Kinematics & Dynamics of Machinery (ME 3320)

Recitation - 2

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1. **Agenda:**
    - Revision(Linkage, Mobility)
    - Problems(Mobility)

2. **Revision:**
    - **What is the mobility of a 4-bar Linkage?**
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    - **If the mobility is equal to "2", that means we need to control** 2 **variables for a given mechanism.**

    - **What are the 6 steps that are essential in the derivation of the vector equation for the position analysis of a 4-bar linkage?**

Step-1: Define the reference frame
Step-2: Define variables and parameters
Step-3: Link length: a, b, h, g
Step-4: Angular/joint variables θ $(input\ angle)$,$\varnothing$ $(coupler\ angle)$,Ψ $(output\ angle)$
Step-5: Pivots: O, A, B, C
Step-6:Vector coordinates of points

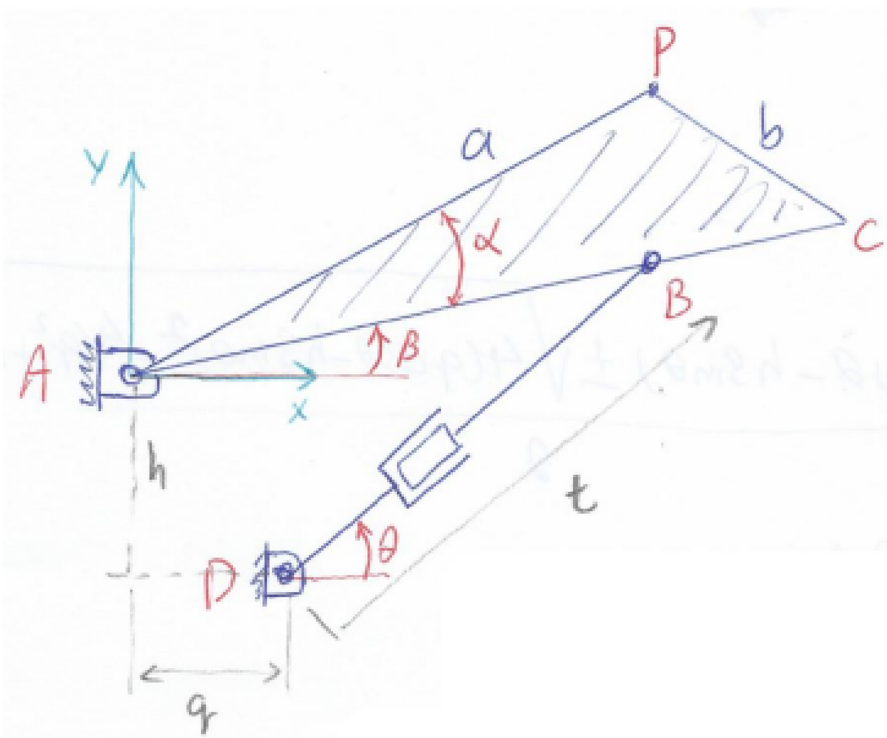
- **What are the two ways of finding the dependent variables in a 4-bar linkage problem?**
    1. Distance Constraints
    2. Loop Equations

- **What idea is the 'distance constraint' method based on?**
  The distance between two points in a link is fixed.

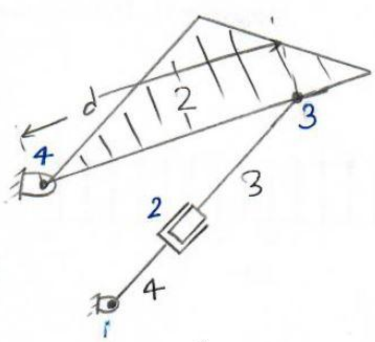
- **What idea is the 'loop equations' based on?**
  We can express the vector coordinates of the point using different paths along the linkage.

