Preventing Runtime Errors of Redis at Compile Time

Abstract

Programmers often interact with database systems by sending queries through libraries or packages in some programming languages. People, however, make mistakes. While some of the syntactic and semantic errors can be prevented by the language at compile time, or caught by the package at runtime, most semantic errors are just being ignored, causing problems at the database system.

In this paper, we demonstrate how to prevent those runtime errors at compile time, thus allowing users to write more reliable database queries without runtime overhead, by exploiting type-level programming techniques such as indexed monad, type-level literals and closed type families in Haskell.

The database system and the package we are targeting are *Redis* and *Hedis* respectively, and our implementation is available as *Edis* on *Hackage*.

Categories and Subject Descriptors

Keywords Haskell; type-level programming; database query language; Redis;

1. Introduction

1.1 Redis

Redis¹ is an open source, in-memory data structure store, often used as database, cache and message broker. A Redis datatype can be think of as a set of key-value pairs, where each value is associated with a binarysafe string key to identify and manipulate with. Redis supports many different kind of values, such as strings, hashes, lists, and sets, etc, and provides a collection of of atomic *commands* to manipulate these values.

For an example, consider the following sequence of commands, entered through the interactive interface of Redis. The keys some-set abd another-set are both associated to a set. The two call to command SADD respectively adds three and two values to the two sets, before SINTER takes their intersection:

```
redis> SADD some-set a b c
(integer) 3
redis> SADD another-set a b
(integer) 2
redis> SINTER some-set another-set
1) "a"
2) "b"
```

Note that the keys some-set and another-set, if not existing before the call to SADD, are created on site. The call to SADD returns the size of the set after completion of the command.

1.2 Hedis

Many third party library exist to allow general purpose programmings to access Redis databases through its TCP protocol. The most popular such library for Haskell is Hedis².

The following program is how the previous example looks like in Hedis:

```
program :: Redis (Either Reply [ByteString])
program = do
    sadd "some-set" ["a", "b"]
    sadd "another-set" ["a", "b", "c"]
    sinter ["some-set", "another-set"]
```

The function sadd takes a key and a list of values as arguments, and returns an Integer on success, or returns a Reply, a low-level representation of replies from the Redis server, in case of failues. All wrapped in Redis³, the context of command execution.

¹https://redis.io

[[]Copyright notice will appear here once 'preprint' option is removed.]

² https://hackage.haskell.org/package/hedis

Note that keys and values, being nothing but binary strings in Redis, are represented using Haskell ByteString. Values of other types must be encoded as ByteStrings before being written to the database, and decoded after being read back.

2. Motivation

All binary strings are equal, but some binary strings are more equal than others.

Although everything in Redis is essentially a binary string, these strings are treated differently. Redis supports many different kind of data structures, such as strings, hashes, lists, etc. While they are all encoded as binary strings before being written to the databse, most commands, much like how the C language treats a piece of data, only works with data of certain types.

Problem 1 The command SET, by its definition, associates a key to a string. In the following example, the key some-string is associated to string foo. Subsequent calls to SADD causes runtime errors, since the value of some-string is not a set, but a string.

```
redis> SET some-string foo
OK
redis> SADD some-string bar
(error) WRONGTYPE Operation against a key
holding the wrong kind of value
```

Problem 2 Even worse, not all strings are equal! The call INCR some-string parses the string associated with key some-string to an integer, increments it by one, and store it back as a string. If the string can not be parse as an integer, a runtime error is raised.

```
redis> SET some-string foo
OK
redis> INCR some-string
(error) ERR value is not an integer or out
  of range
```

In Hedis Hedis, being only a simple wrapper on top of the TCP protocol of Redis, inherits all the problems mentioned above. The following program yields the same error as that in the Redis client.

```
program :: Redis (Either Reply Integer)
program = do
    set "some-string" "foo"
    sadd "some-string" ["a"]
```

```
Left (Error "WRONGTYPE Operation against a key holding the wrong kind of value")
```

The Cause Every key is associated with a value, and every value has it's own type. But Most commands in Redis only work with a certain type of value. When a command is used on a wrong type of key, a runtime error occurs. The problems illustrated above arise from the absence of type checking, with respects to the type of a value that associates with a key. These problems could have been avoided, if we could know the type every key associates with in advance, and prevent programs with invalid commands from executing.

2.1 Hedis as an embedded DSL

Haskell makes it easy to build and use Domain Specific Languages (DSLs), and Hedis can be regarded as one of them. What makes Hedis peculiar is that, it has variable bindings (between keys and values), but with very little or no semantic checking, neither dynamically nor statically.

