Project Technical Memo:
1 DOF Linear Mass-Spring System Design w/ 2 DOF Vibration Absorber

ME 4440-01: Vibrational Analysis
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Submitted: 12/7/2018

Executive Summary:

Our responsibility for the project is to design one DOF and two DOF. Develop a mechanical vibration test. A vibration absorber should be designed with a mass and two springs in the system. A result for one DOF mass and spring system. Mass = 0.67kg and for spring constant- 101 n/m and 58n/m respectively. Results for two DOF add pen spring= 368 N/m. In addition for adding a mass and spring will switch the physical system from vibration absorbing. The amplitude for one DOF the minimum value was = 300RPM. For calibrating the variable speed controller the degree that we choose (45, 30, 0, and -30 degrees). We used the iPhone app to measure motor speed. Overall, we changed the mass value but the system was working successfully.

Shereen Aljumah

Table of Contents: (Jose Nunez)

1.0 Title Page . . . . .Cover

2.0 Executive Summary . . . . .Page 1

3.0 Table of Contents . . . . .Page 2-3

4.0 Problem Definition . . . . .Page 4-7

4.1 Problem Scope . . . . .Page 5

4.2 Technical Review . . . . Page 5-6

4.3 Design Requirements . . . . .Page 6-7

5.0 Design Description . . . . Page 6-9

5.1 Overview. . . . . .Page 6

5.2 1 DOF . . . . Page 6-7

5.2.1 Computations . . . . .Page 7

5.3 2 DOF . . . . Page 8

5.3.1 Computations. . . . . .Page 9

6.0 Evaluation . . . . .Page 10-18

6.1 Overview . . . . Page 10

6.2 Prototyping. . . . . Page 11

6.3 Testing . . . . Page 12-15

6.4 Assessment . . . . .Page 15-16

6.5 Action After Evaluation. . . . . .Page 16-17

7.0 References . . . . .Page 18

8.0 Appendices. . . . . .Page 19-20

8.1 Appendix A: Computations . . . . Page 21-25

8.2 Appendix B: Matlab Code. . . . . .Page 21-25

8.3 Appendix C: Plots . . . . .Page 25-29

List of Tables:

List of Figures:

Problem Definition

Problem Scope:

The main goal of this project is to design a mechanical vibration absorber and investigate one DOF vibration system by using rubber bands. In addition, we will compare the response of the constructed system with the simulation results. We are going to use the Matlab program to compute the equation of motion in parametric form and the plot as well. The purpose of doing this part of the project is to build up a mass-spring system (original sliding mass). The force will be created by the DC motor. The equation that we used to determine the displacement is written in the engineering paper. Two springs in parallel are used for the whole system each with the same spring constant. We also used frictional damping.

Equation of motion:

$$Keq = K1 + K2$$

$$MTotal = Msystem + Meccentricity$$

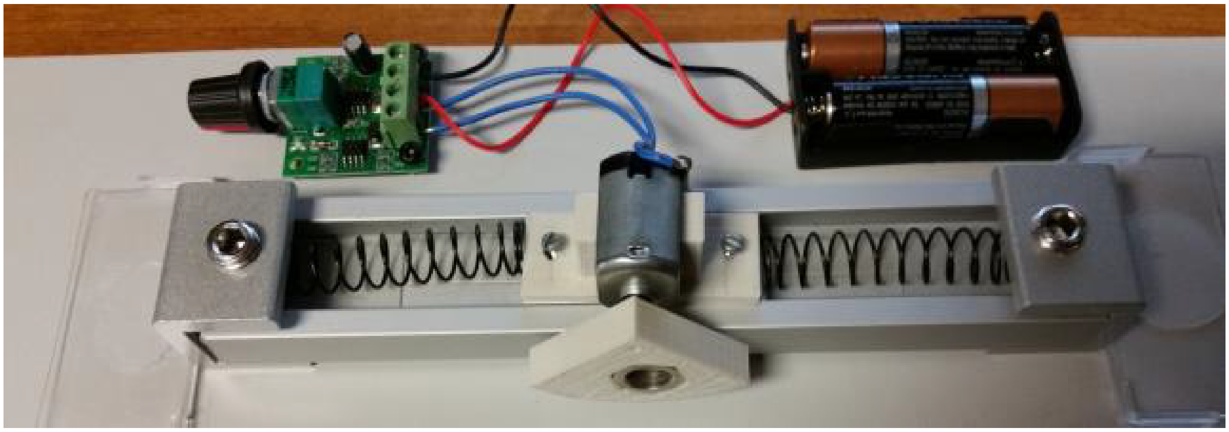
$$Equation\ of\ Motion\ for\ frictional\ dampening:$$

$$M\ddot{x} + Keq \pm \mu N = Fo\sin(wft) \quad \text{(Rao, 6th Edition)}$$

By using an equation of motion we can determine the forces of the system.

