Cyclic Pitch control of helicopter

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EL6243: System Theory and Feedback control
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This is the project for fulfilling the course requirements for EL6243 System Theory and Feedback Control. The project involves implementation of a Weiner Hopf design on a third order system and developing stabilizing controllers for the same.

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Abstract

This project involves developing stabilizing controller for controlling the cyclic pitch dynamics of the helicopter. Initially the set of all stabilizing controllers are found and then one of them is selected to eliminate steady state error for a step response and give a finite value for ramp response. The frequency response of the sensitivity function of this controller is then plotted. It's seen that the response goes to 1 as frequency goes to infinity. In the next part we design an optimal controller. Here we give equal weight to the input and output cost. Simulations for various input responses are given for both the controllers

I. INTRODUCTION

The system considered here is a helicopter and stabilizing controllers are developed for it's dynamics at hover. More specifically the dynamics for its pitch attitude are analyzed and a controller is developed for controlling it. At hover the longitudinal and vertical dynamics are decoupled and the transfer function for pitch control looks as follows.

$$\frac{\theta}{\mathrm{B}_{LC}} = \mathrm{M}_{BL_c}\left[\frac{s + \frac{X_{BLc}M_u}{M_{BLc}} - X_u}{s^3 - (X_u + M_q)s^2 + M_q X_u s + gM_u}\right]$$

Here θ is the pitch attitude in deg and B_{LC} is pitch control in deg

II. Designing set of all stabilizing controller

We first find the set of all stabilizing by finding values of $\delta(r), \delta(M), \delta(L)$. As we are designing this controller to give zero steady state error for step response we assume $\delta(f) = 1$ also $\mu\nu$ hence we choose $\mu = 2$ as we have a first oreder numerator and third order denominator. With this we get:

$$\delta(M) = 5, \delta(L) = 6, \delta(r) = 3$$

A. Finding values of X and Y

To find the values of A and B we first consider $G_P H$ which comes out as

$$G_p H = \frac{-47.24s - 1.711}{s^3 + 3.16s^2 + 0.186s + 1.324}$$

Hence $A = s^3 + 3.16s^2 + 0.186s + 1.324$ and $B = -47.24s - 1.711$. Factorizing this to get the value of X and Y such that $AX + BY = 1$ we get

$$X = 0.75677 \text{ and } Y = 0.016s^2 + 0.05s + 0.0012$$

B. Finding the set of all stabilizing controller

we assume L to be $L = s^6 + 5s^5 + 11s^4 + 14s^3 + 11s^2 + 5s + 1$ a strict hurwitz polynomial. Now the equation of the controller is given by

$$G_c = \frac{YL + AM}{XL - BM}$$

where M is given by $M = M_5s^5 + M_4s^4 + M_3s^3 + M_2s^2 + M_1s + M_0$. Now sublstituing the given values inside the first eqution $YL + AM$ we solve for M by setting the degrees above $\delta(r)$ to 0. Additionally we require a finite value of Kv to get 0 steady state error. For this we need $sG_c(0)$ to be finite. This gives us a condition on M_0 Solving this we get the following values of M

$$M_5 = -0.0160$$

$$M_4 = -0.07954$$
$$M_3 = -0.17327$$
$$M_2 = -0.197064$$
$$M_1 = -0.12933$$
$$M_0 = -0.4414$$

Substituting the values in the equation of G_c we get

$$G_c = \frac{-0.4692s^3 - 1.4007s^2 - 0.1975s - 0.5832}{s^3 + 1.877s^2 - 17.2892s}$$

C. Simulation Result

Firat the open loop operation of the is presented here.

