Kinematics & Dynamics of Machinery (ME 3320)

Recitation - 3

GTA: Shishir Khanal, khanshis@isu.edu

---

1. **Agenda:**
   - Revision(Linkage, Mobility)
   - Problems(Mobility)

2. **Revision:**
   - **The equation to determine the number of independent loops is:**

     $L = j - n + 1$
   - **What is the link length condition for the shortest link of 4-bar linkage to fully rotate?**

     $s + l < p + q$
   - **What is the condition to form a foldable linkage?**

     $s + l = p + q$
   - **What does it mean if we get the boundary value of the input angle as a complex value?**
     4-bar link cannot produce that theta angle
     - **What can be done in such a situation?**
       If possible, reduce the length of one of the linkage parameters (as will be seen in the example later)
   - **For a given triangle below, write the law of sines and cosines.**

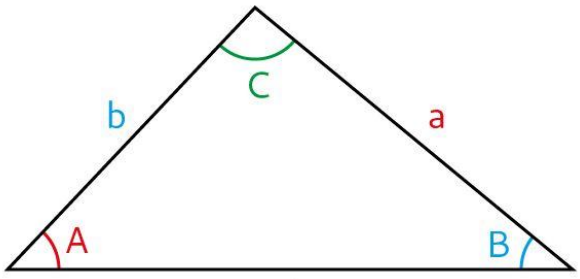
Law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c},$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines:

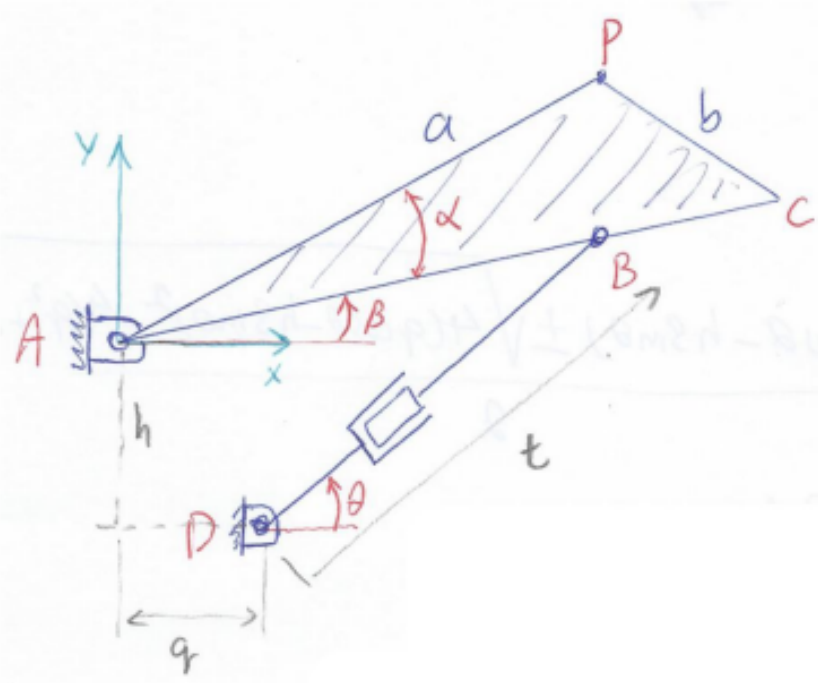
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



- **Match the following:**
  (Related to the Door Mechanism Problem)

Distance Constraint	->	t(θ)
Loop Equation	->	β(θ)
Law of Cosines	->	θ
Derivative of Position	->	Velocity
Second Derivation of Position	->	Acceleration

3. **Problems:**
    - **The linkage shown below is a kinematic sketch of a closing door mechanism with given dimensions. The acceptable value of the prismatic joint is 5 < t < 15.**



