Kinematics & Dynamics of Machinery (ME 3320)

Recitation - 7

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1. **Agenda:**
   - Revision(Cams)
   - Cam Design Problem
   - Matlab for Cam Design
   - Solidworks Cam Toolbox

2. **Revision:**
   - **What is the mobility of a Cam Follower mechanism?**

   - **What are the 3 major motions associated with the cam-follower mechanism?**

     1. ………………………………………………………………..
     2. ………………………………………………………………..
     3. ………………………………………………………………..

   - **How is follower-displacement-motion defined?**

   - **How can we define the rise/return?**

   - **How can we ensure the continuity of the y(θ)?**

   - **If we want to fully define a line, we need 2 points, if we want to fully define a parabola, we need 3 points and if we want to fully define a cubic curve, then we need 4 points. Based on this**

information, how many parameters do we need to define an n^{th} degree polynomial?

- **What is the problem with defining a parabola with 3 points for cams?**

- **What can be done to guarantee smoothness at both ends?**

- **Which order polynomials do we need to use to make the following parameters continuous?**

  Displacement ______________________

  Velocity ______________________

  Acceleration ______________________

  Jerk ______________________

- **How to make sure the displacement of the cam is smooth?**

- **If I want the acceleration to be continuous, it is mathematically similar to wanting _______ to be smooth.**

- **How is Cam's profile defined?**

3. **Cam Design Problem:**
    - **Design a displacement function using polynomial rise and return so that the acceleration is continuous at the boundaries of all rise and return functions.**

      The follower must:
      - Dwell at y = 1 in for 90°
      - Rise 3 in for 60°
      - Dwell at y = 4 in from 150° to 220°
      - Return (fall) to y = 1 in from 220° to 300°
      - Dwell for the remaining 60° of cam rotation.

3.1 Based on the information provided above, hand sketch the follower displacement function as a function of angle 'Θ'.

3.2 The problem requires the acceleration to be continuous at both ends of the rise and return functions. What does it mean to us as cam designers?

3.3 For the rise and return, write the boundary conditions and choose the appropriate polynomial functions.

3.3.1 What order of the polynomial do we need?

3.3.2 How many boundary conditions do we need to fully define this polynomial?

3.3.3 State the boundary conditions

3.3.4 State the displacement polynomial function.

3.3.5 Differentiate the displacement function to obtain the function for velocity, acceleration, and jerk.
(Assume a constant angular velocity)

3.4 Use the boundary condition on the displacement function to solve the coefficients for:

3.4.a. Rise

3.4.b Return

For (B)

$$z(\theta_1) = C_0 = 4$$

$$z(\theta_2) = 1 = C_0 + C_1 + C_2 + C_3 + C_4 + C_5 \qquad \therefore C_1 + C_2 + C_3 + C_4 + C_5 = -3$$

$$v(\theta_1) = 0 \qquad \therefore C_1 = 0$$

$$v(\theta_2) = 0 \qquad C_1 + 2C_2 + 3C_3 + 4C_4 + 5C_5 = 0$$

$$\therefore 2C_2 + 3C_3 + 4C_4 + 5C_5 = 0$$

$$a(\theta_1) = 0 \qquad C_2 = 0$$

$$a(\theta_2) = 0 \qquad 3C_3 + 6C_4 + 10C_5 = 0$$

$$C_3 + C_4 + C_5 = -3$$
$$3C_3 + 4C_4 + 5C_5 = 0$$
$$3C_3 + 6C_4 + 10C_5 = 0$$

$$\begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 3 & 6 & 10 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} -30 \\ 45 \\ -18 \end{bmatrix}$$

$$\therefore z(\theta) = 4 - 30\left(\frac{\theta - \frac{11\pi}{9}}{\frac{4}{9}\pi}\right)^3 + 45\left(\frac{\theta - \frac{11\pi}{9}}{\frac{4}{9}\pi}\right)^4 - 18\left(\frac{\theta - \frac{11\pi}{9}}{\frac{4}{9}\pi}\right)^5$$

(5) $$\therefore z(\theta) = 4 - 30\left(\frac{9\theta - 11\pi}{4\pi}\right)^3 + 45\left(\frac{9\theta - 11\pi}{4\pi}\right)^4 - 18\left(\frac{9\theta - 11\pi}{4\pi}\right)^5 \text{ — (For (B))}$$

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3.5 Use the coefficients and the angle values to evaluate the velocity, acceleration, and jerk functions for:

