Beyond Contrastive Learning: Synthetic Data Enables List-wise Training with Multiple Levels of Relevance

¹Brown University ²Microsoft ³University of Tübingen {reza_esfandiarpoor,ruochen_zhang,macton_mgonzo,stephen_bach}@brown.edu gzerveas@microsoft.com c.eickhoff@acm.org

Abstract

Recent advancements in large language models (LLMs) have allowed the augmentation of information retrieval (IR) pipelines with synthetic data in various ways. Yet, the main training paradigm remains: contrastive learning with binary relevance labels and the InfoNCE loss, where one positive document is compared against one or more negatives. This objective treats all documents that are not explicitly annotated as relevant on an equally negative footing, regardless of their actual degree of relevance, thus (a) missing subtle nuances that are useful for ranking and (b) being susceptible to annotation noise. To overcome this limitation, in this work we forgo real training documents and annotations altogether and use open-source LLMs to directly generate synthetic documents that answer real user queries according to several different levels of relevance. This fully synthetic ranking context of graduated relevance, together with an appropriate list-wise loss (Wasserstein distance), enables us to train dense retrievers in a way that better captures the ranking task. Experiments on various IR datasets show that our proposed approach outperforms conventional training with InfoNCE by a large margin. Without using any real documents for training, our dense retriever significantly outperforms the same retriever trained through self-supervision. More importantly, it matches the performance of the same retriever trained on real, labeled training documents of the same dataset, while being more robust to distribution shift and clearly outperforming it when evaluated zeroshot on the BEIR dataset collection. Code: https://github.com/BatsResearch/sycl

1 Introduction

The ability of information retrieval (IR) methods to rank a collection of documents based on their

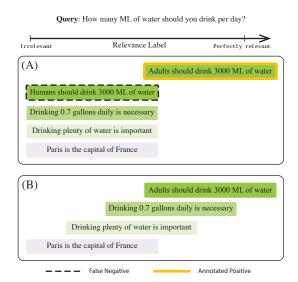


Figure 1: A) Standard contrastive training with real data is vulnerable to false negatives. More importantly, it treats all passages except the explicitly annotated positive passage the same, on a binary basis, regardless of their actual similarity to the given query. B) SyCL generates a synthetic multi-level ranking context and trains the model to rank passages based on their degree of relevance to the given query.

relevance to a given query is critical for many applications like web search, recommendation systems, and more recently, retrieval augmented generation (RAG) (Lewis et al., 2020). However, since existing large-scale IR datasets only provide binary relevance labels, recent work predominantly trains retrievers to simply separate relevant from irrelevant documents. This approach implicitly assumes that during inference the learned similarity metric is precise enough to correctly rank multiple relevant documents with only nuanced differences. Here, we use open source LLMs to generate multiple synthetic documents with graduated relevance levels for each query, which enables us to explicitly guide retrievers to rank a collection of documents during training.

Existing large-scale IR datasets like MS

^{*}Equal contributions.

MARCO provide a small set of annotated documents for each query (ranking context), with binary labels dividing documents into relevant ("positive") and irrelevant ("negative"). This contains very few, and most often only one, annotated relevant document(s) per query (Bajaj et al., 2018). These limitations are reflected in the predominant training paradigm: contrastive learning with the InfoNCE loss (van den Oord et al., 2019). However, this contrastive learning objective differs from ranking in that all documents other than a single annotated positive are treated as negatives of equal non-relevance, regardless of their actual semantic similarity to the query. Additionally, it only takes into account a single relevant document at each training step. By contrast, an effective retriever is expected to rank a collection of documents with potentially multiple positives according to nuanced semantic differences.

On the other hand, the benefits of a rich ranking context are well established in the learning-to-rank (L2R) literature, which often uses datasets with graduated relevance labels for training (Cao et al., 2007; Ai et al., 2019, 2018). Most L2R works date before the advent of transformers and rely on small datasets with engineered features (Qin et al., 2010b; Chapelle and Chang, 2011; Dato et al., 2016), which are not suitable for training contemporary dense retrievers. Unfortunately, existing large-scale datasets suitable for the latter do not provide such graduated relevance labels. As part of this work, we therefore propose to generate large-scale datasets with multiple relevance levels using open-source LLMs.

Many recent works are dedicated to offsetting the limitations of existing datasets within the framework of contrastive learning. To better delineate the positive and negative regions, it is common to use a large number of random negatives through large batch sizes (Qu et al., 2021) and also use existing retrievers to mine negative documents that are harder to distinguish from positives (Gao and Callan, 2021a). To address the false negative problem (large number of unannotated positive documents) in IR datasets, many works use existing retrievers to filter false negatives or pseudo-label the training documents (Thakur et al., 2021a; Hofstätter et al., 2021; Qu et al., 2021; Ren et al., 2021). These techniques lead to complex training pipelines and are limited by the quality of available retrievers. To avoid these problems, Wang et al. (2023a) use LLMs to generate synthetic (query, positive,

negative) triplets. Despite their progress, these efforts remain within the bounds of binary relevance labels and do not address the fundamental discrepancy between contrastive learning and the ranking objective.

In this paper, we propose SyCL (Synthetic ranking Context for List-wise training), a novel approach that enables training large transformerbased retrievers with graduated relevance labels. First, we create a large-scale IR dataset (~2M samples) that provides several documents with different relevance levels for each query. To avoid data annotation problems (e.g., sparsity and noise) while maintaining diversity and scale, we forgo real documents and use open-source LLMs to generate synthetic documents with four different relevance levels for training queries of the MS MARCO dataset. Our dataset allows us to penalize the model's scoring choices differently depending on the relative degree of label disagreement during training. Second, we propose to use the Wasserstein distance as a list-wise loss function that can effectively leverage graduated relevance labels to optimize large transformer-based retrievers.

Without using any real documents, SyCL significantly improves the performance of self-supervised retrievers in both in-domain evaluation on MS MARCO and zero-shot evaluation on the BEIR benchmark. We show that SyCL utilizes the available data more effectively than contrastive learning: using the same queries and synthetic documents, SyCL achieves an average nDCG@10 of 43.2 on BEIR datasets, significantly improving over 36.8 for contrastive learning. In fact, SyCL achieves comparable performance to contrastive learning with real data on MS MARCO and clearly outperforms it in zero-shot evaluation on BEIR. Additionally, SyCL can benefit from existing datasets: we successfully integrate real annotated documents into the synthetic ranking context and further boost performance. Through extensive analytical experiments, we show the individual significance of the Wasserstein loss and graduated relevance labels. Finally, we analyze our data generation pipeline and find that even small, 32B LLMs can generate high-quality training data. We summarize our main contributions as following:

We introduce SyCL, a novel method for training dense retrievers, which (a) uses open-source LLMs to generate a large corpus of synthetic documents with graduated relevance

labels and (b) uses Wasserstein distance as a list-wise loss function for training with multiple relevance levels.