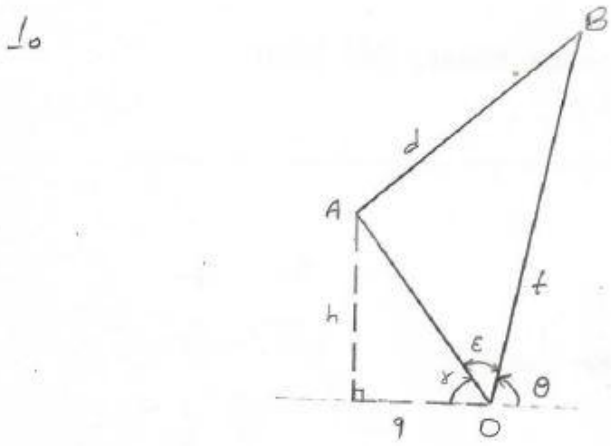
q = 3; h = 4; a = 10; d =|AB| = 8; $\alpha = 30^0$

M = 1; A =<0,0>; B = <q+t*cos(θ),-h+t*sin(θ)> = <d*cos(β), d*sin(β)>; D = <q,-h>; d = |AB|

$t(\theta) = \{-2(q*\cos(\theta) - h*\sin(\theta)) \pm \sqrt{4*(q*\cos(\theta) - h*\sin(\theta))^2 - 4(q^2 + h^2 - d^2)}\} / 2$

$\beta(\theta) = \tan^{-1}((-h + t*\sin(\theta)) / (q + t*\cos(\theta)))$

1)Using triangle ABD and cosine law, compute the limits of angle θ for the limits of the prismatic joint t



$\theta + \gamma + \varepsilon = \pi$

$\therefore \theta = \pi - \gamma - \varepsilon$

For γ;

$\tan(\gamma) = \frac{h}{q}$

$\therefore \gamma = \tan^{-1}\left(\frac{h}{q}\right)$

For ε,

$AB^2 = BO^2 + AO^2 - 2(BO)(AO)\cos(\varepsilon)$

$|AB| = d,\ |BO| = t;\ |AO| = \sqrt{h^2+q^2}$

$d^2 = t^2 + h^2 + q^2 - 2t\sqrt{h^2+q^2}\cos\varepsilon$

$\cos\varepsilon = \frac{t^2+h^2+q^2-d^2}{2t\sqrt{h^2+q^2}}$

$\therefore \varepsilon = \cos^{-1}\left(\frac{t^2+h^2+q^2-d^2}{2t\sqrt{h^2+q^2}}\right)$

