Investigating the Stability of Binary Star Systems

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The dynamical stability of circumbinary planets in binary star systems was investigated by means of the Runge-Kutta fourth-order approach for the numerical solution of the restricted three-body problem. We determine configurations under which a circumbinary planet remains stable by treating the planet's mass as unity and iteratively solving for the celestial positions under different initial conditions. By calibrating our model against the well-studied Kepler-16 system, the circumbinary planet's habitable zone (HZ) was determined accurately. We determine values of 0.374AU and 0.539AU for the inner and outer limits of the habitable zone. Where the total luminosity of both stars was $0.1537\ L_{\odot}$. Other key findings include the determination of a minimum stable separation distance, $a_{\min}$, and found the impact of the mass ratio between the binary stars on system stability. For a mass ratio of 0.25 we find that $a_{\min}$ was 7.75×10^{11} m and 8×10^{11} m for a mass ratio of 0.5 (equal mass stars). To ensure the robustness of our findings, we conducted extensive simulations across various time steps, against a standard time-step of 20s, demonstrating the convergence of our results and visualizing the residuals. This investigation enhances our understanding of the conditions necessary for the stability of planets in binary star systems, contributing valuable insights toward the search for habitable exoplanets in such complex celestial configurations.

I. INTRODUCTION

Can planets exist in multiple stellar systems? In binary star systems, they do. A binary star system is one where two stars orbit the same centre of mass. The term "binary" was coined by William Herschel in 1802 for double stars which are bound gravitationally and orbit together [4]. Such a system is an example of the three body problem. A classic problem in celestial mechanics, the three-body problem, entails estimating the movements of three celestial objects in relation to one another based on their mutual attraction. This puzzle arises from attempting to comprehend how three objects move when gravity is the sole external force.The gravitational pull between any two objects influences the velocity of the other, creating a highly dynamic and unpredictable system that gives rise to complexity. By considering the mass of the third body to be negligible, the three-body problem is simplified to 'restricted' three-body problem. In the past, Johannes Kepler and Isaac Newton solved the two-body problem, which involved just two bodies, by providing equations that accurately described their orbits. Nevertheless, the issue becomes considerably more complex with the addition of a third body since the system's behavior becomes chaotic and sensitive to the initial parameters, making long-term predictions almost impossible. Such sensitivity leads to extremely different outcomes at the smallest change in initial position or velocity [10]. The restricted form of the three body problem is simplified because it allows us to consider the mass of the third object—the planet orbiting a binary star system—to be negligible [5]. Due to its complexity, the R3BP cannot be solved analytically; we therefore depend on a numerical solution. Using mathematical implementations such as the Runge-Kutta method, we are able to investigate the stability of planetary orbits in a three-body system.

Different orbital configurations are present in binary star systems, which revolve around a common center of mass (COM) in either an elliptical or circular path. The initial velocities and the gravitational interactions between the stars determine the nature of these orbits. The more massive star in these systems usually orbits closer to the barycenter due to conservation of angular momentum and gravitational equilibrium. The heavier star has a smaller orbital radius. These dynamics provide information about the gravitational forces operating in binary systems, which affects how we perceive the behaviors and evolution of stars. Exoplanets can orbit a system in three ways: The libration-type (L-type) orbits, which correspond to librations around the Lagrangian equilibrium points L4 and L5, the satellite type (S-type) internal orbits, which revolve around one of the two stars, and the planet-type (P-type) external orbits around both stars in the binary. P-type and S-type orbits have significant astronomical implications and merit in-depth investigation into their features, but L-type orbits are typically not of concern for exoplanets in binary systems [5]. In this investigation, the configuration focused on is that of a planet orbiting the binary system in a circular p-type orbit. This configuration describes a subclass of planets residing in binary star systems called circumbinary planets. Where orbits encircle both stars at a sufficient distance to avoid being destabilized by the binary's motion. It is important to mention that the planet follows a prograde orbit. In a binary star system, if the stars are rotating counterclockwise when viewed from above, a planet with a prograde orbit would also move counterclockwise in its orbit around the center of mass of the system. For the circular orbits, we assume they have a quasi-circular eccentricity. Therefore, it is considered that the acceptable planetary and stellar eccentricity is $e \leq 0.05$. The objective of this project is to investigate the dynamic stability of planetary orbits within binary star systems using the RK4 method. We will analyze how the distance between two stars and a planet can affect the planet's orbit, examining conditions that could allow for stable orbits that support the potential for life. An example of a three-body system is Kepler-16. It was the first discovered transiting circumbinary planet. It is an eclipsing binary star system in the Cygnus constellation that the Kepler spacecraft targeted. Both stars, Kepler-16A and Kepler-16B, are smaller than the Sun, separated by 0.22 AU and have an orbital period of 41 days. The system hosts one extrasolar planet known as Kepler-16b [15]. The configuration of the system shown in Figure 1 is similar to how the three bodies were modelled in our simulations. Therefore, we use the parameters of the system as reference values for the simulation to ensure the investigation is as realistic as possible.