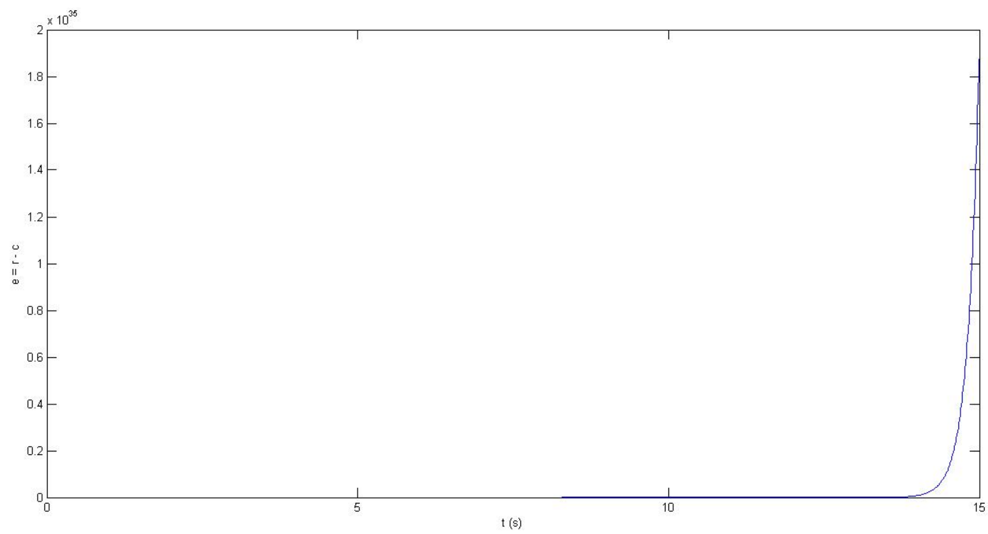

Figure 1: Open loop response of the system.

It can be seen that the system is unable to track a step response without a stabilizing controller. Next we simulate the response of the system with the stabilizing controller found in the previous section for a step input in Figure 2. There is also an added component of sinusoidal disturbance for this.

It is seen that the controller succesfully stabilizes the system for a step input. In addition to this we also simulate the system for a ramp and sinusoidal input.

Addtionally the frequency response of the senitivity of the given controller is given by the bode plot. It is observed that the magnitude of the sensitivity goes to 0 at 0 frequency and goes to 1 for high frequencies

III. Finding the optimal controller

In this section we find the optimal controller. The optimal controller can be found by integrating the square of the error and input. We can also put different weights on the input and error to decide what is penalized more. The error function is given as

$$J = \frac{1}{2\pi j}\int_{-j\,\mathrm{inf}}^{j\,\mathrm{inf}} [E(s)E(-s) + kU(s)U(-s)]ds$$

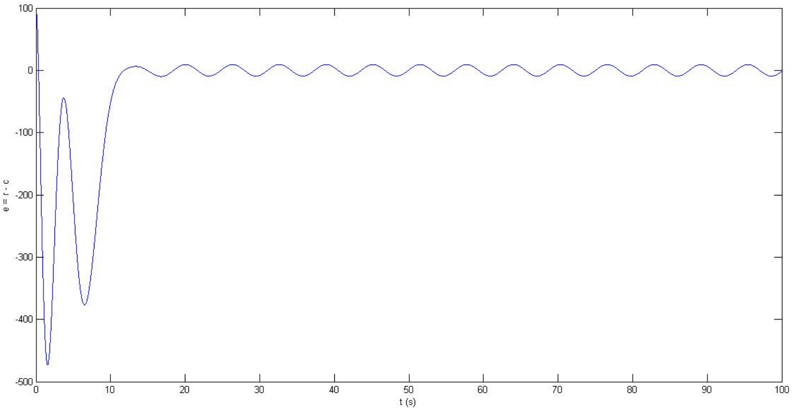

Figure 2: Step response.

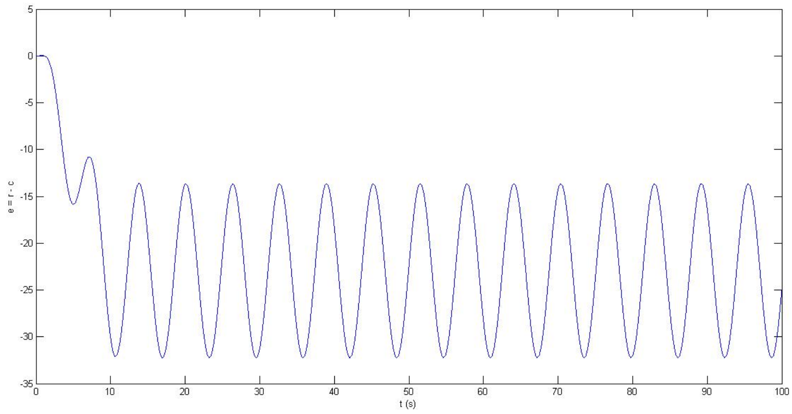

Figure 3: Ramp Response.

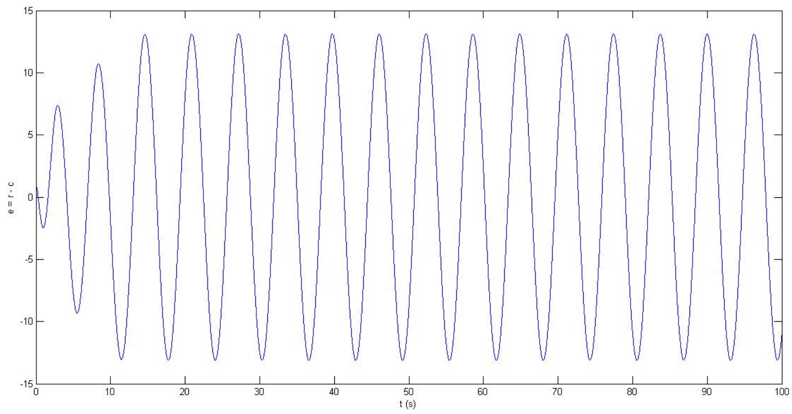

Figure 4: Sinusoidal Response

For this we assume the reference input is $R = \frac{1}{s+2}$ and $k = 1$ additionally $G_u G_{u*} = 1$ and $HH_* = 1$ It is observed that all the assumptions A1 to A6 for designing set of sensitivity functions of stabilizing controllers is satisfied.