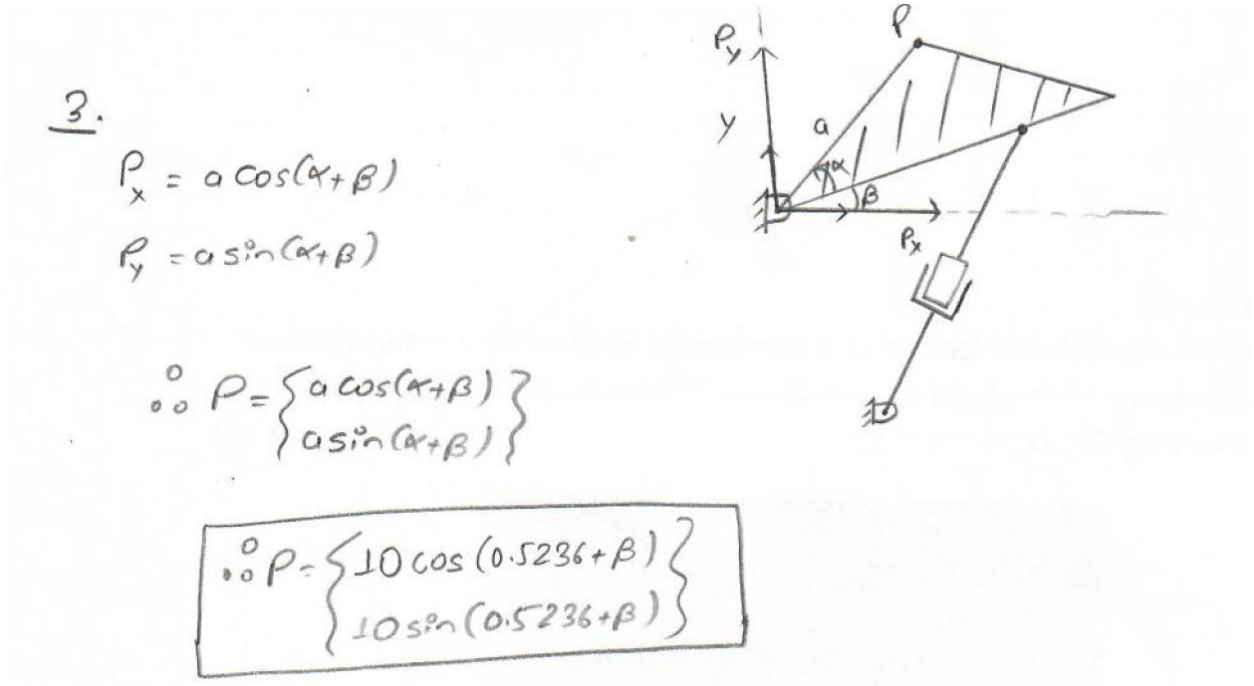
$$\theta = \pi - \tan^{-1}\left(\frac{h}{q}\right) - \cos^{-1}\left(\frac{t^2+h^2+q^2-d^2}{2t\sqrt{h^2+q^2}}\right)$$

From Matlab

A real 'θ' do not exist $\forall\ 5 < t < 13$.

$\forall\ 5 \le t \le 13 \rightarrow \quad 20.6^\circ \le \theta \le 126.9^\circ$

2) Find the position vector of point P and plot it versus the min and max of θ.



3) Matlab:

3.1 Use Matlab to plot $t(\theta)$ versus $0 < \theta < 2\pi$. Highlight part of the plot that is corresponding to $\theta_{min} < \theta < \theta_{max}$. Do the same for $\beta(\theta)$.

(Matlab Code: evaluate_t.m)

3.2 Plot the position vector P for the allowable values of theta.

(Matlab Code: evaluate_t.m)

Bibliography:

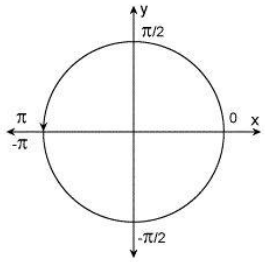
- **Dr. Hedari's HW 2**
- **Dr. Deemyad's Notes**

Miscellaneous:

- Meaning of atan2(From Matlab Documentation):

Four-Quadrant Inverse Tangent

The four-quadrant inverse tangent, `atan2(Y,X)`, returns values in the closed interval `[-pi,pi]` based on the values of `Y` and `X`, as shown in the graphic.



In contrast, `atan(Y/X)` returns results that are limited to the interval `[-pi/2,pi/2]`, shown on the right side of the diagram.

IEEE Compliance

For real inputs, `atan2` has a few behaviors that differ from those recommended in the IEEE®-754 Standard.

	MATLAB®	IEEE
`atan2(0,-0)`	`0`	`pi`
`atan2(-0,-0)`	`0`	`-pi`

- Why is it a good idea to construct more functions while composing a code script?
  - It is, in general, a good idea to write functions than hand computations as they are prone to human error.
  - Functions are separately implementable working pieces of a code. Hence, they form reusable blocks for future projects.
  - Usage of functions breaks the code into sections and allows adding more comments. Hence, the code becomes more readable.

- Are there any other code formatting techniques?
  - Finite-State Machines (Used in Mechatronics)
  - Object-Oriented Programming(Abstraction, Encapsulation, Inheritance, Polymorphism)