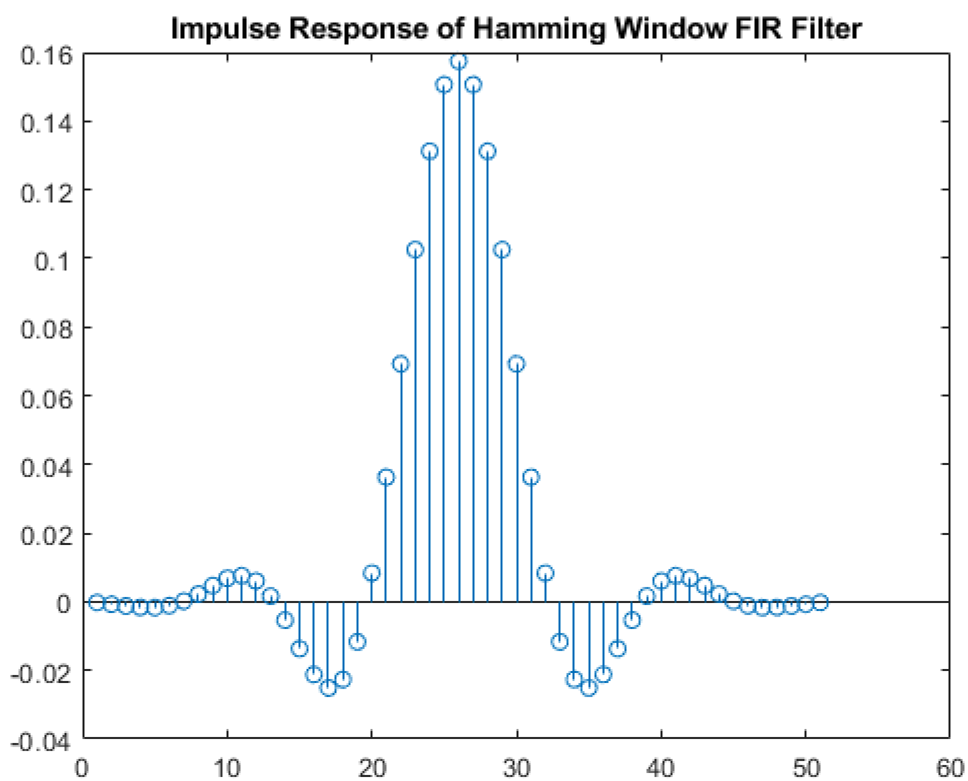

Pole-Zero Plot of Rectangular Window FIR Filter
Imaginary Part
Real Part
50
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1

