Mechanical Control Systems (ME 4473)

Recitation - 7

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(For the best learning experience, students are encouraged to build their own simulink models as they are being worked out in the class)

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1. **Agenda:**
    - Revision
    - Problems(Discrete Time PID: evolution of output signal with a PID Controller, implementation of PID controllers in the digital electronics)

2. **Questions:**

- **PID:** Proportional Integral Derivative

- **What is the mechanism of a PID controller?**

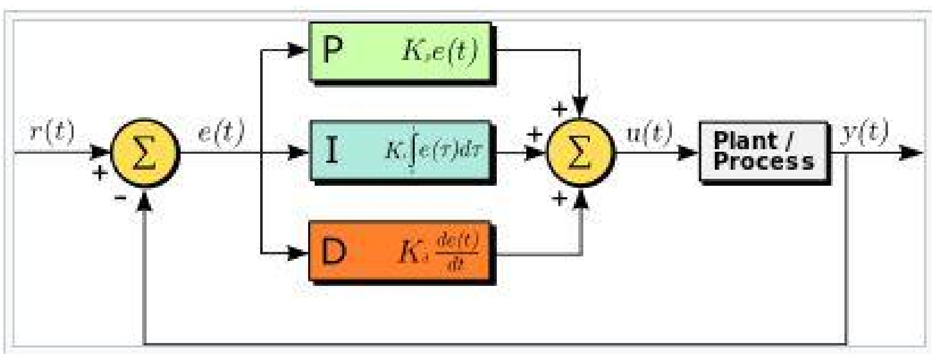
  For a running system, the error function is:

  P: Scaled by Kp

  I : Integrated and then scaled by Ki

  D: Differentiated and then scaled by Kd

  And, each of their resultant is summed. Finally this sum is fed into the plant as an input.



- **A PID controller is a trio of 3 different controllers that compliment each other's shortcomings. Based on the error functions what are the best and worst working conditions for a PID controller?**

	Pros	**Cons**
Proportional Controller	Lower the Error Faster	Overshoot might make the system unstable

Integral Controller	Eliminate Steady state Error	Essentially have Integral windup ; also really slow response
Derivative controller	Resist the errors due to the impulsive forces	Makes the noise components of the signal worse

- **On Lecture 19 and 20, Dr Schoen went in depth over the PID controller tuning techniques. What does it mean to tune a PID Controller?**
  Identify the appropriate values for Kp, Ki and Kd so as to generate an input signal for the plant to produce a desired output.

- **What are the methods used for PID Controller Tuning?**
  1. Model-based Tuning (Design for Disturbance, Set point)
  2. Continuous Cycling Method (Ziegler-Nichols Method)
  3. Matlab Autotuning

3. **Problems:**

Because of the advent of inexpensive Digital electronics, it has become a current trend to design controllers using 'by-wire' controls. These controllers are also more efficient and typically have better performance than the mechanical controllers. However, digital electronics are discrete time systems and hence our continuous time systems need to be interpreted to discrete time domain in order to implement the digital controllers.

A. **For a system defined in continuous time, how do we change it to a discrete time:**

   The differential equation equivalent in the discrete time is called the difference equation. This can be achieved by taking a z-transform of the differential equation.

B. **For a transfer function in matlab defined in continuous-time, how do we change it to discrete time?**

   tf_discrete = c2d(tf_continuous, samplingtime)

C. **What is the relation between s of the Laplace transform and z of the z-transform?**

   z = exp(s*t) where t = sampling time

D. **In the discrete-time, what is the equation for the transfer function of a PID controller?**

Gc(z) = $$Kp + \frac{K_I Tz}{z-1} + K_D \frac{z-1}{Tz}$$ -----------

(13.57)

