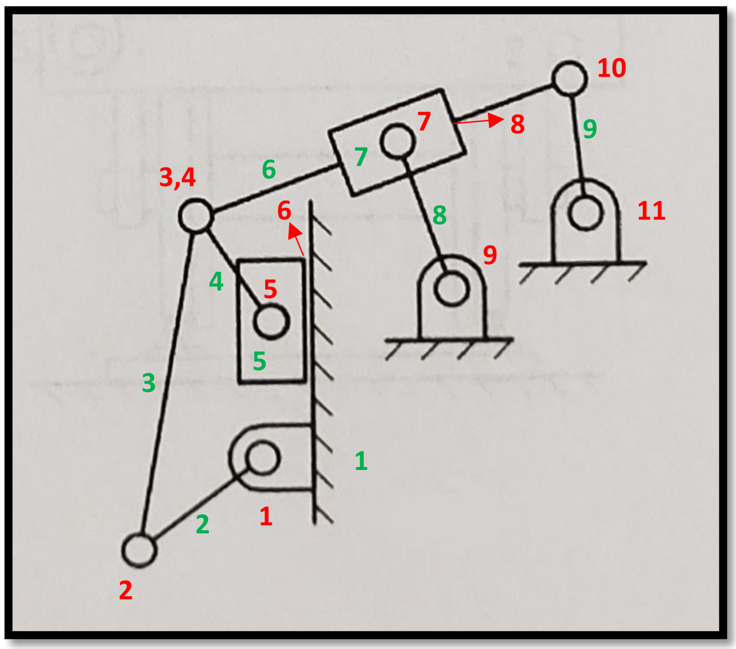
ME 3320 Midterm

Fall 2021

Total: 110 Points (10 points extra credit)

Problem-1: Find the D.O.F. for the following mechanisms **(20 Points)**

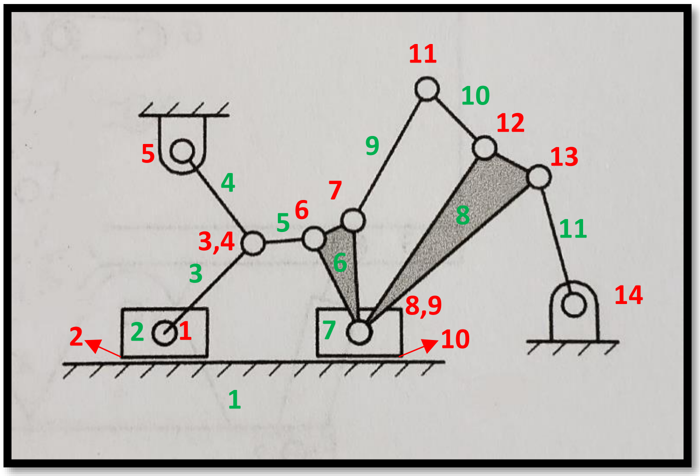
a)



n = 9 , j = 11 , $f_i = 1$ (all joints)

$$M = 3(n-1) - \sum_{i=1}^{j}(3 - f_i) = 3 \times (9-1) - 11 \times (3-1) = 24 - 22 = 2$$

b)



n = 11 , j = 14 , $f_i = 1$ (all joints)