II. THEORY

A. The 4th-Order Runge Kutta Method

It is generally recognized that the set of equations describing the motion of a gravitational dynamical system with respect to three or more celestial bodies cannot have an analytical solution. Numerous techniques for numerical integration have been developed, particularly in the field of space dynamics, to solve systems of ordinary differential equations in dynamical systems. One such technique is the fourth-order Runge-Kutta method [1].

The method outlined computes velocities and accelerations at four time step points: the beginning, two midpoints, and the end. By averaging these computations, this method greatly improves the accuracy of forecasting an object's future position and velocity [3]. With a fourth-order error margin, a^5, which indicates that the local error of the RK4 method is proportional to the fifth power of the time step size, (a). In other words, if you make the step size a smaller by a factor of 10, the error in the final result becomes 10,000 times smaller. The method exhibits good accuracy and is appropriate for lengthy orbital movement simulations. Selecting a small enough time step is necessary for both the RK4 and the Taylor expansion methods to guarantee precise orbital tracking. These techniques make use of starting location and velocity data to enable the tracking of an object's trajectory over time.The laws of motion are used to calculate the acceleration for each step based on the current position. The RK4 technique, in particular, calls for the sequential computation of intermediate steps, each of which is reliant on the previous one's result. Due to its optimal balance between computational load and solution accuracy, RK4 is very popular for solving ordinary differential equations. This makes it perfect for applications in celestial mechanics, including the restricted three-body problem. RK4 requires that the equations of motion be transformed into first-order differential equations in order to explain changes in the object's position and velocity. Within each time step, RK4 approximates these changes in four steps, computing intermediate values to precisely forecast the system's state at the conclusion of the time step. The effectiveness of RK4 is found in its capacity to provide precise outcomes without requiring unreasonably small time steps, which makes it perfect for the limited three-body problem. Its accuracy, however, is dependent upon the time step size used; smaller steps increase processing needs while boosting accuracy. In order to efficiently maintain accuracy, it could be required to adjust the time step sizes for chaotic systems or situations where gravity forces vary quickly. The following equations are the iterative steps of the RK4 method. These equations would be used to update the positions and velocities of the three objects in our dynamical system.

$$Z_1 = X_n + \frac{a}{2}\dot{X}_n, \dot{Z}_1 = \dot{X}_n + \frac{a}{2}\ddot{X}_n, \tag{1}$$

$$Z_2 = X_n + \frac{a}{2}\dot{Z}_1, \dot{Z}_2 = \dot{X}_n + \frac{a}{2}\ddot{Z}_1, \tag{2}$$

$$Z_3 = X_n + a\dot{Z}_2, \dot{Z}_3 = \dot{X}_n + a\ddot{Z}_2, \tag{3}$$

$$X_{n+1} = X_n + \frac{a}{6}\left(\dot{X}_n + 2\dot{Z}_1 + 2\dot{Z}_2 + \dot{Z}_3\right), \tag{4}$$

$$\dot{X}_{n+1} = \dot{X}_n + \frac{a}{6}\left(\ddot{X}_n + 2\ddot{Z}_1 + 2\ddot{Z}_2 + \ddot{Z}_3\right), \tag{5}$$