We began with making Hedis a dynamically checked embedded DSL, and implemented a runtime type checker that keeps track of types of each variable. But then we found that things can be a lot easier, by leveraging the host language's type checker. We encode variable bindings with type-level lists and strings, and control the effects on the bindings with indexed monad. In contrast to the former approach, we embedded our type checker into Haskell's type system, without having to build a standalone one on the term level.

2.2 Contributions

To summarize our contributions:

- We make Hedis statically type-checked, without runtime overhead.
- We demonstrates how to model variable bindings of an embedded DSL with language extensions like type-level literals and data kinds.
- We provide (yet another) an example of encoding effects and constraints of an action in types, with indexed monad[1] and other language extensions such as closed type-families[?] and constraints kinds[15].
- Edis, a package we built for programmers. This package helps programmers to write more reliable Redis programs, and also makes Redis polymorphic by automatically converting back and forth from values of arbitrary types and boring ByteStrings.

3. Type-level dictionaries

To check the bindings between keys and values, we need a *dictionary-like* structure, and encode it as a *type* somehow.

³ Hedis provides another kind of context, RedisTx, for *transactions*, united with Redis under the class of RedisCtx. For brevity, we demonstrate only Redis in this paper.

3.1 Datatype promotion

Normally, at the term level, we could express the datatype of dictionary with *type synonym* like this.⁴

```
type Key = String
type Dictionary = [(Key, TypeRep)]
```

To encode this in the type level, everything has to be promoted[19] one level up. From terms to types, and from types to kinds.

Luckily, with recently added GHC extension data kinds, suitable datatype will be automatically promoted to be a kind, and its value constructors to be type constructors.

```
data List a = Nil | Cons a (List a)
```

Give rise to the following kinds and type constructors: 56

```
List k :: BOX
Nil :: List k
Cons :: k -> List k -> List k
```

Haskell sugars lists [1, 2, 3] and tuples (1, 'a') with brackets and parentheses. We could also express promoted lists and tuples in types like this with a single quote prefixed. For example: '[Int, Char], '(Int, Char).

3.2 Type-level literals

Now we have type-level lists and tuples to construct the dictionary. For keys, String also has a type-level correspondence: Symbol.

```
data Symbol
```

Symbol is defined without a value constructor, because it's intended to be used as a promoted kind.

```
"this is a type-level string literal" :: Symbol
```

3.3 Putting everything together

With all of these ingredients ready, let's build some dictionaries!

```
type DictEmpty = '[]
type Dict0 = '[ '("key", Bool) ]
type Dict1 = '[ '("A", Int), '("B", "A") ]
```

These dictionaries are defined with *type synonym*, since they are *types*, not *terms*. If we ask GHCi what is the kind of Dict1, we will get Dict1 :: [(Symbol, *)]

The kind * (pronounced "star") stands for the set of all concrete type expressions, such as Int, Char or even a symbol "symbol", while Symbol is restricted to all symbols only.

4. Indexed monads

Redis commands are *actions*. We could capture the effects caused by an action, by expressing the states it affects, before and after. That is, the *preconditions* and *postconditions* of an action. In such way, we could also impose constraints on the preconditions.

Indexed monads (or monadish, parameterised monad)[1] can be used[11][9] to model such preconditions and post-conditions in types. An indexed monad is a type constructor that takes three arguments: an initial state, a final state, and the type of a value that an action computes, which can be read like a Hoare triple[14].

```
class IMonad m where
  unit :: a -> m p p a
  bind :: m p q a -> (a -> m q r b) -> m p r b
```

Class IMonad comes with two operations: unit for identities and bind for compositions, as in monads.

We define a new datatype Edis, which is basically just the context Redis indexed with more information in types. We make it an instance of IMonad.

```
newtype Edis p q a = Edis { unEdis :: Redis a }
instance IMonad Edis where
   unit = Edis . return
   bind m f = Edis (unEdis m >>= unEdis . f )
```

The first and second argument of type Edis is where the dictionaries going to stay.

To execute a Edis program, simply apply it to unEdis to get an ordinary program of type Redis, with type information erased.