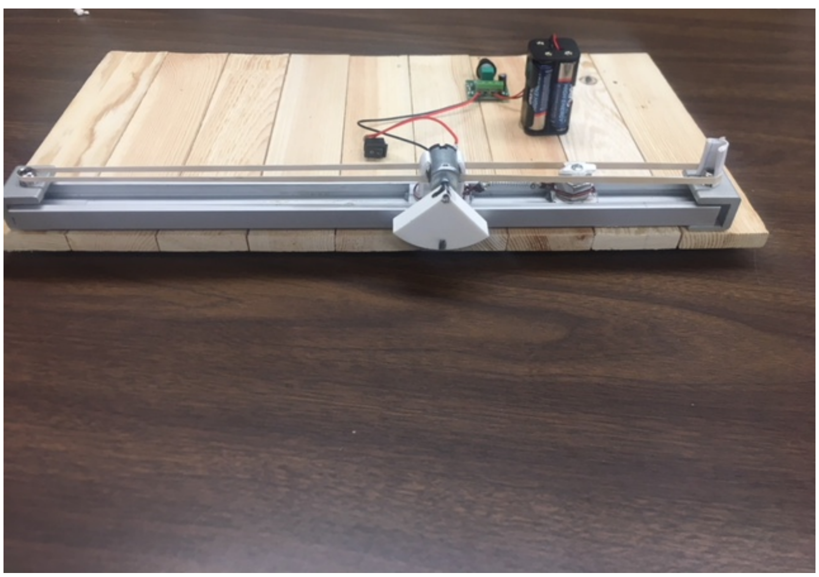
Shereen Aljumah

Technical Review:



Here is the picture that shows the one DOF for the project. This for vibration analyze. Using batteries., springs , wires and a some plot of mass.

Shereen Ajumah



This picture describe a 2 DOF of the project. We a a mass on device. We have changed the spring at the beginning it was a normal spring for the one DOF. in addition, for two DOF we have changed the spring to the rubber band instead of the normal one. Because of, the diameter of the spring was very big we could not continue the project. We used different elements to build up the physical system.

1. A wood board.
2. A normal spring for one DOF.
3. A rubber band springs for two DOF.
4. 4 AA batteries 1.4v for each battery.
5. 4 wires (2 red wires and 2 black wires).
6. A pen spring.
7. A 3D motor.
8. A plot mass.
9. Container to protect the plot.
10. Glue gun.

The design of the vibration system used the provided single DC motor, 1 black and red-colored wires, 1 switch, 1 aluminum ramp with to aluminum slider masses, 1 four pcs battery holder, 1 motor holder, and one small tuner circuit. Glue guns and Gorilla Glue were used to fix the motor, ramp battery box, circuit board, and switch. 2 Metal springs were used to connect the motor slider with the ramp poles which were later replaced by the rubber bands. Loops of copper wires were wrapped around both the motors and a pen spring was soldered to the loops. A 3D printed load was inserted in the shaft of the motor. The switch of the motor was connected to the midsection of the red wire.

Design Requirements:

The design requirements we have to design:

1. Create an equation of motion for the system.
2. Design one DOF.
3. Design two DOF.
4. Calibration of the physical system.
5. Implementation of design.
6. Explain the plot and how the simulation compares to the physical system.

Shereen Aljumah

Design Description (Shishir Khanal):

Overview:

______

1 DOF System:

An eccentric load of mass 0.018 gms was put on the motor that was provided in the project package. This motor was connected with a spring on each side. When the motor was turned on, it started vibrating back and forth. The friction between the sliders and the ramp acted as the vibration damper for the system. The petroleum jelly was used to decrease the friction between the ramp and the sliders so that the amplitude of the vibration can be optimized.

Computations Used:

The First of all, an equation of motion for the system was developed using Newton's laws. After that, Equations for the equivalent spring constant, damping constant, and amplitude were calculated which are shown below. After that equation for the phase angle was calculated which are given below.

$$M\ddot{X} + C_{eq}\dot{x} + k_{eq}x = F_0 * sin(wt)$$

$$k_{eq} = k_1 + k_2$$

$$X = \left(\frac{F_0}{k_{eq}}\right)\left[\frac{1-\left(\frac{4\mu N}{(Pi)*F_0}\right)^2}{\left(1-(w_f/wn)^2\right)^2}\right]$$

$$\boldsymbol{\Phi} = aTan\left(\left(\frac{\frac{4\mu N}{(Pi)*k_{eq}*X}}{1-(w_f/wn)^2}\right)\right)$$

Similarly, simulations and computations were performed in Matlab. The code used in the computation and simulations obtained is provided in the appendix below.

2 Dof System

A pen spring was used to connect the vibrating load with the second slider. After that load on the second slider was increased. For the same angular frequency, the amplitude of the first vibrating system decreased with increasing load. At the natural frequency of the system, the amplitude of the vibrating system almost became zero, and the second mass vibrated with a big amplitude. This was the resonance.

Computations Used:

______The lagrangian was developed for the 2 degrees of freedom system and the system of equations of motion was evaluated. After that, the resonating frequency was calculated using the equation given below using Matlab.

$$w_f = \sqrt{\frac{k_3}{m_2}}$$

Evaluation

Overview: (Jose Nunez)

______A 1 DOF linear mass-spring system with components consisting of a motor, a mass with slider, and two springs were utilized to create a working vibrational system whose objective was to physically model displacements due to vibration. A 2 DOF linear-mass spring system which added a vibration absorber to the previous system via a metallic pen spring was created in order to physically model vibrational absorption and thus impact displacement of the mass in the first system. The construction process of the first system, the 1 DOF system, involved testing for spring constants, voltage input calibration, and the knob calibration for a variable speed controller. The system was simulated theoretically via Matlab code such that theoretical values for amplitude were calculated based on previous rpm calibrations. The actual change in amplitudes were compared to theoretical values resulting in an understanding of external factors which impacted displacement. Further action necessitated to get values close to theoretical values resulting in a need for lubrication, and adjusting of springs to avoid contact with surfaces.