Where we define the variables as follows: X_n is the position of the particle at the n-th time step, $\dot{X}_n$ is the velocity of the particle at the n-th time step, and $\ddot{X}_n$ acceleration of the particle at the n-th time step, Z_1, Z_2, Z_3 are the intermediate variables for the position of the particle within the time step; $\dot{Z}_Pr, \dot{Z}_2, \dot{Z}_3$ are the intermediate variables for the velocity of the particle within the time step; and a is the time step size. [6]

Generally, the equation of motion of a planet in a circumbinary orbit is as follows:

$$\frac{d^2}{dt^2}\mathbf{r}_{\mathrm{Pl-Pr}} = -G\left(\frac{M_{\mathrm{Pr}}}{r^3_{\mathrm{Pl-Pr}}}\mathbf{r}_{\mathrm{Pl-Pr}} + \frac{M_{\mathrm{Sec}}}{r^3_{\mathrm{Pl-Sec}}}\mathbf{r}_{\mathrm{Pl-Sec}}\right). \tag{6}$$

Where M_{Pr} (primary) and M_{Sec} (secondary) refer to the stellar masses, $r_{\mathrm{Pl}-i}$ the vectors that link the planet to the binary stars, M_{Pl} the planet mass and G is the gravitational constant [7].

B. Stability

The objective of this research is to investigate the stability of the planetary orbit. What is stability, and how is it achieved? And how can we quantify it? The capacity of a planet to stay in orbit around a star or binary star system for extended periods of time without being noticeably disturbed or expelled is known as planetary orbit stability. Planetary systems' ability to support life and long-term survival depend on their stability as well as their ability to maintain liquid water.

We take into account the initial semi-major axis of the planet's orbit as well as the mass ratio and eccentricity of the binary star system as free parameters. The definition of the mass ratio is:

$$\mu = \frac{M_{\mathrm{Sec}}}{M_{\mathrm{Pr}} + M_{\mathrm{Sec}}} \tag{7}$$

[12]. Where M_{Pr} refers to the heavier star. For stars of identical mass, $M_{\mathrm{Pr}} = M_{\mathrm{Sec}}$, and $\mu = 0.5$. The gravitational perturbation caused by the secondary star decreases

with decreasing μ. Long-term integration increases one's confidence in the stability of a planetary orbit, but it undoubtedly comes at a computing cost. The question of whether the planetary orbit stays constrained for extended periods of time—typically billions of years—is known as long-term dynamic stability. It indicates that the gravitational interactions between the planet and the binary stars do not cause major perturbations to the orbit if it stays stable over such time-frames. We consider an orbit unstable in two cases: if the planet is ejected from the binary system or if it collides with one of the two stars within a certain amount of time. In these situations, we log the orbit lifetime, also known as the survival time, which is the amount of time that has passed between the start of the orbit and the collision or escape. We do not specifically mention chaos in this categorization, which is the sensitivity to initial conditions that prevents precise long-term predictions of the position and velocity of the system particles. Even stable orbits are capable of chaotic activity. Stable orbits are generally distinguished by their degree of deviation from the unperturbed orbit; orbits near the primary star are nearly circular and largely unaffected by the secondary star's gravity [12].