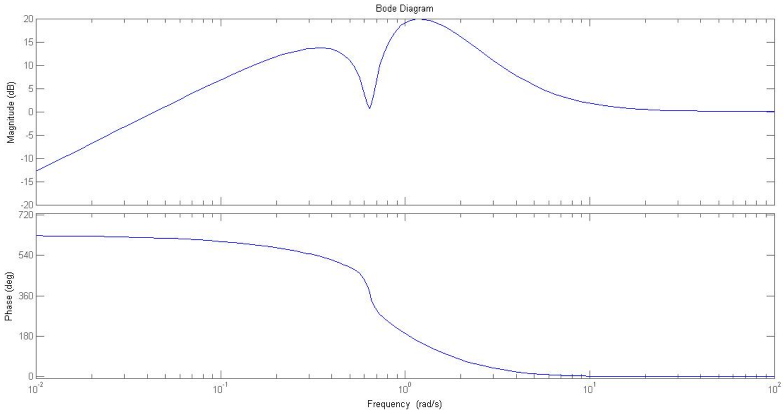

Figure 5: The bode plot of the sensitiviy function

A. Finding Φ and χ

Implementing this in MATLAB we can find the values of ϕ and χas

$$\Phi1 = \frac{-s^8 + 13.61s^6 - 2262s^4 + 8898s^2 - 18.72}{2232s^6 - 1.786e04s^4 + 3.573e04s^2 - 46.84}$$

$$\Phi2 = \frac{s^6 - 9.614s^4 - 8.333s^2 - 1.753}{2232s^4 - 8929s^2 + 11.71}$$

Additionally for this χ_r is 1 hence we can find the values of Ω and Ω_* as

$$\Omega = \frac{(s + 0.04588)(s^2 + 10.19s + 47.15)}{(s + 2)(s + 0.03622)}$$

$$\Omega_* = \frac{-0.00044811(s - 0.04588)(s^2 - 10.19s + 47.15)}{(s - 2)(s - 0.03622)}$$

Now we need to calculate the value of Γ_l. This can be done by taking the partial fractions of $\frac{\Phi2\chi_r\chi_r}{\Omega_*}$. Taking the terms with poles in the left half plane we get.

$$\Gamma_l = \frac{0.3194s - 0.2491}{s^2 1.721s + 0.06805}$$

Now the formula for sensitivity function is given by:

$$S_o = \frac{f(s) - \Gamma_l}{\Omega}$$

Now the degree of $f(s)$ is 1 as $\delta(f(s)) = \delta(n_\Omega) - \delta(d_\Omega)$
Substituting the value of Γ_l in the above equation we get

$$S_o = \frac{(f_1 s + f_0)(s^2 + 2.036s + 0.07244) - 0.3258s0.2471}{(s + 0.04588)(s^2 + 10.19s + 47.15)}$$

Now we need $1 - S_o = \left(\frac{1}{\omega^{l+1}}\right)$. This will decide the top $l + 1$ coefficients of S_o. Solving we get

$$f_1 = 1 \text{ and } f_0 = 8.2035$$

Solving we get the optimal sensitivity function as

$$S_o = \frac{s^3 + 10.24s^2 + 16.45s + 0.842}{s^3 + 10.24s^2 + 47.62s + 2.163}$$

The corresponding stabilizing controller is

$$G_c = \frac{-0.6595s^4 - 2.113s^3 - 0.211s^2 - 0.8787s - 0.03703}{s^4 + 10.28s^3 + 16.82s^2 + 1.438s + 0.0305}$$

