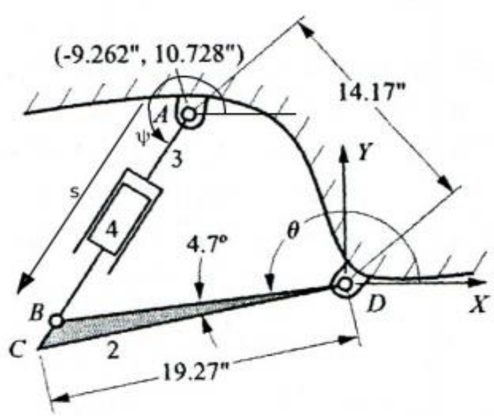
ME 3320 HOMEWORK #3

Due Date: 10/05/2021 (5 pm)

1. Linkage analysis: The linkage shown below is the kinematic sketch for the rear suspension of a motorcycle. The dimensions are given in the drawing; consider BC = 2" along the AB direction for the given configuration. For the analysis, assume that the angular velocity of the input link (link 2) is constant and operating in the CW direction (corresponding to the motorcycle going over a bump).
    a. Calculate the mobility of the linkage. How many loop equations are needed to solve for the dependent joint variables?
    b. Formulate the loop equations.
    c. Solve the loop equations and give explicit expressions for the dependent variables as a function of the input angle θ.
    d. Compute the limits for the input angle θ. Is the linkage going to work as expected? (is the range of motion of θ enough?)
    e. Write the position vector of point C. Use Maple, GIM or similar software to plot the trajectory of point C over the range of θ calculated in d).
    f. Use Maple, GIM, or similar software to plot ψ and s as a function of θ.
    g. Compute the velocity vector for point C. Give the value of the velocity for the configuration shown in the kinematic sketch ($\theta = 200^{o}$), for an input angular velocity of 200 rpm.
    h. Compute the velocity s of the slider. Plot this velocity as a function of θ for a constant input angular velocity of 200rpm.
    i. Plot the acceleration of the slide, s, as a function of θ, for the same constant input angular velocity.



Approach:

(a) → Kinematic Diagram → # Links, joints, dof of joints
& Use mobility eq^n

→ 2 dependent joint variables (figure out how many unknown dependent variables are there)
at least need 2 equations

(b) - Write down loop eq^n

(c) - Use loop eq^n
- might have to combine the equations in a way that cancels one dependent variable
(Using Algebra)

(d) Eq^ns for dependent variables & see if we can make the depent variables complex or ph?

(e) Evaluate the C coordinate from the sketch

(f) Plot the eq^ns in Part (c) for $\theta \Rightarrow [0, 2\pi]$

(g) Derivative of (f)

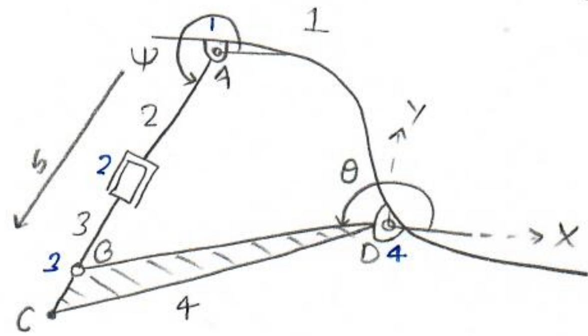
(h) - We have 's'
- Take derivative of 's'

(i) - Take derivative of (h)
- Plot for $\theta \Rightarrow [0, 2\pi]$

(a)

$n = 4$

$j = 4$



$$M = 3(n-1) - \sum_{i=1}^{j}(3-f_i) = 3\times(4-1) - 4(3-1)$$

$$\boxed{\therefore M = 1}$$

Independent variable : θ

Dependent variable : Ψ, s

- Since, the dependent variables are 2, we need 2 equations
- Since 1 loop equation in the planar case (2D) can have 2 equations, we need 1 loop equations.

