Kinematics & Dynamics of Machinery (ME 3320)

Recitation - 7

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1. **Agenda:**
    - Revision(Cams)
    - Cam Design Problem
    - Matlab for Cam Design
    - Solidworks Cam Toolbox

2. **Revision:**
    - **What is the mobility of a Cam Follower mechanism?**
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    - **What are the 3 major motions associated with the cam-follower mechanism?**
        1. Rise(Increase displacement of follower)
        2. Dwell(No change in the displacement of the follower)
        3. Fall(Decrease in the displacement of the follower)
    - **How is follower-displacement-motion defined?**
      As a piecewise continuous function y(θ) where θ-> [0,2π]
    - **How can we define the rise/return?**
      By defining a set of points along the path
      (By model fitting)
    - **How can we ensure the continuity of the y(θ)?**
      By imposing boundary conditions
    - **If we want to fully define a line, we need 2 points, if we want to fully define a parabola, we need 3 points and if we want to fully define a cubic curve, then we need 4 points. Based on this information, how many parameters do we need to define an n^{th} degree polynomial?**
      N+1 parameters
    - **What is the problem with defining a parabola with 3 points for cams?**

It cannot guarantee smoothness at both edges of the curve

- **What can be done to guarantee smoothness at both ends?**
  (Note: First-degree polynomial can guarantee continuity for displacement but only at one end)
  Use higher-order polynomials

- **Which order polynomials do we need to use to make the following parameters continuous?**

  Displacement ______1st Order________________

  Velocity _______3rd Order_______________

  Acceleration _______5th Order________________

  Jerk _______7th Order________________

- **How to make sure the displacement of the cam is smooth?**
  Make sure the velocity function is continuous

- **If I want the acceleration to be continuous, it is mathematically**

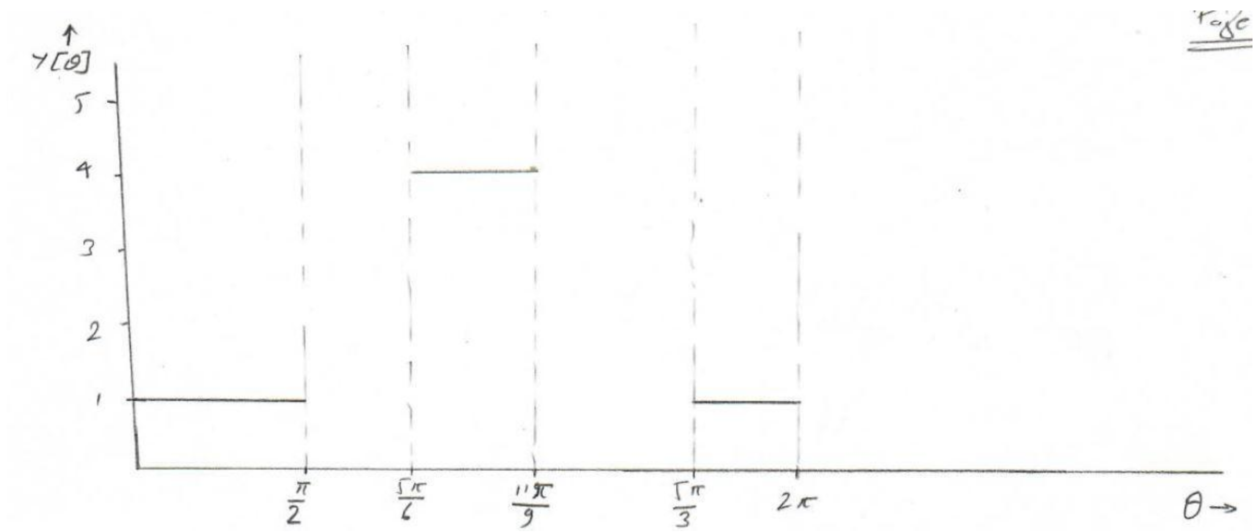
  similar to wanting velocity **to be smooth.**

- **How is Cam's profile defined?**
  As a radius vector i.e. R(θ)
  (Cam profile is a polar coordinate plot of y(θ))

3. **Cam Design Problem:**
   - **Design a displacement function using polynomial rise and return so that the acceleration is continuous at the boundaries of all rise and return functions.**

     The follower must:
     - Dwell at y = 1 in for 90°
     - Rise 3 in for 60°
     - Dwell at y = 4 in from 150° to 220°
     - Return (fall) to y = 1 in from 220° to 300°
     - Dwell for the remaining 60° of cam rotation.

