Mechanical Control Systems (ME 4473)

Recitation - 4

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1. **Agenda:**
    - Revision
    - Problems(OLTF, CLTF, Sensitivity)

2. **Questions:**
    - **What is the meaning of sensitivity?**

The sensitivity is the study of how changing a certain parameter affects the Stability of the control system. Hence, by studying the sensitivity, it is possible to design a better control system that is less sensitive towards the process variations.

- **What is the sensitivity of an open loop system?**

  1

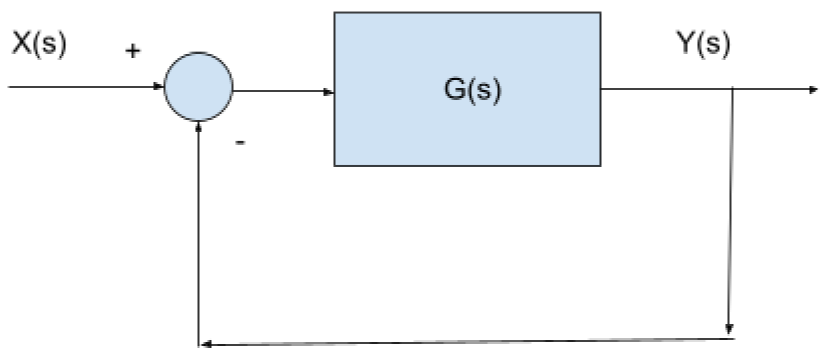
- **What is the expression for sensitivity?**

  $S_k^T = \frac{k}{T} * \frac{\delta T}{\delta k}$

<u>*Instructions for the Questions below:*</u>

- *In the breakout room, work in the pair of 4*
    - *1: A,B,C 2: E,F   3: J,K   4: Work on Matlab*
- *Check each other's work by working on*
    - *1: Matlab 2:A,B   3. E,F 4. J,K*
- *For the person working on Matlab: Also check to see how the sections in the matlab code corresponds to the problems in the worksheet*
- *Choose one from team to share their screen and work on the matlab*
- *Fill the blank spaces with the appropriate transfer function from your notes*
- *Then complete the rest of the discussion problems together*

3. **Problems:**
    1. **For a given block diagram below:**



<u>Case-1:</u> Your G(s) = 0.38(s^2+ 0.1s + 0.55)/(s*(s+1)*(s^2 + 0.06s + 0.5)) .

A. What is the loop transfer function(OLTF) for your system without substitution for G(s)?

(A) $OLTF = \underline{\underline{G(s)}}$

B. What is the closed loop transfer function(CLTF) for your system without substitution for G(s)?

(B) $CLTF = \underline{\underline{\dfrac{G(s)}{1+G(s)}}}$

C. What is the sensitivity of the CLTF with respect to OLTF?

(C) $$S^{CLTF}_{OLTF} = \left(\frac{\partial CLTF}{\partial OLTF}\right)\left(\frac{OLTF}{CLTF}\right) = \left\{\frac{\partial}{\partial G(s)}\left(\frac{G(s)}{1+G(s)}\right)\right\} \dfrac{\frac{\cancel{G(s)}}{1}}{\frac{\cancel{G(s)}}{1+G(s)}}$$

$$= \left\{\frac{1}{1+G(s)} + \frac{-G(s)}{(1+G(s))^2}\right\}(1+G(s))$$

$$= \frac{(-1)\,G(s)}{1+G(s)} + 1 = \frac{-\cancel{G(s)} + 1 + \cancel{G(s)}}{1+G(s)} = \underline{\underline{\frac{1}{1+G(s)}}}$$

D. **Plot the Bode plot for the sensitivity using provided code. What is the peak magnitude?**

Magnitude: 11.2 dB

(Note: This is called nominal Sensitivity peak)

E. **Using the equation given in Bode plot appendix, find the magnitude of the sensitivity at the peak magnitude:**

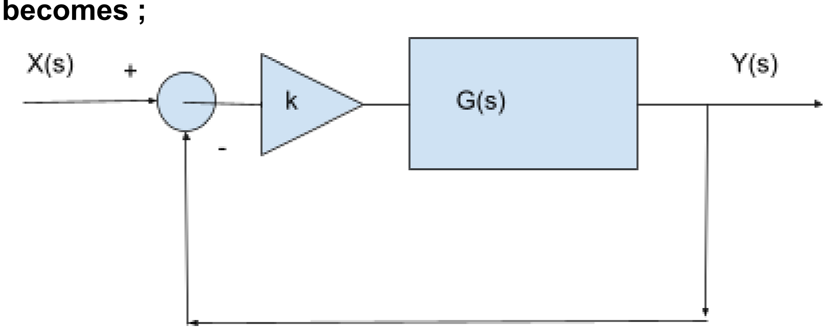
(Before you input for Magnitude in the equation, round the Magnitude to floor value: eg floor(1.99) = 1)

