Douglass-Peucker Algorithm

Polyline simplification

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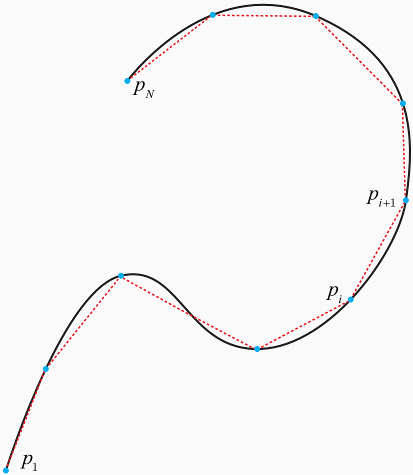
Introduction

Polyline simplification

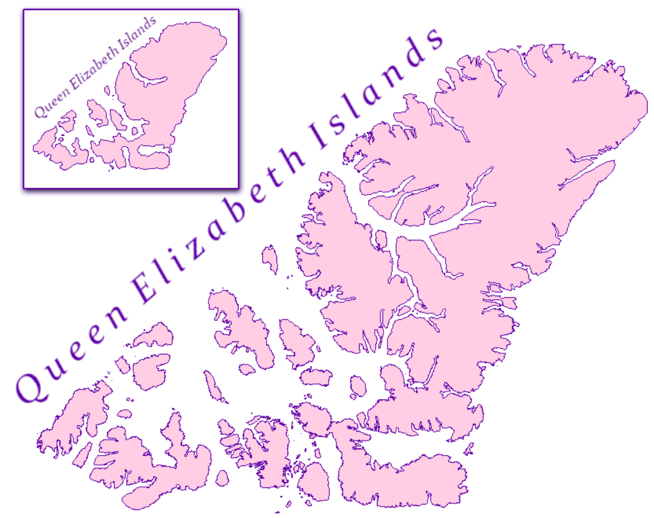


- A polygonal chain (polyline) $P = \{p_1, ..., p_N\}$ is a connected series of line segments $\overline{p_i p_{i+1}}$ $(i = \overline{1, N})$
- Polyline simplification is the process of reducing the resolution of a polyline. This is achieved by removing vertices and edges, while maintaining a good approximation of the original curve.

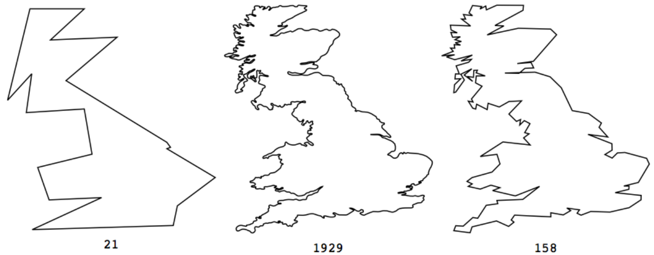
Application

- **Cartography:** map generalization.
- **Vector graphics.**
- **Robotics:** denoising of baseSegment data acquired by a rotating baseSegment scanner.

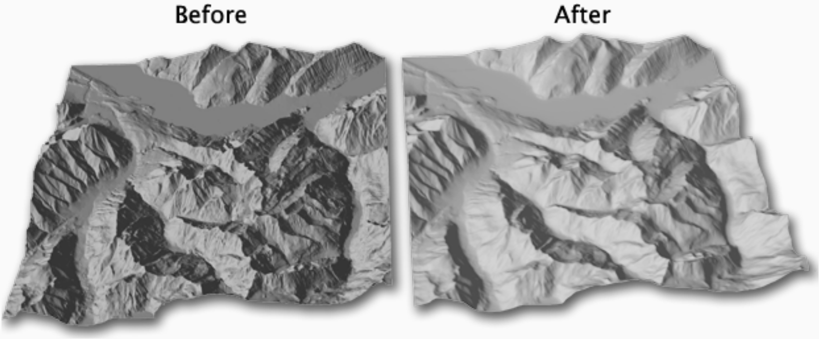
Map Generalization



Map Generalization



Map Generalization



Resources

- *C++ polyline simplification library and demo application* – `http://psimpl.sourceforge.net/`
- *JavaScript polyline simplification library* – `http://mourner.github.io/simplify-js/`
- *Java implementation of DPA* – `https://github.com/hgoebl/simplify-java`
- *Matlab implementation of DPA* – `https://www.mathworks.com/matlabcentral/fileexchange/21132-line-simplification/content/dpsimplify.m`