(b) Choose loop DABD

$$\bar{DA} + \bar{AB} = \bar{DB}$$

$$\begin{bmatrix} DA_x \\ DA_y \end{bmatrix} + \begin{bmatrix} s\cos\Psi \\ s\sin\Psi \end{bmatrix} = \begin{bmatrix} DB\cos\theta \\ DB\sin\theta \end{bmatrix}$$

$$\therefore \begin{bmatrix} DA_x + s\cos\Psi \\ DA_y + s\sin\Psi \end{bmatrix} = \begin{bmatrix} DB\cos\theta \\ DB\sin\theta \end{bmatrix} \quad \text{— (A)}$$

(c) Need to find s & Ψ in-terms of θ,

$$s\cos\Psi = DB\cos\theta - DA_x \quad \text{— (i)}$$

$$s\sin\Psi = DB\sin\theta - DA_y \quad \text{— (ii)}$$

(i)^2 + (ii)^2,

$$s^2 = (DB\cos\theta - DA_x)^2 + (DB\sin\theta - DA_y)^2$$

Choosing +ve for length,

$$s = \sqrt{(DB\cos\theta - DA_x)^2 + (DB\sin\theta - DA_y)^2}$$

(ii)/(i),

$$\therefore \tan\Psi = \frac{DB\sin\theta - DA_y}{DB\cos\theta - DA_x}$$

(d) Can we define 'Ψ' & 's' for all values of θ?

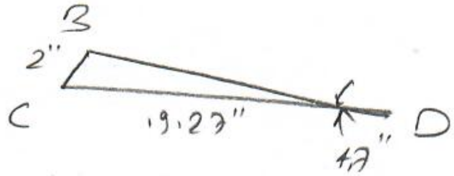
- $\tan^{-1}(x)$ is defined $\forall$ values of 'x'.
- s is defined for $s = \sqrt{x}$ where $x \geq 0$

$$(DB\cos\theta - DA_x)^2 + (DB\sin\theta - DA_y)^2 = 0 \quad \text{— (B)}$$

$BC^2 = DB^2 + CD^2 + 2 \times DB \times CD \times \cos\theta$

$2^2 = DB^2 + (19.27)^2 - 2 \times DB \times 19.27 \times \cos(4.7°)$

$DB^2 - 38.45 DB + 367.33 = 0$



Matlab :

roots([1 -38.45 367.33])

$\therefore DB = 17.71''$ or $20.73''$

- Based on the mechanism, we choose 17.71"

