Assignment 2(Group)

(Hình thức tự luận)

Mỗi nhóm viết tay rồi scan thành file, nộp bài (link file đính kèm) qua hệ thống LMS mới.

Thời hạn nộp bài từ 12.03.2022 đến 19.03.2022.

Bài 1:

. Find a, b, c, and d if

a. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c-3d & -d \\ 2a+d & a+b \end{bmatrix}$

b. $\begin{bmatrix} a-b & b-c \\ c-d & d-a \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix}$

Bài 2:

Exercise _ Simplify the following expressions where A, B, and C are matrices.

a. $2[9(A-B)+7(2B-A)]$
$-2[3(2B+A)-2(A+3B)-5(A+B)]$

b. $5[3(A-B+2C)-2(3C-B)-A]$
$+2[3(3A-B+C)+2(B-2A)-2C]$

Bài 3:

Show that $A+A^T$ is symmetric for *any* square matrix A.

Bài 4:

In each case find the matrix A.

a. $\left(A+3\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{bmatrix}$

b. $\left(3A^T+2\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

c. $(2A-3\begin{bmatrix} 1 & 2 & 0 \end{bmatrix})^T = 3A^T + \begin{bmatrix} 2 & 1 & -1 \end{bmatrix}^T$

d. $\left(2A^T-5\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}\right)^T = 4A-9\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$

Bài 5:

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a transformation. In each case show that T is induced by a matrix and find the matrix.

a. T is a reflection in the x-y plane.

b. T is a reflection in the y-z plane.

Bài 6:

Compute AB, using the indicated block partitioning.

$$A = \left[\begin{array}{cc|cc} 2 & -1 & 3 & 1 \\ 1 & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad B = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ -1 & 0 & 0 \\ \hline 0 & 5 & 1 \\ 1 & -1 & 0 \end{array}\right]$$

Bài 7:

In each case give formulas for all powers $A, A^2, A^3, \ldots$ of A using the block decomposition indicated.

a. $A = \left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 1 & 1 & -1 \\ 1 & -1 & 1 \end{array}\right]$

b. $A = \left[\begin{array}{cc|cc} 1 & -1 & 2 & -1 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Bài 8:

The **trace** of a square matrix A, denoted $\operatorname{tr} A$, is the sum of the elements on the main diagonal of A. Show that, if A and B are $n \times n$ matrices:

a. $\operatorname{tr}(A + B) = \operatorname{tr} A + \operatorname{tr} B$.

b. $\operatorname{tr}(kA) = k \operatorname{tr}(A)$ for any number k.

c. $\operatorname{tr}(A^T) = \operatorname{tr}(A)$.

d. $\operatorname{tr}(AB) = \operatorname{tr}(BA)$.

Bài 9:

If A and B are $n \times n$ matrices, show that:

a. $AB = BA$ if and only if

$$(A+B)^2 = A^2 + 2AB + B^2.$$

b. $AB = BA$ if and only if

$$(A+B)(A-B) = (A-B)(A+B).$$

Bài 10:

Verify that $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ satisfies $A^2 - 3A + 2I = 0$, and use this fact to show that $A^{-1} = \frac{1}{2}(3I - A)$.

Bài 11:

Let A be an $n \times n$ matrix and let I be the $n \times n$ identity matrix.

a. If $A^2 = 0$, verify that $(I - A)^{-1} = I + A$.

b. If $A^3 = 0$, verify that $(I - A)^{-1} = I + A + A^2$.

c. Find the inverse of $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

d. If $A^n = 0$, find the formula for $(I - A)^{-1}$.

Bài 12:

Define $T: \mathbb{R}^n \to \mathbb{R}$ by $T(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \cdots + x_n$. Show that T is a linear transformation and find its matrix.

Bài 13:

a. Find b if $\det \begin{bmatrix} 5 & -1 & x \\ 2 & 6 & y \\ -5 & 4 & z \end{bmatrix} = ax + by + cz$.

b. Find c if $\det \begin{bmatrix} 2 & x & -1 \\ 1 & y & 3 \\ -3 & z & 4 \end{bmatrix} = ax + by + cz$.

Bài 14:

Find the real numbers x and y such that $\det A = 0$ if:

a. $A = \begin{bmatrix} 0 & x & y \\ y & 0 & x \\ x & y & 0 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & x & x \\ -x & -2 & x \\ -x & -x & -3 \end{bmatrix}$

c. $A = \begin{bmatrix} 1 & x & x^2 & x^3 \\ x & x^2 & x^3 & 1 \\ x^2 & x^3 & 1 & x \\ x^3 & 1 & x & x^2 \end{bmatrix}$

Bài 15:

In each case determine whether U is a subspace of $\mathbb{R}^3$. Support your answer.

$U = \{(1, s, t) \mid s \text{ and } t \text{ in } \mathbb{R}\}$.

$U = \{(r, 3s, r - 2) \mid r \text{ and } s \text{ in } \mathbb{R}\}$.

$U = \{(r, 0, s) \mid r^2 + s^2 = 0, r \text{ and } s \text{ in } \mathbb{R}\}$.

$U = \{(2r, -s^2, t) \mid r, s, \text{ and } t \text{ in } \mathbb{R}\}$.

Bài 16:

Use Theorem 5.2.3 to determine if the following sets of vectors are a basis of the indicated space.

$\{(5, 2, -1), (1, 0, 1), (3, -1, 0)\}$ in $\mathbb{R}^3$.

$\{(2, 1, -1, 3), (1, 1, 0, 2), (0, 1, 0, -3), (-1, 2, 3, 1)\}$ in $\mathbb{R}^4$.

$\{(1, 0, -2, 5), (4, 4, -3, 2), (0, 1, 0, -3), (1, 3, 3, -10)\}$ in $\mathbb{R}^4$.

Bài 17:

In each case find the characteristic polynomial, eigenvalues, eigenvectors, and (if possible) an invertible matrix P such that $P^{-1}AP$ is diagonal.

a. $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

b. $A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$

c. $A = \begin{bmatrix} 7 & 0 & -4 \\ 0 & 5 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

d. $A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 0 & 6 \\ 1 & -1 & 5 \end{bmatrix}$