where , K_p , K_I, and K_D are Tuning Parameters for PID Controller

T = sampling Time

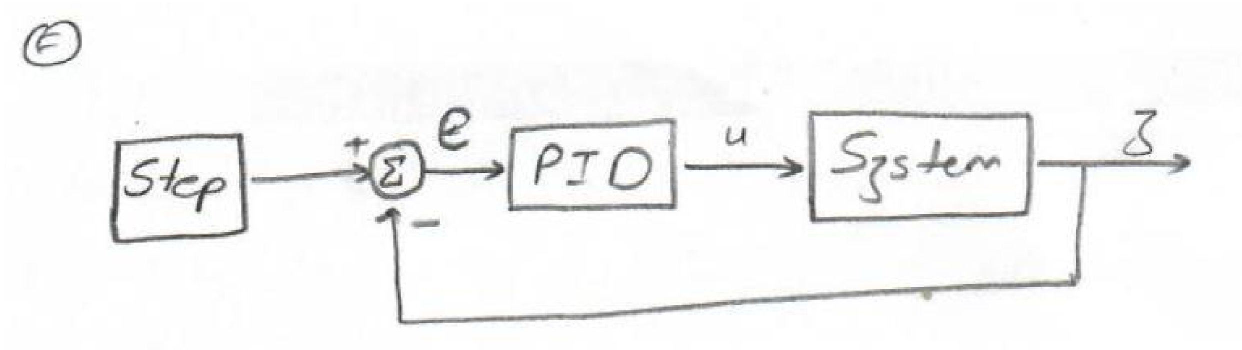
E. **What is the equation for the input signal for the system generated by a discrete PID Controller?**

$$u(k) = K_p * x(k) + K_I[u(k-1) + T*x(k)] + (K_D/T)[x(k) - x(k-1)]$$

x(k) = current error signal
u(k) = current plant input signal
x(k-1) = previous error signal

---------------------- (13.58)

F. **With this information, let's build a discrete-time PID controller for a continuous time system. Consider the following system:**



Let's assume that the sampling rate is 100 Hz. Hence, The sampling time is 0.01 sec.

1. **Discrete Difference Equation: Let's assume that the transfer function for the system is ;**

   System = $\frac{s}{s^2+3s+6}$

   a. **For Ts = 0.01 sec, write the discrete transfer function using Matlab.**

```
dtf =

  0.009851 z - 0.009851
  ---------------------
  z^2 - 1.97 z + 0.9704

Sample time: 0.01 seconds
Discrete-time transfer function
```

   b. **Using this discrete transfer function, evaluate the difference equation for the system:**

1.b.

$$\frac{Y(k)}{X(k)} = \frac{0.009851 z - 0.009851}{z^2 - 1.97 z + 0.9704}$$

$$\frac{Y(k)}{X(k)} = \frac{0.009851 z^{-1} - 0.009851 z^{-2}}{1 - 1.97 z^{-1} + 0.9704 z^{-2}}$$

$$Y(k)\left[1 - 1.97 z^{-1} + 0.9704 z^{-2}\right] = X(k)\left[0.009851 z^{-1} - 0.009851 z^{-2}\right]$$

$$Y(k) - 1.97\, Y(k-1) + 0.9704\, Y(k-2) = 0.009851\, x(k-1) - 0.009851\, x(k-2)$$

$$\boxed{Y(k) = 1.97\, Y(k-1) - 0.9704\, Y(k-2) + 0.009851\left[x(k-1) - x(k-2)\right]}$$

c. **Discrete PID: Assuming Kp = K_I = K_D = 1, write down the transfer function and difference equation for the Discrete PID Controller.**

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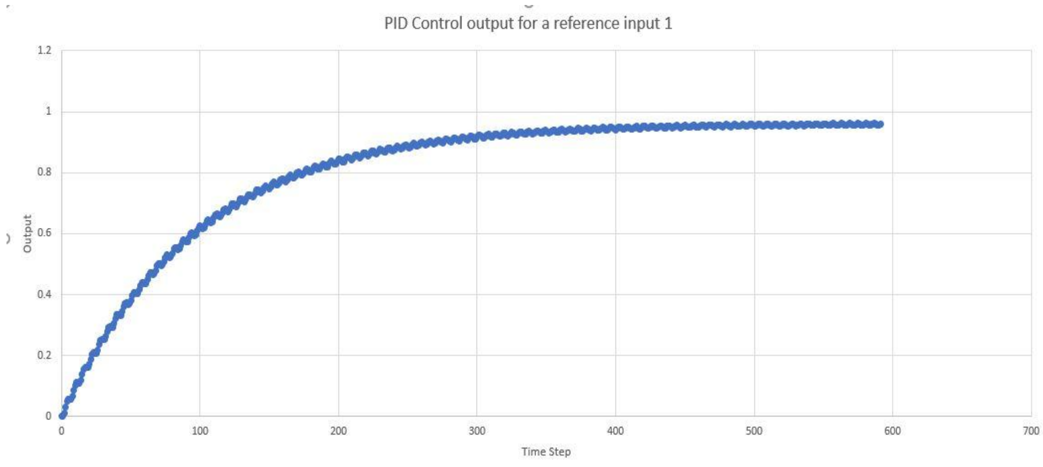
$$\boxed{G_c(z) = 1 + \frac{0.01 z}{z-1} + \frac{z-1}{(0.01) z}}$$