B. Simulation of Optimal Controller

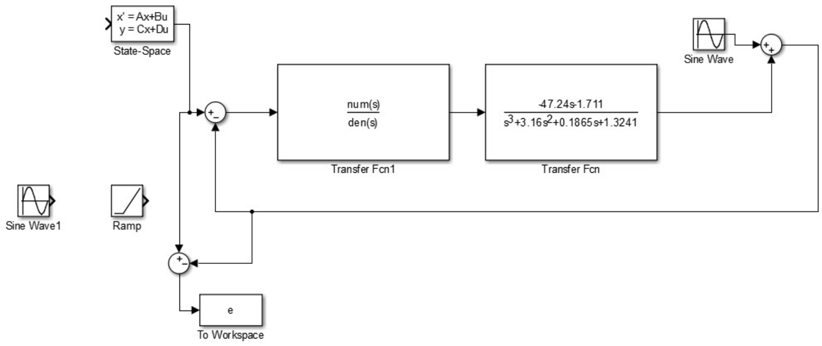

Figure 6: The Simulink Model used to simulate the system

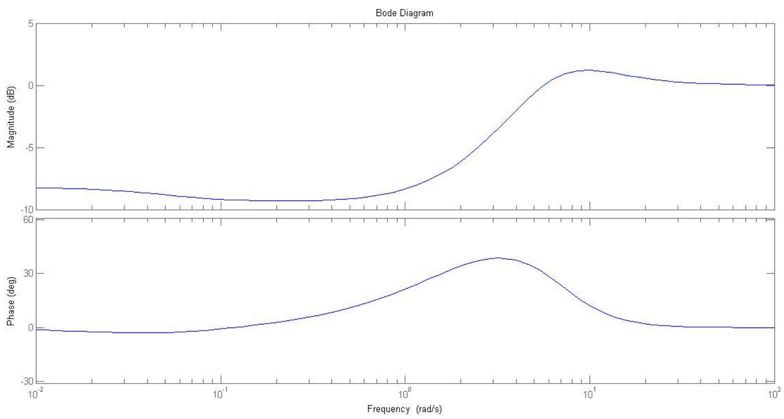

Figure 7: The Bode plot of the sensitivity function

These simulations are done with additional sinusoidal disturbance input as shown in the simulink figure.

From the Bode plot it is seen that the sensitivity function goes to 1 as s goes to ∞. Additionally it can be seen that the controller is able to track the given reference trajectory, which is a decay from an initial value to 0 with 0 steady state error. However it does not track a ramp output properly.

IV. Conclusion

It is observed that the two controllers designed to control the cyclic pitch of the helicopter are able to control the cyclic pitch for the given reference trajectory with minimal error. Additionally the optimal

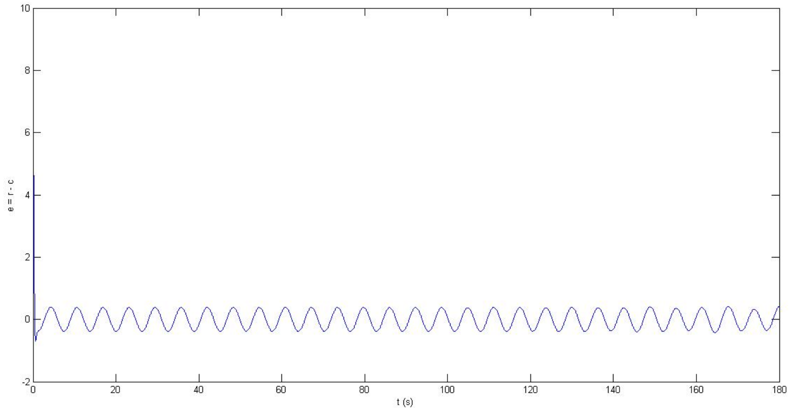

Figure 8: Tracking of the reference trajectory

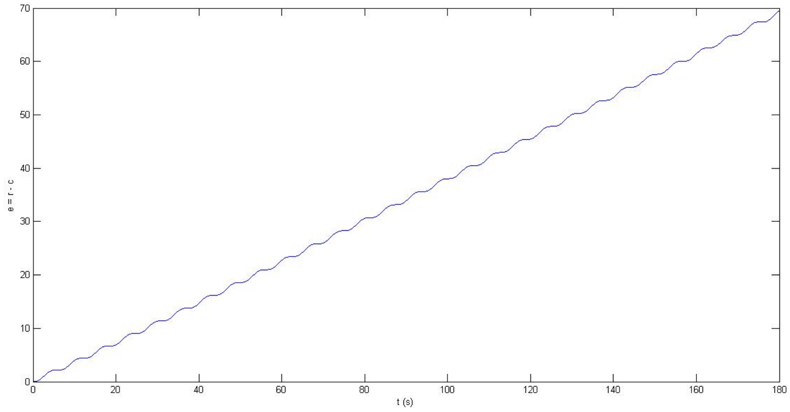

Figure 9: Error while tracking a ramp reference

control is able to achieve this at a reduced overall cost.

It should be noted however that we weren't able to design the optimal controller for a step response. This is because even though one of the set of all stabilizing controller stabilized a step response, that wasn't the optimal controller.