3.1 Based on the information provided above, hand sketch the follower displacement function as a function of angle 'Θ'.



3.2 The problem requires the acceleration to be continuous at both ends of the rise and return functions. What does it mean to us as cam designers?

5th order polynomial is needed to make the acceleration continuous. Also, to fully define the 5th order polynomial, we need 6 points.

3.3 For the rise and return, write the boundary conditions and choose the appropriate polynomial functions.

3.3.1 What order of the polynomial do we need?

5th order polynomial

3.3.2 How many boundary conditions do we need to fully define this polynomial?

6 boundary conditions

3.3.3 State the boundary conditions

a $\frac{\pi}{2} \le \theta < \frac{5\pi}{6} : \begin{cases} y(\frac{\pi}{2}) = 1 & v(\frac{\pi}{2}) = 0 & a(\frac{\pi}{2}) = 0 \\ y(\frac{5\pi}{6}) = 4 & v(\frac{5\pi}{2}) = 0 & a(\frac{5\pi}{2}) = 0 \end{cases}$ — A

$\theta_1 = \frac{\pi}{2} ; \theta_2 = \frac{5\pi}{6}$

$\ddot{\theta} = 0$

$\frac{11\pi}{9} \le \theta < \frac{5\pi}{3} : \begin{cases} y(\frac{11\pi}{9}) = 4 & v(\frac{11\pi}{9}) = 0 & a(\frac{11\pi}{9}) = 0 \\ y(\frac{5\pi}{3}) = 1 & v(\frac{5\pi}{3}) = 0 & a(\frac{5\pi}{3}) = 0 \end{cases}$ — B

$\theta_1 = \frac{11\pi}{9} ; \theta_2 = \frac{5\pi}{3}$

Correction: For A, the theta2 angle is 5pi/6 not 5pi/2.

3.3.4 State the displacement polynomial function.

$$y(\theta) = C_0 + C_1\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) + C_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^2 + C_3\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^3 + C_4\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^4 + C_5\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^5 \text{ — T}$$

3.3.5 Differentiate the displacement function to obtain the function for velocity, acceleration, and jerk.
(Assume a constant angular velocity)

$$v(\theta) = \left[\frac{C_1}{\theta_2-\theta_1} + 2C_2\frac{(\theta-\theta_1)}{(\theta_2-\theta_1)^2} + 3C_3\frac{(\theta-\theta_1)^2}{(\theta_2-\theta_1)^3} + 4C_4\frac{(\theta-\theta_1)^3}{(\theta_2-\theta_1)^4} + 5C_5\frac{(\theta-\theta_1)^4}{(\theta_2-\theta_1)^5}\right]\dot{\theta}$$

$$a(\theta) = \left[\frac{2C_2}{(\theta_2-\theta_1)^2} + 6C_3\frac{(\theta-\theta_1)}{(\theta_2-\theta_1)^3} + 12C_4\frac{(\theta-\theta_1)^2}{(\theta_2-\theta_1)^4} + 20C_5\frac{(\theta-\theta_1)^3}{(\theta_2-\theta_1)^5}\right]\dot{\theta}^2$$

$$J(\theta) = \left[\frac{6C_3}{(\theta_2-\theta_1)^3} + 24C_4\frac{(\theta-\theta_1)}{(\theta_2-\theta_1)^4} + 60C_5\frac{(\theta-\theta_1)^2}{(\theta_2-\theta_1)^5}\right]\dot{\theta}^3$$