- We empirically show that list-wise training with a multi-level ranking context outperforms standard contrastive learning with the InfoNCE loss and binary relevance labels.
- We show that without using real documents, SyCL significantly boosts the performance of self-supervised retrievers and performs on par with training on real, labeled documents through contrastive learning. SyCL even outperforms training with real data when zeroshot evaluated on the BEIR benchmark, which shows a more robust ranking behavior that can perform better in real-world scenarios.

Our results support the deployment of our method for many applications where only a collection of user queries and optionally a handful of demonstration documents are available. More importantly, our findings encourage future work to go beyond the limitations of existing training settings and explore novel data generation approaches that are more appropriate for training retrieval models.

2 Related Work

Dense Retrieval Training Retrieval training pipelines have improved significantly by addressing various limitations of IR datasets. Since most datasets provide very few, if any, annotations for negative documents, it is common to use a large number of random in-batch negatives (i.e., positive documents for other queries in the batch) through large batch sizes (Karpukhin et al., 2020; Qu et al., 2021). Similarly, many works use existing retrievers to search ("mine") the corpus for highly-ranked negative documents, which are hard to distinguish from positives (called hard negatives) (Moreira et al., 2024; Qu et al., 2021). Some suggest using the latest training checkpoint to mine new hard negatives periodically or at each step during training (Xiong et al., 2020; Zhan et al., 2021).

Because IR datasets like MS MARCO are sparsely annotated, many unannotated positives are mistakenly used as negative samples during training. Consequently, several works have leveraged existing retrievers to filter false negatives or create pseudo-labels for training documents, as in a teacher-student distillation scheme (Ren et al.,

2021; Qu et al., 2021). As mentioned in Section 1, these techniques lead to complex training pipelines, require additional compute resources, and are limited by the quality of existing retrievers. Most importantly, all these methods ignore the nuances of ranking and assume a binary definition of relevance. By contrast, we avoid these problems by generating synthetic documents, and simultaneously expand the definition of relevance labels to multiple levels.

Ranking Context The benefits of a rich ranking context are well established in the learning-to-rank (L2R) literature before the advent of transformerbased retrievers (Cao et al., 2007; Ai et al., 2019, 2018). Most L2R works use 4 to 6 levels of relevance during training (Qin et al., 2010b; Chapelle and Chang, 2011; Dato et al., 2016), and hundreds of annotated documents per query, compared to current large-scale datasets, which only provide a binary definition of relevance, and mostly a single positive document. As a result, the impact of multiple relevance levels for training large transformer models is largely unexplored, except for a few limited attempts: motivated by L2R literature, Zerveas et al. (2022, 2023) use a large number of mined documents per query and label propagation based on a custom metric to show that even modern retrievers benefit from a rich ranking context. However, their progress is fundamentally constrained by the limitations of available datasets.

Although it is feasible to create IR datasets with graduated relevance labels using search engine click logs (Rekabsaz et al., 2021), it comes with significant technical and practical challenges. Technically, extensive search engine logs are only available for popular domains, leaving out niche applications (e.g., climate research). Even for popular domains, given the nature of click-through models, graduated relevance labels are only possible for frequent queries, and rare queries are left with sparse binary annotations (Rekabsaz et al., 2021). Practically, search engine logs are valuable business assets and are only selectively released by large companies, which limits the coverage and quality of the resulting datasets. By contrast, our method uses open-source LLMs to generate large datasets with graduated relevance labels, which is applicable to all domains and queries while being publicly accessible.

Synthetic Data Generation for IR Recent works have used LLMs in different ways to improve the quality of data for information retrieval.

Instruction:

(omitted) Given a text query, your mission is to write four different passages, each with a different level of relevance to the given query.

- Perfectly relevant: a passage that is dedicated to the query and contains the exact answer.
- Highly relevant: a passage with some answer for the query, but the answer may be unclear, or hidden amongst other information.
- Related: a passage that seems related to the query but does not answer it.
- Irrelevant: a passage that has nothing to do with the query.

Passage generation instructions

- All passages should be about {{sentences}} sentences long.
- All passages require {{difficulty}}} level education to understand.
- The very first sentence of the passage must NOT completely answer the query. * (omitted)

Example Input:

Query: {{example query}}

Example Output:

[Perfectly relevant passage]

{{example perfectly relevant passage}}}

(omitted)

Figure 2: To create a multi-level ranking context for dense retrieval training, we prompt the LLM to sequentially generate four passages with graduated relevance levels for each query. To generate diverse passages, we randomly sample the value of {{sentences}} and {{difficulty}} for each prompt. To avoid easy-to-identify passages, we include the instruction with "*" in the prompt for a random subset of queries. See Appendix A for details.

For instance, generating synthetic queries for existing unannotated passages is a popular approach for creating training data (Dai et al., 2022; Bonifacio et al., 2022; Jeronymo et al., 2023; Alaofi et al., 2023; Lee et al., 2024). Another approach is to use LLMs to enhance the quality of existing queries (Wang et al., 2023b; Shen et al., 2023; Jagerman et al., 2023; Rajapakse and de Rijke, 2023; Anand et al., 2023; Li et al., 2024; Dhole and Agichtein, 2024; Zhang et al., 2024). More recently, synthetic data has played an important role in training dense retrievers with the ability to follow instructions (Weller et al., 2024; Asai et al., 2022; Wang et al., 2024). Although LLMs do not inherently impose such limitations on generated data, existing work generates synthetic data only with binary relevance levels and inherits many of the problems of real training data discussed in Section 1. By contrast, we use the flexibility of LLMs to overcome the limitations of real data and generate rich ranking contexts with graduated levels of relevance, which is more suitable for training dense retrievers.

