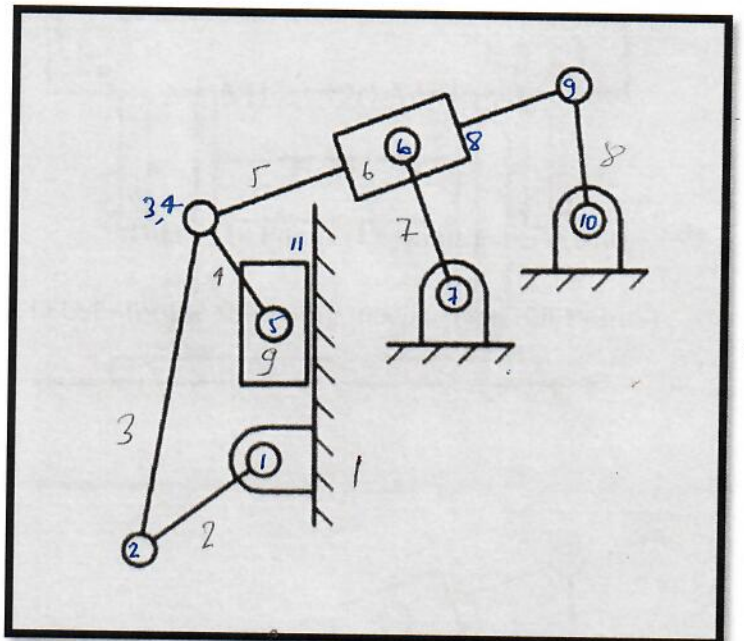
ME 3320 Midterm

Fall 2021

Total: 110 Points (10 points extra credit)

Problem-1: Find the D.O.F. for the following mechanisms **(20 Points)**

a)



$n = 9$

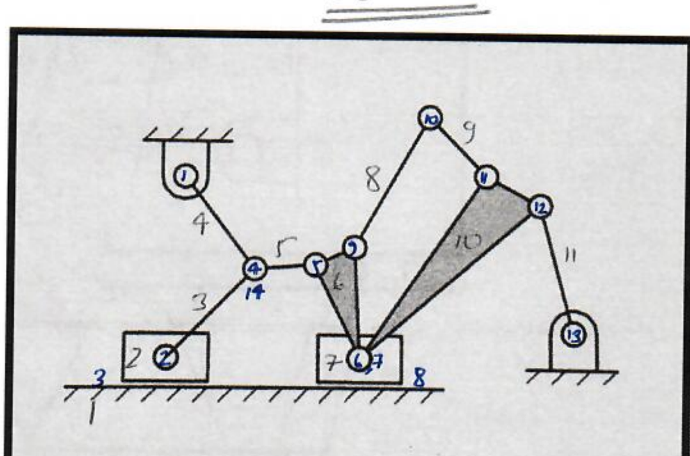
$j = 11$

$f_i = 1$ (all joints)

$$M = 3(n-1) - \sum_{i=1}^{j}(3-f_i) = 3(9-1) - 11(3-1) = 24-22$$

$$\therefore M = 2$$

b)