Stability depends on the mass of the planet, the initial semi-major axis of the planetary orbit, the mass ratio, and the distance ratio [12]. In fact, it is the mass ratio itself that is relevant for defining the zones of stability and not the individual masses of the stars themselves, which is a key point in the characterization of the dynamics of stellar binary systems [10]. The distance ratio is defined as:

$$\rho = \frac{R_0}{D} \tag{8}$$

[12]. Where R_0 is the initial planet separation from the COM and D the distance between the primary and secondary stars. It gives an estimate of the separations between the orbiting planets and stars. For the purpose of assessing whether the planet's orbit is stable, this ratio aids in quantifying the gravitational interactions that occur within the system.

After enduring the intricate and dynamical ordeal of near double stars, the planets manage to reach long-term, dynamically stable orbits that reach a minimal critical distance from the center of mass binary, denoted as a_min. In many cases, the phrase "critical radius" describes the separation of a planet's stable orbit from the system's center of mass. The latter is dependent on the orbital eccentricity and binary mass ratio and is a function of the binary separation a_bin. This is how the stability criteria are characterized.

The gravitational pull of the stars helps to sustain the orbital dynamics of a planet in a stable orbit without significantly altering its path. On the other hand, differences in the planet's distance from the COM over time may cause variations in gravitational forces, which may cause the orbit to become unstable. Gravitational forces are taken into consideration in the RK4 calculations by incorporating the acceleration due to gravity into the differential equations that control the motion of the system as shown by equation (6). Based on Newton's law of universal gravitation, the derivation of acceleration is the first step in this process. According to this law, the gravitational force between two bodies is expressed as a function of their squared distance apart and the product of their masses. Taking into account the positions of the bodies and the gravitational forces applied by every other body, RK4 assesses acceleration at intermediate points inside each time step. The gravitational forces experienced by the planet depend on the planet's separation from the COM of the system and, therefore, the stars. As the planet moves closer or farther away from the COM, the gravitational forces exerted by the stars change accordingly.

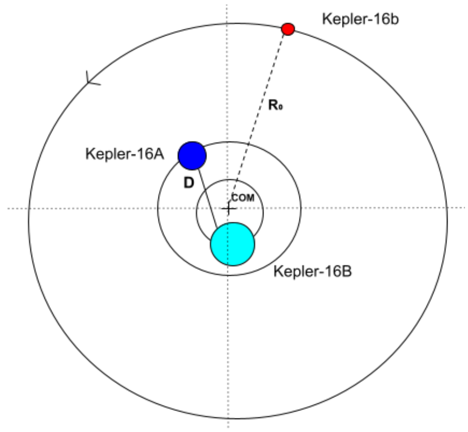

FIG. 1: Schematic showing the configuration of the Kepler-16 system. The planet,16b, orbits the system in a p-type, circumbinary orbit.Kepler 16A and 16B are the binary stars, with heavier primary star orbiting closer to the center of mass. Data extracted from the Extrasolar Planets Encyclopaedia [14].

To meet the stability condition, the planet must orbit outside of the binary semi-major axis for a duration of approximately 3-8 times the binary period. The known circumbinary planets have binaries with periods around 10-40 days, therefore, the planets' periods will be slightly longer than 30-320 days. Because the Kepler targets are predominantly G (like the sun) and K type stars, this orbital period is quite close to the binary's habitable zone (HZ).

K and G-type stars are thought to be cooler than stars of earlier spectral types like O, B, and A. For example, G-type stars like the Sun have surface temperatures ranging from around 5,000 to 6,000 $^\circ$C but K-type stars have slightly colder surface temperatures ranging from about 3,500 to 5,000°C. [15]

The habitable zone around these stars refers to the region where conditions may be sufficient for liquid water and, perhaps, life as we know it on planetary surfaces, which can be found on both the inner and outer edges. The inner edge or boundary is determined by how close a planet can orbit without its surface water evaporating due to intense stellar radiation, while the outer edge or boundary is determined by factors such as atmospheric composition, greenhouse effects, and cloud cover. All of which affect temperature stability and prevent water from freezing completely [11].