References

- Urs Ramer "An iterative procedure for the polygonal approximation of plane curves", *Computer Graphics and Image Processing*, 1(3), 244256 (1972)
- David Douglas & Thomas Peucker, "Algorithms for the reduction of the number of points required to represent a digitized line or its caricature", *The Canadian Cartographer* 10(2), 112122 (1973)
- J.L.G. Pallero Robust line simplification on the plane *Computers & Geosciences* 61 (2013)

Basic idea

Problem

Given a simple polygonal chain P (a polyline) defined by a sequence of points $p_1, p_2, ..., p_n$ in the plane.

Goal

Produce a simplified polyline $Q = \{p_{i_1}, p_{i_2}, ..., p_{i_k}\}$ that for each point $p_l \in P$ the distance ρ between p_l and the segment $\overline{p_{i_j} p_{i_{j+1}}}$ $(i_j < l < i_{j+1})$ is less or equal a given tolerance ε.

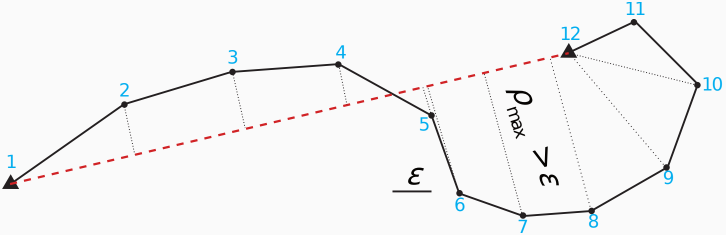
Alternative Goal

Produce a simplified polyline $Q = \{p_{i_1}, p_{i_2}, ..., p_{i_k}\}$ that for each point $p_l \in P$ the distance ρ between p_l and the segment $\overline{p_{i_j} p_{i_{j+1}}}$ $(i_j < l < i_{j+1})$ is minimal for a given number k.

Problem Solving

- The first and the last point (p_s and p_e) in the original polyline linked form a straight line $\overline{p_s p_e}$ that will be called a base segment.
- The point p_k $(s < k < e)$ with a maximal distance ρ_{max} to the base segment defined in the previous step is found.
- If $\rho_{max} \geq \varepsilon$ the resulting polyline is computed as a union of algorithm results on $\overline{p_s p_k}$ and $\overline{p_k p_e}$ base segment.
- Otherwise the algorithm returns the base segment $\overline{p_s p_e}$.

Step by Step



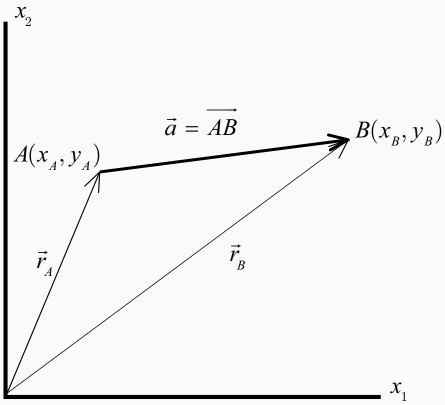
Solving of the Alternative Problem

The solving of the alternative problem is similar to the initial technique.

- At each iteration the algorithm finds the base segment with a maximal distance and divides it, as in the original algorithm. The resulting base segments are kept for the subsequent iterations.
- It is useful to use a priority queue for storage of base segments.