The second system, the 2 DOF system with vibrational absorber, utilized previous displacement values for the linear mass-spring system without the secondary mass. Calibration was done to the second system to ensure that appropriate vibration absorption took place. This involved a replacement of a metal spring to a smaller metallic pen spring, the addition of a lubricant to mitigate wear on the bottom of the slider for the second mass, experimentation on the vibration absorber mass to reduce lateral movement affecting displacement, and an increase in weight in the secondary mass to increase vibrational absorption.

Prototyping:(Jose Nunez)

For the 1st degree of freedom linear mass-spring vibrational system, a setup was created utilizing the initial components received for the project, which included: a battery case, variable speed controller, dc motor, two sliders, and a slider rail measuring about fourteen inches. Other than calibration of the voltage, variable speed controller, and springs, the prototyping focused on an offload mass with an eccentricity that was to be created for the project.

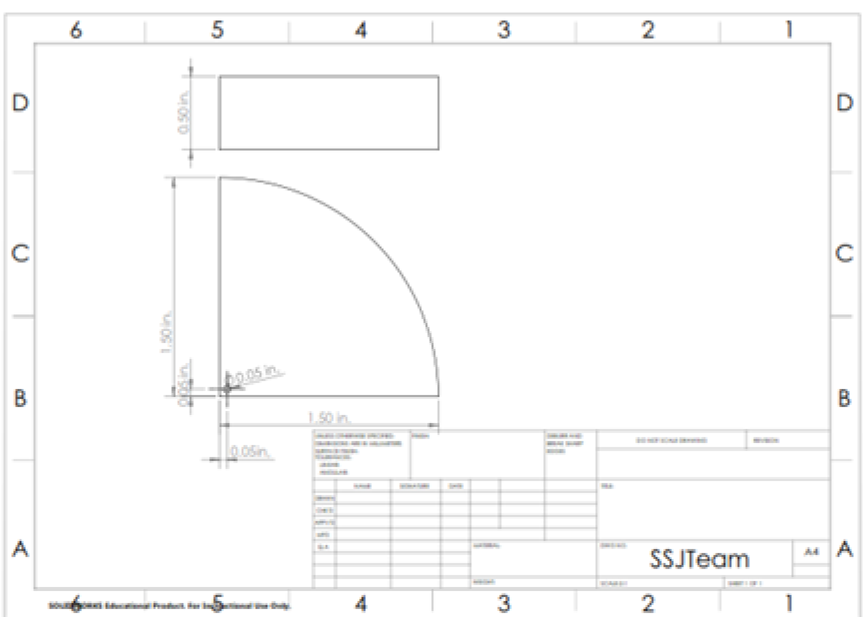


Figure 1: Solidworks Eccentricity Mass Design

Figure 1 shows the design of the eccentricity mass attached to the mass slider in the 1 DOF system. The dimensions chosen for the mass were a thickness of 0.5 inches with a radius of 1.5 inches as a quarter-circle.

Initially, the 1 DOF system had two metal springs rather than rubber bands whose spring constants were 3.45 and 6.75 N/m, respectively. The metal springs proved to have a non-negligible mass which affected the displacements and were thus scrapped in favor of two rubber bands, whose spring constants are about 101 and 58 N/m, respectively.

______For the 2 DOF system with vibrational absorber, the mass slider that acted as the vibrational absorber was at first a rectangular block of wood with height of about 3 inches with mass of 0.01 kg. The spring connected to the mass-slider that held the motor was a metal spring with a 6.57 N/m spring constant. The problems that arose came once again with the weight of the metal spring with the addition of an excess length. Thus, a correction was made to the design to utilize a metal pen spring with a spring constant of about 368 N/m that was soldered onto two metal wires to ensure a connection that wouldn't break, as such breaking occurred during testing. The new vibrational absorber included a metal bolt placed on the mass slider belonging to some sort of torque wrench. The new mass-slider system measured at around 0.04 kg, with a negligible height to avoid lateral displacements along the height of the vibrational absorber.

The system as a whole was screwed into a wooden plank board support in order to avoid external vibrations, the possible damping is unknown.

Testing:(Jose Nunez)

______Testing of the the spring was done utilizing the general formula for spring constants, which is force over displacement, across various iterations of mass to ensure a suitable average. The two rubber springs in the final design had spring constants of about 101 and 58 N/m, respectively. The metal pen spring had a spring constant of about 368 N/m. The testing of the spring constants involved a process of testing the rubber bands using three known masses: a 200g, 300g, and 400g mass; the subsequent displacements of the rubber bands across a fixed reference point were used to calculate and average for the spring constants.

Column1	Mass(g)	Weight(N)	x1(cm)	x2(cm)	dx(m)	K = W/dx	Avg K (N/m)	Column2
Spring-1(R)	200	1.962	9.2	10.8	0.016	122.625	101.0645604	
	300	2.943	9.2	12	0.028	105.107143		
	400	3.924	9.2	14.4	0.052	75.4615385		
Spring -2(R)	200	1.962	9.4	12.2	0.028	70.0714286	57.59494232	
	300	2.943	9.4	14.3	0.049	60.0612245		
	400	3.924	9.4	18.6	0.092	42.6521739		
Spring-3(M)	200	1.962	2.8	3.4	0.006	327	368.0519481	
	300	2.943	2.8	3.5	0.007	420.428571		
	400	3.924	2.8	3.9	0.011	356.727273		
								R = Elastic Rubber
								M = Metal Spring

Table 1: Spring Constant Measurements **(Shishir Khanal)**

Testing was done to calibrate the variable speed controller in the 1 DOF and 2 DOF systems such that a good assessment of speeds could be obtained via the turning of a knob in the speed control unit.