III. METHOD

The RK4 method was used to solve for the positions of all three bodies at a time-step,dt of 20. We run the simulation for different mass ratios to determine different values for a_min. Initially, we have a system with a mass ratio of 0.25; to achieve a stable orbit, the simulation is run countless times with a "perturbation factor" set on the R_0 function. And we varied the perturbation factor until the planet approximately returned to its initial position. The mass ratios were calculated using equation (7). Initial values are shown in the table below for the two cases investigated:

M_{Pr} / $M_\odot$	$M_{\mathrm{Sec}}/M_\odot$	D (m)	Eccentricity e_{Bin}
6	2	1.5×10^{11}	0.05
6	6	1.5×10^{11}	0.05

TABLE I: Parameters of the primary and secondary stars used in the simulation. The investigation was carried out for the two different mass ratios, as shown. The semi-major axis of each star was approximated as half the distance between them.

The mass of the planet is set to be negligible per the requirements of the R3BP. The aim was to find a stability criterion for this specific set of initial parameters—a range of values that the separation of the planet must fall within in order for the planet to not deviate from its predicted path. With the now stable orbits, we continue the investigation by varying the μ and finding the critical threshold of stability, a_min. For equal-mass stars, μ is 0.5, another case that was investigated in this research. Another criteria that needs to be fulfilled in order for the orbit to be considered stable is the simulation timescale. To ensure the reliability of a_min we run the simulation for t_max 50 years. If the planet still returns to its initial position after a long period of time, we consider its orbit stable.

By modeling our system after the Kepler-16 system, we are able to calculate the HZ. With parameters: , and using the inner and outer boundry equations.The HZ in P-type systems is dynamic, with its limits changing based on the luminosity of the binary stars, their spectral types, and the binary eccentricity as the stars orbit each other.The positions of the HZ borders, and hence a planet's ability to maintain habitable conditions, are determined by the total flux received at the top of the planet's atmosphere. It is in the region where the planet's effective temperature T_e lies between: $273 < T/K < 373$ [7].

$$L_* = 4\pi R_*^2 \sigma T_*^4 \tag{9}$$

L_* represents the star's luminosity, R_* is the radius of the star, and σ stands for the Stefan-Boltzmann constant, which is used to calculate the energy emitted by a blackbody in terms of its temperature. Lastly, T_* is the effective surface temperature of the star [13]. The inner boundary is the closest distance to the star that allows the planet to maintain water, whereas the outer boundary is the farthest distance from the star where a planet could maintain liquid water on its surface. Within the inner boundary (IB) the stellar radiation is too powerful causing any water to evaporate, and the water freezes when the planet is in the outer boundary (OB) as seen in equations (10) and (11) below. For stars in the main sequence:

$$r_{\mathrm{IB}} = \sqrt{L_{\mathrm{T}}/1.1} \tag{10}$$

$$r_{\mathrm{OB}} = \sqrt{L_{\mathrm{T}}/0.53} \tag{11}$$

[16]. Where L_{T} is the total luminosity (summation of both stars')and the values (1.1) and (0.53) represent stellar fluxes at the inner and outer boundary based on the work of Kasting. (0.53) is the value where carbon dioxide starts to condense and increase the albedo of the planet [9].

Parameter	Values for 16-A	Values for 16-B
Mass	$0.6897 \pm 0.0035\ M_\odot$	$0.20255 \pm 0.00065\ M_\odot$
Radius	$0.6489 \pm 0.0013\ R_\odot$	$0.22623 \pm 0.00059\ R_\odot$
Luminosity	$0.1489 \pm 0.0004\ L_\odot$	$\sim 0.0057 \pm 0.0004\ L_\odot$

TABLE II: Parameters for the spectroscopic binaries 16-A and 16-B. Data extracted from SIMBAD [2]

IV. RESULTS

μ	a_{min} (m)	Distance Ratio ρ
0.25	7.75×10^{11}	5.17
0.50	8×10^{11}	5.33

TABLE III: Parameters of the primary and secondary stars used in the simulation. The investigation was carried out for the two different mass ratios, as shown. The semi-major axis of each star was approximated as half the distance between them.