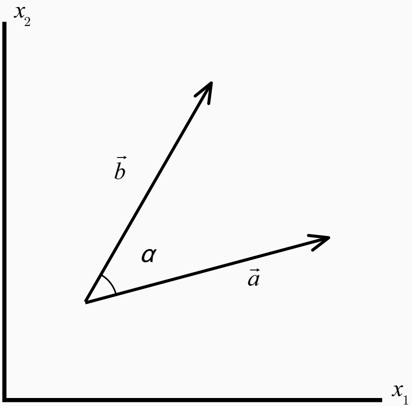
Distance to a segment

Vectors



$$\begin{aligned}\overrightarrow{a} &= \overrightarrow{AB} = \overrightarrow{r_B} - \overrightarrow{r_A} \\ &= (x_B - x_A, y_B - y_A) = (x_a, y_a)\end{aligned}$$

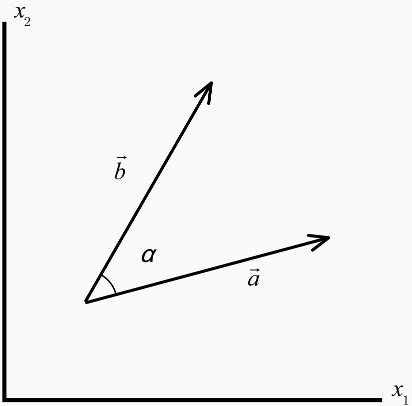
Scalar Product



$$\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}| \cdot \cos\alpha = x_a x_b + y_a y_b$$

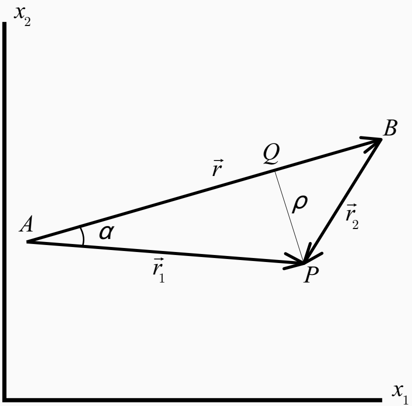
$$|\overrightarrow{a}| = \sqrt{\overrightarrow{a} \cdot \overrightarrow{a}} = \sqrt{x_a^2 + y_a^2}$$

Vector Product



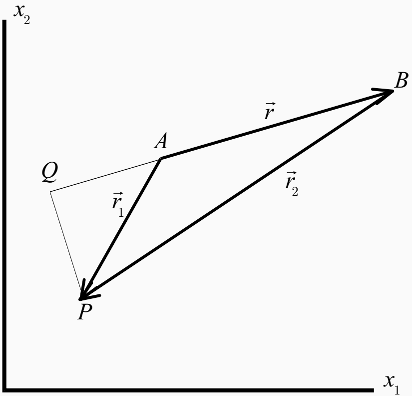
$$\overrightarrow{a} \times \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}| \cdot \sin\alpha$$
$$= x_a y_b - y_a x_b$$

Distance



$$\rho = |\overrightarrow{r_1}| \sin \alpha = |\overrightarrow{r_1}| \frac{|\overrightarrow{r} \times \overrightarrow{r_1}|}{|\overrightarrow{r}||\overrightarrow{r_1}|} = \frac{|\overrightarrow{r} \times \overrightarrow{r_1}|}{|\overrightarrow{r}|}$$

Improving



- if $\overrightarrow{r_2}^2 >= \overrightarrow{r_1}^2 + \overrightarrow{r}^2$ then $\rho = |\overrightarrow{r_1}|$
- if $\overrightarrow{r_1}^2 >= \overrightarrow{r_2}^2 + \overrightarrow{r}^2$ then $\rho = |\overrightarrow{r_2}|$
- else $\rho = |\frac{|\overrightarrow{r} \times \overrightarrow{r_1}|}{|\overrightarrow{r}|}$

Algorithm features

Complexity

The expected complexity of this algorithm can be described by the linear recurrence

$$T(n) = 2T(n/2) + O(n)$$

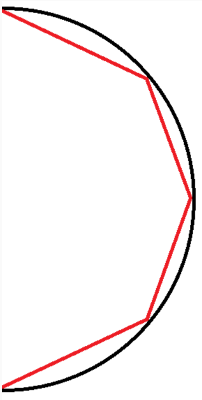
Using the **Master Theorem** we can find that the complexity is

$$T(n) \in \Theta(n \log(n))$$

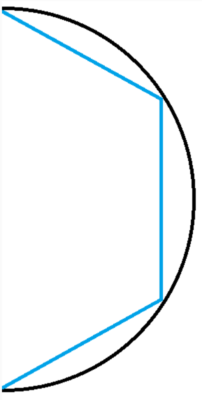
The **worst-case** complexity is $O(n^2)$

Optimality

The Douglas-Peucker algorithm is not optimal with respect to the number of points.



