Minimum Cost Flow: Simplified Version

Time Limit: 2 seconds

Problem Description

A weighted flow network is a directed graph $G = (V, E)$ where each edge $e \in E$ has a capacity $c(e) > 0$ and a weight $w(e) \in \mathbb{R}$. A flow f from source s to sink t is a function $E \to \mathbb{R}^+ \cup \{0\}$ such that:

- $f(e) \leq c(e)$ for $e \in E$.
- $f(e) = 0$ for $e \notin E$.
- $\sum_{u \in V} f(u, v) = \sum_{u' \in V} f(v, u')$ for $v \in V \backslash \{s, t\}$.

Given a flow network and a positive number r, write a program to compute the minimum cost flow f^* from s to t such that $|f^*| = \sum_{v \in V} f^*(s, v) = r$ and $\sum_{e \in E} f^*(e)w(e)$ is minimized. You may assume there is only one edge between any pair of distinct vertices.

Technical Specifications

1. The number of test cases is no more than 20.
2. Basic: $n \leq 20$, $m \leq 50$.
3. Hard: $n \leq 50$, $m \leq 200$.
4. For $(u, v) \in E$, $1 \leq c(u, v) \leq 10^6$, and c_i's are integral.
5. For $(u, v) \in E$, $0 \leq w(u, v) \leq 10^6$, and w_i's are integral.
6. $\sum_{e \in E} f^*(e)w(e) \leq 10^9$.

Input Format

The first line of the input file contains an integer indicating the number of test cases. The first line of each test case contains r integers n, m, s, t, r where $V = \{0, \ldots, n-1\}$, $|E| = m$, s is the source, t is the sink, r is the amount of the flow. The i-th of the following m lines contains 4 integers u_i, v_i, c_i, w_i representing that the i-th edge is from u_i to v_i, $c(u_i, v_i) = c_i$, and $w(u_i, v_i) = w_i$.

Output Format

For each test case, output $\sum_{e \in E} f^*(e)w(e)$ if there is a flow f such that $|f| \geq r$. If there is no such flow, then output `no solution`.

Sample Input

```
5
4 5 0 3 3
0 1 12 1
0 2 2 1
1 2 11 1
1 3 1 1
2 3 1 1
4 5 0 3 2
0 1 100000 1
0 2 100000 3
1 2 1 1
1 3 100000 4
2 3 100000 1
4 5 0 3 100001
0 1 100000 1
0 2 100000 3
1 2 1 1
1 3 100000 4
2 3 100000 1
6 9 0 5 20
0 1 16 1
0 2 13 2
2 1 4 3
1 3 12 4
3 2 9 5
2 4 14 6
4 3 7 7
3 5 20 8
4 5 4 9
5 7 0 4 2
0 1 1 2
0 2 1 1
1 2 1 1
1 3 1 4
2 3 1 1
3 4 1 3
2 4 1 2
```

Sample Output

```
no solution
7
400004
316
10
```