These findings highlighted in III show how the distribution of stellar masses affects the gravitational stability of planets orbiting binary stars. A planet can orbit closer and still remain stable the larger the mass difference between the two stars. On the other hand, in equal mass systems, the planet must orbit slightly farther in order to find a stable path.

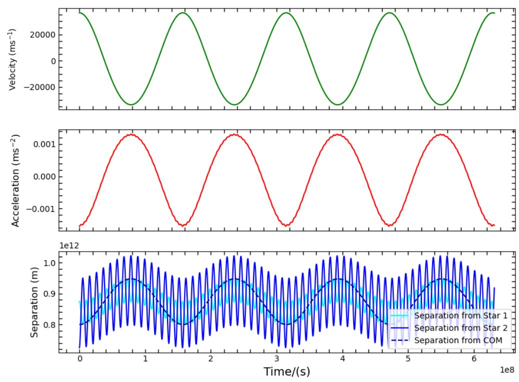

FIG. 2: The x-components of velocity and acceleration of the planet. The separation of the planet from COM and the primary and secondary stars.

This plot shows how the x-component of acceleration and velocity of an object changes over the duration of the planet's orbit. When you integrate the acceleration function over time, you obtain the velocity function. This plot shows how the velocity of the object changes over time. It indicates whether the object is speeding up, slowing down, or moving at a constant velocity. The slope of this plot at any point represents the acceleration at that moment. By observing these plots simultaneously we can see how changes in acceleration influence the velocity of the object. Acceleration is out-of-phase with velocity.

In the contour plot,the stable regions are between 0.22 and 0.55 on the normalized separation distance scale. At the minimum values, the planet has just reached a stable orbit, any position closer to the systme would cause it to be sucked in by the gravitational fields of either star and collide. The maximum values represent the positions where

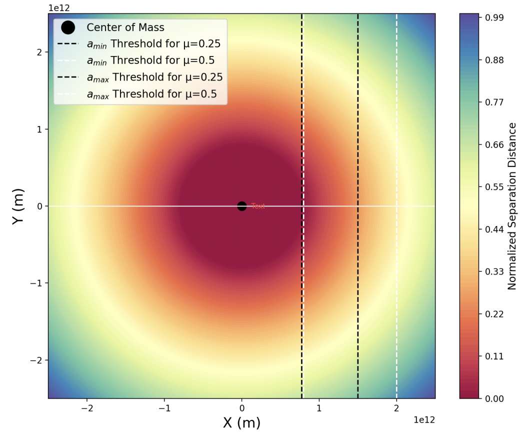

FIG. 3: Contour plot to show the minimum and maximum separation for each mass ratio case.

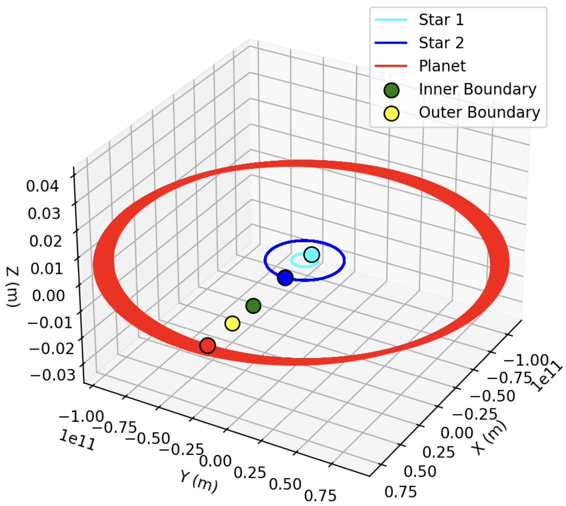

FIG. 4: Visualization of the Kepler-16 system with the inner and outerb boundaries of the habitable zones clearly marked.

the planet reaches orbital stability with the least amount of perturbations. For a primary and secondary star with parameters as shown in table **??**, we determine values of 0.374AU for r_{IB} and 0.539AU for r_{OB}. Where the total luminosity of both stars was 0.1537 $L_{\odot}$.