3.4 Use the boundary condition on the displacement function to solve the coefficients for:

3.4.a. Rise

For ④

$$\zeta(\theta_1) = C_0 + C_1\left(\frac{\theta_1-\theta_1}{\theta_2-\theta_1}\right) + C_2\left(\frac{\theta_1-\theta_1}{\theta_2-\theta_1}\right)^2 + C_3\left(\frac{\theta_1-\theta_1}{\theta_2-\theta_1}\right)^3 + C_4\left(\frac{\theta_1-\theta_1}{\theta_2-\theta_1}\right)^4 + C_5\left(\frac{\theta_1-\theta_1}{\theta_2-\theta_1}\right)^5$$

$$1 = C_0 \qquad \therefore C_0 = 1$$

$$\zeta(\theta_2) = C_0 + C_1\left(\frac{\theta_2-\theta_1}{\theta_2-\theta_1}\right) + C_2\left(\frac{\theta_2-\theta_1}{\theta_2-\theta_1}\right)^2 + C_3\left(\frac{\theta_2-\theta_1}{\theta_2-\theta_1}\right)^3 + C_4\left(\frac{\theta_2-\theta_1}{\theta_2-\theta_1}\right)^4 + C_5\left(\frac{\theta_2-\theta_1}{\theta_2-\theta_1}\right)^5$$

$$C_0 + C_1 + C_2 + C_3 + C_4 + C_5 = 4$$

$$C_1 + C_2 + C_3 + C_4 + C_5 = 4 - 1 = 3$$

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$$v(\theta_1) = \left[\frac{C_1}{\theta_2-\theta_1}\right]\dot\theta = 0 \qquad \therefore C_1 = 0$$

$$v(\theta_2) = \left[\frac{C_1}{\theta_2-\theta_1} + 2C_2\frac{\theta_2-\theta_1}{(\theta_2-\theta_1)^2} + 3C_3\frac{(\theta_2-\theta_1)^2}{(\theta_2-\theta_1)^3} + 4C_4\frac{(\theta_2-\theta_1)^3}{(\theta_2-\theta_1)^4} + 5C_5\frac{(\theta_2-\theta_1)^4}{(\theta_2-\theta_1)^5}\right]\dot\theta = 0$$

$$\left[\overset{0}{\cancel{C_1}} + 2C_2 + 3C_3 + 4C_4 + 5C_5\right]\frac{\dot\theta}{\theta_2-\theta_1} = 0$$

$$2C_2 + 3C_3 + 4C_4 + 5C_5 = 0$$

$$a(\theta_1) = \frac{2C_2}{(\theta_2-\theta_1)^2} + 0 + 0 + 0 = 0 \qquad \therefore C_2 = 0$$

$$a(\theta_2) = \left[\frac{2C_2}{(\theta_2-\theta_1)^2} + 6C_3\frac{(\theta_2-\theta_1)}{(\theta_2-\theta_1)^3} + 12C_4\frac{(\theta_2-\theta_1)^2}{(\theta_2-\theta_1)^4} + 20C_5\frac{(\theta_2-\theta_1)^3}{(\theta_2-\theta_1)^5}\right]\dot\theta^2 = 0$$

$$\overset{0}{\cancel{2C_2}} + 6C_3 + 12C_4 + 20C_5 = 0$$

$$3C_3 + 6C_4 + 10C_5 = 0$$

Matlab:

A = [1 1 1; 3 4 5; 3 6 10]

B = [3; 0; 0]

X = inv(A) * B

$$C_3 + C_4 + C_5 = 3$$
$$3C_3 + 4C_4 + 5C_5 = 0$$
$$3C_3 + 6C_4 + 10C_5 = 0$$

$$\begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 3 & 6 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 30 \\ -45 \\ 18 \end{bmatrix}$$

$$\zeta(\theta) = 1 + 30\left(\frac{\theta - \frac{\pi}{2}}{\frac{\pi}{3}}\right)^3 - 45\left(\frac{\theta - \pi/2}{\pi/3}\right)^4 + 18\left(\frac{\theta - \pi/2}{\pi/3}\right)^5$$

$$\therefore \zeta(\theta) = 1 + \frac{405}{4}\left(\frac{2\theta-\pi}{\pi}\right)^3 - \frac{3645}{16}\left(\frac{2\theta-\pi}{\pi}\right)^4 + \frac{2187}{16}\left(\frac{2\theta-\pi}{\pi}\right)^5 \text{ — (For ④)}$$

