Mechanical Control Systems (ME 4473)

Recitation - 8

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(Students are advised to actively use Matlab™ during the Recitation session)

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1. **Agenda:**
   - Revision
   - Problems(Bode Plot Based Design)

2. **Questions:**

   Bode Basics:

   - **What information does a Bode Plot provide?**
     Response Curve of our system of interest

   - **Why do we look at a Gain and Phase plot in a Bode plot?**
     LTI system can only change magnitude and phase of an input signal

   - **Magnitude equation for Bode Plot**
     Mag = $20\log_{10}(K)$

   - **What are the factors that we individually evaluate during the construction of a Bode Plot?**
     - Constant Gain
     - Poles or zeros at origin
     - Complex Conjugate Poles

   - **Fill in the blanks:**
     - A unit change in ($\log_{10}(w)$) in the rectangular coordinate system = 1 dec of variation in w in semi log coordinates.
     - Number of decades between two frequencies = $\log_{10}(w_2/w_1)$

   Properties of Transfer Function Components:

   - **How does a constant gain affect the Bode Plot?**
     Magnitude: Constant at K_{dB}
     Phase: Either 0 or -180^0(for negative gain)

- **How do poles/zeros at origin affect the Bode Plot?**

  Magnitude: Magnitude changes as , $\pm$ 20p $\log_{10}(w)$ with a slope of $\pm$ 20p dB/dec for every poles or zeros

  Phase: Every zero increases the phase by 90^0. Every pole decreases the phase by 90^0.

  (Zero makes the signal lead ahead, pole makes the signal lag behind)

- **How do complex conjugate poles/zeros affect the Bode Plot?**

  Magnitude: 40 db/dec(+ zeros, -poles)

  Underdamped systems show bump at the resonant frequency

  Phase:180^0 (+ zeros, -poles)

<u>Bode Plot & Stability:</u>

- **What happens in Magnitude & Phase Plot at the corner Frequency?**

  Magnitude: Magnitude change occurs at the corner frequency

  Phase: Half of the remaining phase change occurs after the corner frequency (within a decade of corner frequency)

  (Total Phase change occurs within the 2 decades: one before and one after the corner frequency)

- **Gain crossover point:** Point where magnitude graph crosses frequency axis.i.e. Frequency of zero gain a.k.a. Input Magnitude = Output Magnitude
- **Gain Margin & Phase Margin:**
  - Gain Margin is measured downward from freq axis
  - Phase margin is measured upwards from deg = -180^0 axis.

- Matlab code for the Bode plot: bode(tf)

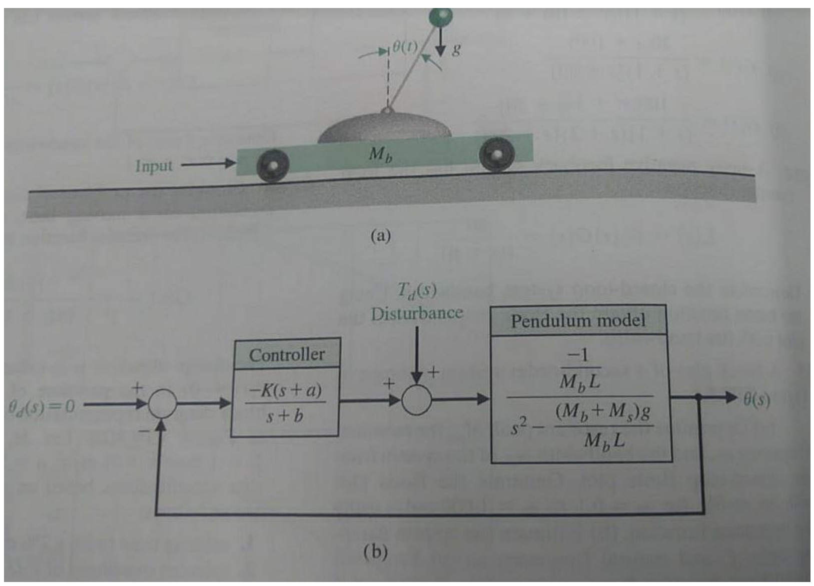
3. **Problems:**

CP8.8 Consider a problem of controlling an inverted pendulum on a moving base, as shown in the fig below:

a. **The transfer function of the system is:**

$$G(s) = \frac{-1/(M_b L)}{s^2 - (M_b + M_s)g/(M_b L)}.$$

The design objective is to balance the pendulum (i.e. Θ(t) ≈ 0) in the presence of disturbance inputs. A block diagram of the system is given below.