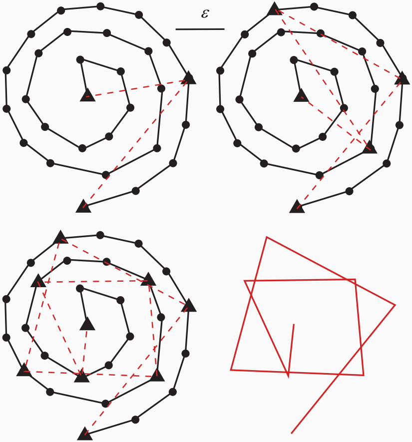
The answer of Douglas-Peucker Algorithm



The optimal answer with respect to the number of points

Robust

The Douglas-Peucker technique would have to be considered a non-robust algorithm since the resulting polyline may contain self-intersections.



Robust algorithm

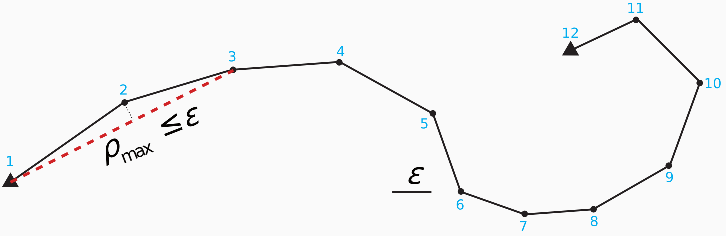
Non-Recursive Non-Robust Variant

- The last point (N in the original line) added to the output line, is joined to vertex $N + 2$ to generate the base segment. Then the distance between vertex $N + 1$ and the base segment is computed and compared with the established tolerance.
- If the computed distance $\rho < \varepsilon$, a new base segment will be created between vertices N and $N + 3$ and the distances between the base segment and all the intermediate points between N and $N + 3$ will be computed again. As long as the greatest of these distances does not exceed the predefined tolerance, new base segments will be created between points N and $N + 4 \ldots N + n$ from the original line. In the extreme case the last point from the original line could be reached.

Non-Recursive Non-Robust Variant

- When a base segment $N/N+k$ is found for which the distance to the farthest intermediate point is greater than the tolerance, the vertex $N+k-1$ will be added to the output line. Hence the tolerance of all vertices between N and $N+k-1$ will be ensured since the base segment $N/N+k-1$ was checked at the previous step.
- The algorithm goes back to the first step and considers now the point $N+k-1$ as the initial vertex for the new base segment.
- The algorithm ends as the last working base segment has the last point from the original polyline as final vertex and all intermediate vertices included in it are within tolerance.

Step by Step

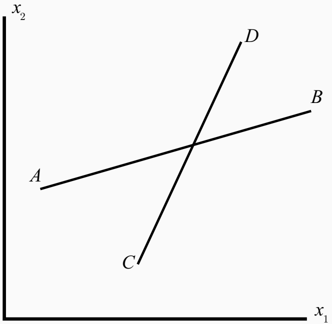


Robust Variant

The robust modification of the non-recursive Douglas-Peucker technique involves the detection of the possible intersections between:

- The new segments of the output line and the non-processed segments of the original line.
- The new segments of the output line and the previous segments of the same output line.

Segment Intersection



$$
\begin{aligned}
&\overrightarrow{AB} \times \overrightarrow{AC} \cdot \overrightarrow{AB} \times \overrightarrow{AD} \leq 0 \\
\text{and } &\overrightarrow{CD} \times \overrightarrow{CA} \cdot \overrightarrow{CD} \times \overrightarrow{CB} \leq 0 \\
\text{and } &\overline{AB} \subset \overline{CD} \text{ or } \overline{CD} \subset \overline{AB}
\end{aligned}
$$

Example