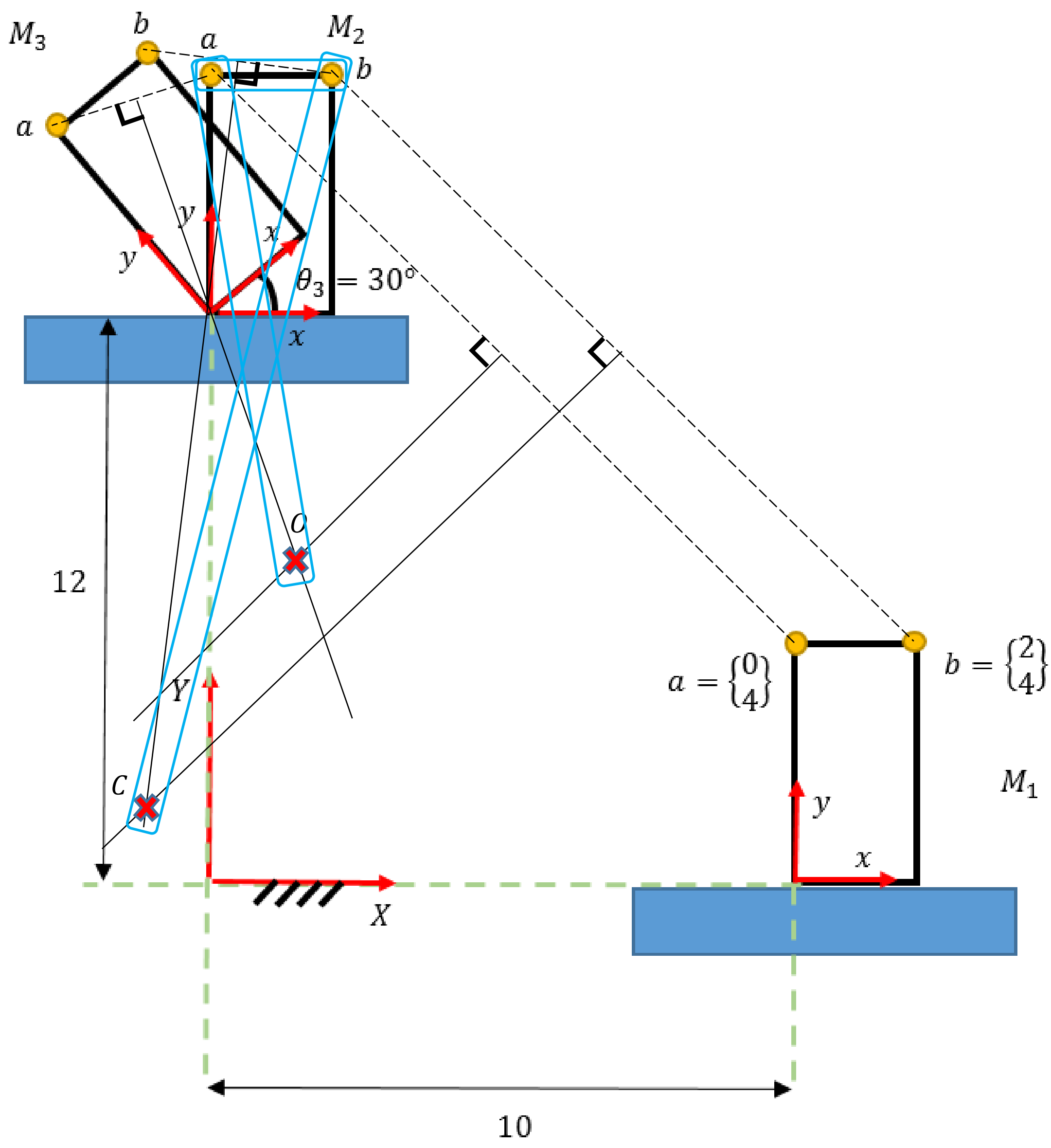
$$M = 3(n-1) - \sum_{i=1}^{j}(3 - f_i) = 3 \times (11-1) - 14 \times (3-1) = 30 - 28 = 2$$

Problem-2: Design a four-bar linkage to move the object through three positions shown in the figure. Using points “a” and “b” on the object for moving pivot. **(40 Points)**

a) Graphical synthesis in the plane **(20 Points)**
b) Algebraic synthesis in the plane (**Only** find pivot “O” based on moving pivot “a”) **(20 Points)**

Solution

a)



M_3
b
a
M_2
b
a
y
y
x
$\theta_3 = 30°$
x
O
12
Y
C
$a = \begin{Bmatrix} 0 \\ 4 \end{Bmatrix}$
$b = \begin{Bmatrix} 2 \\ 4 \end{Bmatrix}$
M_1
y
x
X
10

b)

$$a = \begin{Bmatrix} 0 \\ 4 \end{Bmatrix}$$

$\bar{d}_1 = \begin{Bmatrix} 10 \\ 0 \end{Bmatrix}$, $[R(\theta_1)] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\theta_1 = 0°)$