$n = 11$

$j = 14$

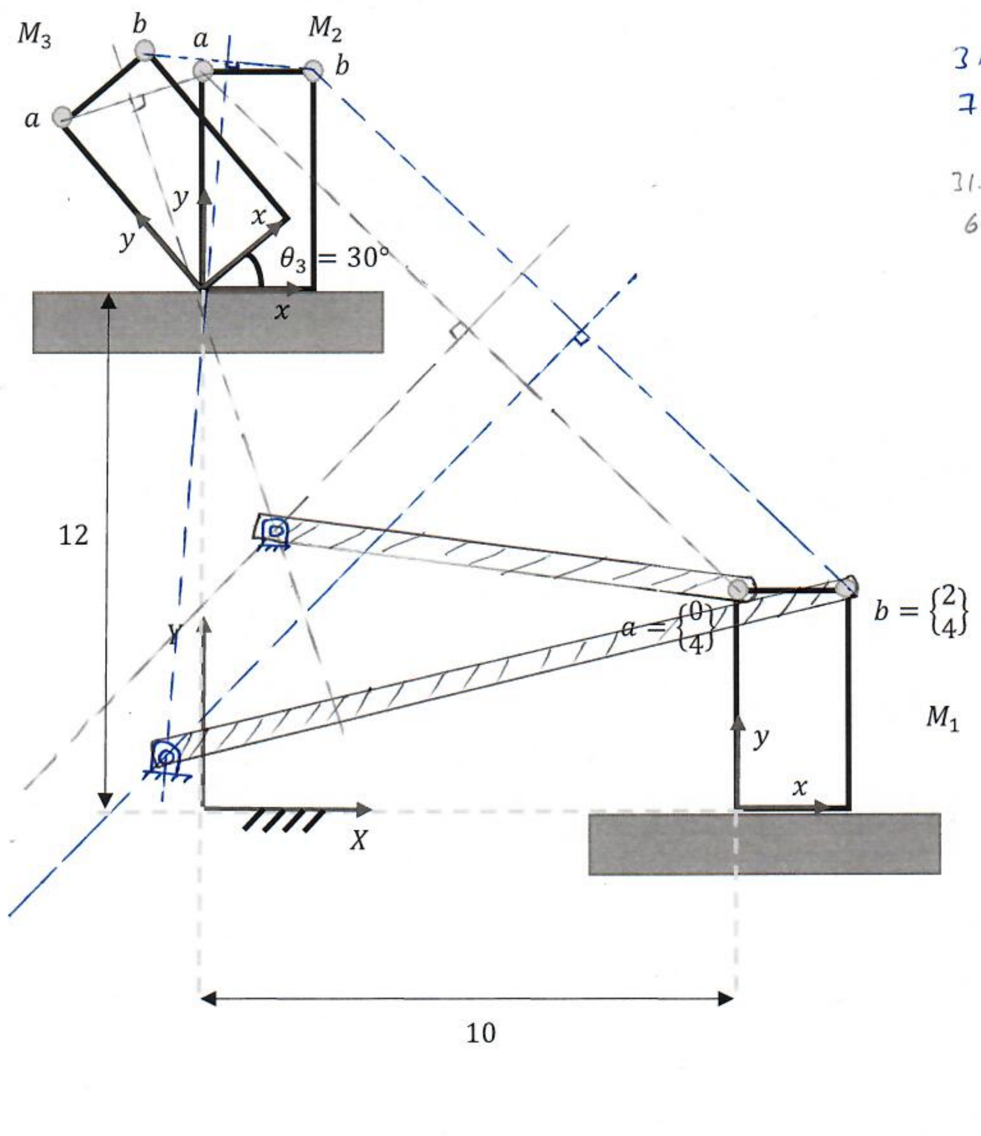
$f_i = 1$ (all joints)

$$M = 3(n-1) - \sum_{i=1}^{j}(3-f_i) = 3(11-1) - 14(3-1) = 30-28$$

$$\therefore M = 2$$

Problem-2: Design a four-bar linkage to move the object through three positions shown in the figure. Using points "a" and "b" on the object for moving pivot. **(40 Points)**

a) Graphical synthesis in the plane **(20 Points)**
b) Algebraic synthesis in the plane (**Only** find pivot "O" based on moving pivot "a") **(20 Points)**



31 steps
7 steps

31.5 steps
6.5 steps

2) (b)

$$\bar{a} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$d_1 = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \quad R[\theta_1 = 0^\circ] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$d_2 = \begin{bmatrix} 0 \\ 12 \end{bmatrix} \quad R(\theta_2 = 0^\circ) = \begin{bmatrix} \cos 0 & -\sin 0 \\ \sin 0 & \cos 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$d_3 = \begin{bmatrix} 0 \\ 12 \end{bmatrix} \quad R(\theta_3 = 30^\circ) = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\bar{A}_1 = \bar{d}_1 + [R(\theta_1)]\,\bar{a} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$$

$$\bar{A}_2 = \bar{d}_2 + [R(\theta_2)]\,\bar{a} = \begin{bmatrix} 0 \\ 12 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 16 \end{bmatrix}$$

$$\bar{A}_3 = \bar{d}_3 + [R(\theta_3)]\,\bar{a} = \begin{bmatrix} 0 \\ 12 \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \end{bmatrix} + \begin{bmatrix} -2 \\ 2\sqrt{3} \end{bmatrix} = \begin{bmatrix} -2 \\ 12+2\sqrt{3} \end{bmatrix} = \begin{bmatrix} -2 \\ 15.46 \end{bmatrix}$$