3 Synthetic Ranking Context for List-wise Training (SyCL)

To better approximate the inference objective, we propose to train dense retrievers on passages with multiple levels of relevance, thus creating a rich *multi-level ranking context* for each query. Since most available large-scale IR datasets that can be

used for training transformer-based dense retrievers only provide passages with binary ground truth labels, we use LLMs to generate passages with graduated relevance levels for each query (Section 3.1). We additionally propose to use the Wasserstein distance as a list-wise loss function that, unlike contrastive learning, can take advantage of this ranking context (graded ground truth relevance levels and multiple non-negative passages per query) and effectively optimize dense retrievers (Section 3.2).

3.1 Multi-level Ranking Context

We leverage open-source LLMs to generate multilevel ranking contexts for the MS MARCO training queries at scale. We use the official TREC Deep Learning¹ relevance guidelines to prompt LLMs for each query to write passages that answer it at four different levels of relevance: perfectly relevant, highly relevant, related, and irrelevant (Fig. 2). See Appendix A for the exact prompt.

The *relative* relevance of passages is most important for ranking them correctly. Even with clear instructions, when asked to generate passages of a specified relevance level without references, the LLM is not aware of how they will compare to other, independently generated documents of the same, higher or lower specified relevance to the same query. Thus, we prompt the LLM to generate all four passages for each query sequentially, in the same inference session. This allows the LLM to

¹https://trec.nist.gov/data/deep2019.html

gradually decrease the relevance of each generated passage relative to already generated passages in its context in order to achieve the correct ranking order. To help the LLM better understand the task, we provide one in-context example consisting of a query and four passages, i.e., one passage for each relevance level.

Corpus Diversity The in-context example additionally serves to reduce the distribution shift between the synthetic and real passages in terms of attributes like style; without examples, our synthetic documents tend to be distinctly clearer and more direct than real passages. Hence, to increase the diversity of synthetic passages, for each prompting instance, we randomly sample one in-context example from a small pool of 82 queries, each with four annotated passages, which we source from TREC DL 2023. This requires a very small number of ground truth labels (328 labeled passages in total), and compared to the scale of annotation in MS MARCO (more than 500k annotated queries), the incurred cost is negligible.

Moreover, similar to Wang et al. (2024), for each prompt, we use templates to specify a randomly sampled passage length and difficulty level. As we noticed that the LLM has a tendency to provide the exact answer to the query in the very first sentence of the perfectly relevant passage, making them easily identifiable, we also explicitly instruct the LLM to avoid this in a random subset of prompts. See Appendix A for more details.

We use simple text processing to extract the four passages from the LLM response and assign them sequential labels, {3,2,1,0}, based on the specified relevance level for which they were generated.

3.2 Training with Multiple Levels of Relevance

To effectively leverage the multi-level ranking contexts we generate, we propose using the 2-Wasserstein distance. Although it has been used as a distance as part of retrieval pipelines in different roles, e.g., regularization (Yu et al., 2020), to the best of our knowledge, we are the first to propose it as a relevance loss function for training dense retrievers. The Wasserstein distance is non-differentiable; to utilize it as a loss function, we use its differentiable analytical expression and algorithm for computation that can be derived when comparing two Gaussian distributions (Mathiasen and Hvilshøj, 2020), although neither our ground-

truth nor estimated score distributions are actually Gaussian. However, we find that this approximation outperforms the most popular list-wise functions used in the learning-to-rank literature (Table 3).

Compared to the KL-divergence, which has been used as a multi-level list-wise loss function for dense retrievers (Zerveas et al., 2022, 2023), the Wasserstein distance in this role has the following main advantages.

First, distributing probability mass over candidate documents is penalized according to their ground-truth score distance from the ground-truth target document; assigning some probability mass to a document with g.t. label 0 instead of the correct document with g.t. 3 is penalized more strongly than assigning it to a document with g.t. label 2. By contrast, the KL-divergence is insensitive to this relative distance in the estimated score distribution. As long as the g.t. relevant document (or any other document) is not assigned its due g.t. probability mass in the estimated score distribution, it will be penalized the same regardless of where this probability mass goes.

Moreover, it is computed by comparing the groundtruth and estimated score distributions across documents of the entire batch, not only across those in the context of a single query; we hypothesize that this acts as a regularization, e.g., granting resilience to the range of score values or outliers.

4 Experiments

Our experiments demonstrate the effectiveness of a synthetic multi-level ranking context and the Wasserstein loss for training dense retrievers. First, without using any real documents or annotations for training, SyCL fine-tuning improves the performance of self-supervised dense retrievers. Second, we show that the Wasserstein loss with multiple levels of relevance outperforms InfoNCE using the same queries and passages. Third, we find that SyCL training only on synthetic documents performs similarly to contrastive training with real data of the same size on TREC DL, while on average, it outperforms it in terms of out-of-domain generalization (BEIR). Overall, the best ranking effectiveness is achieved when incorporating existing real data into our synthetic multi-level ranking context. Through additional analytical experiments, we show the individual impact of the Wasserstein loss and graduated relevance labels. Finally, we

nDCG@10	DL19	DL20	MM Dev	FEVER	Hotpo	tQA	FiQA	NQ	Quora	Touche
Base Contriever (BC	45.5	44.8	20.6	66.8	48.	2	24.6	25.4	83.5	18.6
BC + InfoNCE Synth. BC + WS Synth.	55.3 59.6	51.5 59.8	26.3 30.2	68.0 81.8	46. 57.		26.8 27.3	33.2 41.9	75.8 83.3	15.0 20.3
BC + InfoNCE Real BC + WS Synth. + Real	63.0 63.2	61.2 61.6	34.2 32.9	69.6 80.6	59. 59.		29.1 30.0	42.8 42.5	81.7 83.7	14.6 16.8
nDCG@10	CQADup Android	Scidocs	Climate FEVER	DBPedia	TREC COVID	Scifa	ct NF0	Corpus	ArguAn	a BEIR Avg
Base Contriever (BC)	37.5	15.1	15.2	29.4	27.7	63.9) 3	32.4	31.4	37.1
BC + InfoNCE _{Synth} . BC + WS _{Synth} .	35.0 39.0	15.1 16.4	21.4 27.0	32.0 36.7	26.6 52.7	62.5 62.0	_	31.5 31.8	26.4 28.2	36.8 43.2
BC + InfoNCE Real BC + WS Synth. + Real	38.2 40.5	16.2 16.0	18.3 25.5	37.6 38.9	34.0 51.2	65.1 66.0	_	31.5 33.0	33.6 33.0	40.9 44.2

Table 1: Ranking effectiveness (nDCG@10). Base Contriever (BC): self-supervised Contriever model. 'BC +' denotes the fine-tuning setting in terms of **loss function**: InfoNCE / Wasserstein (WS), and **type of data**: real data from the MS MARCO training set with annotated positives and mined hard negatives (Real) / fully synthetic multi-level documents (Synth.) / combination. DL19, DL20, and MM Dev are the TREC DL 2019, TREC DL 2020, and Dev evaluation sets of MS MARCO. Evaluation on the rest of sets is zero-shot. Purple: SyCL, our method.

inspect different components of our data generation pipeline and find that even smaller, 32B-scale LLMs can generate high-quality data with comparable quality to larger, 70B-parameter models.