4.1 PING: the first attempt

In Redis, PING does nothing but replies with PONG if the connection is alive. In Hedis, PING has type:

```
ping :: Redis (Either Reply Status)
```

Now we have $\mathsf{Edis},\ \mathsf{let's}\ \mathsf{make}\ \mathsf{our}\ \mathsf{own}\ \mathsf{version}\ \mathsf{of}\ \mathsf{ping}^7$

```
ping :: Edis xs xs (Either Reply Status)
ping = Edis Hedis.ping
```

The dictionary xs in the type remains unaffected after the action, because ping does not affect any key-value bindings. To encode other commands that modi-

 $^{^4\,\}mbox{TypeRep}$ supports term-level representations of data types, available in Data. Typeable

 $^{^6}$ To distinguish between types and promoted constructors that have ambiguous names, prefix promoted constructor with a single quote like 'Ni1 and 'Cons

⁶ All kinds have sort BOX in Haskell[7]

 $^{^{\}overline{7}}$ ping from Hedis is qualified with Hedis to prevent function name clashing in our code.

fies key-value bindings, we need type-level functions to annotate those effects on the dictionary.

5. Type-level functions

5.1 Closed type families

Type families have a wide variety of applications. They can appear inside type classes [4][3], or at toplevel. Toplevel type families can be used to compute over types, they come in two forms: open [17] and closed [6].

We choose *closed type families*, because it allows overlapping instances, and we need none of the extensibility provided by open type families. For example, consider both term-level and type-level &&:

The first instance of And could be subsumed under a more general instance, And a b. But the closedness allows these instances to be resolved in order, just like how cases are resolved in term-level functions. Also notice that how much And resembles to it's term-level sibling.

5.2 Functions on type-level dictionaries

With closed type families, we could define functions on the type level. Let's begin with dictionary lookup.

Another benefit of closed type families is that type-level equality can be expressed by unifying type variables with the same name. Get takes two type arguments, a dictionary and a symbol. If the key we are looking for unifies with the symbol of an entry, then Get returns the corresponding type, else it keeps searching down the rest of the dictionary.

```
Get '['("A", Int)] "A" evaluates to Int.
But Get '['("A", Int)] "B" would get stuck.
That's because Get is a partial function on types, and
these types are computed at compile-time. It wouldn't
```

these types are computed at compile-time. It wouldn't make much sense for a type checker to crash and throw a "Non-exhaustive" error or be non-terminating.

We could make Get total, as we would at the term level, with Maybe.

Other dictionary-related functions are defined in a similar fashion.

```
-- inserts or updates an entry
type family Set
    (xs :: [(Symbol, *)]) -- old dictionary
                            -- key
    (s :: Symbol)
    (x :: *)
                            -- type
    :: [(Symbol, *)] where -- new dictionary
    Set '[]
                       s x = '['(s, x)]
    Set ('(s, y) ': xs) s x = ('(s, x) ': xs)
    Set ('(t, y) ': xs) s x =
        '(t, y) ': (Set xs s x)
-- removes an entry
type family Del
    (xs :: [(Symbol, *)]) -- old dictionary
    (s :: Symbol)
                           -- kev
    :: [(Symbol, *)] where -- new dictionary
   Del '[] s
                         = '[]
    Del ('(s, y) ': xs) s = xs
    Del ('(t, y) ': xs) s = '(t, y) ': (Del xs s)
-- membership
type family Member
    (xs :: [(Symbol, *)]) -- dictionary
    (s :: Symbol)
                           -- key
    :: Bool where
                           -- exists?
    Member '[]
                           s = False
   Member ('(s, x) ': xs) s = True
   Member ('(t, x) ': xs) s = Member xs s
```

5.3 Proxies and singleton types

Now we could annotate the effects of a command in types. DEL removes a key from the current database, regardless of its type.

KnownSymbol is a class that gives the string associated with a concrete type-level symbol, which can be

retrieved with symbolVal. Where encodeKey converts Proxy s to ByteString.

```
encodeKey = encode . symbolVal
```

Since Haskell has a phase distinction[13], types are erased before runtime. It's impossible to obtain information directly from types, we can only do this indirectly, with $singleton\ types[5]$.

A singleton type is a type that has only one instance, and the instance can be think of as the representative of the type at the realm of runtime values.

Proxy, as its name would suggest, can be used as singletons. It's a phantom type that could be indexed with any type.

```
data Proxy t = Proxy
```

In the type of del, the type variable s is a Symbol that is decided by the argument of type Proxy s. To use del, we would have to apply it with a clumsy termlevel proxy like this:

```
del (Proxy :: Proxy "A")
```

Making Redis polymorphic

Redis supports many different datatypes, these datatypes as can be viewed as *containers* of strings. For example, lists (of strings), sets (of strings), and strings themselves.