(E) $M = 20\log_{10}(|\text{Sensitivity}|)$

$$\log_{10}(|\text{Sensitivity}|) = \frac{M}{20}$$

$$|\text{Sensitivity}| = 10^{\left(\frac{M}{20}\right)} = 10^{(11/20)} = \underline{\underline{3.5}}$$

<u>Case-2:</u> **The sensitivity around 3.54 is too high. Typical desired sensitivity is between 1.3 & 2. Now, let's modify our system and add a gain element 'k' such that the block diagram of our system becomes ;**



F. **What is the OLTF & CLTF for the system without substitution for k & G(s)?**

(F) $OLTF = k\,G(s)$

$$CLTF = \frac{k\,G(s)}{1 + k\,G(s)}$$

G. **What is the expression for sensitivity of the CLTF due to the variations in the new OLTF (without substitution for k & G(s))?**

(G)

$$S^{CLTF}_{OLTF} = \left\{\frac{\partial}{\partial kG}\left(\frac{k\cdot G}{1+k\cdot G}\right)\right\}\left\{\frac{\frac{k\cdot G}{1}}{\frac{k\cdot G}{(1+k\cdot G)}}\right\} = \left\{\frac{\partial}{\partial kG}\left(\frac{k\cdot G}{1+kG}\right)\right\}\left\{(1+kG)\right\}$$

$$= \left\{\frac{1}{1+kG} - \frac{k\cdot G}{(1+kG)^2}\right\}(1+kG) = 1 - \frac{kG}{1+kG} = \frac{1}{1+kG}$$

H. **Evaluate F, G & H for k = 0.5.**

(H) $OLTF = 0.5\,G(s)$

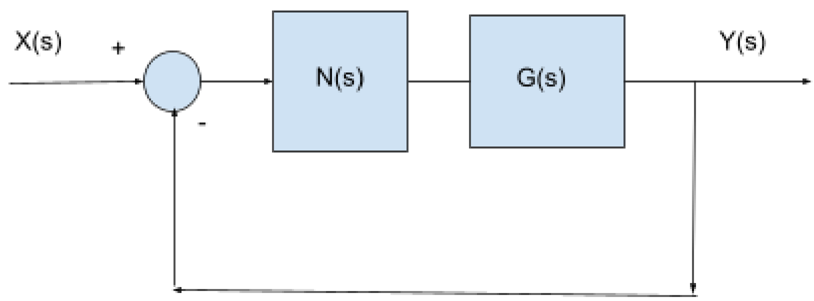
$$CLTF = \frac{0.5 * G(s)}{1 + 0.5 * G(s)}$$

$$S^{CLTF}_{OLTF} = \frac{1}{1 + 0.5 * G}$$

I. **Using the provided code plot the parameters for case-2.**
   a. **Did the magnitude of the Nominal sensitivity peak in the Closed Loop Sensitivity go down?**
      Yes

   b. **In the step response, does the new system take more time or less time to get to the steady state region than the last one?**
      Takes more time to get to steady state position, rise time is more

Case-3: Lets try to approach this issue using this using a different approach (We don't want to slow the rise time). Instead of Gain Element, Let's add a notch filter such that the block diagram becomes:



(notch N(s) = (s^2 + 0.7^2)/(s^2 + 0.35*s + .49))
(This Notch filter has a notch at 0.7 rad/s with width of Q = 2)

J. **What is the OLTF & CLTF for the new system?**

J

$$OLTF = N(s)\,G(s)$$

$$CLTF = \frac{N(s)\,G(s)}{1 + N(s)\,G(s)}$$

K. **What is the expression for the sensitivity for the new CLTF with respect to OLTF?**

(k) $$S_{OLTF}^{CLTF} = \left\{ \frac{\partial}{\partial NG} \left(\frac{NG}{1+NG} \right) \right\} \frac{\frac{NG}{1}}{\frac{\cancel{G}NG}{1+NG}}$$

$$= \left\{ \frac{1}{1+NG} - \frac{NG}{(1+NG)^2} \right\} (1+NG)$$

$$= \frac{1}{1+NG}$$

L. **Using the provided code plot the parameters for case - 3.**

   a. **What happens to the Nominal sensitivity peak?**
      Smears out to a lower frequency, peak approximately same as gain

   b. **In the step response, is the rise time of the system improved using the Notch filter than in Case II?**
      Yes

Conclusion:

