Mechanical Control Systems (ME 4473)

Recitation - 5

GTA: Shishir Khanal, khanshis@isu.edu

---

1. **Agenda:**
    - Revision(System Representation, Block Diagram, State Space)

2. **Things we learnt so far is representation of system using:**
    a. **Differential Equations**
        i. **Higher order coupled differential equations**
        ii. **First order system of differential equations**
    b. **Block Diagram representation**
    c. **Transfer Function representation**

Modeling using Differential Equations	
Pro	**Cons**

Analogous Systems:

Modeling using Block Diagrams	
Pro	**Cons**

Modeling using Transfer Functions	
Pro	**Cons**

3. **Write the matlab keywords to evaluate:**
    a. **Transfer function:**
    b. **Step Response:**
    c. **Closed Loop Transfer function:**

Problem-1: For a given transfer function:

$$G(s) \;=\; \frac{s(s-a)(s+b)}{(s+c)(s+d-ej)}$$

where, a, b, c, d, e <0

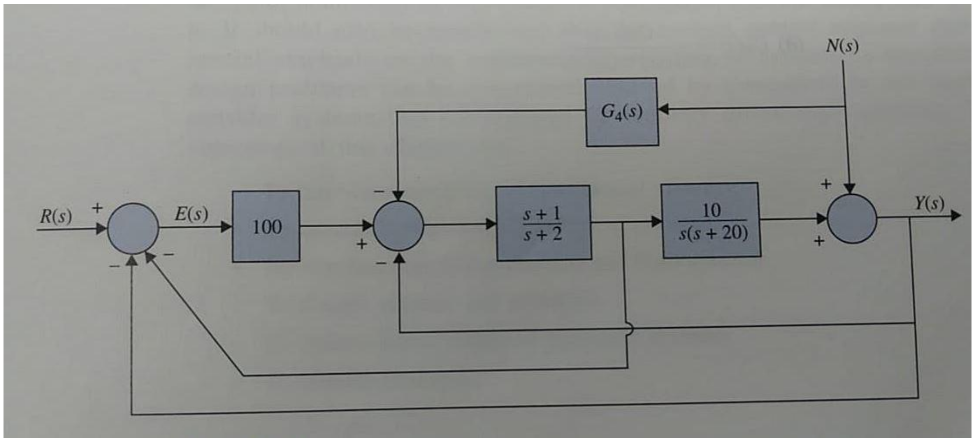
a. **What are the poles of the transfer function?**

b. **What are the zeros of the transfer function?**

c. **Represent the poles and zeros in the σ vs. jw graph.**

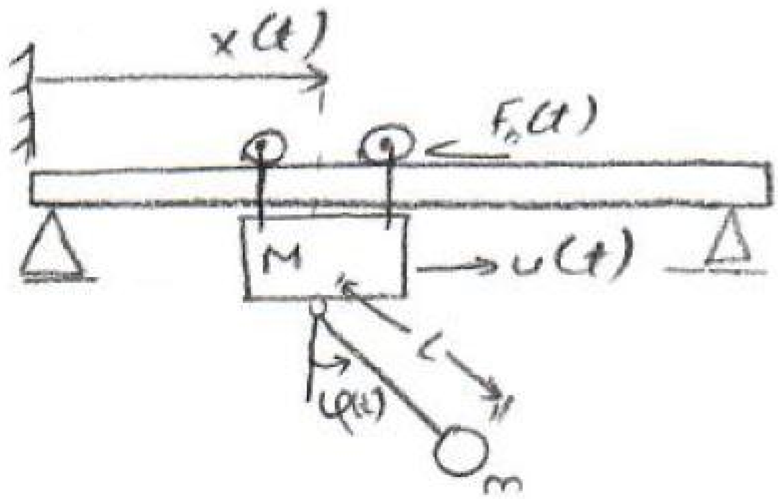
d. **Is the system stable?**

Problem 2: The block diagram below represents a Feedback system with noise. Transfer function G4(s) is for the reduction of effect of Noise.