4.1 Setup

Training We use Llama 3.3 70B (Dubey et al., 2024) to generate one passage for each level of relevance (i.e., total ranking context size of four) for training queries of the MS MARCO dataset. We use the 83 queries from TREC DL 2023, each with four annotations, to create the pool of in-context examples for generation (Section 3.1). We use them to simulate a very small-scale manual annotation effort, as TREC DL 2023 is meant for the new version of the MS MARCO dataset (v2) and is not used for training or evaluation by either recent work or ours. During training, we use all passages corresponding to other queries in the batch as level zero passages in the multi-level ranking context of a given query. We use the unsupervised Contriever (Izacard et al., 2021) as our base model. See Appendix D for our experiments with other models. All real data used for training are sourced from the MS MARCO training set.

Evaluation For in-domain evaluations, we use the TREC DL 2019, TREC DL 2020, and Dev set of the MS MARCO dataset. For out-of-domain evaluations, we use the 14 publicly avail-

able datasets in the BEIR benchmark (Thakur et al., 2021b). This provides a more realistic measure of how well the model can perform in real-world scenarios where it encounters unseen data (Thakur et al., 2021b). To simplify our BEIR evaluation experiments for duplicate question retrieval, we only use the Android subforum of the CQADupStack dataset.

4.2 Results

Table 1 shows our main results on the effectiveness of using a synthetic multi-level ranking context with the Wasserstein loss to train dense retrievers.

Our approach significantly improves the performance of the unsupervised Contriever model for both in-domain evaluation on the MS MARCO dataset and out-of-domain evaluation on datasets in the BEIR benchmark. In terms of nDCG@10, our method improves the base model performance by 6.2 across BEIR, 14.1 on TREC DL19, and 14.9 on TREC DL20.

Notably, for in-domain evaluation, the performance boost for the DL19 and DL20 sets is more significant than that of the Dev set (9.7). This is expected: MS MARCO Dev is extremely sparsely annotated (mostly, one positive per query), missing most real positive documents. Compared to contrastive training with a single positive, a training method like ours teaches the model to distribute

relevance scores among more documents in the ranking context (see Fig. 4). Therefore, it has a much higher probability of assigning a high score to documents other than the annotated positive, and the chance for the latter to be displaced to lower ranks increases. The question therefore is, whether the documents displacing the ground-truth positive are indeed relevant. Qualitative inspection of ranked documents (Table 9) and evaluation on more densely annotated sets (Table 1) indicate that the answer is affirmative and may explain the difference in performance improvements. DL19 and DL20 additionally provide multi-level relevance labels, which helps to better evaluate the fine-grained ranking capabilities of retrievers.

Multi-level ranking context with Wasserstein loss uses the same data more effectively than InfoNCE. For an apples-to-apples comparison with the standard contrastive training, we train the model with InfoNCE loss using the same synthetic passages (InfoNCE _{Synth}. in Table 1). For this, we use the passages from levels 3 and 2 as positives and passages from levels 1 and 0 as negatives. Although both setups use the same queries and passages, multi-level ranking context with Wasserstein loss uses the data more effectively and clearly outperforms contrastive training.

To evaluate contrastive training with real data, we use the human-annotated positives and two hard negatives mined by BM25 to match the number of negatives in synthetic data (InfoNCE Real in Table 1). Although training with real, labeled documents leads to slightly better performance for indomain evaluation on the MS MARCO dataset, we find that training exclusively on synthetic documents through SyCL performs comparably. On the other hand, SyCL better generalizes to outof-domain datasets in the BEIR benchmark and outperforms real data by 2.3 nDCG@10 on average. This indicates better robustness to distribution shift and unseen data, which has been argued to be the most important attribute of IR methods for real-world applications (Thakur et al., 2021b).

Augmenting real data with multi-level synthetic passages further improves performance. To benefit from both real and synthetic data, we assign relevance levels 3 and 1 to positive and negative real passages, respectively, and incorporate them into the synthetic multi-level ranking context. Adding real data to the synthetic ranking context improves SyCL's ranking effectiveness on DL19 and DL20 from 59.6 and 59.8 to 63.2 and

61.4, respectively. Compared to training with real data and the InfoNCE loss, training with SyCL on the combined data improves nDCG@10 scores from 40.9 to 44.2 on the BEIR benchmark. Adding real data seems to slightly degrade performance on MS MARCO Dev, which we attribute to its extremely sparse annotation (see discussion at the beginning of this section).

4.3 Additional Analysis

Fine-grained relevance levels are necessary for achieving good performance. To separate the impact of using multiple relevance levels from the Wasserstein loss, we repeat our main experiment with the Wasserstein loss but with binary ground truth relevance levels. For this, we assign relevance levels 1 and 0 to more relevant (levels 3 and 2) and less relevant (levels 1 and 0) synthetic passages, respectively (Table 2). We find that even with the same data and loss function, fine-grained relevance levels are necessary for achieving good performance: using binary relevance levels instead decreases the boost in performance by 4.0 nDCG@10 on average across all sets.

Wasserstein loss is more effective than other **list-wise loss functions.** We compare our proposed Wasserstein loss against other list-wise loss functions that can take advantage of multiple levels of relevance (Table 3). We evaluate the Approximate NDCG (a smooth, differentiable approximation of the nDCG metric) (Qin et al., 2010a), RankNet (Burges et al., 2005), and ListNet (Cao et al., 2007) loss functions, which have been used extensively in learning-to-rank approaches before the advent of dense retrieval. We also evaluate the KL divergence, which is often used for model distillation, but has also been used for training with a multi-level ranking context (Zerveas et al., 2022, 2023). Except for RankNet, all other loss functions seem to take advantage of multiple levels of relevance and outperform the binary InfoNCE loss. However, we find that the Wasserstein loss is the most effective and provides significant gains over the next best loss function (ListNet).

