Mechanical Control Systems (ME 4473)

Recitation - 1

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1. Agenda:

- Office Hours Poll
- Revision
- Problems(Differential Equation, Block Diagrams, Transfer Function)

2. Match the Following

Transient Response	---->	Homogeneous solution
Steady State Response	---->	Particular solution
Error	---->	Desired signal - Measured Signal
Duhamel Integral	---->	Convolution Operation
Impulse Response	---->	Unit Impulse * Transfer Function

3. Fill in the Blanks

- Convolution in Time Domain is <u>Multiplication</u> in Frequency domain.

- <u>Step Response:</u> Output of a system to a step function input

- <u>Frequency Response:</u> Output of a system to a sine signal

- <u>Impulse Response:</u> Output of a system to a step function input

- <u>Transfer Function:</u> Output signal divided by Input signal at 0 initial conditions in Frequency domain

4. Introduction to Simulink

@Matlab& Simulink

5. Problems:

- **Write following differential equation in the block diagram**

$$2y''(t) + 5y'(t) + 4y(t) = x(t)$$

$$y(t) = \frac{x(t)}{4} - \frac{5}{4}y'(t) - \frac{1}{2}y''(t)$$

Procedure:

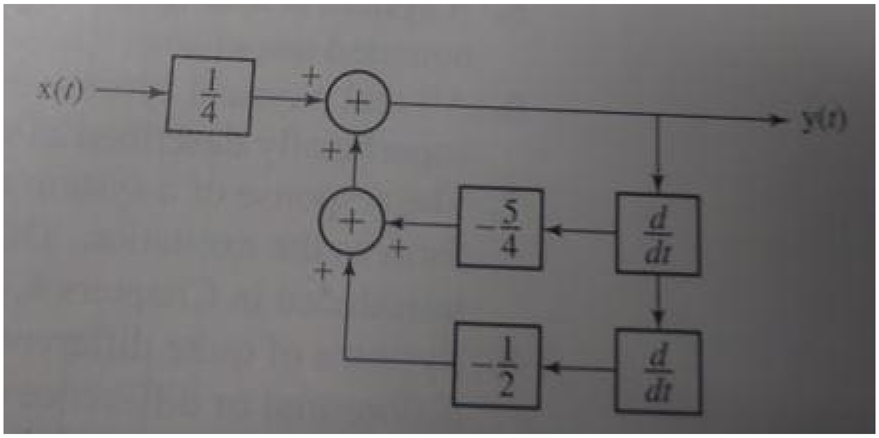
a. **Identify the Input & Output**

Input: x(t)

Output: y(t)

b. **Identify the required operators**
Operators: Differentiator, Summer

c. **Evaluate the Constants**
Constants: 1/4, -5/4, -1/2
d. **Figure out the appropriate connections**
a. **Draw a block diagram for the following differential equation using differentiators**



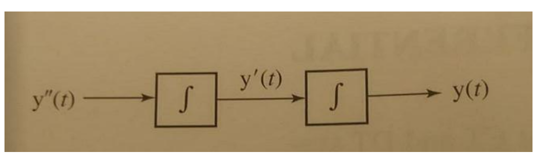
(Sign of Constant can be switched with sign of summer)

(Step Response in Simulink)

b. **For the differential equation above, draw the block diagram using integrators**

As we saw in the Simulink example, the differentiators are problematic for practical implementation(Emphasis of High Frequency noise of differentiators). Hence, we would like to use integrators.

$$2y''(t) + 5y'(t) + 4y(t) = x(t)$$



$$y''(t) = \frac{x(t)}{2} - 2y(t) - \frac{5}{2}y'(t)$$

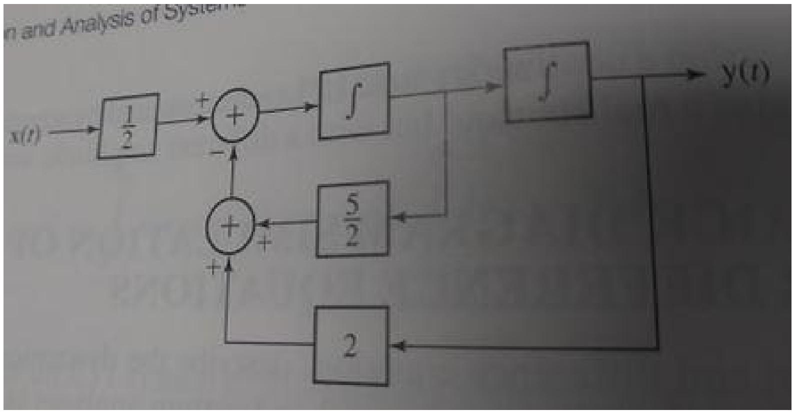
i. **Identify the Input & Output**
Input: x(t)
Output: y(t)