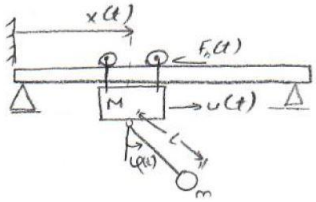
a. **Evaluate the transfer function Y(s)/R(s) when N(s) = 0**

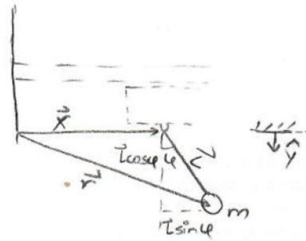
Problem-3: Consider the hanging crane structure in the fig below. The mass of the cart is M, the mass of the payload is m, massless rigid connector has length 'L' and the friction is modeled as $F_b(t) = b\dot{x}$ where x(t) is the distance travelled by the cart.



a. **Write the equations of the motion describing the motion of the cart and the payload.**

1



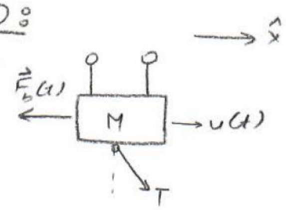


$$\vec{x}_p = \vec{x} + \vec{L}\sin\varphi$$
$$y_p = -L\cos\varphi$$

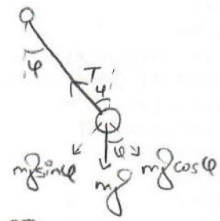
$$\dot{x}_p = \dot{x} + L\dot{\varphi}\cos\varphi$$
$$\dot{y}_p = L\dot{\varphi}\sin\varphi$$

$$\ddot{x}_p = \ddot{x} + L(\ddot{\varphi}\cos\varphi - (\dot{\varphi})^2\sin\varphi)$$
$$\ddot{y}_p = L(\ddot{\varphi}\sin\varphi + (\dot{\varphi})^2\cos\varphi)$$

FBO:



A



B

A $\sum F_x = m\ddot{x}$

$$\vec{u}(t) - \vec{T}\sin\varphi - \vec{F}_b(t) = M\ddot{\vec{x}}$$

$$\vec{T}\sin\varphi = \vec{u}(t) - M\ddot{\vec{x}} - \vec{F}_b(t) \quad \text{— 1}$$

B

$$\sum F_{x_p} = m\ddot{x}_p$$

$$\vec{T}\sin\varphi = m(\ddot{\vec{x}} + L(\ddot{\varphi}\cos\varphi - (\dot{\varphi})^2\sin\varphi)) \quad \text{— 2}$$

$$\sum F_{y_p} = m\ddot{y}_p$$

$$-\vec{T}\cos\varphi + mg = mL(\ddot{\varphi}\sin\varphi + (\dot{\varphi})^2\cos\varphi) \quad \text{— 3}$$

Now, Eliminate $\vec{T}$ from eq^s,

I. Substitute $\vec{T}\sin\varphi$ in ② from ①,

$$\vec{u}(t) - M\ddot{\vec{x}} - \vec{F}_b(t) = m\ddot{\vec{x}} + mL\ddot{\varphi}\cos\varphi - mL(\dot{\varphi})^2\sin\varphi$$

$$(M+m)\ddot{x} + mL\ddot{\varphi}\cos\varphi - mL(\dot{\varphi})^2\sin\varphi + F_b(t) = u(t) \text{ — } ④$$

II Multiply ② by $\cos\varphi$ & ③ by $\sin\varphi$ & Add ②&③,

$$-\cancel{T\cos\varphi\sin\varphi} + mg\sin\varphi = mL\ddot{\varphi}\sin^2\varphi + \cancel{mL(\dot{\varphi})^2\cos\varphi\sin\varphi}$$

$$(+)\quad \cancel{T\sin\varphi\cos\varphi} \quad = m\ddot{x}\cos\varphi + mL\ddot{\varphi}\cos^2\varphi - \cancel{mL(\dot{\varphi})^2\sin\varphi\cos\varphi}$$

$$mg\sin\varphi = mL\ddot{\varphi}\sin^2\varphi + mL\ddot{\varphi}\cos^2\varphi + m\ddot{x}\cos\varphi$$

$$mg\sin\varphi = mL\ddot{\varphi} + m\ddot{x}\cos\varphi$$

$$\therefore\ mL\ddot{\varphi} + mg\sin\varphi + m\ddot{x}\cos\varphi = 0 \text{ — } ⑤$$

$$\boxed{\begin{array}{l}(M+m)\ddot{\vec{x}} + mL\ddot{\vec{\varphi}}\cos\varphi - mL(\dot{\vec{\varphi}})^2\sin\varphi + b\dot{\vec{x}}(t) = \vec{u}(t)\\ mL\ddot{\varphi} + mg\sin\varphi + m\ddot{x}\cos\varphi = 0\end{array}}$$

b. Linearize the system about small angles around stable equilibrium of the pendulum.

c. Decouple the linearized coupled differential equation.

d. Write the state space representation of the system.

Bibliography:

- Modern Control Systems, 13th ed
- Automatic Control Systems, 9th ed