It is a 3-point synthesis.
So, we have 2-design equations

$$\bar{A}_2 \cdot \bar{A}_2 - \bar{A}_1 \cdot \bar{A}_1 - 2(\bar{A}_2 - \bar{A}_1)\cdot \bar{O} = 0 \quad \text{— (I)}$$

$$\bar{A}_3 \cdot \bar{A}_3 - \bar{A}_1 \cdot \bar{A}_1 - 2(\bar{A}_3 - \bar{A}_1)\cdot \bar{O} = 0 \quad \text{— (II)}$$

$$\vec{O} = \begin{bmatrix} O_x \\ O_y \end{bmatrix}$$

$$\bar{A}_2 \cdot \bar{A}_2 = \begin{bmatrix} 0 \\ 16 \end{bmatrix}\begin{bmatrix} 0 \\ 16 \end{bmatrix} = 16^2 + 0 = 256$$

$$\bar{A}_1 \cdot \bar{A}_1 = \begin{bmatrix} 10 \\ 4 \end{bmatrix}\begin{bmatrix} 10 \\ 4 \end{bmatrix} = 100 + 16 = 116$$

$$\bar{A}_3 \cdot \bar{A}_3 = \begin{bmatrix} -2 \\ 15.46 \end{bmatrix}\begin{bmatrix} -2 \\ 15.46 \end{bmatrix} = 243.14$$

$$\bar{A}_2 - \bar{A}_1 = \begin{bmatrix} 0 \\ 16 \end{bmatrix} - \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} -10 \\ 14 \end{bmatrix}$$

$$\bar{A}_3 - \bar{A}_1 = \begin{bmatrix} -2 \\ 15.46 \end{bmatrix} - \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} -12 \\ 11.46 \end{bmatrix}$$

Take (I),

$$256 - 116 - 2(-10O_x + 14O_y) = 0$$

$$\boxed{\therefore -10O_x + 12O_y = 70} \quad \text{— (A)}$$

Take (II),

$$243.14 - 116 - 2(-12O_x + 11.46O_y) = 0$$

$$\boxed{\therefore -12O_x + 11.46O_y = 63.57} \quad \text{— (B)}$$

Solving (A) & (B),

$$\boxed{\begin{aligned} O_x &= 1.34 \\ O_y &= 6.95 \end{aligned}}$$

Problem-3: (50 Points)

Follower displacement function: Design a displacement function.

The follower must:

- Dwell at y = 2 cm for 90°
- Rise 3 cm for 45° with continuous velocity → 2rd order Polynomial with 4 BC.
- Dwell at y = 5 cm from 135° to 225°
- Return (fall) to y = 2 cm from 225° to 270° with continuous displacement → 1st order polynomial with 2 B.C.
- Dwell for the remaining 90° of cam rotation.

a. Write the boundary conditions and choose the degree of the polynomial to satisfy them.

