Mechanical Control Systems (ME 4473)

Recitation - 2

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- **Agenda:**
  - Revision (Definitions & Terms)
  - Problems(State Space)

- **Answer the following questions:**
  - **What is a state?**

Set of variables together with input signals and governing equations provide a full description of system

  - **What is the meaning of state space description of the system?**

A control system can be represented as a operation which maps output to the input

  - **What is the matrix equation for the state space representation of a system?** (No Noise, No Uncertainty)

x'(t) = Ax + Bu

A = System Matrix

B = Input Matrix

y(t) = Cx+Du

C = Output Matrix

D= Feedthrough Matrix

  - **How many state equations can a 5th order ODE be represented in terms of?**

    5

  - **Which of the following techniques are used to linearize a system?**
    1. Taylor Series Representation
    2. Small Angle Approximation
    3. Jacobian Linearization
    4. **All of the Above**
    5. None of the Above

- **Problems:**
  - **For the simple pendulum, the nonlinear equations of motion are given by:**

$$\Theta''(t) + \frac{g}{L}sin(\Theta) + \frac{k}{m}\Theta'(t) = 0$$

(Written against actual convention: dot notation for time derivative and prime notation for length derivative)

where g is gravity, L is the length of the pendulum, m is the mass attached at the end of the pendulum (we assume the rod is massless), and k is the coefficient of friction at the pivot point.

a. **Linearize the equations of motion about the equilibrium condition Θ= 0°.**

$$\ddot{\theta}(t) + \frac{g}{l}\sin\theta + \frac{k}{m}\dot{\theta}(t) = 0$$

<u>Small angle Approximation</u>,

$$\sin\theta \approx \theta$$

($\theta \le 14°$ (Relative error exceeds 1%)

(<u>Source</u>: Wikipedia

$$\ddot{\theta} + \frac{g}{l}\theta + \frac{k}{m}\dot{\theta} = 0$$

$$\boxed{\therefore\ \ddot{\theta}(t) + \frac{k}{m}\dot{\theta}(t) + \frac{g}{l}\theta(t) = 0}$$

b. **Obtain a state variable representation of the system.**

$$\dot{x}_1 = \dot{\theta} = x_2$$

$$\dot{x}_2 = \left(-\frac{k}{m}\right)x_2 + \left(-\frac{g}{l}\right)x_1$$

$$\dot{X} = \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 \\ -\frac{k}{m} & -\frac{g}{l}\end{bmatrix}}_{\substack{A \\ \text{"State Matrix"}}} \underbrace{\begin{bmatrix}x_1 \\ x_2\end{bmatrix}}_{\substack{X \\ \text{"State Vector"}}}$$

$$Y = \underbrace{[1\ \cancel{0}\ 1]}_{C}\ X$$

Correction: On matrix ‘A’ the elements of (2,1) & (2,2) should be reversed

c. **Come up with parameters for the model of a pendulum**

-g/l = -10 (approx 1m length of string)
-k/m = -0.1 (k = 0.1; m =1)

Now, let's change our system by providing an input such that our original ODE becomes:

$$\Theta''(t) \ + \frac{g}{L}sin(\Theta) \ + \frac{k}{m}\Theta'(t) \ = u(t)$$

(Written against actual convention: dot notation for time derivative and prime notation for length derivative)

d. Linearize the equations of motion about the equilibrium condition Θ= 0°.

$$\ddot{\Theta}(t) + \frac{g}{l}\sin(\Theta(t)) + \frac{k}{m}\dot{\Theta}(t) = u(t)$$

<u>Linearization:</u>

$$\ddot{\theta} + \frac{g}{l}\theta + \frac{k}{m}\dot{\theta} = u(t)$$

e. Obtain a state variable representation of the system.

Let,

$$x_1 = \theta$$
$$x_2 = \dot{\theta}$$

$$\dot{x}_1 = \dot{\theta} = x_2$$
$$\dot{x}_2 = \ddot{\theta} = \left(-\frac{k}{m}\right)\dot{\theta} + \left(-\frac{g}{l}\right)\theta + u(t)$$
$$= \left(-\frac{k}{m}\right)x_2 + \left(-\frac{g}{l}\right)x_1 + u(t)$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{g}{l} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \underbrace{[1 \quad 1]}_{C} X$$

([1 1] outputs sum of x1 & x2)

f. Evaluate a step response of this system using simulink.

@Simulink

Bibliography:

- **Modern Control Systems, 13th edition, Chapter 3**