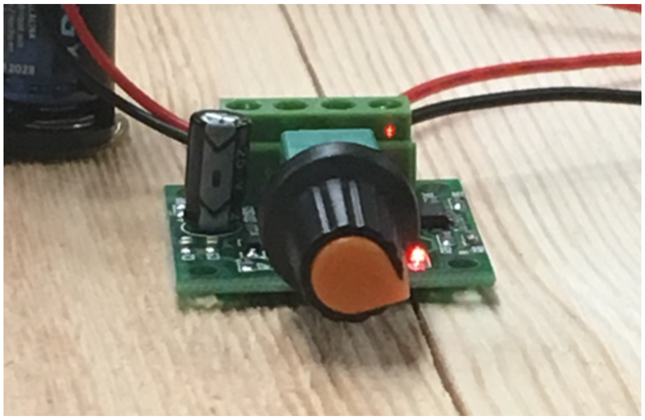


Figure 2: Photo of variable speed knob

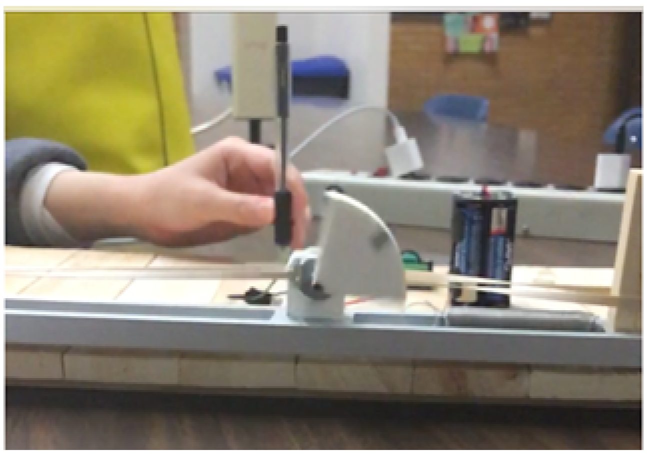
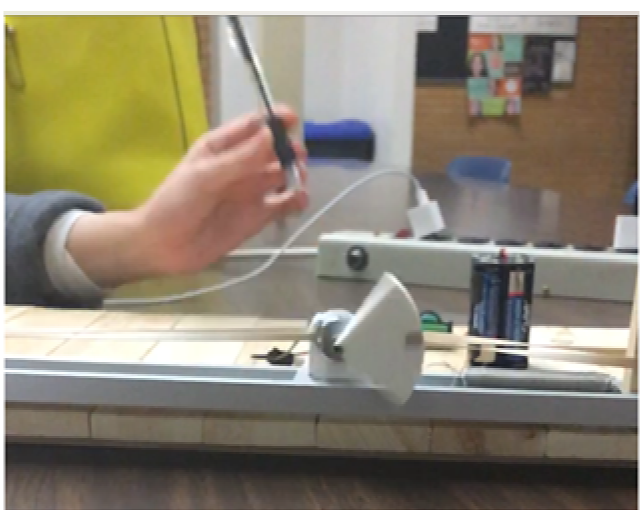


Figure 3 (above): Stopwatch Start

Figure 3(below): Stopwatch Stop



In order to calibrate the variable speed controller, an overview of speeds, calculated in rpm's, was taken at positions 45 degrees, 30 degrees, 0 degrees, and -30 degrees from the horizontal, where the reference point utilizes the orange line on the knob. To calculate motor speeds, the eccentricity mass was utilized as a reference for the speed. A "slow-mo" setting was selected on an Iphone 5s, measuring at 10 second intervals, taken via the stopwatch application. One member of the project team verified the ten second intervals, while a second person utilized the video of the ten second interval in "slow-mo" to count the number of revolutions using the removal and addition of the pen, which functioned as a reference, given complications in the "slow-mo" video settings.

Further testing was done on the 1 DOF and 2 DOF system to measure actual displacements so as to compare to theoretical values. The measurements for displacements were calculated using a ruler placed at the center of the mass-slider sitting at a resting position, wherein the actual displacement/amplitude was measured using the maximum displacement shown across various speeds; calculated at the previous positions.

Assessment:(Jose Nunez)

______The prevailing theme of the project was the measurement of amplitude and thus displacements of the linear mass-spring systems, given a calculated forced frequency and a vibrational absorber, respectively. Actual and Theoretical values for the 1 DOF system and 2 DOF system were calculated as follows:

1 DOF System

Position	RPM	Amplitude (Theory)	Amplitude (Actual)	% ERROR
45	426	12.4 cm	1.5 cm	N/A
30	444	4.11 cm	2.3 cm	78.7
0	462	2.6 cm	2.9 cm	3.7
-30	456	3 cm	2.7 cm	3.4

2 DOF System

Position	RPM	Amplitude (Theoretical)	Ammplitude (Actual)
45	426	0	1 mm
30	444	0	1.5 mm
0	462	0	1.5 mm
-30	456	0	2 mm