V. DISCUSSION

$$a_{\min} = D(1.60 + 5.10e_{\mathrm{Bin}} + 4.12\mu - 2.22e_{\mathrm{Bin}}^2 - 4.27\mu e_{\mathrm{Bin}} - 5.09\mu^2 + 4.61\mu^2 e_{\mathrm{Bin}}^2) \quad (12)$$

According to [7], equation (12) can be used to calculate the minimum planet separation required as opposed to running the simulations to acquire a value. The minimum separation via calculations is then: 3.80×10^{11} m for the case of μ 0.5 and 3.76×10^{11} m for μ 0.25, see. However if in the simulation the calculated $a_{\min}$ values are utilized we see the planet return to its initial position but with great perturbations in its orbit. Therefore we consider its orbit instable. For the Kepler-16 system with a mass ratio of 0.23, star distance 3.36×10^{10} , and eccentricity o.159 $a_{\min}$ is 7.28×10^{10} m.

The separation from COM of the planet was calculated for different time steps to show the importance of choosing the correct time step and how the final position of the planet varies for each time-step. Because our system is so sensitive to initial parameters and its stability depends on it returning to its original position then choosing the lowest possible dt is of great importance. The residuals subplot demonstrates the difference in the final position of the planet for each time step where for $dt = 20s$ it is the "expected" value and the rest are the "observed" values.

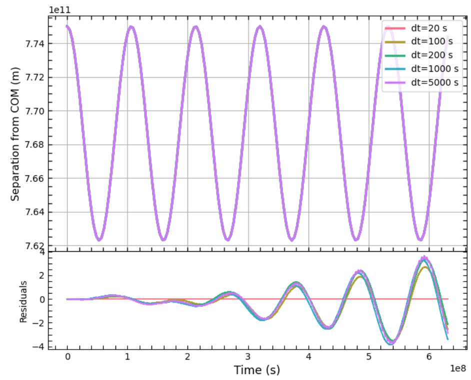

FIG. 5: Separation of the planet from COM as a function of time. The plot demonstrates the difference in the final position calculated for different time-steps.Simulation time of 50 years.

In the future, it would be beneficial to expand on this research project by also investigating the orbital stability of planets in an s-type orbit. As mentioned previously, s-type orbits also merit investigation. Alternatively, this project can be developed by using a different numerical integration approach that could potentially yield more accurate results. It may also be beneficial to take into account the mass of the planet instead of considering it to be negligible. It will cause the three-body problem (3BP) to become more complex but will provide a more accurate representation of the system's dynamics and potential stability. The planet's atmosphere should also be taken into consideration when investigating its habitability. A planet's atmosphere has a significant impact on whether or not it is habitable. The medium through which star radiation is first received and the transition from insulation to equilibrium temperature takes place on a planet is its atmosphere. An atmosphere will react differently to radiation from different stars because stars of different spectral types have varied spectral distributions of incident energy. This suggests that the energy received at the top of the planet's atmosphere is influenced differently by stars with different spectral energy distributions. [7]

As seen in other investigations, to simulate gravitational dynamics, the method of the IAS15 integrator was used. It is a 15th order integration scheme that provides an optimal solution for long-term integration. Renowned for its accuracy and reliability it's a part of the REBOUND simulation package and stands out because it automatically adapts its time step to the physical situation it's simulating. This is crucial for systems where forces change rapidly in time. The IAS-15 is generally preferred over the RK4 method for multiple reasons. For starters, RK4 uses a fixed time step and can be inefficient or inaccurate for systems with highly varying force dynamics because it doesn't adjust the step size based on the system's behavior. RK4 can miss or in-accurately simulate critical interactions like close planetary encounters. In comparison, the IAS-15 can maintain high accuracy over a wide range of conditions by adjusting its step size, which makes it highly effective for long-term integration of planetary systems where dynamic encounters can occur unpredictably. [5]