4.4 Synthetic Data Generation and Quality

LLMs successfully follow the definition of relevance levels. To check if synthetic passages adhere to their corresponding relevance level, we use a high-quality embedding model, e5-mistral-instruct(Wang et al., 2023a), to measure the similarity between 10,000 randomly

	DL19	DL20	MS Dev	BEIR
Binary	48.9	48.7	22.1	40.5
Multi-Level	59.6	59.8	30.2	43.2

Table 2: Performance of models trained with Wasserstein loss on the same synthetic data with binary $(\{1,0\})$ and graduated $(\{3,2,1,0\})$ relevance labels.

selected queries and their corresponding synthetic documents. Figure 3 reports the distribution of similarity scores for passages in each relevance level. We find that the generator LLM understands the relevance levels and appropriately decreases the relevance between the query and generated document based on its pre-specified target level and the documents sequentially generated before it.

We provide a sample of the generated passages in Appendix B, which shows that the LLM first generates a positive passage that fully answers the query and then, with some nuanced changes, creates a less relevant positive passage that provides a partial answer to the query, and similarly keeps reducing the relevant content for the other two less relevant passages in the context.

Small LLMs also generate high-quality data. To understand the impact of the LLM on the quality of the synthetic data, we also generate data with two other LLMs, Qwen 2.5 72B and Qwen 2.5 32B (Team, 2024), and use it to train the retriever similar to our main experiments (Table 5). For in-domain evaluation, data generated with larger LLMs leads to better performance on DL20 and Dev splits of MSMARCO. However, for out-ofdomain evaluation on BEIR datasets, data generated with the smaller Qwen 2.5 32B leads to performance similar to data generated with Llama 3.3 70B. Although recent works use 70B scale public or larger proprietary models, our results show that data generated with larger models does not always lead to better performance.

Loss	DL19	DL20	MS Dev	BEIR
Approx. nDCG	54.7	52.5	27.8	39.1
RankNet	54.3	48.9	24.6	35.3
ListNet	56.9	55.4	27.5	42.2
KL-div	56.1	54.9	27.4	42.1
Wasserstein	59.6	59.8	30.2	43.2

Table 3: Performance (nDCG@10) of models trained on multi-level synthetic data with different list-wise losses.

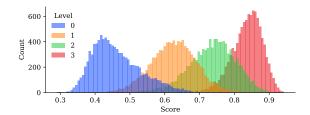


Figure 3: Distribution of the similarity scores between queries and the corresponding synthetic passages for each relevance level.

We investigate the impact of the in-context example and the randomly selected instructions in the prompt (e.g., length) on the quality of the synthetic data. We create two alternative prompts, one without the in-context examples and the other without the randomly selected instructions, and use the resulting data for training (Table 4). We find that although both of these techniques contribute to the quality of synthetic data, in-context examples are more important, especially for out-of-domain generalization to BEIR datasets.

	DL19	DL20	MS Dev	BEIR
Full	59.6	59.8	30.2	43.2
No in-context example	59.5	60.0	29.9	42.8
No random variation	59.9	58.3	29.8	42.8

Table 4: Impact of prompt design on retrieval performance. Full: our main prompt. No IC example: prompt without in-context examples. No random variation: prompt without randomly sampled instructions (e.g. length requirement).

	DL19	DL20	MS Dev	BEIR
Llama 3.3 70B	59.6	59.8	30.2	43.2
Qwen 2.5 72B	60.0	57.9	30.3	42.4
Qwen 2.5 32B	61.1	56.9	29.3	42.9

Table 5: Performance of models trained with synthetic data generated by different LLMs.

5 Conclusion

In this work, we show that transformer-based dense retrievers stand to benefit from training with a multi-level ranking context. As the annotations of large-scale datasets available for dense retrieval do not inherently support multi-level training, we propose to use Large Language Models to sequentially generate for each query a set of passages that

answer it in decreasing degrees of relevance. To leverage this multi-level ranking context, we propose using an approximation of the Wasserstein distance as a loss function, which significantly outperforms InfoNCE and other list-wise losses on the same synthetic documents. Importantly, we show that without training on any real documents, our method attains comparable in-domain performance to training on real, annotated data with InfoNCE, while exhibiting better out-of-distribution performance. This indicates our method is a promising choice in real-world scenarios where the availability of annotated documents is limited and data distributions continuously shift.

Limitations

Our work requires the availability of a collection of user queries in the target domain. For many domains, a large collection of user queries is already provided by existing datasets or can be collected from online forums like Reddit or even from users' conversation history with LLM assistants. However, one can imagine very rare applications where none of these resources is available. Thus, we encourage future work to explore the combination of our work with synthetic query generation techniques (Wang et al., 2024). However, generating a large collection of queries from scratch also comes with it own challenges. While there are many very frequently occurring queries, 70% of (distinct) queries occur only once (Brenes and Gayo-Avello, 2009). Therefore, the LLM would be challenged to imagine representative user queries in most situations.

Similar to other works on synthetic data generation, our work is also limited by the capabilities of LLMs. For instance, data generation for specialized domains could pose a challenge for existing LLMs, especially at smaller scales. Considering the progress in generating synthetic instruction tuning data for specialized domains, we encourage future work to explore opportunities to expand applications of synthetic ranking data to specialized domains as well.

Moreover, we assume that LLMs understand the difference between relevance levels and can generate suitable data accordingly. We show experimentally that this is in fact the case and LLMs successfully generate documents with four different relevance levels. However, we speculate that if we increase the number of relevance levels, after a

certain point, the differences would be too nuanced for existing LLMs to recognize and follow. We encourage future work to first explore the limitations of existing LLMs in terms of understanding nuanced semantic differences through instruction and explore more advanced approaches for controlling the semantic similarity of the generated documents.

Ethical Considerations

Since we use the MS MARCO training queries to guide the data generation process, our synthetic data might inherit the social biases and ethical concerns related to the MS MARCO dataset. Moreover, similar to other works on synthetic data generation, our data also inherits the social biases and ethical concerns related to the LLM used for generating the synthetic documents. Although we did not observe any harmful content during the course of this project, a principled analysis of social biases, factual correctness, and other ethical concerns is needed before use in sensitive real-world applications.