The displacements for the 1 DOF system showed incremental displacements in actual testing while the theoretical values had decreasing amplitudes with higher speeds. This could be due to the fact that the natural frequency was much lower than the forced frequency, thus resulting in a lack of resonance. The rpm calibration showed that the motor had a peak in between the horizontal position an the -30 degree position. (See Appendix C: 1 DOF Amplitude Plots)

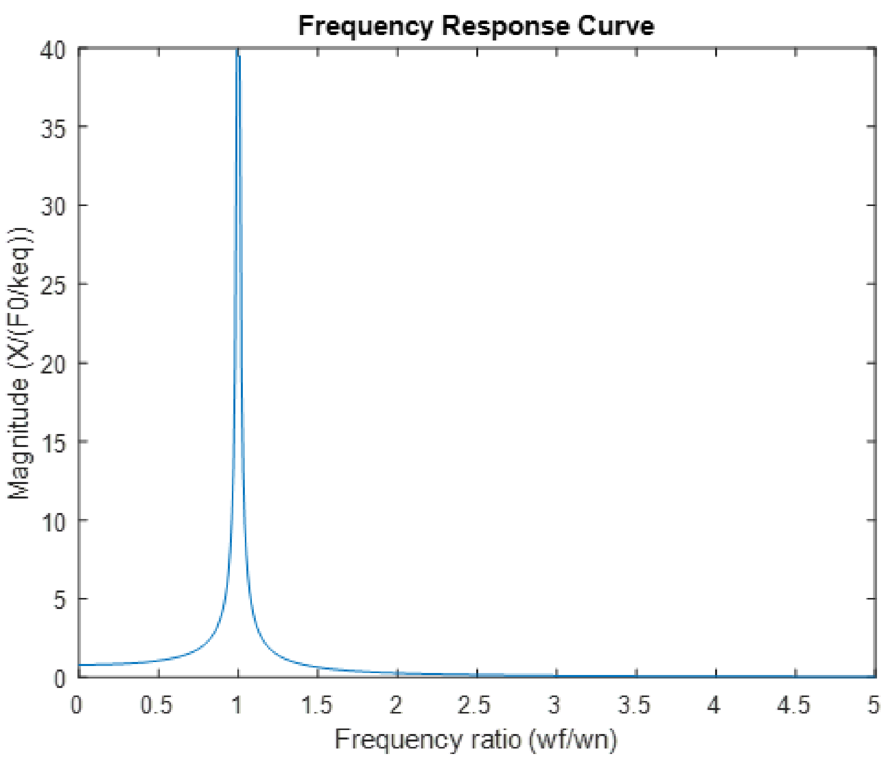
The 2 DOF system showed that after compensation the displacement of the main mass-slider was reduced significantly due to the vibrational absorber. However, both the vibrational absorber and the main mass-slider vibrated at the same speed and with the same displacement, this may show that the spring connected via soldered wires might have created a fixed system such that the metal pen spring influenced the system very little.

Action Items Based on Evaluation: (Jose Nunez)

Possible problems in the system in the beginning of testing included wear on the bottom of the slider in the vibration absorber due to a high coefficient of friction between the aluminum slider and the aluminum rail. To correct the problem, a lubricant in the form of Vaseline was applied to the rail resulting in a friction coefficient of 0.3, down from 1.1.

A loss of amplitude was noted in the initial prototyping of the 2 DOF system with the long vibration absorber due to contact between the springs and the mass. This was corrected by replacing the wood block with a short metal bolt, furthermore a piece of folded paper was added to the end of the spring to create more space between the mass and springs.

A look at the amplitude tables in the for the 1 DOF system resulted in identifying that the motor had a minimum speed of about 300 RPM close to the 15 degree position, however this was still larger than the assumed natural frequency. Any position closer to the vertical resulted in a lack of power to operate the system. The system had 4 AAA batteries however calibration of speeds given a change in batteries was not attempted, yet in theory a lower voltage might have provided a maximum RPM closer to the natural frequency which would have resulted in a larger magnitude.



<u>Plot : Frequency Response Curve for 1 DOF system</u>(**Shishir Khanal**)

References

Rao, Singiresu S. Mechanical Vibrations, Sixth Edition. Pearson, 2017

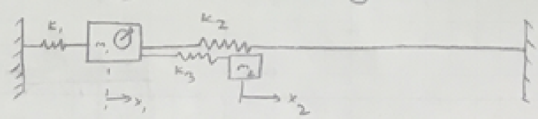
Appendix

Appendix A: Computations

2 DOF

ME 4440 | Shishir Khanal | Group Members: Rose Nunez, Shereen Aljhurah

For 2-Degree of freedom system



x_1: $\mathcal{L} = T - U$

$$= \frac{1}{2} m_1 \dot{x}_1^2 - \frac{1}{2} k_1 x_1^2 - \frac{1}{2} k_2 x_1^2 - \frac{1}{2} k_3 (x_2 - x_1)^2$$

Equivalent viscous damper

$$c_{eq} = \frac{\mu A}{h}$$

x_2: $\mathcal{L} = T - U$

$$= \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} k_3 x_2^2$$

Updated Lagrangian Eq:

$$D = \frac{1}{2} \mu \dot{x}^2$$

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) + \frac{\partial D}{\partial \dot{x}} = F_0 \sin(\omega_f t)$$

For x_1:

$$-k_1 x_1 - k_2 x_1 + k_3 x_1 - m_1 \ddot{x}_1 + \frac{\mu A}{h} \dot{x}_1 = F_0 \sin(\omega_f t)$$