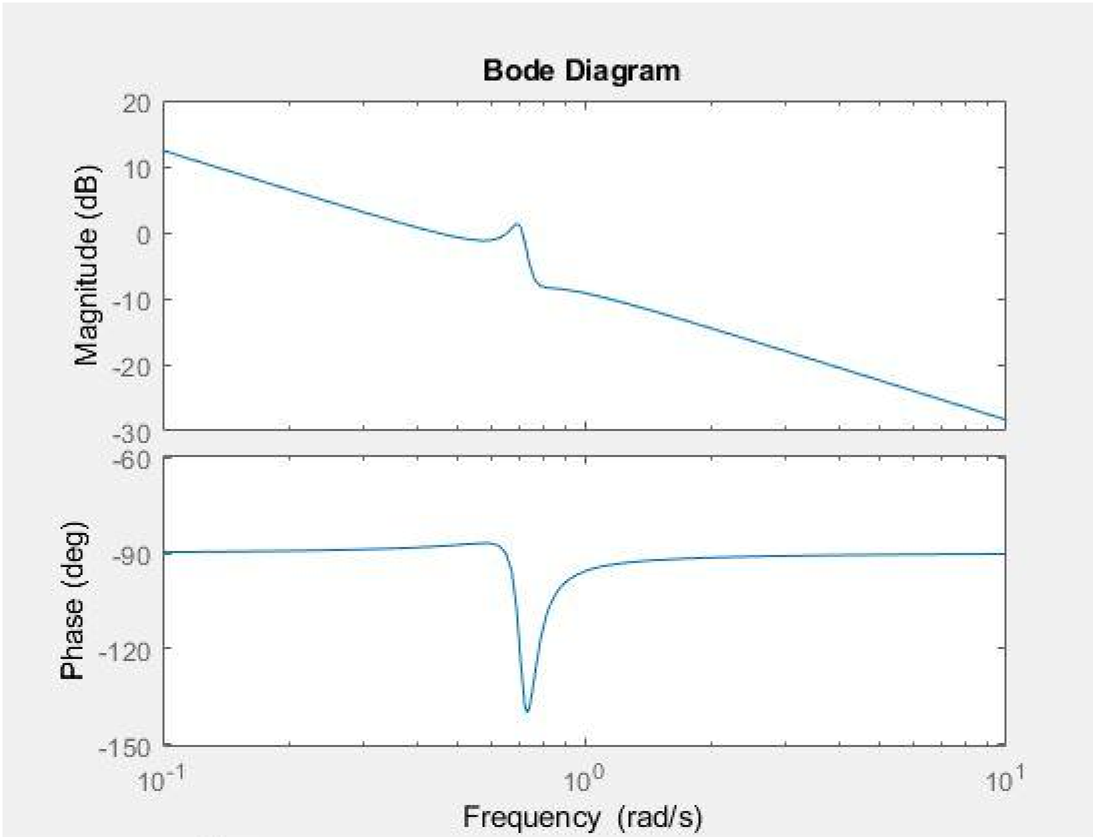
1. By decreasing the gain, the sensitivity decreases but the rise time also becomes slow (i.e. the system is slowed down). This is because reducing the gain of the system also reduces the gain across the low frequencies. This affects our rise time and step response (& our ability to follow really slow moving signals)
2. Notch filter allows to change a narrow range of frequencies without affecting the low frequencies. It doesn't affect the lower frequencies but changes the phase significantly. Hence, the rise time is not affected significantly.

Bibliography:

- Understanding the Sensitivity function, https://www.youtube.com/watch?v=BAWdZvF1O40

Appendix:

1. **Bode Plot:**
   Matlab Example:



- X-axis: Frequency(rad/s)
- Y -axis: Magnitude(dB) , Phase(degree)
- Resonant frequencies are shown by a spike in magnitude in the bode plot
- Matlab's keyword: bode(G) allows zooming in the curve. Also, by clicking on a certain point in the curve, it is possible to see the magnitude and frequency at that point
- For an open loop transfer function G(jw)H(jw),
  Magnitude(dB) = 20 log(base 10)(|G(jw)H(jw)|)
- To find which plot corresponds to which section, click on the plot. Then go to Insert -> Legend

2. **Notch Filter:** Notch filters are used to filter out a narrow band of frequency magnitude from a particular frequency band. In doing this they flip the phase

3. **Why do we need to care about Gain Margin?**

Gain Margin tells you how much pure gain can your system can absorb, before it becomes unstable

4. **Why do we need to care about Phase Margin?**

Phase margin tells you how much pure phase lag your system can absorb, before it becomes unstable

5. **Some systems have a combination of additional gain and phase which can make your system unstable much easily. This is where looking at the sensitivity function comes in handy**

6. **Bode Plot of Cases:**