$\bar{d}_2 = \begin{Bmatrix} 0 \\ 12 \end{Bmatrix}$, $[R(\theta_2)] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\theta_2 = 0°)$

$\bar{d}_3 = \begin{Bmatrix} 0 \\ 12 \end{Bmatrix}$, $[R(\theta_3)] = \begin{bmatrix} \cos(30°) & -\sin(30°) \\ \sin(30°) & \cos(30°) \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ $(\theta_3 = 30°)$

$$\bar{A}_1 = \bar{d}_1 + [R(\theta_1)]\bar{a} = \begin{Bmatrix} 10 \\ 0 \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 4 \end{Bmatrix}$$

$$\bar{A}_2 = \bar{d}_2 + [R(\theta_2)]\bar{a} = \begin{Bmatrix} 0 \\ 12 \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 16 \end{Bmatrix}$$

$$\bar{A}_3 = \bar{d}_3 + [R(\theta_3)]\bar{a} = \begin{Bmatrix} 0 \\ 12 \end{Bmatrix} + \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{Bmatrix} 0 \\ 4 \end{Bmatrix} = \begin{Bmatrix} -2 \\ 12 + 2\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} -2 \\ 15.46 \end{Bmatrix}$$

$\bar{A}_1.\bar{A}_1 = 116$ $\bar{A}_2.\bar{A}_2 = 256$ $\bar{A}_3.\bar{A}_3 = 243.14$

Write two design equations:

Eq1: $\bar{A}_2.\bar{A}_2 - \bar{A}_1.\bar{A}_1 - 2(\bar{A}_2 - \bar{A}_1).\bar{O} = 256 - 116 - 2\left(-10\, O_x + 12\, O_y\right) = 0$

Eq2: $\bar{A}_3.\bar{A}_3 - \bar{A}_1.\bar{A}_1 - 2(\bar{A}_3 - \bar{A}_1).\bar{O} = 243.14 - 116 - 2(-12O_x + 11.46O_y) = 0$

$$-10\, O_x + 12\, O_y = 70$$
$$-12O_x + 11.46O_y = 63.57$$

$$\boxed{\begin{aligned} O_y &= 6.95 \\ O_x &= 1.34 \end{aligned}}$$

Problem-3: (50 Points)

Follower displacement function: Design a displacement function.

The follower must:

- Dwell at y = 2 cm for 90°

- Rise 3 cm for 45° with continuous velocity

- Dwell at y = 5 cm from 135° to 225°

- Return (fall) to y = 2 cm from 225° to 270° with continuous displacement

- Dwell for the remaining 90° of cam rotation.

a. Write the boundary conditions and choose the degree of the polynomial to satisfy them.

Solve for the coefficients of the polynomial for the **rise**. **(10 Points)**

b. Write the boundary conditions and choose the degree of the polynomial to satisfy them.

Solve for the coefficients of the polynomial for the **return**. **(10 Points)**

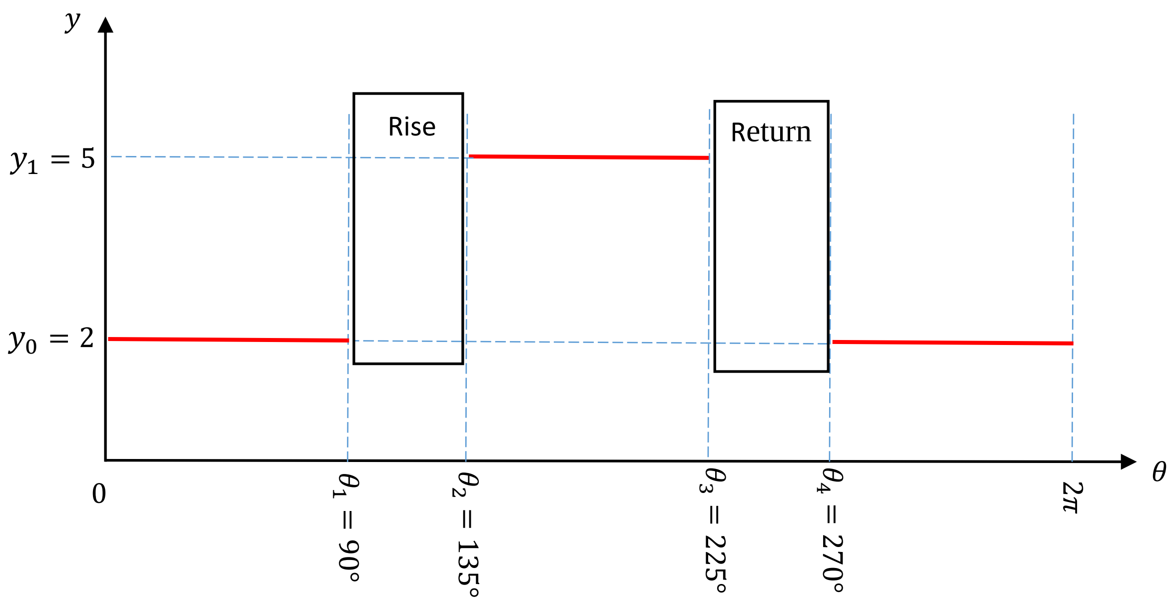
c. Write the equations of $y = y(\theta)$ for each section of the displacement function. **(10 Points)**

