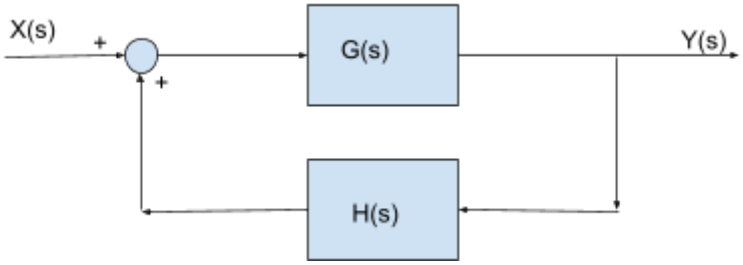
Mechanical Control Systems (ME 4473)

Recitation - 2

GTA: Shishir Khanal, khanshis@isu.edu

- **Agenda:**
  - Revision (Definitions & Terms)
  - Problems(Transfer Function, Block Diagram Reduction)
- **Definitions**
  - Open Loop Transfer Function: Product of all system transfer functions before the feedback.
  - Closed Loop Transfer Function: Ratio of Output to Input signal which accounts for the feedback or Feedforward loops.
  - Time Delay: Time lag between output & input signals

- **Questions**
  - The block diagram below represents a lesser known “Positive Feedback Control System”. Derive the transfer function for the system.



$$E(s) = X(s) + B(s)$$

$$B(s) = H(s) \cdot Y(s)$$

$$Y(s) = G(s) \cdot E(s)$$

$$= G(s)\left(X(s) + B(s)\right)$$

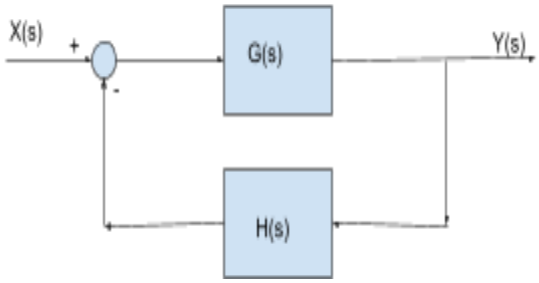
$$= G(s)\left(X(s) + H(s)\,Y(s)\right)$$

$$G(s)X(s) = Y(s) - H(s)\,Y(s)\,G(s)$$

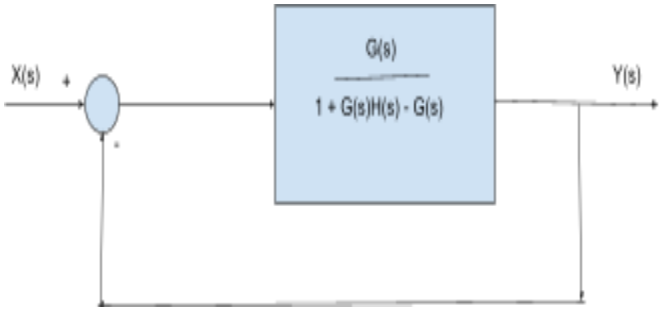
$$G(s)\,X(s) = \left(1 - G(s)H(s)\right)Y(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{G(s)}{1 - G(s)H(s)}}$$

- For a given Block Diagram:



Fig(a)



Fig(b)

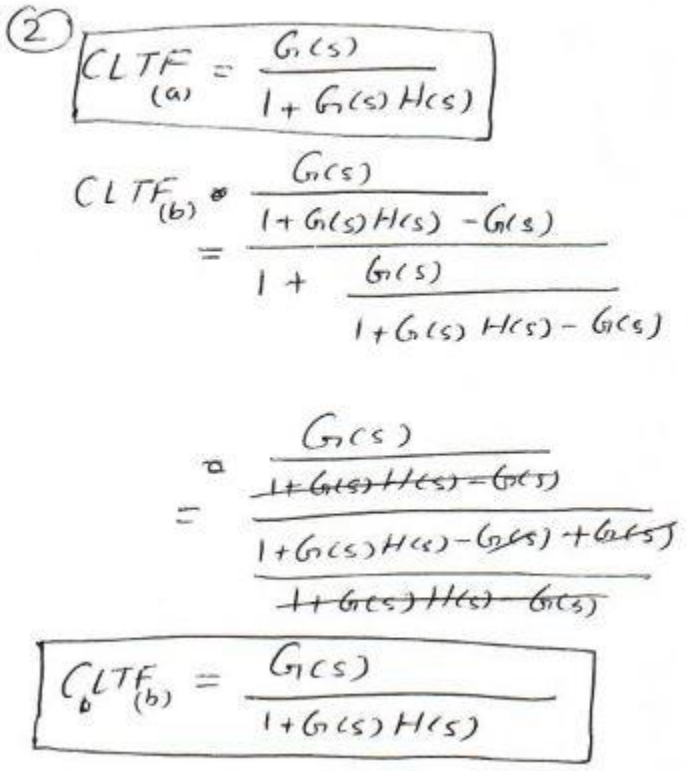
1. What is the Open Loop Transfer Function of a & b?

