Kinematics & Dynamics of Machinery (ME 3320)

Recitation - 9

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1. **Agenda:**
    - Revision(Gear & Overall Kinematics)
    - Cam Design Problem
    - Gear Design Problem

2. **Revision:**
    - **In ME 3320 this semester, you learned the design and synthesis of Linkage and Cam mechanisms. What is their importance in the Mechanical Design Process?[1]**
        1. Function Generation
        2. Path Generation
        3. Motion Generation

    - **Define:**
        1. **Design:** Given a machine, calculate its motion and forces

        2. **Synthesis:** Given a motion/forces, create a machine able to do the task.

    - **What is the most important application of a gear mechanism?**
      Power transmission by converting torque to rpm and vice versa

    - **What is the physics behind a gear mechanism?**

        1. If two objects are rolling without slipping, their linear velocity at the point of contact is the same
        2. For gears with no losses, the power input from one gear is equal to the power output from the other gear.
    - **We have the following equation for the gear ratio:**

$$\boxed{R = \frac{\omega_{in}}{\omega_{out}} = \pm\frac{r_{out}}{r_{in}}}$$

      How do we choose the appropriate sign?
      + for external contact of meshed gears

- for internal contact of meshed gears

- **List the basic parameters that define the geometry of a gear.**

  Base Radius
Pitch Radius
Pressure Angle
Number of Teeth
Thickness of teeth

- **What parameters need to be considered to investigate if two gears will mesh?**
  1. The pressure angle and the pitch needs to be the same
  2. Investigate the clearance of addendum and dedendum (to make sure they do not hit each other)

- **What skills did you build this semester from pursuing Kinematics and Dynamics of Machinery?**
  1. Being able to evaluate mobility allowed to determine how many actuators are needed to output motion from a mechanism.
  2. Design, Synthesize and analyze 4-bar linkage based mechanisms
     a. Case Study of representative 4-bar linkage mechanisms
  3. Design of Cam Profile using follower profile
  4. Gear Basics, Geometry, and gear-based mechanisms

3. **Cam Design Problem:**

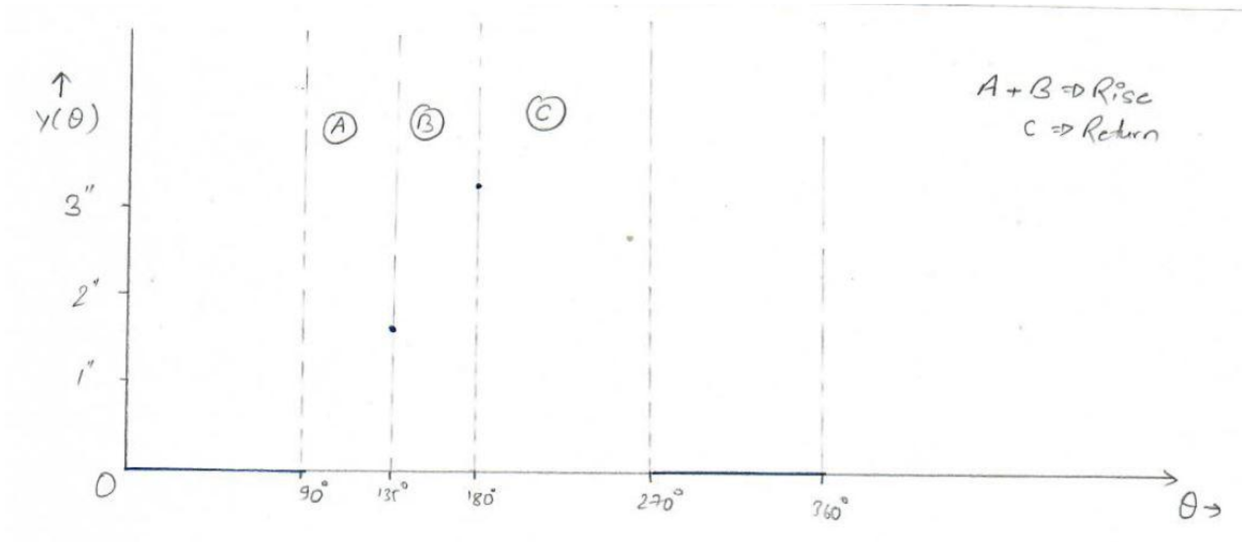
Write the equations of displacement function designed for a single rise and return from y = 0” to y = 3” and back.