(Use 300 rpm for the angular velocity)

3.5.a Rise

3.5.b Return

$$v(\theta) = \left[-90 \frac{\left(\theta - \frac{11\pi}{9}\right)^2}{\left(\frac{4}{9}\pi\right)^3} + 180 \frac{\left(\theta - \frac{11\pi}{9}\right)^3}{\left(\frac{4}{9}\pi\right)^4} - 90 \frac{\left(\theta - \frac{11\pi}{9}\right)^4}{\left(\frac{4}{9}\pi\right)^5} \right] \dot{\theta}$$

$$v(\theta) = \left[-\frac{405}{32} \frac{(9\theta - 11\pi)^2}{\pi^3} + \frac{405}{64} \frac{(9\theta - 11\pi)^3}{\pi^4} - \frac{90 \times 9^5}{4^5 \times 9^4} \frac{(9\theta - 11\pi)^4}{\pi^5} \right] \dot{\theta}$$

$$\therefore v(\theta) = \left[-\frac{405}{32\pi^3} (9\theta - 11\pi)^2 + \frac{405}{64\pi^4} (9\theta - 11\pi)^3 - \frac{405}{512\pi^5} (9\theta - 11\pi)^4 \right] \dot{\theta}$$

$$a(\theta) = \left[-180 \frac{\left(\theta - \frac{11\pi}{9}\right)}{\left(\frac{4}{9}\pi\right)^3} + 540 \frac{\left(\theta - \frac{11\pi}{9}\right)^2}{\left(\frac{4}{9}\pi\right)^4} - 360 \frac{\left(\theta - \frac{11\pi}{9}\right)^3}{\left(\frac{4}{9}\pi\right)^5} \right] \dot{\theta}^2$$

$$a(\theta) = \left[-\frac{180 \times 9^3}{4^3 \times 9\pi^3} (9\theta - 11\pi) + \frac{540 \times 9^4}{4^4 \times 9^2 \pi^4} (9\theta - 11\pi)^2 - \frac{360 \times 9^5}{4^5 \times 9^3 \pi^5} (9\theta - 11\pi)^3 \right] \dot{\theta}^2$$

$$a(\theta) = \left[-\frac{3645}{16\pi^3} (9\theta - 11\pi) + \frac{10935}{64\pi^4} (9\theta - 11\pi)^2 - \frac{3645}{128\pi^5} (9\theta - 11\pi)^3 \right] \dot{\theta}^2$$

$$J(\theta) = \left[\frac{-180}{\left(\frac{4}{9}\pi\right)^3} + 1080 \frac{\left(\theta - \frac{11\pi}{9}\right)}{\left(\frac{4}{9}\pi\right)^4} + \frac{1080 \left(\theta - \frac{11\pi}{9}\right)^2}{\left(\frac{4}{9}\pi\right)^5} \right] \dot{\theta}^3$$

$$J(\theta) = \left[-\frac{32805}{16\pi^3} + \frac{1080 \times 9^4}{4^4 \times 9 \times \pi^4} (9\theta - 11\pi) - \frac{1080 \times 9^5}{4^5 \times 9^2 \times \pi^5} (9\theta - 11\pi)^2 \right] \dot{\theta}^3$$

$$J(\theta) = \left[-\frac{32805}{16\pi^3} + \frac{196830}{64\pi^4} (9\theta - 11\pi) - \frac{98415}{128\pi^5} (9\theta - 11\pi)^2 \right] \dot{\theta}^3$$

4. Matlab for Cam Design

4.1 Complete the provided Cam design code for Matlab.

(In Matlab)

4.2 The code plots the velocity, acceleration, and jerk using numerical differentiation and using the equations. What is the similarity and difference between the plots?

Similarity:

Difference:

5. **Solidworks Cam Toolbox**

5.1 Using the Solidworks Cam Design Toolbox, build the cam that can provide the follower profile described in problem 3.

(In Solidworks)

Bibliography:

Problem Courtesy: Dr. Deemyad

Miscellaneous:

- **It is possible to obtain the plots in Solidworks by performing Motion Simulation on the Cam Follower assembly. This tutorial performs the simulation and the plots in Solidworks(Time 5:25):** https://www.youtube.com/watch?v=zV4rF5X-_4E

- **This link explains all the parameters of the Cam Design Toolbox:** https://www.youtube.com/watch?v=kTziNDFzGz0