For x_2:

$$k_3 x_2 - m_2 \ddot{x}_2 + \frac{\mu A}{h} \dot{x}_2 = F_0 \sin(\omega_f t)$$

$$-m_1 \ddot{x}_1 + \frac{\mu A}{h} \dot{x}_1 - (k_1 + k_2 - k_3) x_1 = F_0 \sin(\omega_f t)$$

$$-m_2 \ddot{x}_2 + \frac{\mu A}{h} \dot{x}_2 + k_3 x_2 = F_0 \sin(\omega_f t)$$

$$\begin{bmatrix} -m_1 & 0 \\ 0 & -m_2 \end{bmatrix} + \begin{bmatrix} \frac{\mu A}{h} & 0 \\ 0 & \frac{\mu A}{h} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 - k_3 & 0 \\ 0 & k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \sin(\omega_f t) \\ F_0 \sin(\omega_f t) \end{bmatrix}$$

For Resulting Amplitude of the second mass,

$$\omega_f = \sqrt{\frac{k_3}{m_2}}$$

$k_3 \rightarrow$ spring constant of third spring

$m_2 \rightarrow$ mass of the vibration absorber.

Appendix B: Matlab Code

<u>1 DOF Amplitude</u> (Shishir Khanal)

```
% ME 4440
% Mechanical Vibrations Project
% Group Members:
% Sheeren Aljumah
% Jose Nunez Torres
% Shishir Khanal
% Date: 12/07/2018
% Purpose : Develop a simulation for the amplitude of the particular
% solution of the vibration system of interest
%-----------------------------------------------------------
clear; clc;
% Clears the screen and workspace for fresh start every time
mu = 0.3;
% Assume coefficient of kinetic friction between two aluminium surfaces
%is taken as
% 0.3
M = .065;
% Total mass of the load and the motor is taken as 0.065 kg
m0 = 0.018;
% mass of load is taken as 0.4 kg
g = 9.81;
% Acceleration due to gravity is 9.81 ms^-2
e = 0.023 ;
% eccentricity of the rotating mass is 0.023 m
wf = 456*(2*pi/60);
% equation for limiting angular frequency that will give the real amplitude,
% Rpm used include: 426 at 45 degree position, 444 at 30 degrees, 462 at 0
% degrees from the vertical, and 456 at -30 degrees
k1 = 101 ;
```

```
k2 = 58;
% Spring constants
keq = k1+k2;
% Assuming the two springs are in parallel and same value of spring constant.
%Keq = k1+k2
wn = (keq/(M+m0))^(1/2);
% equation for natural frequency where Total Mass is used
F0 = m0*e*(wf^2);
% equation for F0
N = (M+m0)*g;
% expression for Normal force
Term1 = 1-((4*mu*N)/(pi*F0))^2;
Term2 = (1-(wf^2/wn^2));
X = (F0/keq)*(Term1/(Term2)^2)^(1/2);
% Amplitude
TermA = (4*mu*N)/(pi*keq*X);
phi = atan(TermA/Term2);
% phase lag
t = 0:0.01:5;
Xp = X * sin(wf*t - phi);
% expression for particular solution
plot(t,Xp)
ylabel('Xp(t)->');
xlabel('t->');
```

1 DOF Frequency Response Curve (Shishir Khanal)

```
% ME 4440
% Mechanical Vibrations Project - Memo 2
```

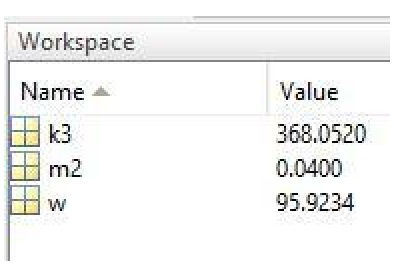
```
 % Group Members:
% Sheeren Aljumah
% Jose Nunez Torre
s % Shishir Khamal
% Date: 12/7/2018
% Purpose : Develop a simulation for the frequency response of the particular
% solution of the vibration system of interest
 %-----------------------------------------------------------
 clear; clc;
% Clears the screen and workspace for fresh start every time
mu = 0.3;
% Assume coefficient of kinetic friction between two aluminiums is taken as 0.3
 M = 0.065;
% Total mass of the load and the motor is taken as 0.065kg
 m0 = 0.018;
 % mass odfload is taken as 0.18 kg
 g = 9.81;
 % Acceleration due to gravity is 9.81 ms^-2
e = 0.023 ;
% eccentricity of the rotating mass is 0.023 m
 wf = ((4*mu*M*g)/(m0*e*pi))^(1/2);
% equation for limiting angular frequency
k1 = 101;
k2 = 58;
 % Spring constanta
 keq = k1+k2;
 wn = (keq/(M+m0))^(1/2);
 % equation for natural frequency
F0 = m0*e*wf^2;
```

```
% equation for F0
 N = M*g;
% expression for Normal force
 Term1 = 1-((4*mu*N)/(pi*F0))^2;
 r = 0:0.01:5;
Term2 = (1-(r.^2)).^2;
 X = (F0/keq)*((Term1./(Term2))).^(1/2); Mag = abs(X/(F0/keq));
% phase lag t = 0:0.01:5;
 % expression for particular solution
 plot(t,Mag);
 title('Frequency Response Curve');
ylabel('Magnitude (X/(F0/keq))');
xlabel('Frequency ratio (wf/wn)');
```