ii. **Identify the required operators**
Operators: Integrator, Summer

iii. **Evaluate the Constants**

Constants: 1/2, 2, 5/2

iv. **Figure out the appropriate connections**



(Step Response in Simulink)

c. **Now draw the block diagram using the transfer function**

$$2y''(t) + 5y'(t) + 4y(t) = x(t),\ y(0) = 0,\ y'(0) = 0$$

i. What is the transfer function of the differential equation?

$$2y''(t) + 5y'(t) + 4y(t) = x(t)$$

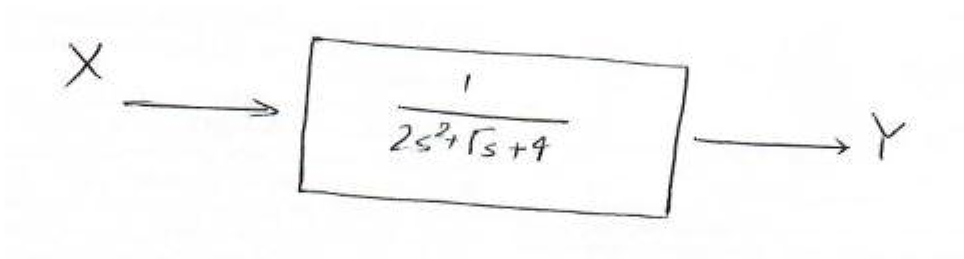
$$\mathcal{L}\{2y''(t)\}(s) + \mathcal{L}\{5y'(t)\}(s) + \mathcal{L}\{4y(t)\}(s) = \mathcal{L}\{x(t)\}(s)$$

$$2\{s^2Y(s) - s\overset{0}{\cancel{y(0)}} - \overset{0}{\cancel{y'(0)}}\} + 5\{sY(s) - \overset{0}{\cancel{y(0)}}\} + 4Y(s) = X(s)$$

$$2s^2Y(s) + 5sY(s) + 4Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{2s^2 + 5s + 4}$$

ii. What is the block diagram of the system?



Conclusion:

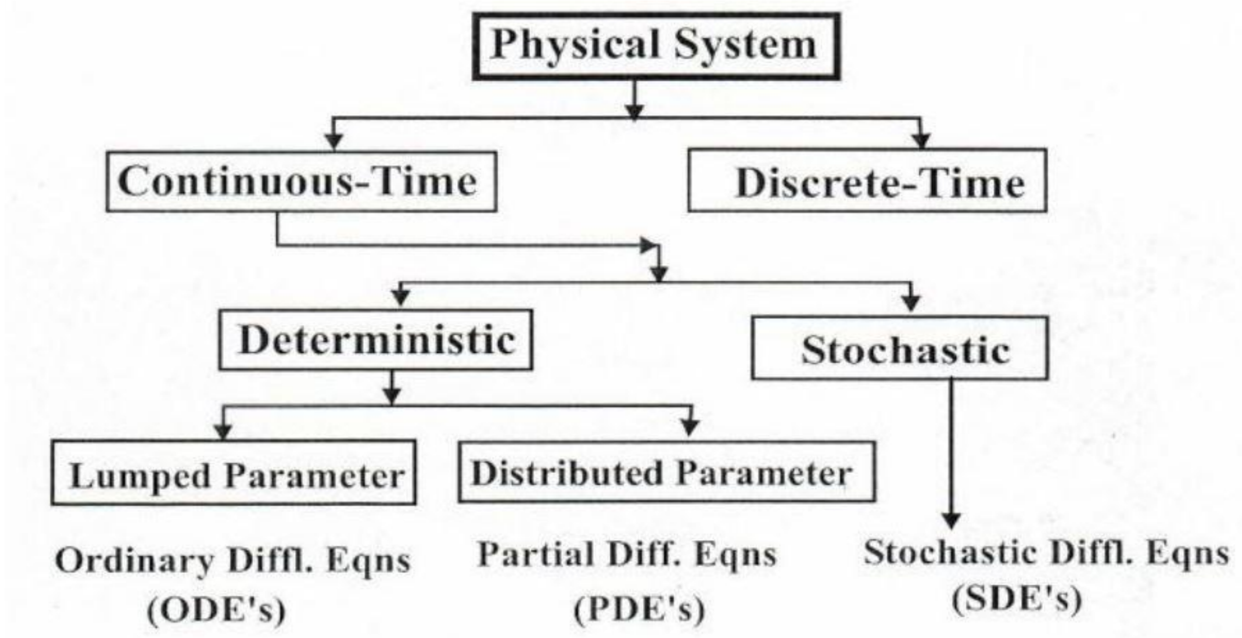
1. **Simulink is Awesome (but is troublesome in debugging)**
2. **Integrators are better to use than differentiators**
3. **Transfer Functions makes our life easy**