- **Follower dwells from $\Theta = 0^0$ to 90^0**
- **Raises for $\Theta = 90^0$**
- **And then returns for $\Theta = 90^0$**
- **And dwells rest of the cycle**

The rise needs to be designed such that :

- **It has constant acceleration for half of the rise**
- **Constant deceleration (of the same value) for the other half of the rise**
- **Velocity function is continuous along with the rise**
- **The return should be such that the velocity function is constant**

a. **Draw the initial displacement sketch for the follower motion**



b. **Provide your reasoning for how should we choose the order of polynomial function for the rise and return.**

Rise: Since the acceleration needs to be constant, that means the velocity has to be linear and the displacement has to be quadratic(i.e. Second-order). Hence, for each half of the rise, we need 3 boundary conditions.

Return: Return needs to have a constant velocity which means displacement can be linear. Let's also assume continuity for the displacement. So, that means we can choose the first-order polynomial for the return. Also, that means we need 2 boundary conditions.

c. **Write the conditions on the displacement, velocity, and acceleration for each section of both rise and return and select the degrees of the polynomial functions to be used.**

BC for Ⓐ

$\theta_1 = 90°$, $\theta_2 = 135°$

$y(90°) = 0''$

$y(135°) = 1.5''$

$v(90°) = 0$ in/s

BC for Ⓑ

$y(135°) = 1.5''$, $\theta_1 = 135°$

$y(180°) = 3''$, $\theta_2 = 180°$

$v(180°) = 0$

BC for Ⓒ

$y(180°) = 3''$, $\theta_1 = 180°$

$y(270°) = 0''$, $\theta_2 = 270°$

Displacement/Velocity function for rise:

$$y(\theta) = C_0 + C_1 \frac{(\theta-\theta_1)}{\theta_2-\theta_1} + C_2 \left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^2 ; \quad v(\theta) = \left[\frac{C_1}{(\theta_2-\theta_1)} + 2C_2 \frac{(\theta-\theta_1)}{(\theta_2-\theta_1)^2}\right]\dot{\theta}$$

Displacement function for return:

$$y(\theta) = C_0 + C_1 \frac{(\theta-\theta_1)}{(\theta_2-\theta_1)}$$

For Rise:

$y(\theta_1) = C_0$ $\quad$ $\boxed{\therefore C_0 = y(\theta_1)}$

$y(\theta_2) = C_0 + C_1 + C_2$ $\quad$ $\boxed{C_1 + C_2 = y(\theta_2) - C_0}$

$v(\theta_1) = \frac{C_1 \dot{\theta}}{\theta_2-\theta_1}$ $\quad$ $\boxed{\therefore C_1 = v(\theta_1)\frac{(\theta_2-\theta_1)}{\dot{\theta}}}$

$v(\theta_2) = \left[\frac{C_1}{\theta_2-\theta_1} + \frac{2C_2}{(\theta_2-\theta_1)}\right]\dot{\theta}$ $\quad$ $\boxed{C_1 + 2C_2 = \frac{v(\theta_2)(\theta_2-\theta_1)}{\dot{\theta}}}$

For Ⓐ $\quad C_0 = 0 \quad \boxed{C_1 + C_2 = 1.5} \quad C_1 = 0 \quad C_2 = 1.5''$

For Ⓑ $\quad C_0 = 1.5'' \quad \boxed{C_1 + C_2 = 1.5''} \quad C_1 = 0 \quad C_2 = 1.5''$

$$\therefore y_{rise}(\theta) = \begin{cases} 1.5\left(\frac{\theta-90}{45}\right)^2 & \text{for region Ⓐ} \\ 1.5 + 1.5\left(\frac{\theta-135}{45}\right)^2 & \text{for region Ⓑ} \end{cases}$$

For Return:

$y(\theta_1) = C_0 \qquad y(\theta_2) = C_0 + C_1$

$C_0 = 3'' \qquad C_0 + C_1 = 0$

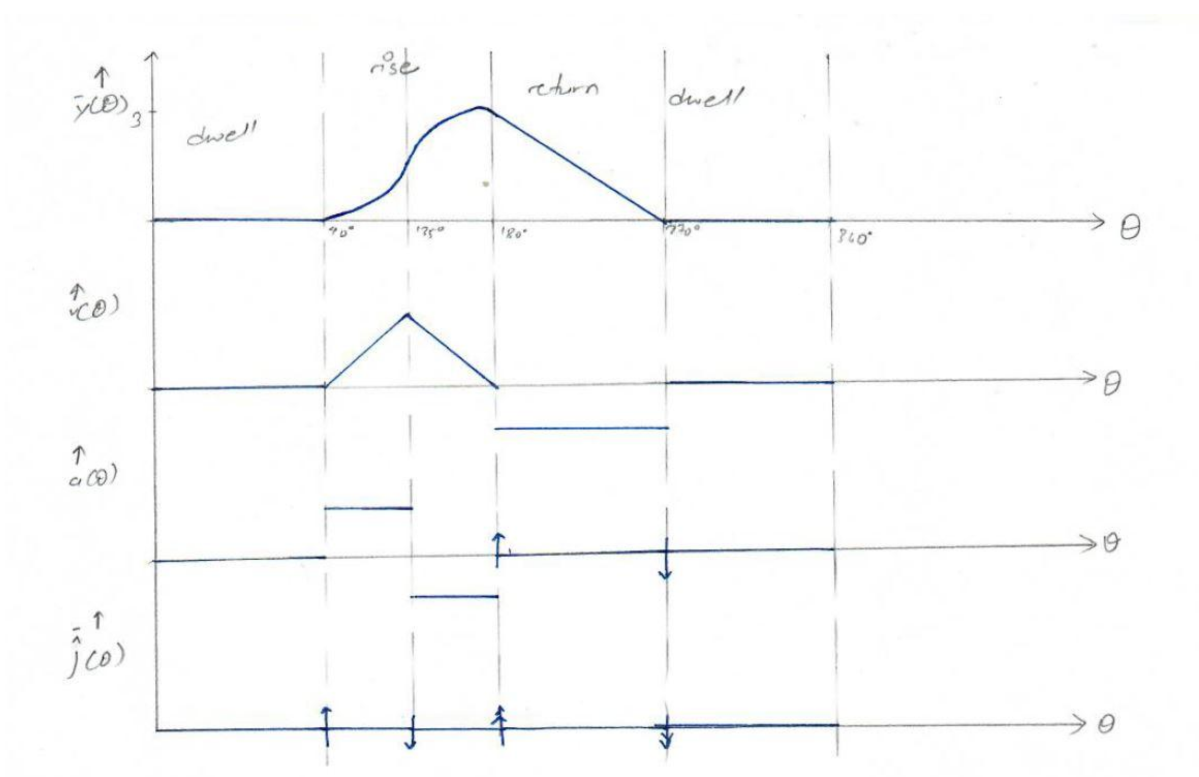
$\therefore C_1 = -3''$

$$\therefore y_{return}(\theta) = 3 - 3\,\frac{(\theta-180)}{90} = 3\left(\frac{90-\theta+180}{90}\right) = \frac{1}{30}(270-\theta) = \underline{\underline{\frac{-1}{30}\theta + 9}}$$

d. Write the displacement equations y = y(Θ) for each section.

$$
y(\theta) = \begin{cases} 0 & 0^\circ \le \theta < 90^\circ \\ 1.5\left(\dfrac{\theta-90}{45}\right)^2 & 90^\circ \le \theta < 135^\circ \\ 1.5\left[1+\left(\dfrac{\theta-135}{45}\right)^2\right] & 135^\circ \le \theta < 180^\circ \\ -\dfrac{1}{30}\theta + 9 & 180^\circ \le \theta < 270^\circ \\ 0 & 270^\circ \le \theta < 360^\circ \end{cases}
$$

e. Using the information discussed above create a hand-drawn sketch of the displacement, velocity, acceleration, and jerk profile for the follower mechanism.