2DOF Natural Frequency

```
clear ;clc;
k3 = 368.052;
% spring constant of Metal spring
m2 = 0.04 ;
% mass of second load 40 gms
w = sqrt(k3/m2);
%natural frequency
```

2DOF Natural frequency Output

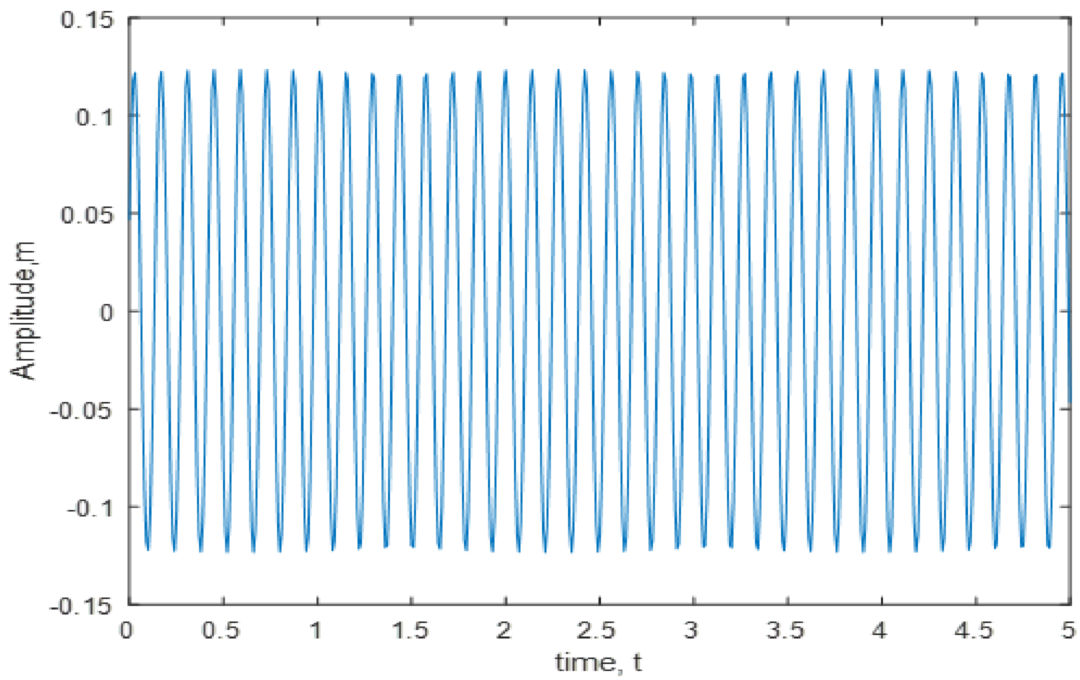

Workspace

Name	Value
k3	368.0520
m2	0.0400
w	95.9234

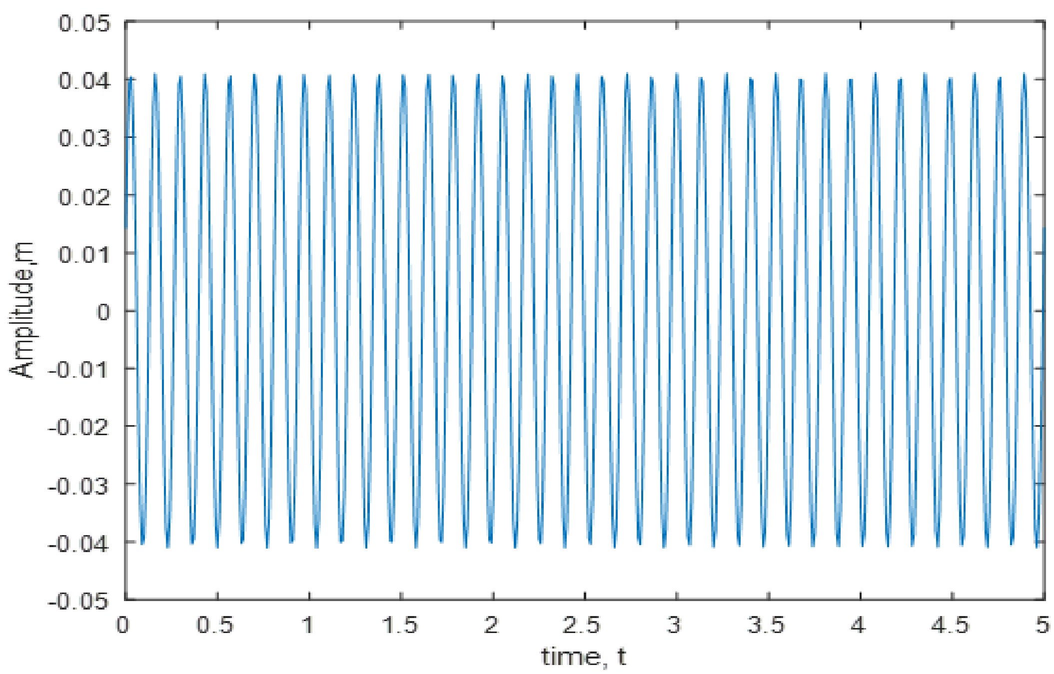
Appendix C: Plots

1 DOF Amplitude Plots (Jose Angel Nunez-Torres)