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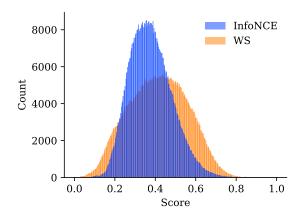


Figure 4: Distribution of the top 100 similarity scores across all Dev queries of MS-MARCO dataset by models trained with Wasserstein and InfoNCE loss. The model trained with multiple relevance levels learns a more fine-grained notion of relevance.

A Prompting Details

Table 10 shows the exact prompt that we used to generate multi-level ranking contexts for training queries of the MS-MARCO dataset. To create incontext examples, we use the annotations in TREC DL 2023 split. For each prompt, we randomly sample one query and four passages (one for each relevance level in TREC DL 2023 annotations) and use them as the in-context example. increase the diversity of the generated passages, for each prompt, we randomly sample the value of {{num_sentences}} from {none, 2, 5, 10, 15} probabilities {0.5,0.1,0.2,0.1,0.1}. Similarly, we randomly sample the value of {{difficulty_level}} from {none, high school, college, PhD} with probabilities $\{0.4, 0.2, 0.2, 0.2\}$. For both variables, if the sampled value is none, we do not include the corresponding instruction in the prompt.

We also noticed the LLM has a tendency to provide the exact answer to the query in the very first sentence of the perfectly relevant passage. To avoid such spurious patterns, in 30% of the prompts we include an additional instruction and explicitly ask the LLM to avoid answering the query in the very first sentence of the perfectly relevant passage.

B Qualitative Examples

Sample Synthetic Ranking Context Table 8 shows an example of the synthetic multi-level ranking context generated for the query "What do cells do?". We observe that LLMs understand the defini-

tion of relevance levels and generate high-quality data with nuanced differences. First, the LLM generates a perfectly relevant passage that provides a complete answer to the query. Then, to create a slightly less relevant positive passage, the LLM makes small changes to leave out some parts of the answer while still answering the query correctly. For the level 1 passage, which is intended to be a hard-to-detect negative, the LLM stays on topic and discusses cells and functionality, but instead of the functionality of the cells, it explains the functionality of the organs, which might seem like a relevant document but in fact, does not answer the query. Finally, for level 0, it generates a passage that is totally off-topic and has nothing to do with cells and thus is completely irrelevant to the query.

Sample Retrieved Passages Table 9 shows the retrieved passages for a sample query by a model trained on binary ranking contexts with InfoNCE and another model trained on multi-level ranking contexts with Wasserstein distance. Although both models identify the most relevant passage correctly, the model trained on multi-level ranking contexts has a better understanding of relevance and retrieves better passages in other ranks.

C Implementation Details

We train our models for only one epoch using the Trainer module in the Huggingface transformers library². For both training and evaluation, we use the maximum length of 256 for both queries and passages. We use a total batch size of 64 across four GPUs (batch size of 16 per device). We set the learning rate to 1e-5, gradient accumulation steps to 4, and warm-up ratio to 0.05. We use the default parameters in version 4.48.0 of the transformers library for all other configurations, e.g., optimizer, learning rate scheduler, etc. Each one of our experiments takes about 2 hours using one machine with four L40s GPUs.

D Other Models

We repeat our main experiments using Condenser (Gao and Callan, 2021a) and CoCondenser-Marco (Gao and Callan, 2021b) as the base retrievers. Condenser is a BERT model with a slight architectural modification during pre-training that makes the learned representations more suitable for retrieval. CoCondenser-Marco is a Condenser

²https://github.com/huggingface/transformers

model fine-tuned on the MS MARCO corpus in an unsupervised manner (i.e., without using any labels). Since these two models do not perform as well as the Contriever model, we train them for three epochs instead of one and also increase the learning rate to 1e-4. As shown in Table 6, synthetic data significantly improves the base unsupervised model in both cases. Moreover, except for Condenser on the DL20 split of MS MARCO, training using multiple relevance labels leads to better performance compared to contrastive training with binary labels using the InfoNCE loss. Notably, the base Condenser model is only trained with a language modeling objective without any retrievalspecific fine-tuning, which could potentially impact its ability to learn the nuanced differences between multiple levels of relevance. Furthermore, we noticed that Wasserstein loss leads to smaller gradient norms than InfoNCE loss (i.e., smaller updates and thus slower convergence). As a result, we speculate that for lower-quality models or models without contrastive pre-training, the difference between InfoNCE and Wasserstein losses will increase with more training steps.

nDCG@10	DL19	DL20	MS Dev	BEIR
Condenser	1.1	3.3	0.6	6.3
+ InfoNCE Synth.	58.1	57.0	28.3	37.1
+ WS _{Synth} .	63.3	55.9	29.7	39.3
CoCondenser-Marco	31.1	33.7	14.0	31.0
+ InfoNCE Synth.	59.6	59.0	29.7	39.4
+ WS _{Synth} .	59.7	59.6	30.5	41.3

Table 6: Self-supervised Condenser (Gao and Callan, 2021a) and CoCondenser trained on MS MARCO. The models are further fine-tuned on our synthetic data using InfoNCE with binarized labels or Wasserstein distance with the original 4-level labels.

E Practical Value

We show that even under a strict interpretation of relevance labels, our method outperforms BM25 without using any real passages or their annotations (Table 7). Following TREC guidelines, we exclude passages with relevance label 1 for the strict evaluation setup. To be a viable approach for practical applications, dense retrievers should at least perform better than BM25, which does not require any training and still achieves strong performance. However, most dense retrieval methods fail to outperform BM25 without additional fine-tuning on labeled training data. Recently, Wang et al. (2022)

managed to outperform BM25 without using any labeled data. However, they resorted to a complex multi-stage training pipeline to achieve this. By contrast, we keep our training pipeline simple: we use synthetic data to better capture the ranking objective during training.

nDCG@10	DL19	DL20
BM25 (Yang et al., 2017) Base Contriever (BC)	41.7 37.6	41.2 36.9
BC + InfoNCE _{Synth} . BC + WS _{Synth} .	48.0 52.6	45.0 53.4
BC + InfoNCE Real BC + WS Synth. + Real	57.7 56.6	55.4 54.6

Table 7: Evaluation excluding passages with label 1 (Related), as per the official TREC guidelines

F Loss Functions

We calculate the similarity between query q and document d as the inner product between their embeddings. Specifically,

$$sim(d, q) = f_{\theta}(d) \cdot f_{\theta}(q)$$
,

where f is the embedding function parameterized by θ .