Solve for the coefficients of the polynomial for the **rise. (10 Points)**

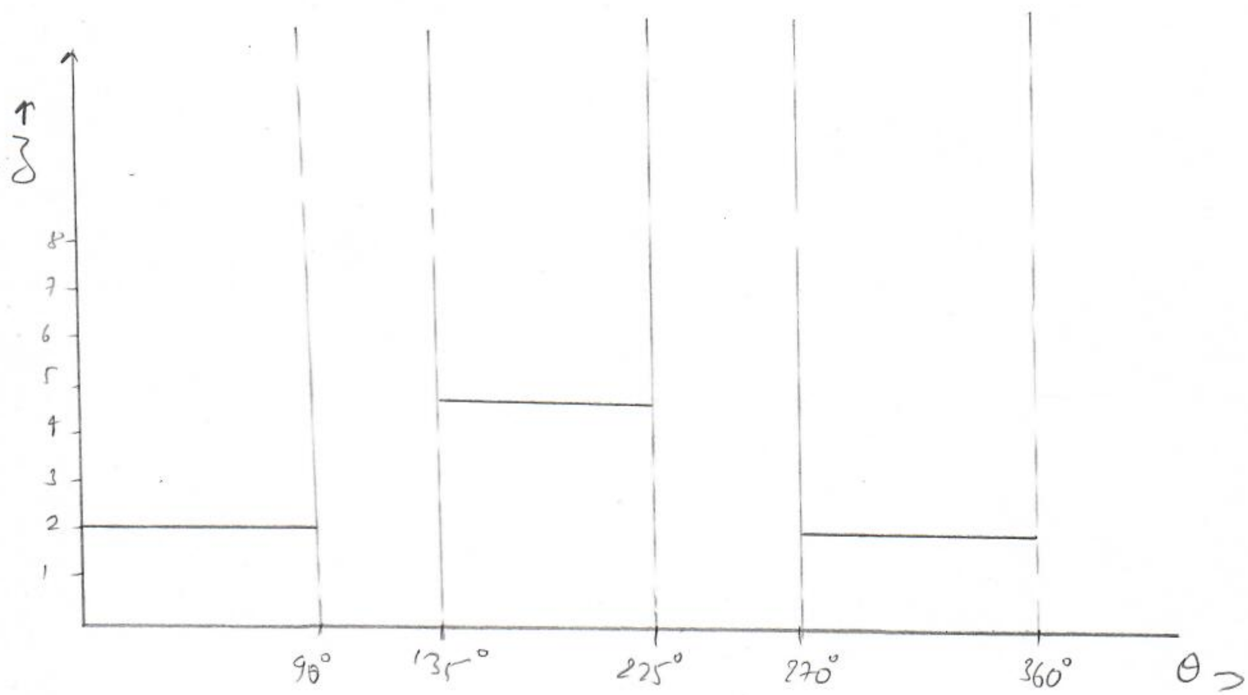
b. Write the boundary conditions and choose the degree of the polynomial to satisfy them.

Solve for the coefficients of the polynomial for the **return. (10 Points)**

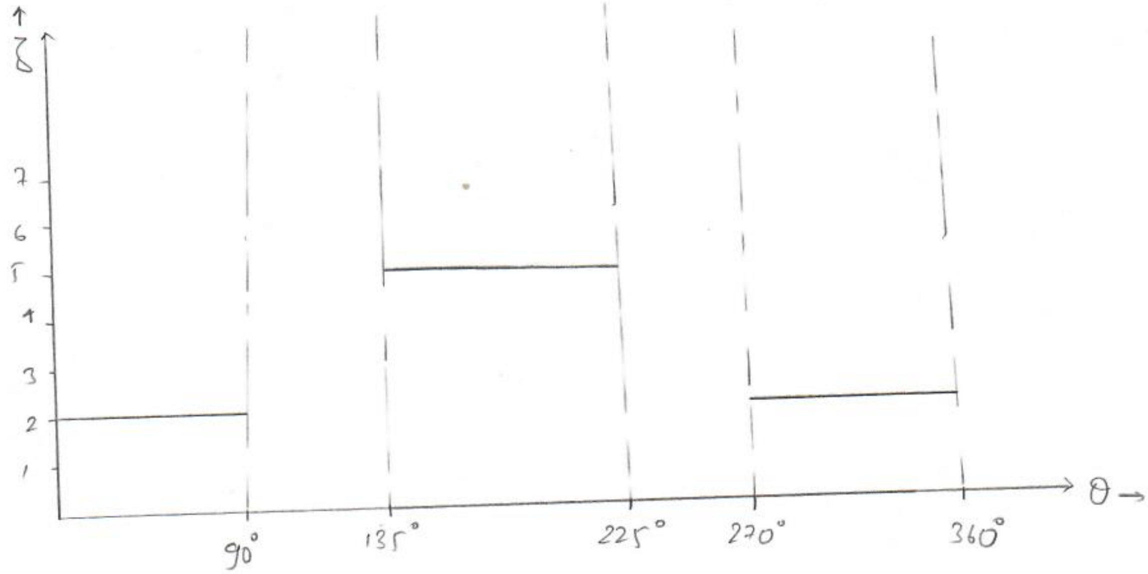
c. Write the equations of $y = y(\theta)$ for each section of the displacement function. **(10 Points)**

d. Write the equations of the velocity, acceleration and jerk as a function of θ and the constant angular velocity of the cam, ω. **(10 Points)**

e. Plot the displacement, velocity, acceleration, and jerk functions (with hand). **(10 Points)**