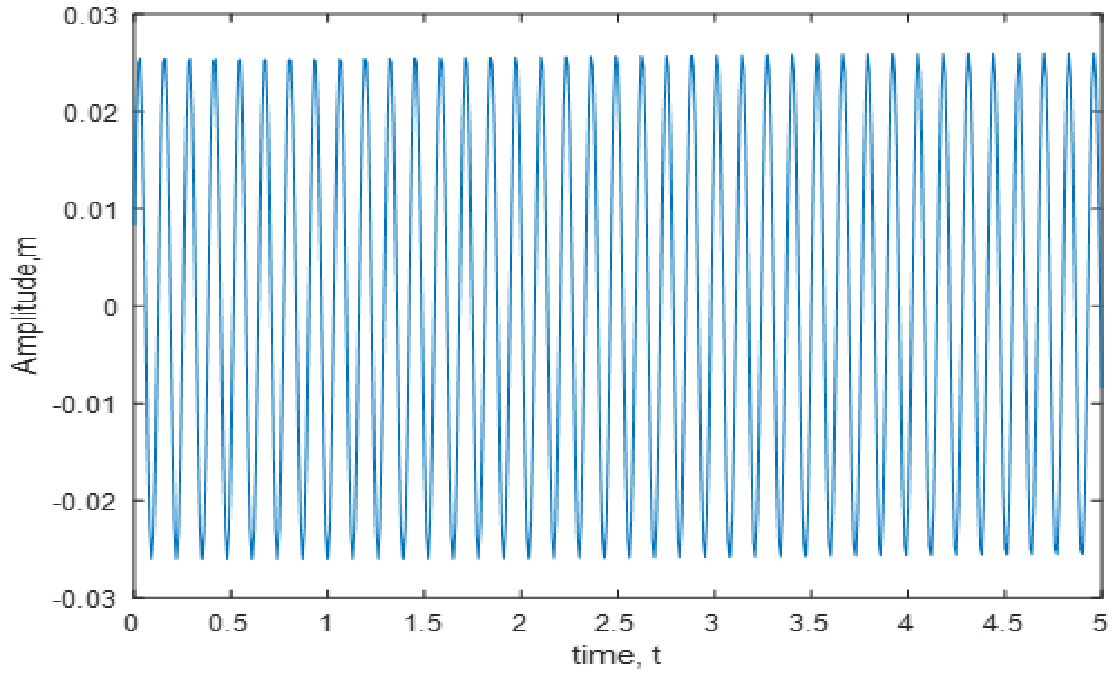
Plot 1: 45 Degree Position, 426 RPM Amplitude



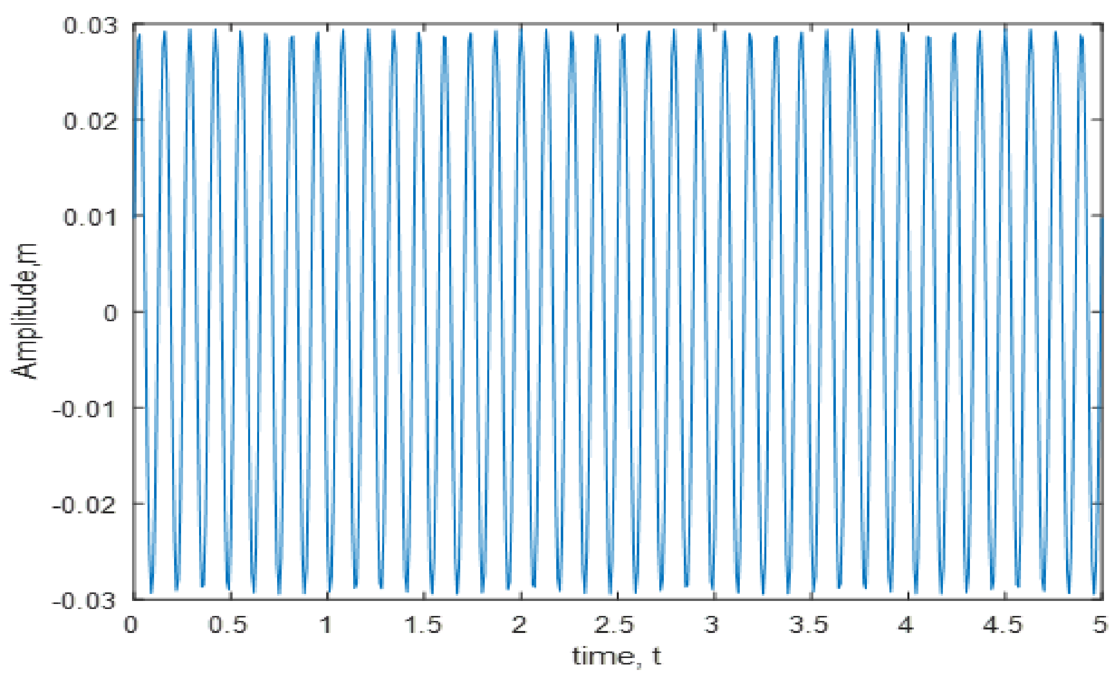
Plot 2: 30 Degree Position, 444 RPM Amplitude



Plot 3: 0 Degree Position, 462 RPM Amplitude



Plot 4: -30 Degree Position, 456 RPM Amplitude



1 DOF Frequency Response Plot (Shishir Khanal)

Plot 5: 1 DOF Frequency Response Curve