

Table 1 Topology cheet sheet 1

| | topological space (X, \mathfrak{D}) | metric space (X, d) |
|-------------------------------|--|---|
| 内部 (内点) | $M^i \quad (M \subset X)$ | $\{x \mid \exists \epsilon > 0, N(x; \epsilon) \subset M\}$ |
| 中心 c の開球 | - | $N(c; \epsilon) = \{x \mid d(x, c) < \epsilon\}$ |
| 閉包 (触点) | $\overline{M} \quad (M \subset X)$ | $\{x \mid \forall \epsilon > 0, N(x; \epsilon) \cap M \neq \phi\}$ $\Leftrightarrow \{x \mid \exists (x_n)_{n \in \mathbb{N}} \in M \text{ s.t. } \lim_{n \rightarrow \infty} x_n = x\}$ |
| 境界 (境界点) | $M^i \setminus \overline{M}$ | $\{x \mid \forall \epsilon > 0, (N(x; \epsilon) \cap M \neq \phi) \wedge (N(x; \epsilon) \cap M^c \neq \phi)\}$ |
| 導集合 (集積点) | $M^a = \{x \mid x \in \overline{M - \{x\}}\}$ | $\{x \mid \forall \epsilon > 0, N(x; \epsilon) \cap (M - \{x\}) \neq \phi\}$ |
| 孤立点集合 (孤立点) | $M \setminus M^a$ | $\{x \mid \exists \epsilon > 0, N(x; \epsilon) \cap M = \{x\}\}$ |
| 基底 \mathfrak{B} : open sets | $\forall O \in \mathfrak{D}, O = \cup_{\lambda \in \Lambda} B_\lambda, B_\lambda \in \mathfrak{B}$ | |
| 近傍系 $V(x)$ | $V(x) = \{V \mid x \in V^i\}$ | |
| 基本近傍系 $V^*(x)$ | $\exists V \in V^*(x) \text{ s.t. } V^* \subset V, \forall V \in V(x)$ | |
| 第二可算公理 | $ \mathfrak{B} \leq \aleph_0$ | |
| 可分空間 | $\exists M \subset X \text{ s.t. } M \leq \aleph_0 \wedge \overline{M} = X$ | |
| 第一可算公理 | $ V^*(x) \leq \aleph_0$ | |
| 連続写像 $f : X \mapsto Y$ | $\forall O \in \mathfrak{D}, f^{-1}(O) \in \mathfrak{D}$ | $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } \forall a \in X, d_X(x, a) < \delta \Rightarrow d_Y(f(x), f(a)) < \epsilon$ |
| 同相写像 $f : X \mapsto Y$ | f is bijection. f, f^{-1} is a continuous function. | |

Table 2 Topology cheet sheet 2

| | topological space (X, \mathfrak{D}) | metric space (X, d) | euclid space $(\mathbb{R}^n, d^{(n)})$ |
|-------------------------|--|---|---|
| 連結 | There is no open set U, V satisfying: 1. $S \subset U \cup V$ 2. $U \cap V = \emptyset$ 3. $U \cap S \neq \emptyset, V \cap S \neq \emptyset$ | | Let Y be subset of \mathbb{R}^n Y is connected. \Leftrightarrow Y is path-connected. |
| 弧状連結 | $f: [0, 1] \mapsto X, f(0) = x, f(1) = y$ f is continuous function. | | |
| コンパクト | Every open cover C of X has a finite subcover . | Complete and Totally bounded | Bounded closed set (Heine-Borel) |
| 点列コンパクト | Every sequence of points in X has a convergent subsequence converging to a point in X . | | |
| 可算コンパクト | Every countable open cover C of X has a finite subcover. | | |
| Lindelöf の性質 | Every open cover C of X has at most countable subcover. | | |
| 局所連結 | $\forall x \in X, \forall V \in \mathfrak{D}(x \in V), \exists U \subset V$ s.t. U is connected. | | |
| 局所コンパクト | $\forall x \in X, \exists V \in \mathfrak{D}(x \in V)$ s.t. V is compact. | | |
| 有界 | - | $\forall m \in M, \forall x \in X, \exists r > 0$ s.t. $r < \infty, d(x, m) < r$ | $\forall k \in \{1, 2, \dots, n\}, \exists L_k \in \mathbb{R}, R_k \in \mathbb{R}$ $X \subset [L_1, R_1] \times [L_2, R_2] \times \dots \times [L_n, R_n]$ |
| 全有界 | - | $\forall \epsilon > 0, \exists C$ s.t. $\forall U \in C, \text{diam}(U) < \epsilon, C$ is finite cover. | |
| 完備 | - | Every cauchy sequence of points in X is a convergent sequence. | $(e.g.) \mathbb{R}^n, []$ |
| 一様連続写像 $f: X \mapsto Y$ | - | $\forall \epsilon > 0, \exists \delta > 0$ s.t. $\forall x, a \in X d_X(x, a) < \delta \Rightarrow d_Y(f(x), f(a)) < \epsilon$ | |
| 一様同相写像 $f: X \mapsto Y$ | - | f is bijection. f, f^{-1} is a uniformly continuous function. | |