6.1 Denoting containers

Most Redis commands only work with a certain type of these containers. To annotate what container a key is associated with, we introduce these container types.

```
data Strings
data Lists
data Sets
```

SET stores a string, regardless the datatype the key was associated with. Now we could implement SET like this:

```
set :: KnownSymbol s
   => Proxy s
    -> ByteString
                        -- data to store
    -> Edis xs (Set xs s Strings) (Either Reply Status)
set key val = Edis $ Hedis.set (encodeKey key) val
```

After SET, the key will be associated with Strings in the dictionary, indicating that it's a string.

6.2 Automatic data serialization

But in the real world, raw binary strings are hardly useful, people would usually serialize their data into strings before storing them, and deserialize them back when in need.

Instead of letting users writing these boilerplates, them. With the help from cereal, a binary serialization library. cereal comes with these two functions:

```
encode :: Serialize a
       => a -> ByteString Source
decode :: Serialize a
       => ByteString -> Either String a
```

Which would do all the works for us, as long as the datatype it's handling is an instance of class Serialize.9

6.3 Extending container types

We rename container types and extend it with an extra type argument, to indicate what kind of encoded value it's holding.

```
data StringOf x
data ListOf x
data SetOf x
```

set reimplemented with extended container types:

```
set :: (KnownSymbol s, Serialize x)
    => Proxy s
               -- can be anything, as long as it's serializable
    -> x
    -> Edis xs (Set xs s (StringOf x)) (Either Reply Status)
set key val = Edis $ Hedis.set (encodeKey key) (encode val)
```

For example, if we execute set (Proxy :: Proxy "A") True, a new entry '("A", StringOf Bool) will be inserted to the dictionary.

6.4 Handling INCR

Commands such as INCR and INCRBYFLOAT, are not only container-specific, they also have some requirements on what types of value they could operate on.

We could handle this by mapping Redis's strings of integers and floats to Haskell's Integer and Double.

```
incr :: (KnownSymbol s, Get xs s ~ Just (StringOf Integer))
     => Proxy s -> Edis xs xs (Either Reply Integer)
incr key = Edis $ Hedis.incr (encodeKey key)
incrbyfloat :: (KnownSymbol s, Get xs s ~ Just (StringOf Double))
     => Proxy s -> Double -> Edis xs xs (Either Reply Double)
incrbyfloat key n = Edis $ Hedis.incrbyfloat (encodeKey key) n
```

Imposing constraints

To rule out programs with undesired properties, certain constraints must be imposed, on what arguments they can take, or what preconditions they must hold.

⁸ They are defined in GHC. TypeLits.

 $^{^9\,\}mathrm{The}$ methods of Serialize will have default generic implementations for all datatypes with some language extensions enabled, no sweat!

Consider the following example: LLEN returns the length of the list associated with a key, else raises a type error.

```
redis> LPUSH some-list bar
(integer) 1
redis> LLEN some-list
(integer) 1
redis> SET some-string foo
OK
redis> LLEN some-string
(error) WRONGTYPE Operation against a key
holding the wrong kind of value
```

Such constraint could be expressed in types with equality constraints[18].

Where (\sim) denotes that Get xs s and Just (ListOf x) needs to be the same.

The semantics of LLEN defined above is actually not complete. LLEN also accepts keys that do not exist, and replies with \emptyset .

```
redis> LLEN nonexistent
(integer) 0
```

In other words, we require that the key to be associated with a list, textbfunless it doesn't exist at all.

7.1 Expressing constraint disjunctions

Unfortunately, expressing disjunctions in constraints is much more difficult than expressing conjunctions, since the latter could be easily done by placing constraints in a tuple (at the left side of =>).

There are at least three ways to express type-level constraints [5]. Luckily we could express constraint disjunctions with type families in a modular way.

The semantics we want could be expressed informally like this:

```
Get xs s \equiv Just (ListOf x) \lor \neg (Member xs s)
```

We could achieve this simply by translating the semantics we want to the domain of Boolean, with typelevel boolean functions such as (&&), (||), Not, (==), etc. 10 To avoid

```
Get xs s == Just (ListOf x) || Not (Member <math>xs s)
```

To avoid addressing the type of value (as it may not exist at all), we defined an auxiliary predicate IsList:: Maybe * -> Bool to replace the former part.

```
IsList (Get xs s) || Not (Member xs s)
```

The type expression above has kind Bool, we could make it a type constraint by asserting equality.

```
(IsList (Get xs s) | Not (Member xs s)) ~ True
```