3 :



(a) Rise :

$\theta_1 \le \theta < \theta_2 \Rightarrow$

$\theta_1 = 90°;\ \theta_2 = 135°$

$$\left.\begin{aligned}\zeta(\theta_1) &= 2\\ \zeta(\theta_2°) &= 5\end{aligned}\right\}-(A) \qquad \left.\begin{aligned}v(\theta_1) &= 0\\ v(\theta_2°) &= 0\end{aligned}\right\}-(B)$$

For continuous velocity we need 3rd order polynomial
↳ We need 4 Boundary Condition

$$\zeta(\theta) = C_0 + C_1\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) + C_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^2 + C_3\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^3;\ a(\theta) = \left[\frac{2C_2}{(\theta_2-\theta_1)^2} + \frac{6C_3(\theta-\theta_1)}{(\theta_2-\theta_1)^3}\right]\dot{\theta}^2$$

$$v(\theta) = \left[\frac{C_1}{\theta_2-\theta_1} + \frac{2C_2(\theta-\theta_1)}{(\theta_2-\theta_1)^2} + \frac{3C_3(\theta-\theta_1)^2}{(\theta_2-\theta_1)^3}\right]\dot{\theta};\ j(\theta) = \left[\frac{6C_3}{(\theta_2-\theta_1)^3}\right]\dot{\theta}^3$$

Imposing Boundary Condition,

(A)

$$\zeta(\theta_1) = C_0 + C_1\left(\frac{\cancel{\theta_1-\theta_1}}{\theta_2-\theta_1}\right)^0 + C_2\left(\frac{\cancel{\theta_1-\theta_1}}{\theta_2-\theta_1}\right)^2 + C_3\left(\frac{\cancel{\theta_1-\theta_1}}{\theta_2-\theta_1}\right)^3$$

$$\zeta(\theta_1) = C_0$$

$$\boxed{\therefore C_0 = 2}$$

$$\zeta(\theta_2) = C_0 + C_1\left(\frac{\cancel{\theta_2-\theta_1}}{\cancel{\theta_2-\theta_1}}\right)^1 + C_2\left(\frac{\cancel{\theta_2-\theta_1}}{\cancel{\theta_2-\theta_1}}\right)^2 + C_3\left(\frac{\cancel{\theta_2-\theta_1}}{\cancel{\theta_2-\theta_1}}\right)^3$$

$$\cancel{\zeta(\theta_2)}^{5} = \cancel{C_0}^{2} + C_1 + C_2 + C_3$$

$$\boxed{\therefore C_1 + C_2 + C_3 = 3} \text{ ——— (I)}$$

(B)

$$v(\theta_1) = \left[\frac{C_1}{\theta_2-\theta_1} + \frac{2C_2\cancel{(\theta_1-\theta_1)}}{(\theta_2-\theta_1)^2} + \frac{3C_3\cancel{(\theta_1-\theta_1)}^2}{(\theta_2-\theta_1)^3}\right]\dot{\theta}$$

$$\cancel{v(\theta_1)}^0 = \frac{C_1}{\theta_2-\theta_1}\ \dot{\theta};\ \dot{\theta} \neq 0$$

$$\boxed{\therefore C_1 = 0}$$

$$v(\theta_2) = \left[\frac{C_1}{\theta_2-\theta_1} + \frac{2C_2\cancel{(\theta_2-\theta_1)}}{(\theta_2-\theta_1)^{\cancel{2}}} + \frac{3C_3\cancel{(\theta_2-\theta_1)^2}}{(\theta_2-\theta_1)^{\cancel{3}}}\right]\dot{\theta}$$

$$\therefore \cancel{v(\theta_2)}^0 = \left[\frac{\cancel{C_1}^0 + 2C_2 + 3C_3}{\theta_2-\theta_1}\right]\dot{\theta},\ \dot{\theta} \neq 0$$

$$\boxed{\therefore 2C_2 + 3C_3 = 0} \text{ ——— (II)}$$

Take (I) & (II)