3. **Problems:**

- **The linkage shown below is a kinematic sketch of a closing door mechanism with given dimensions. The acceptable value of the prismatic joint is: 5 < t < 15.**



1) Calculate the Mobility of this linkage



$n = 4$

$j = 4$

$$M = 3(n-1) - \sum_{i=1}^{j} (3 - f_i)$$

$$= 3(4-1) - 4(3-1)$$

$$\boxed{\therefore M = 1}$$

2) What are the coordinates of points A, B & D?

$$\vec{A} = \langle 0, 0 \rangle$$

$$\vec{B} = \langle q + t\cos\theta,\ t\sin\theta - h \rangle$$

$$\vec{D} = \langle q, -h \rangle$$

$$\vec{B} = \langle d\cos\beta, d\sin\beta \rangle$$

3) For the defined reference frame and given θ, perform the position analysis:

3. a) Using the distance constraint method, compute the dependent variable t(θ) analytically as an explicit function of the input angle θ. Consider the length of $\overline{AB}$ as the fixed distance.

3.a Distance Constraint,

$$\vec{AB}\cdot\vec{AB} = d^2$$

$$(\vec{B}-\vec{A})\cdot(\vec{B}-\vec{A}) = d^2$$

$$\left(\begin{Bmatrix} q+t\cos\theta \\ t\sin\theta-h \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}\right)\cdot\left(\begin{Bmatrix} q+t\cos\theta \\ t\sin\theta-h \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}\right) = d^2$$

$$\begin{Bmatrix} q+t\cos\theta \\ t\sin\theta-h \end{Bmatrix}\cdot\begin{Bmatrix} q+t\cos\theta \\ t\sin\theta-h \end{Bmatrix} = d^2$$

$$(q+t\cos\theta)^2 + (t\sin\theta-h)^2 = d^2$$

$$q^2 + t^2\cos^2\theta + 2qt\cos\theta + t^2\sin^2\theta + h^2 - 2ht\sin\theta = d^2$$

$$t^2\cos^2\theta + t^2\sin^2\theta + 2(qt\cos\theta - ht\sin\theta) + q^2 + h^2 - d^2 = 0$$

$$t^2 + 2(q\cos\theta - h\sin\theta)t + q^2 + h^2 - d^2 = 0$$

$$\boxed{\therefore\ t^2 - B(\theta)t + C = 0} \quad \Rightarrow \quad t = \frac{-B(\theta) \pm \sqrt{(B(\theta))^2 - 4C}}{2}$$

$$\boxed{\therefore\ t(\theta) = \frac{-2(q\cos\theta - h\sin\theta) \pm \sqrt{4(q\cos\theta - h\sin\theta)^2 - 4(q^2+h^2-d^2)}}{2}}$$

Correction: t^2 + B(Θ) + C = 0

3. b) Formulate the loop equation and solve it to find the explicit equation for dependent variable β(θ)

3.b Consider loop ABDA,

$$\vec{AB} + \vec{BO} + \vec{OA} = 0$$

$$\vec{AB} = -\vec{BO} - \vec{OA}$$

$$\boxed{\therefore \vec{AB} = \vec{OB} + \vec{AO}} \quad \text{— (I)}$$

$$\vec{AB} = \vec{B} - \vec{A} = \begin{Bmatrix} d\cos\beta \\ d\sin\beta \end{Bmatrix}$$

$$\vec{OB} = \vec{B} - \vec{O} = \begin{Bmatrix} g + t\cos\theta \\ t\sin\theta - h \end{Bmatrix} - \begin{Bmatrix} g \\ -h \end{Bmatrix} = \begin{Bmatrix} t\cos\theta \\ t\sin\theta \end{Bmatrix}$$

$$\vec{AO} = \vec{O} - \vec{A} = \begin{Bmatrix} g \\ -h \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} g \\ -h \end{Bmatrix}$$

(I) becomes,

$$\begin{Bmatrix} d\cos\beta \\ d\sin\beta \end{Bmatrix} = \begin{Bmatrix} t\cos\theta \\ t\sin\theta \end{Bmatrix} + \begin{Bmatrix} g \\ -h \end{Bmatrix}$$

$$\therefore \begin{Bmatrix} d\cos\beta \\ d\sin\beta \end{Bmatrix} = \begin{Bmatrix} g + t\cos\theta \\ -h + t\sin\theta \end{Bmatrix}$$