4. **Gear Design Problem:**
   a. **Selection of Gear: Using the datasheet for spur gears below, select two spur gears to have an output angular velocity that is 3 times the input angular velocity. These gears have a pressure angle, $\phi = 20^0$. And the diametral pitch of $P_d = 16$.**

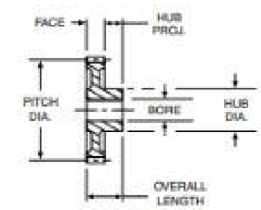
Spur Gears

16 and 12 Diametral Pitch (Steel & Cast Iron)

20° Pressure Angle (will not operate with 14-1/2° spurs)

ALL DIMENSIONS IN INCHES
ORDER BY CATALOG NUMBER OR ITEM CODE

No. of Teeth	Pitch Dia.	Bore	Hub Dia.	Hub Proj.	Style See Page 323	Without Keyway or Setscrew Catalog Number	Without Keyway or Setscrew Item Code	With Keyway and Setscrew† Catalog Number	With Keyway and Setscrew† Item Code
16 DIAMETRAL PITCH						Face = .750" Outside Dia. = Pitch Dia. + .125" Overall Length = .750" + Hub Proj.			
STEEL									
12	.750	.375	.56	.50		YB12	09916	YB12-3/8†	46141
14	.875		.69			YB14	09918	YB14-3/8†	46142
15	.938	.375	.75	.50		YB15	09920	YB15-3/8†	45991
		.500				–	–	YB15-1/2	46143
16	1.000	.500	.81	.50		YB16	09922	YB16-1/2	46144
18	1.125		.94			YB18	09924	YB18-1/2	46145
20	1.250	.625	1.05	.50		YB20	09926	YB20-5/8	46146
24	1.500	.625	1.20	.50		YB24	09928	YB24-5/8	46147
		.750				–	–	YB24-3/4	46148
28	1.750	.625	1.45	.50		YB28	09930	YB28-5/8	46149
		.750				–	–	YB28-3/4	46150
30	1.875	.625	1.58	.50		YB30	09932	YB30-5/8	46151
		.750				–	–	YB30-3/4	46152
		.875			A	–	–	YB30-7/8	46153
32	2.000	.625	1.70	.50		YB32	09934	YB32-5/8	46154
		.750				–	–	YB32-3/4	46155
		.875				–	–	YB32-7/8	46156
		1.000				–	–	YB32-1	46157
36	2.250	.625	1.95	.50		YB36	09936	–	–
40	2.500		2.20	.62		YB40	09938	–	–
48	3.000		2.00			YB48A	10572	–	–
56	3.500		2.50			YB56A	10574	–	–
60	3.750		2.75			YB60A	10576	–	–
64	4.000	.750	2.88	.75		YB64A	10578	–	–
72	4.500		3.38			YB72A	10580	–	–
80	5.000		3.88			YB80A	10582	–	–
CAST IRON									
96	6.000	.750	1.75	.75	D	YB96	10584	–	–
128	8.000		2.00			YB128	10588	–	–
144	9.000		2.00			YB144	10590	–	–
160	10.000	.875	2.00	.75		YB160	10592	–	–
192	12.000			1.00		YB192	10594	–	–
12 DIAMETRAL PITCH						Face = 1.000" Outside Dia. = Pitch Dia. + .167" Overall Length = 1.000" + Hub Proj.			
STEEL									
12	1.000	.500	.75	.62		YD12	09940	YD12-1/2*	46158
13	1.083	.625	.83	.62		YD13	09942	YD13-5/8‡	46159
14	1.167		.92			YD14	09944	YD14-5/8	46160
15	1.250		.99			YD15	09946	YD15-5/8	46161
16	1.333		1.07			YD16	09948	YD16-5/8	46162
18	1.500	.750	1.24	.62		YD18	09950	YD18-3/4	46163
20	1.667		1.32			YD20	09952	YD20-3/4	46164
21	1.750	.750	1.40	.62		YD21	09954	YD21-3/4	46165
		.875				–	–	YD21-7/8	46166
24	2.000	.750	1.65	.62		YD24	09956	YD24-3/4	46167
		.875			A	–	–	YD24-7/8	46168
		1.000				–	–	YD24-1	46169
28	2.333	.750	1.99	.62		YD28	09958	YD28-3/4	46170
		.875				–	–	YD28-7/8	46171
		1.000				–	–	YD28-1	46172
30	2.500	.750	2.15	.62		YD30	09960	–	–
36	3.000		1.94	.88		YD36A	10596	–	–
42	3.500		2.44			YD42A	10598	–	–
48	4.000	.875	2.88	.88		YD48A	10600	–	–
54	4.500		3.38			YD54A	10602	–	–