(a)

$$OLTF_{(a)} = G(s)\,H(s)$$

$$OLTF_{(b)} = \frac{G(s)}{1 + G(s)\,H(s) - G(s)}$$

2. What is the Closed Loop Transfer Function of a & b?



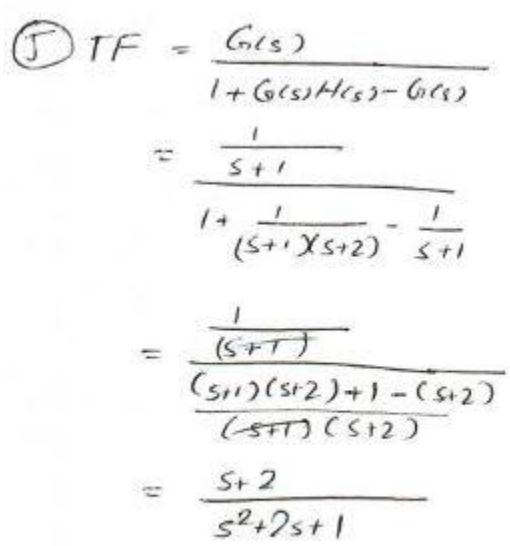
3. Are those two systems equivalent?

   Yes

4. If yes, which one of the above Open Loop Transfer functions is correct?

   Both! , but for their respective systems

5. Assume: G(s) = $\frac{1}{(s+1)}$,H(s) = $\frac{1}{(s+2)}$. Add a gain of k = 1, 3 to the system and compare its step response using Simulink.



$$\text{(5) } TF = \frac{G(s)}{1 + G(s)H(s) - G(s)}$$

$$= \frac{\frac{1}{s+1}}{1 + \frac{1}{(s+1)(s+2)} - \frac{1}{s+1}}$$

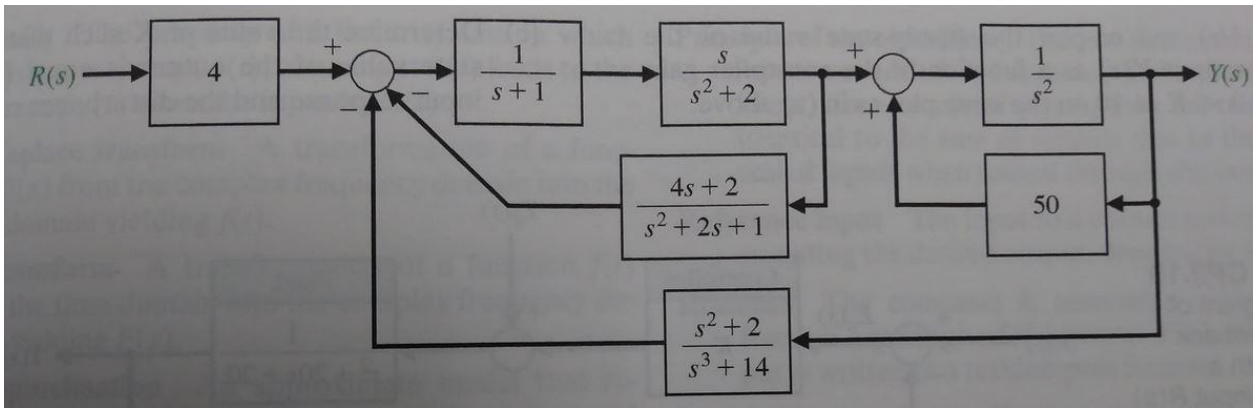
$$= \frac{\frac{1}{(s+1)}}{\frac{(s+1)(s+2)+1-(s+2)}{(s+1)(s+2)}}$$

$$= \frac{s+2}{s^2+2s+1}$$

@ Simulink

- List all the steps for Reducing Block Diagrams
    1. Combine all Cascade blocks
    2. Combine all Parallel blocks
    3. Eliminate all minor feedback loops
    4. Shift summing points to the left & takeoff points to the right
    5. Repeat steps (1) - (4)

- For the Block Diagram Below:



1. Compute the Closed Loop Transfer Function for the system using Matlab.

@ Matlab Code

2. Evaluate the step response of the whole system and equivalent system using Simulink.

@ Simulink

Conclusion:

1. Systems that have the same closed loop Transfer function have the same step response.
2. Merging the block diagram changes the system, hence the controller for the actual system is not the same as the equivalent system.

Bibliography:

1. **Control System Lectures:** https://www.youtube.com/watch?v=rJOZ4dj0hnw

(Open Loop Transfer Function takes account of the whole system and is useful in Frequency response techniques to decide what the controller should do)

2. **Modern Control Systems, Dorf, Bishop, 13th Edition**