$$d\cos\beta = g + t\cos\theta \quad \text{— (A)}$$

$$d\sin\beta = -h + t\sin\theta \quad \text{— (B)}$$

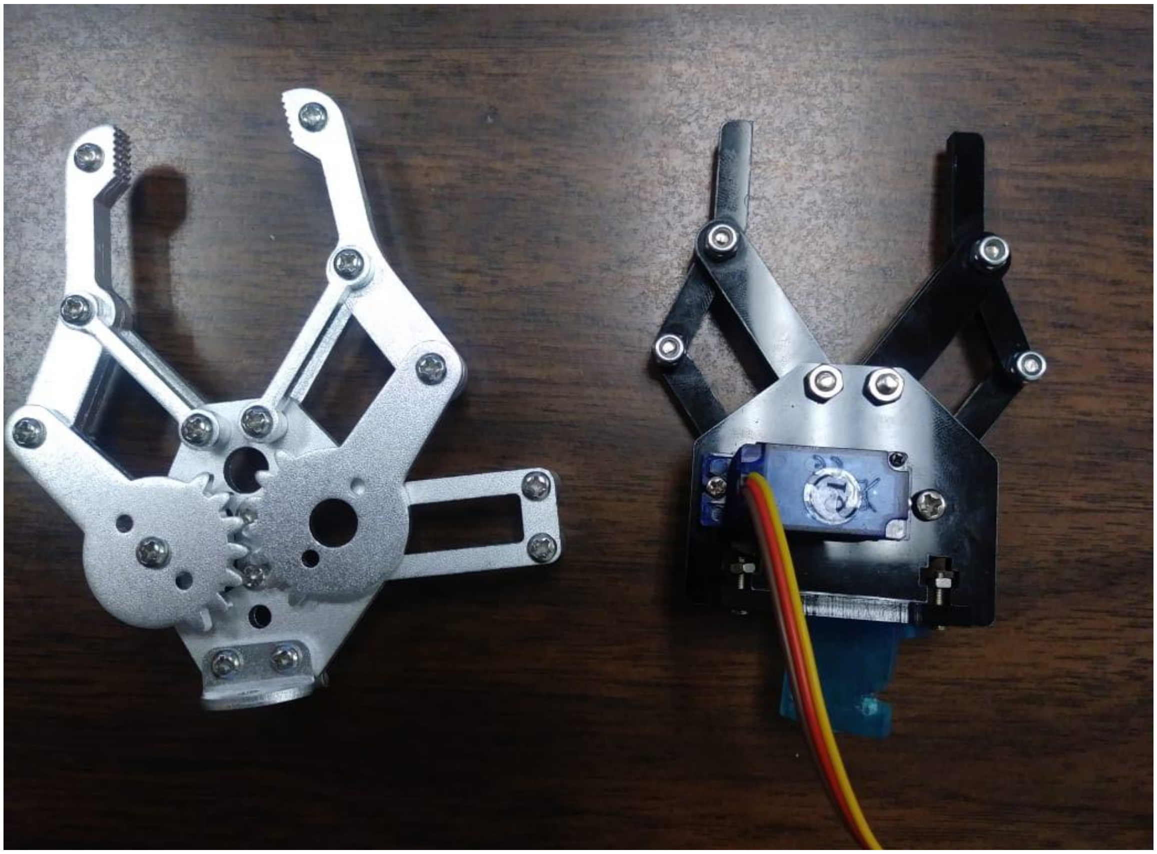
Perform (B)/A,

$$\frac{\sin\beta}{\cos\beta} = \frac{-h + t\sin\theta}{g + t\cos\theta}$$

$$\boxed{\therefore \beta(\theta) = \tan^{-1}\left(\frac{-h + t\sin\theta}{g + t\cos\theta}\right)}$$

(This problem will return in Recitation-3)

- **Demo:**
  There are several applications of Linkage design around us. One common application is given below:



1. **What is this part called?**
   A Gripper

2. **What is its relevance in the Kinematics class?**
   It is composed of two 4-bar linkages.

3. **Can you suggest any approaches to use this to balance between holding and squeezing?**
   a. For same size objects, it is possible to manually calibrate a grasper
   b. It is possible for a human to operate a grasper (eg, surgical robots)
   c. For advanced grasping, the approach is to identify the objects and then compute the grasping points which is used to grasp the object.

(Demo: Run the system using a potentiometer)

Bibliography:

- **Dr. Hedari's HW 2**
- **Dr. Deemyad's Notes**

Miscellaneous:

- For avid googlers like me:
  - Watts Linkage
  - Pantograph
  - Ornithopter
  - Compliant Mechanism
- If you like 3D models:
  - Thangs.com
  - Thingiverse.com
- Even Ironman uses 4-bar Linkage!