3.4.b Return

For (B)

$$\zeta(\theta_1) = C_0 = 4$$

$$\zeta(\theta_2) = 1 = C_0 + C_1 + C_2 + C_3 + C_4 + C_5 \qquad \therefore\ C_1 + C_2 + C_3 + C_4 + C_5 = -3$$

$$v(\theta_1) = 0 \qquad \therefore\ C_1 = 0$$

$$v(\theta_2) = 0 \qquad C_1 + 2C_2 + 3C_3 + 4C_4 + 5C_5 = 0$$

$$\therefore\ 2C_2 + 3C_3 + 4C_4 + 5C_5 = 0$$

$$a(\theta_1) = 0 \qquad C_2 = 0$$

$$a(\theta_2) = 0 \qquad 3C_3 + 6C_4 + 10C_5 = 0$$

$$C_3 + C_4 + C_5 = -3$$

$$3C_3 + 4C_4 + 5C_5 = 0$$

$$3C_3 + 6C_4 + 10C_5 = 0$$

$$\begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 3 & 6 & 10 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} -30 \\ 45 \\ -18 \end{bmatrix}$$

$$\therefore\ \zeta(\theta) = 4 - 30\left(\frac{\theta - \frac{11\pi}{9}}{\frac{4}{9}\pi}\right)^3 + 45\left(\frac{\theta - 11\pi/9}{\frac{4}{9}\pi}\right)^4 - 18\left(\frac{\theta - 11\pi/9}{\frac{4}{9}\pi}\right)^5$$

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(5) $$\therefore\ \zeta(\theta) = 4 - 30\left(\frac{9\theta - 11\pi}{4\pi}\right)^3 + 45\left(\frac{9\theta - 11\pi}{4\pi}\right)^4 - 18\left(\frac{9\theta - 11\pi}{4\pi}\right)^5 \text{ --- (For (B))}$$

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3.5 Use the coefficients and the angle values to evaluate the velocity, acceleration, and jerk functions for:
(Use 300 rpm for the angular velocity)

3.5.a Rise

For velocity,

$$v(\theta) = \left[90\frac{(\theta - \pi/2)^2}{(\pi/3)^3} - 180\frac{(\theta - \pi/2)^3}{(\pi/3)^4} + 90\frac{(\theta - \pi/2)^4}{(\pi/3)^5}\right]\dot{\theta}$$

$$\therefore\ v(\theta) = \left[\frac{1215}{2\pi^3}(2\theta - \pi)^2 - \frac{3645}{2\pi^4}(2\theta - \pi)^3 + \frac{10935}{8\pi^5}(2\theta - \pi)^4\right]\dot{\theta}$$

$$a(\theta) = \left[180\frac{(\theta-\frac{\pi}{2})}{(\frac{\pi}{3})^3} - 540\frac{(\theta-\frac{\pi}{2})^2}{(\frac{\pi}{3})^4} + 360\frac{(\theta-\frac{\pi}{2})^3}{(\frac{\pi}{3})^5}\right]\dot{\theta}^2$$

$$a(\theta) = \left[\frac{2430}{\pi^3}(2\theta-\pi) - \frac{10935}{\pi^4}(2\theta-\pi)^2 + \frac{10935}{\pi^5}(2\theta-\pi)^3\right]\dot{\theta}^2 -$$

$$J(\theta) = \left[\frac{180}{(\frac{\pi}{3})^3} - 1080\frac{(\theta-\frac{\pi}{2})}{(\frac{\pi}{3})^4} + 1080\frac{(\theta-\frac{\pi}{2})^2}{(\frac{\pi}{3})^5}\right]\dot{\theta}^3$$

$$J(\theta) = \left[\frac{4860}{\pi^3} - \frac{43740}{\pi^4}(2\theta-\pi) + \frac{65610}{\pi^5}(2\theta-\pi)^2\right]\dot{\theta}^3 -$$