With constraint kind, a recent addition to GHC. Type constraints now has its own kind: Constraint. That means type constraints are not restricted to the left side of a => anymore, they could appear in anywhere that accepts something of kind Constraint, and any type that has kind Constraint can also be used as a type constraint. ¹¹

As many other list-related commands also have this "List or nothing" semantics, we could abstract the lengthy type constraint above and give it an alias with type synonym.

```
ListOrNX xs s = (IsList (Get xs s) | Not (Member xs s)) ~ True
```

The complete implementation of LLEN with ListOrNX would become:

8. Assertions

Users may need to make assertions about the status of some key-type bindings in a Redis program. For example, declaring the existence of a key and the type of its associating value. We provide these functions, which do nothing but fiddling with types.

8.1 A complete example

The following program increases the value of "A" as an integer, push the result of the increment to list "L", and then pops it out.

```
main :: IO ()
main = do
```

¹⁰ Available in Data. Type. Bool and Data. Type. Equality

 $^{^{11}\,\}mathrm{See}$ https://downloads.haskell.org/~ghc/7.4.1/docs/html/users_guide/constraint-kind.html.

The syntax is pretty heavy, like the old days when there's no *do-notation*[8]. But if we don't need any variable bindings between operations, we could compose these commands with a sequencing operator (>>>).

9. Discussions

Syntax No one could ignore the glaring shortcoming of the syntax, which occurs mainly in two places: *symbol singletons* and *indexed monad*. We are hoping that these issues could be solved with future syntactic extensions.

Returns only determined datatypes All other data structures in Redis also follow the semantics similar to "List or nothing" as LLEN has. Take the case of GET, which has a similar semantics that should be typed:

Since the key may not exist, we don't know what x would be. We could left x ambiguous, and let it be decided by the caller. But users will then be forced to spell out the complete type signature of everything, including the dictionaries, only to specify the desired resulting type.

Instead of allowing the key to be non-existent, we require that the key must exist and it's associating type to be determined at compile time. So our version of get has a stricter semantics:

```
get :: (KnownSymbol s, Serialize x
    , Just (StringOf x) ~ Get xs s)
=> Proxy s -> Edis xs xs (Either Reply (Maybe x))
```

Commands with multiple inputs or outputs Some command may take a variable number of arguments as inputs, and returns more than one value as outputs. To illustrate this, consider sinter in Hedis:

```
Hedis.sinter :: [ByteString] -- keys
-> Redis (Either Reply [ByteString]) -- values
```

(Proxy :: Proxy "A") In Hedis such command could easily be expressed with lists of ByteStrings. But in Edis, things escalate (Proxy :: Proxy "L") qdickly, as the keys and values will have to be expressed (Proxy :: Proxy "L") with heterogeneous lists[10], which would be pratically (Proxy :: Proxy "L") infeasible, considering the cost, if not impossible.

Most importantly, the keys will all have to be constrained by KnownSymbol, which enforces these type literals to be concrete and known at compile time. It's still unclear whether this is possible.

So instead, we are offering commands that only has a single input and output.

Notice that **not all Hedis programs can be typechecked** (even if they might turn out to be type safe). We opted for type safety instead of expressiveness.

Redis Transactions Redis also provides *transactions*, another context for executing commands. Redis trasanctions are atomic in the sense that, all commands in a transaction will executed sequentially, and no other requests issued by other clients will be served in the middle. ¹²In contrast, we cannot make such a guarantee in the ordinary context, which may destroy the assertions we made in types.

At the point of writing, transactions are not supported in our implementation. We are planning to add it in the future, and we are expecting that there wouldn't be much difficulties, since we've implemented a runtime type checker specifically targeting Redis transactions once, before we moved on to the types.

10. Related Work

While Redis can be viewed as a non-relational database system (although Redis seems reluctant to admit this in recent years), there also similar goals on relational database systems, trying to achieve a safer database interface by making them statically checked. *HaskellDB*[12][2] expresses quries with relational algebra-like combinators, wrapped in phantom types. Or as examples, at first to demonstrate the power of dependent types in Agda[16], and then with singletons in Haskell[5].

11. Conclusions

We have demonstrated how a DSL with variablebindings could be embedded in types. With more and more extensions added to the language, Haskell is gradually becoming a dependently-typed language, 13 but it's not that dreadful as many (including us) would have thought.

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¹³ Why not be dependently typed? http://stackoverflow.com/questions/12961651/why-not-be-dependently-typed

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