InfoNCE We calculate the InfoNCE loss as follows:

$$-\log \frac{\exp(\sin(d^+,q))}{\sum_{d \in D_q} \exp(\sin(d,q))},$$

where d^+ is the positive document and D_q is the ranking context for query q (i.e., the collection of positive and negative documents for q). Note that for InfoNCE loss, D_q can contain one and only one positive document and the rest must be negative.

KL Divergence Given the similarity scores between a query and documents in its ranking context, we calculate the KL loss as follows:

$$D_{\mathrm{KL}}(\sigma(Y)||\sigma(\hat{Y})),$$

where σ is the softmax function and $Y \in \mathbb{R}^{|D_q|}$ and $\hat{Y} \in \mathbb{R}^{|D_q|}$ are respectively the ground truth relevance labels and predicted relevance labels (i.e., similarity scores) for documents in the ranking context of query q.

Wasserstein Distance We use the special case of Wasserstein distance between two multivariate gaussian distributed inputs $X \sim \mathcal{N}(\mu_x, C_x)$ and $Y \sim \mathcal{N}(\mu_y, C_y)$, where μ and C are mean and covariance of each distribution, respectively. For gaussian distributions, the 2-Wasserstein distance reduces to

$$D(X,Y) = \|\mu_x - \mu_y\|^2 - \text{tr}(C_x + C_y - 2(C_x C_y)^{\frac{1}{2}}).$$

In our implementation, we calculate the Wasserstein score for the entire batch. Specifically, for each batch, we create matrices $H \in \mathbb{R}^{b \times |D_q|}$ and $\hat{H} \in \mathbb{R}^{b \times |D_q|}$ of shape (batch size, ranking context size) and minimize $\mathrm{D}(H,\hat{H})$ during training. Each row of H corresponds to ground truth relevance labels for one query in the batch. Similarly, one row of \hat{H} corresponds to the predicted similarity scores between one query in the batch and documents in its ranking context. We use the fast implementation proposed by Mathiasen and Hvilshøj (2020).

³https://gist.github.com/Flunzmas/ 6e359b118b0730ab403753dcc2a447df

Query	what do cells do?
Level 3	Cells are the basic structural and functional units of living organisms. They are responsible for carrying out various functions necessary for life, including growth, reproduction, metabolism, and response to stimuli. Cells take in nutrients, convert them into energy, and eliminate waste products. They also contain genetic material, known as DNA, which carries information necessary for the cell's development and function. Additionally, cells communicate with each other to coordinate their activities and maintain overall health.
Level 2	The human body is made up of trillions of cells, each with its own unique characteristics and functions. Some cells, such as muscle cells, are specialized for movement and contraction, while others, like nerve cells, are responsible for transmitting signals and controlling various bodily functions. Cells also have the ability to adapt to their environment, change shape, and even migrate to different parts of the body. For example, immune cells can recognize and respond to foreign substances, helping to protect the body from infection and disease.
Level 1	Tissues are groups of similar cells that work together to perform specific tasks. Organs are structures made up of two or more types of tissues that function together to maintain the overall health of an organism. The study of the structure, behavior, and function of tissues and organs is essential for understanding how the body works and how diseases develop. By examining the relationships between different tissues and organs, researchers can gain insights into the complex processes that occur within living organisms.
Level 0	The city of Paris is known for its stunning architecture, rich history, and vibrant cultural scene. Visitors from around the world flock to the City of Light to marvel at iconic landmarks like the Eiffel Tower, Notre-Dame Cathedral, and the Louvre Museum. The city's charming streets and alleys are lined with quaint cafes, boutiques, and restaurants, offering a unique and unforgettable experience for travelers. Whether you're interested in art, fashion, or food, Paris has something to offer everyone.

Table 8: Synthetic multi-level ranking context generated for one query, which shows that the LLM successfully follows the defined relevance levels and generates passages with correct relative similarity to the given query.