d. Write the equations of the velocity, acceleration and jerk as a function of θ and the constant angular velocity of the cam, ω. **(10 Points)**

e. Plot the displacement, velocity, acceleration, and jerk functions (with hand). **(10 Points)**

Solution

a)



For rise: Continuous velocity $\Rightarrow$ 3rd degree polynomial

$$y(\theta) = c_0 + c_1 \frac{\theta - \theta_1}{\theta_2 - \theta_1} + c_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right)^2 + c_3 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right)^3$$

$$v(\theta) = \left(\frac{c_1}{\theta_2 - \theta_1} + \frac{2\, c_2(\theta - \theta_1)}{(\theta_2 - \theta_1)^2} + \frac{3\, c_3(\theta - \theta_1)^2}{(\theta_2 - \theta_1)^3}\right)\dot{\theta}$$

$$\left.\begin{array}{r} y(\theta_1) = y_0 \\ y(\theta_2) = y_1 \\ v(\theta_1) = 0 \\ v(\theta_2) = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y(90°) = 2 \\ y(135°) = 5 \\ v(90°) = 0 \\ v(135°) = 0 \end{array}\right.$$

$$y(\theta) = c_0 + c_1 \frac{\theta - 90}{135 - 90} + c_2 \left(\frac{\theta - 90}{135 - 90}\right)^2 + c_3 \left(\frac{\theta - 90}{135 - 90}\right)^3$$

$$v(\theta) = \left(\frac{c_1}{135 - 90} + \frac{2\, c_2(\theta - 90)}{(135 - 90)^2} + \frac{3\, c_3(\theta - 90)^2}{(135 - 90)^3}\right)\dot{\theta}$$

$$y(90) = c_0 + c_1 \cancelto{0}{\frac{90-90}{135-90}} + c_2 \cancelto{0}{\left(\frac{90-90}{135-90}\right)^2} + c_3 \cancelto{0}{\left(\frac{90-90}{135-90}\right)^3} = 2$$

$$\boxed{c_0 = 2}$$

$$y(135) = c_0 + c_1 \cancelto{1}{\frac{135-90}{135-90}} + c_2 \cancelto{1}{\left(\frac{135-90}{135-90}\right)^2} + c_3 \cancelto{1}{\left(\frac{135-90}{135-90}\right)^3} = 5$$

$$c_0 + c_1 + c_2 + c_3 = 5 \Rightarrow c_1 + c_2 + c_3 = 3$$

$$v(90) = \left(\frac{c_1}{135-90} + \cancelto{0}{\frac{2\,c_2(90-90)}{(135-90)^2}} + \cancelto{0}{\frac{3\,c_3(90-90)^2}{(135-90)^3}}\right)\dot{\theta} = 0$$

$$\frac{c_1}{135-90}\dot{\theta} = 0 \Rightarrow \boxed{c_1 = 0}$$

$$v(90) = \left(\frac{2\,c_2\cancel{(135-90)}}{(135-90)^{\cancel{2}}} + \frac{3\,c_3(135-90)^{\cancel{2}}}{(135-90)^{\cancel{3}}}\right)\dot{\theta} = 0$$