Other investigations trained a machine learning model capable of rapidly ascertaining whether the circumbinary planetary systems are stable. Furthermore, it was discovered that compared to other machine learning (ML) algorithms, deep neural networks (DNNs) have better accuracy and precision. Utilizing machine learning to forecast the stability of non-coplanar circumbinary planetary systems is a significant additional feature. Previous studies have demonstrated that ML are a good fit for predicting a multi-planetary system's stability. The following are some benefits of ML: First,when compared to N-body simulation, ML is extremely quick. While it takes a while to generate the training data through N-body simulation in this type of work, it only takes thirty minutes to train the DNNs model using laptop computing power. Furthermore, using the trained DNNs to predict more samples only takes a few minutes. Secondly, ML is a more accurate description than polynomial function of the transition regions between orbital stability and instability [8].

The Kepler Project was a NASA space mission started in 2009 with the primary purpose of finding Earth-sized exoplanets orbiting other stars. It accomplished this by continually monitoring the brightness of over 150,000 stars in a fixed area of view in the Cygnus and Lyra constellations. Other than Kepler-16, some of the planets discovered include: Kepler-34 A and B have very eccentric orbits of 0.521, and their contact with the planet is strong enough to cause a notable deviation from the Keplerian solution after only one period. It is now obvious that circumbinary planets can exist in a variety of star configurations. The primary star masses range from 0.69 to 1.53 $M_\odot$ (Kepler-16 A & PH1 H1 Aa), mass ratios from 1.03 to 3.76 $M_\odot$ (Kepler-34 & PH1), and eccentricities from 0.023 to 0.521 (Kepler-47 & Kepler-34). Similarly, the eccentricities of planetary orbits vary greatly, ranging from almost circular (0.007) to large (0.182)(Kepler-16 & Kepler-34). There is no trend for orbital resonance with the binary. It is evident that no particular geometry is favored. [15]

VI. CONCLUSIONS

To review, the stability of a circumbinary planet was investigated by using the RK4 method to numerically solve the restricted three body problem. This was achieved by setting the planet's mass to unity and solving for the positions of the three objects utilizing different initial parameters until the planet achieved stability. Stability was categorized as the planet returning to its initial position when setting it at a minimum separation from the center of mass of the system and continuing to do so for a period of 50 years. The system was then modeled after the Kepler-16 system so that the habitable zone of the planet could be calculated. By calibrating our model against the well studied Kepler-16 system, the circumbinary planet's habitable zone (HZ) was determined accurately. We determine values of 0.374AU for $r_{\rm IB}$ and 0.539AU for $r_{\rm OB}$. Where the total luminosity of both stars was 0.1537 $L_\odot$. Other key findings include the determination of a minimum stable separation distance, a_min , and the impact of the mass ratio between the binary stars on system stability. For a mass ratio of 0.25 we find that $a_{\rm min}$ is 7.75×10^{11} and 8×10^{11} for a mass ratio of 0.5 (equal mass stars).To test the reliability of our results, we ran simulations of the systems for different time steps and showed their convergence 5.

Therefore, the restricted three body problem was solved numerically to determine the minimum separation of the planet from the COM to achieve planetary stability. This was achieved for the two different mass ratios with a simulation time of 50 years and a time-step of 20s. Siumlating our system after the Kepler-16 system we were able to identify the habitable region for a system with a mass ratio of 0.23.

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VII. APPENDIX

$M_\odot$	M_J	G/N×m^2 kg^{-2}	(σ)/W×m^{-2}×K^{-4}	AU
1.99×10^{30} kg	1.90×10^{27} kg	6.67×10^{-11}	5.67×10^{-8}	1.5×10^{-11}

TABLE IV: The values of the constants used in this investigation.