$C_2 + C_3 = 3$

$2C_2 + 3C_3 = 0$

$$\boxed{\therefore C_2 = 9;\ C_3 = -6}$$

$$\zeta(\theta) = 2 + 9\left(\frac{\theta-90}{135-90}\right)^2 - 6\left(\frac{\theta-90}{135-90}\right)^3;\ a(\theta) = \left[\frac{18}{(135-90)^2} - \frac{36(\theta-90)}{(135-90)^3}\right]\dot{\theta}^2$$

$$v(\theta) = \frac{18(\theta-90)}{(135-90)^2} - \frac{18(\theta-90)^2}{(135-90)^3};\ j(\theta) = \left[\frac{-36}{(135-90)^3}\right]\dot{\theta}^3$$

(b) Return:

For continuous displacement

We need 1^{st} order polynomial

↳ 2 Boundary Conditions

$\theta_1 \leq \theta < \theta_2$ $\Rightarrow$ $s(\theta_1) = 5$

$\theta_1 = 225^\circ; \theta_2 = 270^\circ$ $s(\theta_2) = 2$

$$s(\theta) = C_0 + C_1 \frac{(\theta - \theta_1)}{(\theta_2 - \theta_1)}$$

$$v(\theta) = \left(\frac{C_1}{\theta_2 - \theta_1}\right)\dot{\theta}$$

$$a(\theta) = 0$$

Imposing BC

$$\cancel{s(\theta_1)}^{5} = C_0 + C_1 \frac{\cancel{(\theta_1 - \theta_1)}}{(\theta_2 - \theta_1)} \rightarrow 0 \qquad \cancel{s(\theta_2)}^{2} = C_0 + C_1 \frac{\cancel{(\theta_2 - \theta_1)}}{\cancel{(\theta_2 - \theta_1)}} \rightarrow 1$$

$$\boxed{\therefore C_0 = 5}$$

$$2 = 5 + C_1$$

$$\boxed{\therefore C_1 = -3}$$

$$\boxed{\begin{aligned} s(\theta) &= 5 - 3\frac{(\theta - 225)}{(270 - 225)} \\ v(\theta) &= \left(\frac{-3}{(270-225)}\right)\dot{\theta} \\ a(\theta) &= 0 \\ j(\theta) &= 0 \end{aligned}}$$

(c)

$$s(\theta) = \begin{cases} 2 & 0^\circ \leq \theta < 90^\circ \\ 2 + 9\left(\frac{\theta - 90}{135 - 90}\right)^2 - 6\left(\frac{\theta - 90}{135 - 90}\right)^3 & 90^\circ \leq \theta < 135^\circ \\ 5 & 135 \leq \theta < 225^\circ \\ 5 - 3\frac{(\theta - 225)}{(270 - 225)} & 225^\circ \leq \theta < 270^\circ \\ 2 & 270^\circ \leq \theta < 360^\circ \end{cases}$$

(d)

$$v(\theta)=\begin{cases} 0 & 0^\circ\leq\theta<90^\circ \\ \left[\dfrac{18(\theta-90)}{(135-90)^2}-\dfrac{18(\theta-90)^2}{(135-90)^3}\right]\dot{\theta} & 90^\circ\leq\theta<135^\circ \\ 0 & 135^\circ\leq\theta<225^\circ \\ \left(\dfrac{-3}{(270-225)}\right)\dot{\theta} & 225^\circ\leq\theta<270^\circ \\ 0 & 270^\circ\leq\theta<360^\circ \end{cases}$$

$$a(\theta)=\begin{cases} 0 & 0^\circ\leq\theta<90^\circ \\ \left[\dfrac{18}{(135-90)^2}-\dfrac{36(\theta-90)}{(135-90)^3}\right]\dot{\theta}^2 & 90^\circ\leq\theta<135^\circ \\ 0 & 135^\circ\leq\theta<225^\circ \\ 0 & 225^\circ\leq\theta<270^\circ \\ 0 & 270^\circ\leq\theta<360^\circ \end{cases}$$

$$j(\theta)=\begin{cases} 0 & 0^\circ\leq\theta<90^\circ \\ \left[\dfrac{-36}{(135-90)^3}\right]\dot{\theta}^3 & 90^\circ\leq\theta<135^\circ \\ 0 & 135^\circ\leq\theta<225^\circ \\ 0 & 225^\circ\leq\theta<270^\circ \\ 0 & 270^\circ\leq\theta<360^\circ \end{cases}$$

(e)