$$\frac{2\,c_2 + 3\,c_3}{(135-90)}\dot{\theta} = 0 \Rightarrow 2\,c_2 + 3\,c_3 = 0 \Rightarrow c_2 = -\frac{3}{2}\,c_3$$

$$c_2 + c_3 = -\frac{3}{2}\,c_3 + c_3 = -0.5\,c_3 = 3 \Rightarrow \boxed{c_3 = -6} \Rightarrow \boxed{c_2 = 9}$$

$$y(\theta) = 2 + 9\left(\frac{\theta-90}{135-90}\right)^2 - 6\left(\frac{\theta-90}{135-90}\right)^3$$

$$v(\theta) = \left(\frac{18(\theta-90)}{(135-90)^2} - \frac{18(\theta-90)^2}{(135-90)^3}\right)\dot{\theta}$$

$$v(\theta) = \left(\frac{18(\theta-90)}{(135-90)^2} - \frac{18(\theta-90)^2}{(135-90)^3}\right)\dot{\theta}$$

$$a(\theta) = \left(\frac{18}{(135-90)^2} - \frac{36(\theta-90)}{(135-90)^3}\right)\dot{\theta}^2$$

$$j(\theta) = \left(-\frac{36}{(135-90)^3}\right)\dot{\theta}^3$$

b)

For return: Continuous displacement $\Rightarrow$ 1st degree polynomial

$$y(\theta) = k_0 + k_1\frac{\theta-\theta_3}{\theta_4-\theta_3}$$

$$\left.\begin{array}{l} y(\theta_3) = y_1 \\ y(\theta_4) = y_0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y(225°) = 5 \\ y(270°) = 2 \end{array}\right.$$

$$y(\theta) = k_0 + k_1\frac{\theta-225}{270-225}$$

$$y(225) = k_0 + k_1\cancelto{0}{\frac{225-225}{270-225}} = 5$$

$$\boxed{k_0 = 5}$$

$$y(270) = k_0 + k_1\cancelto{1}{\frac{270-225}{270-225}} = 2$$

$$k_0 + k_1 = 5 + k_1 = 2 \Rightarrow \boxed{k_1 = -3}$$

$$y(\theta) = 5 - 3\frac{\theta-225}{270-225}$$

$$v(\theta) = -\frac{3}{(270-225)}\dot{\theta}$$

c)

$$y(\theta)=\begin{cases} 2 & 0\le\theta<90 \\ 2+9\left(\dfrac{\theta-90}{135-90}\right)^2-6\left(\dfrac{\theta-90}{135-90}\right)^3 & 90\le\theta<135 \\ 5 & 135\le\theta<225 \\ 5-3\dfrac{\theta-225}{270-225} & 225\le\theta<270 \\ 2 & 270\le\theta<2\pi \end{cases}$$

d)

$$v(\theta)=\begin{cases} 0 & 0\le\theta<90 \\ \left(\dfrac{18(\theta-90)}{(135-90)^2}-\dfrac{18(\theta-90)^2}{(135-90)^3}\right)\dot{\theta} & 90\le\theta<135 \\ 0 & 135\le\theta<225 \\ -\dfrac{3}{(270-225)}\dot{\theta} & 225\le\theta<270 \\ 0 & 270\le\theta<2\pi \end{cases}$$

$$a(\theta)=\begin{cases} 0 & 0\le\theta<90 \\ \left(\dfrac{18}{(135-90)^2}-\dfrac{36(\theta-90)}{(135-90)^3}\right)\dot{\theta}^2 & 90\le\theta<135 \\ 0 & 135\le\theta<225 \\ 0 & 225\le\theta<270 \\ 0 & 270\le\theta<2\pi \end{cases}$$

$$j(\theta)=\begin{cases} 0 & 0\le\theta<90 \\ \left(-\dfrac{36}{(135-90)^3}\right)\dot{\theta}^3 & 90\le\theta<135 \\ 0 & 135\le\theta<225 \\ 0 & 225\le\theta<270 \\ 0 & 270\le\theta<2\pi \end{cases}$$

e)