STANDARD TOLERANCES

DIMENSION		TOLERANCE
BORE	All	±.0005

16 D.P. 12 D.P.

REFERENCE PAGES

Alterations — 322
Horsepower Ratings — 58, 59
Lubrication — 322
Materials — 323
Selection Procedure — 49

†3/8" bore have one setscrew. No keyway.

YB15-1/2 and larger have standard keyway at 90° to setscrew. See page 323.

*YD12-1/2 has one setscrew. No keyway.

‡YD13-5/8 has one setscrew. No keyway.

YD14-5/8 bore and larger have standard keyway at 90° to setscrew.

a.1. Assume that the gears have an external contact write down the relevant equation that can be used to select the gears. What will this gear system do to the torque?

$$\omega_{out} = -3\ \omega_{in}$$

$$\boxed{R = \frac{\omega_{in}}{\omega_{out}} = -\frac{\omega_{in}}{3\ \omega_{in}} = -\frac{1}{3}}$$

Similarly, $R = T_{out}/T_{in}$

Hence, investigating the teeth numbers of the gears in the same classification of the diametral pitch will provide us with our desired meshing gears.

Now, output angular velocity is 3 times the input angular velocity. If we assume 100% efficient power transmission, the torque output is reduced by 3 times to keep the power constant.

a.2. Provide a list of gear sets that fit our purpose. Assume that we are only concerned with the gears that have a diametral pitch of 16.

(36 & 12), (60 & 20), (72 & 24), etc.

a.3. Choose a single set of gear and evaluate the other gear geometry parameters(Let's choose 36 & 12 for us).

1. **Addendum**

$$a = \frac{1.0}{P_d} = 0.0625\ in$$

2. **Dedendum**

$$b = \frac{1.25}{P_d} = 0.078\ in$$

3. **Pitch Radius**

$$P_d = \frac{N}{2r_p}$$

$$r_{p_{in}} = \frac{N}{2P_d} = \frac{36}{2\times16} = 1.125$$

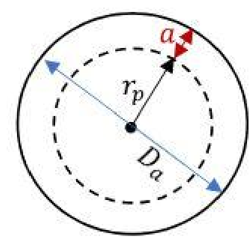
$$r_{p_{out}} = \frac{N}{2P_d} = \frac{12}{2\times16} = 0.375$$

4. **Outside Diameter**

$$D_a = 2r_p + 2a$$

$$D_{a_{in}} = 2r_{p_{in}} + 2a_{in} = 2\times1.125 + 2\times0.0625 = 2.375$$

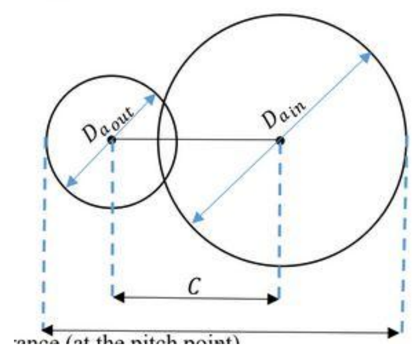
$$D_{a_{out}} = 2r_{p_{out}} + 2a_{out} = 2\times0.375 + 2\times0.0625 = 0.875$$



5. **Center Distance**

$$C = r_{p_{in}} + r_{p_{out}} = 1.125 + 0.375 = 1.5$$

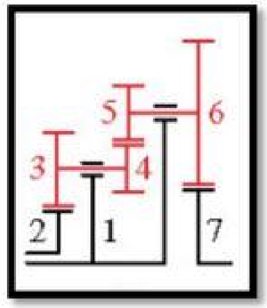
$$L = 2r_{p_{in}} + 2r_{p_{out}} + 2a = 2\times(1.125 + 0.375 + 0.0625) = 3.125$$ (Overall dimension of the system)