3.5.b Return

$$v(\theta) = \left[-90\frac{(\theta-\frac{11\pi}{9})^2}{(\frac{4}{9}\pi)^3} + 180\frac{(\theta-\frac{11\pi}{9})^3}{(\frac{4}{9}\pi)^4} - 90\frac{(\theta-\frac{11\pi}{9})^4}{(\frac{4}{9}\pi)^5}\right]\dot{\theta}$$

$$v(\theta) = \left[-\frac{405}{32}\frac{(9\theta-11\pi)^2}{\pi^3} + \frac{405}{64}\frac{(9\theta-11\pi)^3}{\pi^4} - \frac{90\cdot 9^5}{4^5\cdot 9^4}\frac{(9\theta-11\pi)^4}{\pi^5}\right]\dot{\theta}$$

$$\therefore v(\theta) = \left[-\frac{405}{32\pi^3}(9\theta-11\pi)^2 + \frac{405}{64\pi^4}(9\theta-11\pi)^3 - \frac{405}{512\pi^5}(9\theta-11\pi)^4\right]\dot{\theta} -$$

$$a(\theta) = \left[-180\frac{(\theta-\frac{11\pi}{9})}{(\frac{4}{9}\pi)^3} + 540\frac{(\theta-\frac{11\pi}{9})^2}{(\frac{4}{9}\pi)^4} - 360\frac{(\theta-\frac{11\pi}{9})^3}{(\frac{4}{9}\pi)^5}\right]\dot{\theta}^2$$

$$a(\theta) = \left[-\frac{180\times 9^3}{4^3\times 9\pi^3}(9\theta-11\pi) + \frac{540\times 9^4}{4^4\times 9^2\pi^4}(9\theta-11\pi)^2 - \frac{360\times 9^5}{4^5\times 9^3\pi^5}(9\theta-11\pi)^3\right]\dot{\theta}^2$$

$$a(\theta) = \left[-\frac{3645}{16\pi^3}(9\theta-11\pi) + \frac{10935}{64\pi^4}(9\theta-11\pi)^2 - \frac{3645}{128\pi^5}(9\theta-11\pi)^3\right]\dot{\theta}^2 -$$

$$J(\theta) = \left[\frac{-180}{(\frac{4}{3}\pi)^3} + 1080\,\frac{(\theta - \frac{11\pi}{9})}{(\frac{4}{3}\pi)^4} + \frac{1080\,(\theta - \frac{11\pi}{9})^2}{(\frac{4}{3}\pi)^5}\right]\dot{\theta}^3$$

$$J(\theta) = \left[-\frac{32805}{16\pi^3} + \frac{1080\times 9^4}{4^4\times 9\times \pi^4}(9\theta - 11\pi) - \frac{1080\times 9^5}{4^5\times 9^2\times \pi^5}(9\theta - 11\pi)^2\right]\dot{\theta}^3$$

$$J(\theta) = \left[-\frac{32805}{16\pi^3} + \frac{196830}{64\pi^4}(9\theta - 11\pi) - \frac{98415}{128\pi^5}(9\theta - 11\pi)^2\right]\dot{\theta}^3$$

4. **Matlab for Cam Design**

4.1 Complete the provided Cam design code for Matlab.

(In Matlab)

4.2 The code plots the velocity, acceleration, and jerk using numerical differentiation and using the equations. What is the similarity and difference between the plots?

Similarity: Both the curves have the same profile

Difference: The curve evaluated using the equations is scaled by a factor of angular velocity.

5. **Solidworks Cam Toolbox**

5.1 Using the Solidworks Cam Design Toolbox, build the cam that can provide the follower profile described in problem 3.

(In Solidworks)

Bibliography:

Problem Courtesy: Dr. Deemyad

Miscellaneous:

- **It is possible to obtain the plots in Solidworks by performing Motion Simulation on the Cam Follower assembly. This tutorial performs the simulation and the plots in Solidworks(Time 5:25):** https://www.youtube.com/watch?v=zV4rF5X-_4E

- **This link explains all the parameters of the Cam Design Toolbox:** https://www.youtube.com/watch?v=kTziNDFzGz0