

Midterm (RR)

Please write down your student ID and your name on your answer sheets.

1. (25%) Solving recurrences:

(a) (5%) $T(n) = 13T(\frac{n}{4}) + \Theta(n^2) = \Theta(g(n))$. Solve $g(n)$.

(b) (5%) $T(n) = 7T(\frac{n}{3}) + \Theta(n^2) = \Theta(g(n))$. Solve $g(n)$.

(c) (5%) $T(n) = 4T(\frac{n}{4}) + \Theta(n) = \Theta(g(n))$. Solve $g(n)$.

(d) (5%) $T(n) = 3 + T(n-1) = \Theta(g(n))$. Solve $g(n)$.

(e) (5%) $T(n) = T(\frac{n}{2}) + \Theta(\log_2 n) = \Theta(g(n))$. Solve $g(n)$.

2. (10%) What is $\min_{0 \leq x \leq 10000} |100 - x| + |101 - x| + |102 - x| + |105 - x| + |1 - x|$?

3. (10%) Compute all longest non-decreasing subsequences of $\langle 1, 9, 8, 2, 8, 7, 6, 7, 6, 5, 7 \rangle$.

4. (10%) Matrix-chain multiplication: Solve the instance $(5, \langle 4, 1, 6, 9, 3, 5 \rangle)$, and give the optimal way to multiply $A_1 A_2 A_3 A_4 A_5$.

5. (10%) Suppose you have to process n tasks T_1, \dots, T_n . It takes t_i minutes to complete processing task T_i , and no two tasks can be processed simultaneously. The tasks can be processed in any order, so you may process T_p either before or after processing T_q where $1 \leq p < q \leq n$. Let c_i be the completion time of T_i , i.e., you complete processing task T_i at time c_i . For example, there are two tasks T_1 and T_2 . Let $t_1 = 2$ and $t_2 = 4$. If you process T_1 first, then $c_1 = 2$ and $c_2 = 2 + 4 = 6$. If you process T_2 first, then $c_2 = 4$ and $c_1 = 4 + 2 = 6$. The average completion time is defined as $\frac{1}{n} \sum_{i=1}^n c_i$. What is the minimum average completion time of the instance: $n = 5, t_1 = 3, t_2 = 1, t_3 = 5, t_4 = 6, t_5 = 8$.

6. (10%) There are n companies c_1, \dots, c_n in the ALGO group. Recently, Mr. Rhythm, the chairman of the ALGO group, determined to merge all these n companies into one. However, according to companies laws, he can only merge two companies into a new one at a time. In addition, the new company must issue new ID cards to its employees. If it has x employees, then it has to issue x new ID cards. I.e., if Mr. Rhythm merges companies A and B into C where A has x_A employees and B has x_B employees, then C has to issue $x_A + x_B$ new ID cards. Assume there are 5 companies c_1, c_2, c_3, c_4, c_5 having $x_1 = 5, x_2 = 14, x_3 = 7, x_4 = 20, x_5 = 28$ employees. What is the minimum number of the ID cards issued during the merging process?