6. **Clearance at the pitch point**

$$c = b - a = 0.078 - 0.0625 = 0.0155$$

b. **Planetary Gear Train Analysis: Following figure shows a planetary gear train. Compute the planetary gear train ratio. Then considering the sun gear (2) as fixed, and the input arm (1) angular velocity is 360 rpm, compute the output (ring gear) angular velocity.**



ω_{in} = gear 1 (carrier)

ω_{out} = gear 7 (ring gear)

ω_s = gear 2 (fixed $\rightarrow \omega_s = 0$)

$$R = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_{s/c}}{\omega_{r/s}} = \frac{\omega_{2/1}}{\omega_{7/2}} = \left(\frac{\omega_2}{\omega_3}\right)\cancelto{1}{\left(\frac{\omega_3}{\omega_4}\right)}\left(\frac{\omega_4}{\omega_5}\right)\cancelto{1}{\left(\frac{\omega_5}{\omega_6}\right)}\left(\frac{\omega_6}{\omega_7}\right)$$

$$\frac{\omega_2}{\omega_3} = -\frac{N_3}{N_2} \quad ; \quad \frac{\omega_4}{\omega_5} = -\frac{N_5}{N_4} \quad ; \quad \frac{\omega_6}{\omega_7} = -\frac{N_7}{N_6}$$

$$R = \left(-\frac{N_3}{N_2}\right)(-1)\left(-\frac{N_5}{N_6}\right)(-1)\left(\frac{-N_7}{N_6}\right) = \underline{\underline{\frac{-N_3 N_5 N_7}{N_2 N_4 N_6}}}$$

$$R = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_{2/1}}{\omega_{7/2}} = \frac{\omega_2 - \omega_1}{\omega_7 - \omega_1} = -\frac{N_3 N_5 N_7}{N_2 N_4 N_6}$$

$$-\omega_1\left(-\frac{N_2 N_4 N_6}{N_3 N_5 N_7}\right) = \omega_7 - \omega_1$$

$$\therefore \omega_7 = \left(1 + \left(\frac{N_2 N_4 N_6}{N_3 N_5 N_7}\right)\right)\omega_1$$

$$\boxed{\therefore \omega_7 = \left(1 + \left(\frac{N_2 N_4 N_6}{N_3 N_5 N_7}\right)\right) 360\,\text{rpm}}$$

Bibliography:
Problem Courtesy: Dr. Deemyad

[1] https://www.cs.cmu.edu/~rapidproto/mechanisms/examples.html

Miscellaneous:

- With the first design class completed this semester, you can start working on Professional Networking and Internships.
    - You can **create a Github repository** for the ME 3320
        - Create a Github account
        - Download Github for Desktop application
        - Use the app to upload the Project files to the cloud(Pdf file for Hand calculations and Solidworks files, code scripts can be put in their respective formats)

(This will help you keep track of past projects and provide you a convenient way to share your past projects to the employers)

- **Create a Linkedin account.** Everyone is on Linkedin.
    - Create a Linkedin Account
    - Update your account and use it as a resume
    - Try to be active(you can post interesting videos, presentation slides, and even share your publications)
    - Search for people and the jobs that you are interested in
        - Keywords: Mechatronics, Perception, Kinematics, Manufacturing, Design, etc.
        - Look at the profile of the engineers, you will get an idea of how to improve your professional branding
        - Look at the job postings, you will get an idea of what skills are relevant to the area that you are interested in. You can choose your electives using this information.
        - Grow your network by adding relevant people on Linkedin. This will help you in the long run.

- Spend some time building and updating **your resume**. Take help from ISU Career Center. They specialize in mock interviews and resume writing.
- Websites like **Udemy, Udacity, and Coursera** provide access to learn skills that might be relevant to you but might not be accessible when you want to learn them. They also have some tutorials to build certain machines like Drones or a vehicle simulation, computer vision,

ROS, etc. This is especially useful if you are looking for project ideas for classes like Senior Design, Mechatronics, Robotics, etc.

- There are **Reddit and Discord groups** for several branches of Mechanical Engineering. These group members are active and can provide you with expert advice for free on the problems of your projects. Since engineering design problems do not come with a specific right answer, the insight of experts is really helpful in optimizing a design.