rank	InfoNCE, binary relevance labels	Wasserstein loss, 4-level relevance labels
1	How many ml of water should you drink in a day? The Institute of Medicine recommends that men drink 3000 ml of water each day and women drink 2100 ml. This equals approximately 13 cups of liquid for men and 9 cups from women. An individual often requires more water to stay hydrated in hot weather or due to strenuous exercise. < continued >.	How many ml of water should you drink in a day? The Institute of Medicine recommends that men drink 3000 ml of water each day and women drink 2100 ml. This equals approximately 13 cups of liquid for men and 9 cups from women. An individual often requires more water to stay hydrated in hot weather or due to strenuous exercise.
2	Since 2,000 mL of fluid are needed daily for normal body functions, first determine how many mL each patient has consumed so far today. Identify which patients need to be encouraged to consume more fluids to meet the 2,000 mL intake standard.	How many ml of water should you drink in a day? The Institute of Medicine recommends that men drink 3000 ml of water each day and women drink 2100 ml. This equals approximately 13 cups of liquid for men and 9 cups from women. An individual often requires more water to stay hydrated in hot weather or due to strenuous exercise. < continued >
3	How many ml of water should you drink in a day? The Institute of Medicine recommends that men drink 3000 ml of water each day and women drink 2100 ml. This equals approximately 13 cups of liquid for men and 9 cups from women. An individual often requires more water to stay hydrated in hot weather or due to strenuous exercise.	The recommendation stated that a person should have 1 ml (about 1/5 of a teaspoon) of water for each calorie he or she consumes. The average diet at the time was approximately 1900 calories, meaning you needed about 64 ounces of water per day. Now the Institute of Medicine sets general guidelines for total water intake.It recommends that women consume a total of 91 ounces (that's about 2.7 liters) per day from all food and beverages combined. For men, it's about 125 ounces a day (or 3.7 liters).Depending on your diet, about 25% of the water you consume comes from your food. Most of us healthy folks get enough water in the foods and liquids we consume. That includes any liquid we drink even caffeinated beverages like soda, coffee and teat recommends that women consume a total of 91 ounces (that's about 2.7 liters) per day from all food and beverages combined. For men, it's about 125 ounces a day (or 3.7 liters).
4	How many quarts of water should you drink each day? The recommended minimum amount of water you should have each day is 8 cups, which is equal to 2 quarts.	The recommendation stated that a person should have 1 ml (about 1/5 of a teaspoon) of water for each calorie he or she consumes. The average diet at the time was approximately 1900 calories, meaning you needed about 64 ounces of water per day. Now the Institute of Medicine sets general guidelines for total water intake. It recommends that women consume a total of 91 ounces (that's about 2.7 liters) per day from all food and beverages combined. For men, it's about 125 ounces a day (or 3.7 liters). Depending on your diet, about 25% of the water you consume comes from your food. Most of us healthy folks get enough water in the foods and liquids we consume. That includes any liquid we drink even caffeinated beverages like soda, coffee and tea. or men, it's about 125 ounces a day (or 3.7 liters). Depending on your diet, about 25% of the water you consume comes from your food. Most of us healthy folks get enough water in the foods and liquids we consume. That includes any liquid we drink even caffeinated beverages like soda, coffee and tea.
5	If you are overweight or workout vigorously, this number will increase. And then, if you want to lose weight, you can add 500 ml water to your regular water intake to burn around 23 calories per day that will help you lose upto 5 pounds of weight per year.	The Institute of Medicine recommends an average of 3.7 liters (125 ounces) per day for healthy adult men and 2.7 liters (91 ounces) per day for healthy adult women, allowing adjustments for activity and health levels, climate and elevation, and the amount of water consumed from food and other drinks.
6	Presuming you're awake for approximately 16 hours per day, you'll have to drink between 4.65 and 6.25 fluid ounces per hour. That may seem like a lot, but it isn't much more than four to eight sips per hour (depending on how much you take in).	How much you would need to drink daily isn't clear, I would suggest just a bit more than 1 liter a day instead of the often quoted 2 - 3 liters a day. Metabolic processes will generate about 300 ml of water a day, your food contains about 800 ml of water daily. The rest of your intake is what you drink.
7	It means your normal urine output per hour should be anywhere between 33.3 and 83.3 ml. If it's not within this range, there's something wrong. However, you need to ensure that you're drinking no less than 2 liters of fluid per day. These numbers may change a bit considering your unique circumstances.	The Institute of Medicine advises that men consume roughly 3.0 liters (about 13 cups) of total beverages a day and women consume 2.2 liters (about 9 cups) of total beverages a day.
8	However, each drinking session of three pints is at least six units, which is more than the safe limit advised for any one day. Another example: a 750 ml bottle of 12% wine contains nine units. If you drink two bottles of 12% wine over a week, that is 18 units.	How much should you drink: It's said we need 8-10 glasses of water a day (8 oz. glasses). That's at least 2 quarts of water. This is just to provide the water we need to wash away the acidity from our bodily functions and remove our own wastes.
9	A 10 ml bottle contains 1000 units There are 100 units in a mL. 1 cc equals 100 units, so to figure how long a 10mL bottle, (1000 units) will last, you divide the number of units you use per day into 1000, and there you have it. Actually it depends on the concentration of the bag of solution you have. 10 ml bottle contains 1000 units There are 100 units in a mL. 1 cc equals 100 units, so to figure how long a 10mL bottle, (1000 units) will last, you divide the number of units you use per day into 1000, and there you have it.	How much water does one person need to drink per day? you should drink at least 7 to 10 average sized glasses of water each day. One average sized glass is about eight ounces. There are 16 ounces in a pint, 2 pints in a quart, and 4 quarts in a gallon, so, mathematically, there are about 128 ounces in a gallon.
10	When I got to work, I filled up my 16-ounce water bottle and drank it through a straw. For some reason, drinking though a straw helped me to drink more because I would take sips without thinking. I had to fill this up six times per day to get the 3 liters. For the first few days, I made a conscious effort to keep up with this, but after day four, I started to write down how many ounces I drank just to keep track.	How many ml of water should you drink in a day? A: The Institute of Medicine recommends that men drink 3000 ml of water each day and women drink 2100 ml. This equals approximately 13 cups of liquid for men < continued >

Table 9: Retrieved MS MARCO (real) passages for a sample query by a Contriever trained on synthetic documents using binary labels with InfoNCE (left) and the same model trained on the same documents using multi-level ranking contexts with the Wasserstein distance as a loss (right). In MS MARCO only one of these documents is labeled as relevant, although in fact many documents are relevant or even near-duplicates; that makes them false negatives.

Type	Content
System	# Task
	You are a data engineer whose goal is to generate synthetic passages that teach a ranking system to sort a collection of passages based on how relevant they are to the user's search query (similar to a web search engine). Given a text query, your mission is to write four different passages, each with a different level of relevance to the given query. Specifically, you should write one passage for each of the following relevancy levels: - "Perfectly relevant passage": a passage that is dedicated to the query and contains the exact answer. - "Highly relevant passage": a passage that has some answer for the query, but the answer may be a bit unclear, or hidden amongst extraneous information. - "Related passage": a passage that seems related to the query but does not answer it. - "Irrelevant passage": a passage that has nothing to do with the query.
	<pre>## Passage generation instructions - All passages should be about {{num_sentences}} sentences long All passages require {{difficulty_level}} level education to understand {{The very first sentence of the passage must NOT completely answer the query.}} - Avoid copying the query verbatim. It's acceptable if some parts of the "Perfectly relevant passage" are not topically related to the query How related each passage is to the given query should closely adhere to the corresponding relevancy level Passages can be less relevant to a given query for different reasons. For example, they might be less useful, less accurate, less comprehensive, etc. Explore different ways for writing less relevant passages. Be creative! - Do not provide any explanation in any passage on why it is relevant or not relevant to the query.</pre>
	<pre>## Evaluation criteria To double check if you have successfully accomplished the task, you should imagine how a search engine like Google Search would rank the generated passages if you search for the given query. To accomplish the task successfully, a search engine like Google Search should rank your passages in the same order that you generated them. In other words: - the perfectly relevant passage should fully answers the query the highly relevant passage should be less relevant to the query than the perfectly relevant passage the related passage should be less relevant to the query than the highly relevant passage the irrelevant passage should not provide any useful information about the query.</pre>
User	Do not explain yourself or output anything else. Be creative! ## Query: {{IC_example_query}}
Assistant	[Perfectly relevant passage]
	{{IC_example_perfectly_relevant_passage}}
	[Highly relevant passage]
	{{IC_example_highly_relevant_passage}}
	[Related passage]
	{{IC_example_related_passage}}
	[Irrelevant passage]
	{{IC_example_irrelevant_passage}}
User	<pre>## Query: {{main_query}}</pre>

Table 10: Our full prompt template used to generate synthetic multi-level ranking contexts for each query. See Appendix A for more details.