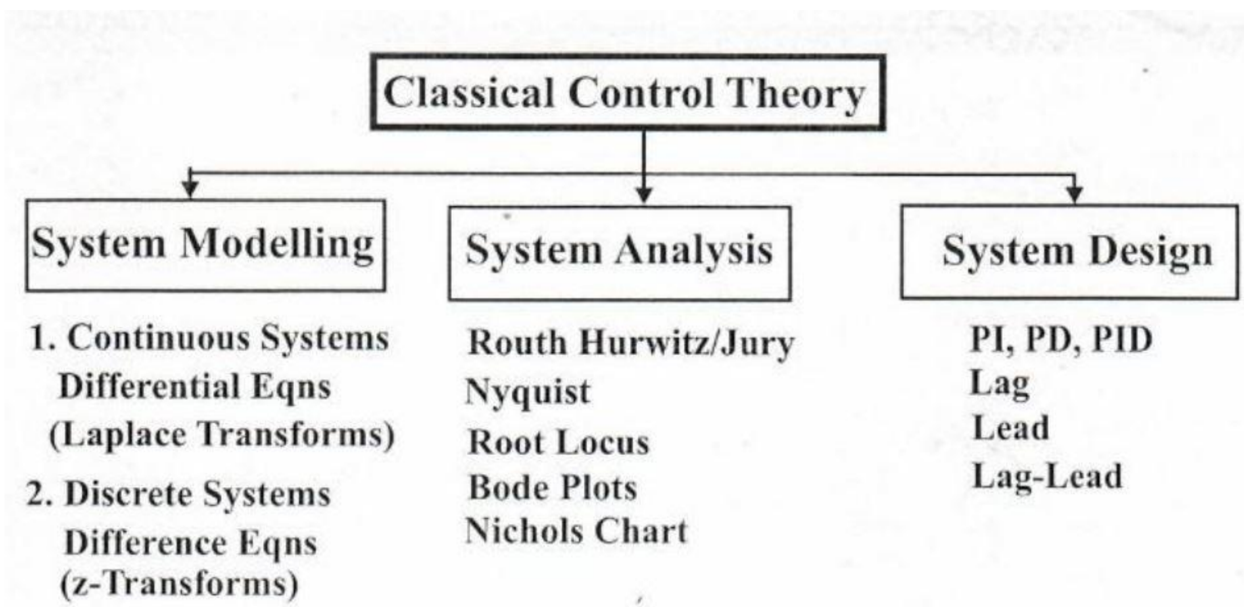
Appendix:

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{\left(s^2+a^2\right)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{\left(s^2+a^2\right)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{\left(s^2+a^2\right)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{\left(s^2+a^2\right)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s\left(s^2-a^2\right)}{\left(s^2+a^2\right)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s\left(s^2+3a^2\right)}{\left(s^2+a^2\right)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

Big Picture of 4473:





- **Bibliography:**

- Signals & Systems, Roberts, Pg 189, Block Diagram Simulation of Differential or Difference Equations
- **Table of Laplace Transform: https://www.pinterest.com/pin/417216352964290194/**