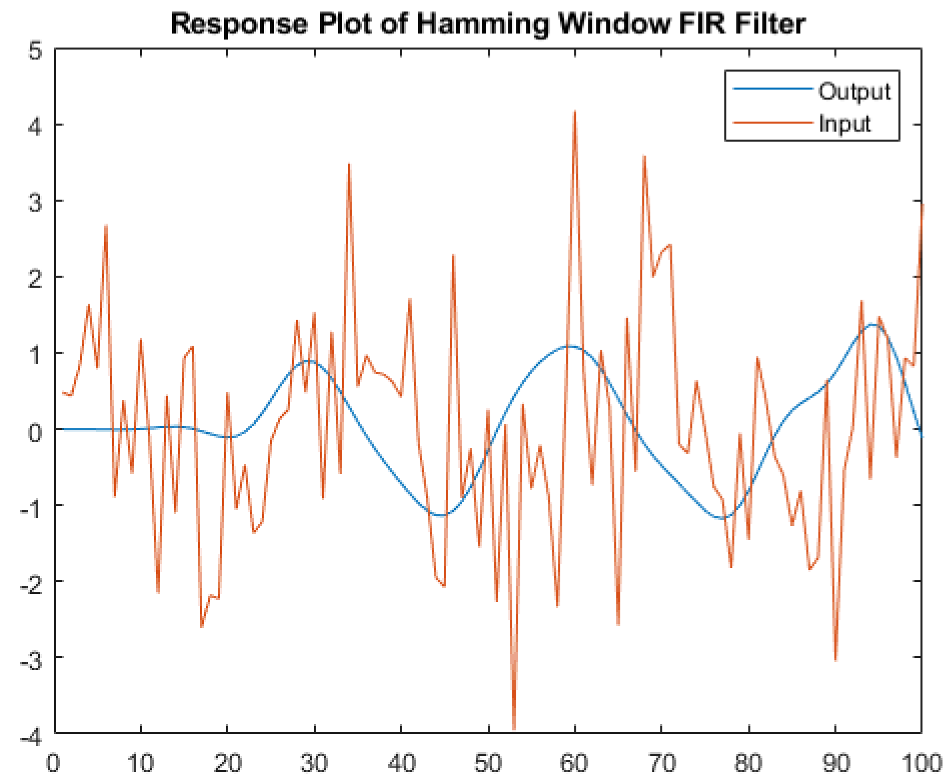

Impulse Response of Rectangular Window FIR Filter

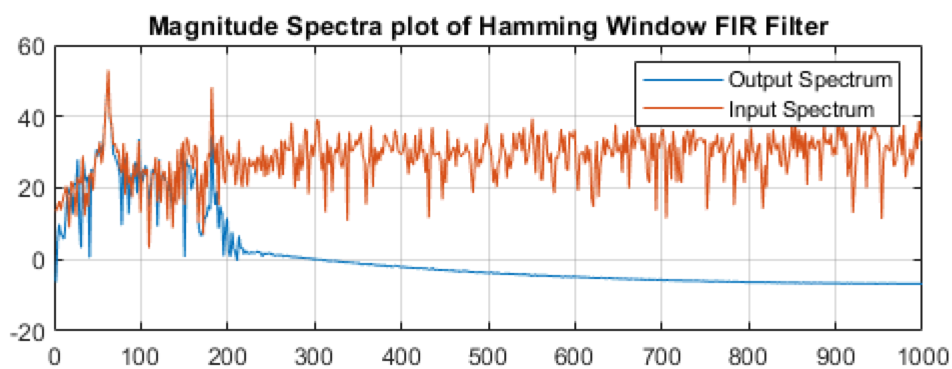

Magnitude Response of Hamming Window FIR Filter

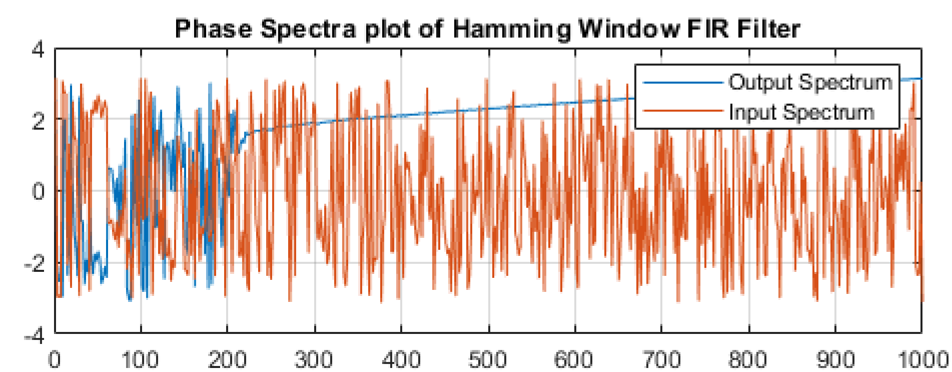

Phase Response of Hamming Window FIR Filter

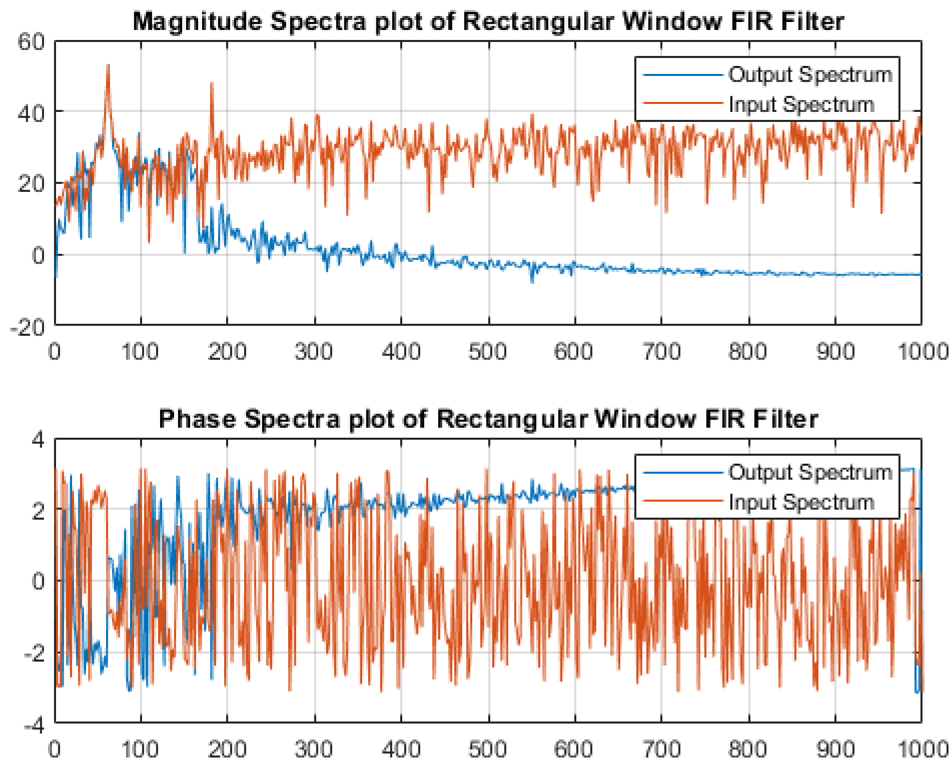

Pole-Zero Plot of Hamming Window FIR Filter
Imaginary Part
Real Part
50
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5


Impulse Response of Hamming Window FIR Filter
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0
10
20
30
40
50
60


Response Plot of Hamming Window FIR Filter
Output
Input
5
4
3
2
1
0
-1
-2
-3
-4
0
10
20
30
40
50
60
70
80
90
100


Magnitude Spectra plot of Hamming Window FIR Filter
Output Spectrum
Input Spectrum
60
40
20
0
-20
0
100
200
300
400
500
600
700
800
900
1000


Phase Spectra plot of Hamming Window FIR Filter
Output Spectrum
Input Spectrum
4
2
0
-2
-4
0
100
200
300
400
500
600
700
800
900
1000



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