$\therefore DB = 17.71''$

Eq^n (B) becomes

$(17.71\cos\theta + 9.26)^2 + (17.71\sin\theta - 10.73)^2 = 0$

$327.99\cos\theta - 380.06\sin\theta + 514.52 = 0$

Matlab:

```
syms 'x'
reslult = solve ( 327.99*cos(x)-380.06*sin(x)+514.52 == 0, x);
double (result)
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$\therefore \theta = 2.28 \pm 0.22i$

θ is complex. Hence there is no 'θ' that can make 's' complex

$\therefore$ there is no limit for theta for both 'ψ's'

(e) $\vec{C} = \begin{bmatrix} 19.27\cos(\theta + 4.7°) \\ 19.27\sin(\theta + 4.7°) \end{bmatrix}$ Plot Matlab/GIM

(f) Matlab/GIM

(8) $\vec{C} = \begin{bmatrix} 19.27\cos(\theta+4.7^\circ) \\ 19.27\sin(\theta+4.7^\circ) \end{bmatrix}$

$\dot{\vec{C}} = \begin{bmatrix} -19.27\sin(\theta+4.7^\circ)\cdot\dot{\theta} \\ 19.27\cos(\theta+4.7^\circ)\cdot\dot{\theta} \end{bmatrix}$ $\qquad \theta = 200^\circ \qquad \dot{\theta} = 200 \text{ rpm} = 200\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ sec}}$

$= 20.94 \text{ rads}^{-1}$

$\dot{C} = \begin{bmatrix} -19.27(\sin 204.7^\circ)\times 20.94 \text{ in/s} \\ 19.27(\cos 204.7^\circ)\times 20.94 \text{ in/s} \end{bmatrix}$

$\therefore \dot{C} = \begin{bmatrix} 168.62 \\ -366.60 \end{bmatrix} \text{ rads}^{-1}$

(h) $s = \sqrt{(17.71\cos\theta+9.26)^2+(17.71\sin\theta-10.73)^2}$

$\dot{s} = \dfrac{1}{2\sqrt{327.99\cos\theta-380.06\sin\theta+514.52}} \times \left\{2(17.71\cos\theta+9.26)(-17.71\sin\theta)+2(17.71\sin\theta-10.73)\cdot(17.71\cos\theta)\right\}\dot{\theta}$

$\dot{s} = \dfrac{(-163.99\sin\theta-190.03\cos\theta)\dot{\theta}}{\sqrt{327.99\cos\theta-380.06\sin\theta+514.52}}$

(i) $\ddot{s} = \dot{\theta}\dfrac{d}{dt}\left((-163.99\sin\theta-190.03\cos\theta)(327.99\cos\theta-380.06\sin\theta+514.52)^{-\frac{1}{2}}\right)$ $\qquad \ddot{\theta} = 0$ constant angular velocity

$= \dot{\theta}\Big\{(-163.99\sin\theta-190.03\cos\theta)\dfrac{d}{dt}(327.99\cos\theta-380.06\sin\theta+514.52)^{-\frac{1}{2}}$

$+ (327.99\cos\theta-380.06\sin\theta+514.52)^{-\frac{1}{2}}\dfrac{d}{dt}(-163.99\sin\theta-190.03\cos\theta)\Big\}$

$= \dot{\theta}\Big\{\dfrac{163.99\sin\theta-190.03\cos\theta}{2(327.99\cos\theta-380.06\sin\theta+514.52)^{3/2}} \times (-327.99\sin\theta-380.06\cos\theta)\times\dot{\theta}$

$+ (327.99\cos\theta-380.06\sin\theta+514.52)^{-\frac{1}{2}} \times (-163.99\cos\theta+190.03\sin\theta)\times\dot{\theta}\Big\}$

$= \dot{\theta}^2\left\{\dfrac{(163.99\sin\theta-190.03\cos\theta)(-327.99\sin\theta-380.06\cos\theta)}{2(327.99\cos\theta-380.06\sin\theta+514.52)^{3/2}} + \dfrac{190.03\sin\theta-163.99\cos\theta}{(327.99\cos\theta-380.06\sin\theta+514.52)^{\frac{1}{2}}}\right\}$

$= \dot{\theta}^2\left\{\dfrac{(163.99\sin\theta-190.03\cos\theta)(-327.99\sin\theta+380.06\cos\theta)+2(327.99\cos\theta-380.06\sin\theta+514.52)(190.03\sin\theta-163.99\cos\theta)}{2(327.99\cos\theta-380.06\sin\theta+514.52)^{3/2}}\right\}$

(i) In Matlab

$\dot{\theta} = 20.94\ \text{rads}^{-1}$

For numerator,

expand(vpa(numerator))

$$\ddot{s} = \dot{\theta}^2 \left\{ \frac{-198232.68\sin^2\theta + 373961.94\sin\theta\cos\theta + 19544.67\sin\theta - 179796.96\cos^2\theta - 168748.99\cos\theta}{2\left(327.99\cos\theta - 380.06\sin\theta + 514.52\right)^{3/2}} \right\}$$

$$\ddot{s} = 219.24\left(\frac{-198232.68\sin^2\theta - 179796.96\cos^2\theta + 373961.94\sin\theta\cos\theta + 19544.67\sin\theta - 168748.99\cos\theta}{\left(327.99\cos\theta - 380.06\sin\theta + 514.52\right)^{3/2}}\right)$$