b. **Let M_s = 10kg, M_b = 100 kg, L = 1m, g = 9.81m/s^2, a = 5 and b = 10. The design specifications, based on the unit step disturbance are :**
   i. **Settling time(with 2% criterion) of $T_s \leq 10$ s**
   ii. **Percentage overshoot ≤ 40% &**
   iii. **Steady-state tracking error less than 0.1^0 in the presence of disturbance**

c. **Using the information provided above, compute:**

i. **Closed-Loop transfer function from disturbance to the output with K as an adjustable parameter.**

$$TF_{pend} = \frac{\frac{-1}{100}}{s^2 - \frac{110 \times 9.81}{100}} = \frac{-1}{100s^2 - 1079.1} = \frac{-0.01}{s^2 - 10.79}$$

$$TF_c = \frac{-K(s+a)}{(s+b)} = -\frac{K(s+5)}{(s+10)}$$

$$CLTF = \frac{G}{1+GH} = \frac{\frac{-0.01}{s^2-10.79}}{1 + \frac{-0.01}{s^2-10.79}\left(\frac{-K(s+5)}{s+10}\right)} = \frac{-0.01(s+10)}{(s^2-10.79)(s+10)+0.01K(s+5)}$$

$$= \frac{-0.01(s+10)}{s^3+10s^2-10.79s-107.9+(0.01K)s+0.05K}$$

$$\therefore CLTF = \frac{-0.01(s+10)}{s^3+10s^2+(0.01K-10.79)s+(0.05K-107.9)}$$

ii. **Range of K that will satisfy the steady state output requirement**

For Requirement-3:

$$e_{ss} < 0.1^\circ = 0.001745$$

Using FVT,

$$e_{ss} = \lim_{s\to 0} s\,\theta(s) = \lim_{s\to 0} s(CLTF) \cdot T_d(s)$$

$$= \lim_{s\to 0} (s)\left(\frac{-0.01(s+10)}{s^3+10s^2+(0.01K-10.79)s+(0.05K-107.9)}\right)\cdot\left(\frac{1}{s}\right)$$

$$0.00175 > \frac{-0.1}{0.05K-107.9}$$

$$0.0000875K - 0.1883 > -0.1$$

$$0.0000875K > 0.088255$$

$$\therefore K > 1011.9$$

iii. **Expression to compute output maximum magnitude(M_{pw}) and resonant frequency(w_r) using equations (8.36) and (8.37)**

From 8.37,

$$M_{pw} = \left(2\zeta\sqrt{1-\zeta^2}\right)^{-1}$$

$$= \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$M_{pw}^2 = \frac{1}{4\zeta^2(1-\zeta^2)}$$

$$4\zeta^2(1-\zeta^2) = \frac{1}{M_{pw}^2}$$

$$4\zeta^2 - 4\zeta^4 = \frac{1}{M_{pw}^2}$$

$$a = \zeta^2$$

$$4a^2 - 4a + \frac{1}{M_{pw}^2} = 0$$

$$a = \frac{4 \pm \sqrt{16 - 16/M_{pw}^2}}{8}$$

$$\zeta^2 = \frac{1 \pm \sqrt{1 - \frac{1}{M_{pw}^2}}}{2}$$

choose −

$$\zeta = \pm\sqrt{\frac{1 \pm \sqrt{1 - \frac{1}{M_{pw}^2}}}{2}}$$

choose '+'

$$\therefore \zeta = \sqrt{\frac{1 - \sqrt{1 - \frac{1}{M_{pw}^2}}}{2}}$$

for $\zeta < 0.707$

$\zeta \equiv$ zeta

From 8.36,

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}$$

$$\therefore \omega_n = \frac{\omega_r}{\sqrt{1 - 2\zeta^2}}$$

d. Write a matlab code to evaluate:

i. Closed-Loop transfer function from disturbance to the output with K as an adjustable parameter. Bode Plot of the closed loop system
@Matlab

ii. Automatically compute and output maximum magnitude(M_{pw}) and resonant frequency(w_r)
@Matlab

e. If the performance specification is not satisfied, change K and iterate the design using the first two scripts. For the third script:

i. Plot the response Θ(t) to a unit step disturbance with K as an adjustable parameter
@Matlab

ii. Label the plot properly
@Matlab

Bibliography:

- **Dorf, Modern Control Systems, 13th ed**