$$u(k) = e(k) + u(k-1) + 0.01\, e(k) + 100\left[e(k) - e(k-1)\right]$$

$$\boxed{u(k) = 101.01\, e(k) - 100\, e(k-1) + u(k-1)}$$

d. **Evolution of the System: Using the excel sheet, evaluate the evolution of the system.**



2. **Now, let's make it a little bit fun. Lets change our system to:**

$$\text{System} = \frac{s}{s^2 - 3s + 6}$$

What did we do here that makes our system interesting?

Changed the system such that there are poles in the right hand plane.

a. **Discrete Time Transfer Function (T = 0.01):**

```
dtf =

  0.01015 z - 0.01015
  -------------------
  z^2 - 2.03 z + 1.03

Sample time: 0.01 seconds
Discrete-time transfer function.
```

3. **Now, let's design a discrete time & Continuous time controller in Simulink.**
   @Simulink.
   1. **Step response comparison with the continuous time model**

If the step response of continuous time and discrete time system are the same (qualitatively), then those transfer functions represent the equivalent system. However, sampling time of the loop should be the same as that for computing discrete transfer function.

2. **PID Controller comparisons at $Kp = K_I = K_D = 1$**

The coefficients are not enough to make the system stable

3. **PID Tuning of Discrete PID using Autotuner**

4. **Copy the Discrete PID coefficients and use it on the continuous PID**

The actual Discrete PID coefficients also have a factor of sampling Time. Also the system becomes stable, the same coefficients in the discrete time and continuous time systems do not produce identical outputs.

5. **In the self tuner, tuning using the response time, transient behavior and controller effort**

Tuning PID is an engineering design process. So, there are no right answers. However, there are some wrong answers:

1. PID parameters that still do not make system stable
2. PID parameters that request the amount of gain from the actuator that the system actuator can't produce

6. Finally, the selected result still has an overshoot. Now, we could see if we could internally modify the damping ratio of the system so that we could reduce the overshoot. However, after every change in the transfer function, autotuning should be done separately.

Conclusion:

1. PID Controllers are awesome.
2. PID controllers are 3 different types of controllers that complement each other.
3. Process of Implementation of a Digital PID to a continuous-time system:
    a. Figure out the Sampling Time
    b. Build the appropriate Control Loop using Discrete functions
    c. For each function imported from the Library, change the sampling time from the default to the one corresponding to your system.
    d. For the simulation:
        i. Convert the Continuous-time transfer function to discrete time transfer function
        ii. Click the ‘Tune’ button inside the Discrete PID Controller to generate parameters for the P, I and D controllers.
        iii. Make sure the output response meets the requirement for the system.
        iv. If the output doesn't meet the requirements, manually change the control parameters until it does.
    e. For the controller implementation:
        i. Remove the transfer function but keep the control loop
        ii. Feed the input signal from controller to the appropriate pin for the actuator to provide the control signal.
        iii. For the output measurement and evaluation of error function, connect your sensor that measures the output to the appropriate pin and feed that information to the scope and the summer.
        iv. You may have to re-tune the controller if the actual system doesn’t provide the same response as the simulated system.

Bibliography:

- **PID Picture: https://en.wikipedia.org/wiki/PID_controller**
- **Simple Examples of PID Control: https://www.youtube.com/watch?v=XfAt6hNV8XM**
- **Modern Control Systems, 13th ed, 13.9 Implementation of Digital Controllers**