Tutorial-2

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Sec:- A

1.
```
void fun(int n){
    int j=1, i=0;
    while(i<n){
        i=i+j;
        j++; }}
```

If i is increasing at the rate of j.

→ If K is total no. of iterations, while loop terminates if,

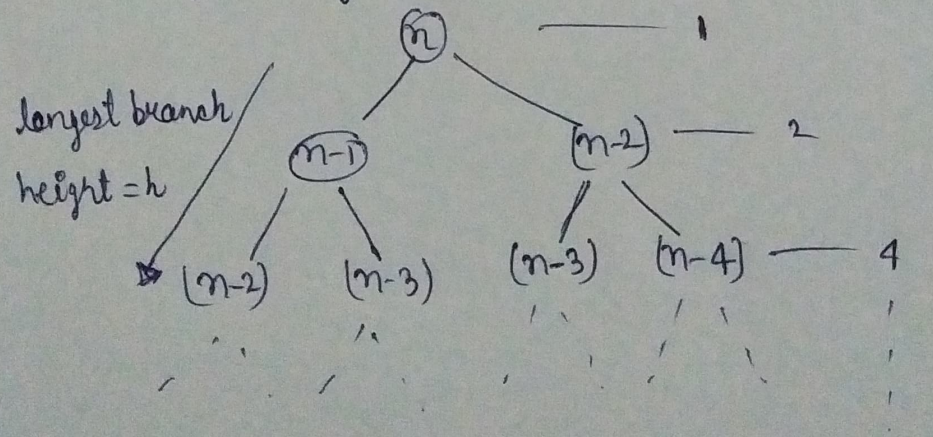
$$0+1+\cdots+K = \frac{K(K+1)}{2} > n$$

T.C → $O(\sqrt{n})$.

2. The recurrence rel^n for the recursive method of fibonacci series is —

$$T(n) = T(n-1) + T(n-2) + 1$$

Solving using tree method —



$T.C = 1+2+\cdots+2^n$ or 2^n

$a=1, \quad r=2 \qquad S = \frac{a(r^{\text{terms}}-1)}{r-1}$

$= \frac{1(2^{n+1}-1)}{(2-1)}$

$T.C = O(2^{n+1}) = O(2 \cdot 2^n) = O(2^n)$

Space complexity $= O(n)$ [∵ Stack size never exceeds the depth of the call's tree shown above]

Ans 3:- Program with complexity –

i) $n \log n$ –

```
void fun(int n) {
    for(int i=1; i<=n; i++) {
        for(int j=1; j<=n; j+=i)
            printf("*");
    }
}
```

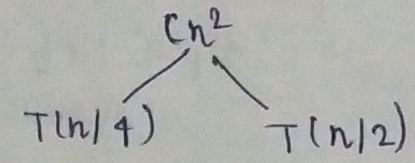
ii) n^3

```
void function(int n) {
    for(i=1; i<=n; i++){
        for(int j=1; j<=n; j++){
            for(int k=1; k<=n; k++) {
                printf("#");
            }
        }
    }
}
```

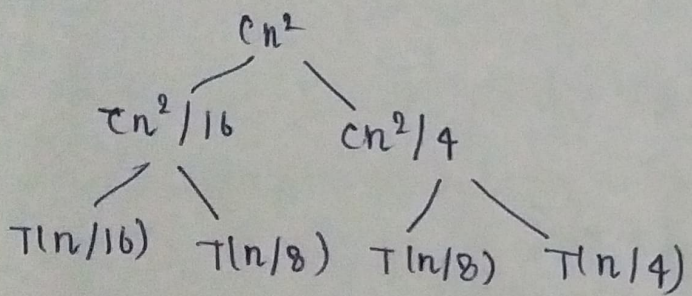
iii) $\log(\log n)$ → for(int i=2; i<=n; i = pow(i,k)){ //O(1)};
also, interpolation search has this complexity

Ans#4:- $T(n) = T(n/4) + T(n/2) + cn^2$

Following is the initial recursion tree,



on further breaking down,



To know the value of T(n) we need to calculate the sum of tree nodes level by level.

$\Rightarrow T(n) = cn^2 + 5n^2/16 + 25n^2/256 + \dots$

GP with ratio 5/16

$S_\infty = \frac{n^2}{1 - 5/16} \Rightarrow T.C = O(n^2)$

Ans#5:- Same as ques 9
→ $O(n \log n)$

Ans#6:-
```
for(int i=2; i<= n; i= pow(i,k))
{
   //O(1) expression
}
```

In this case i takes value $2, 2^k, (2^k)^k, (2^{k^2})^k = 2^{k^3} \dots$

$2^{k \log_k(\log(n))}$

The last term must be less than or equal to n, we have,

$$2^{k \log_k(\log(n))} = 2^{\log n} = n, \text{ It's True}$$

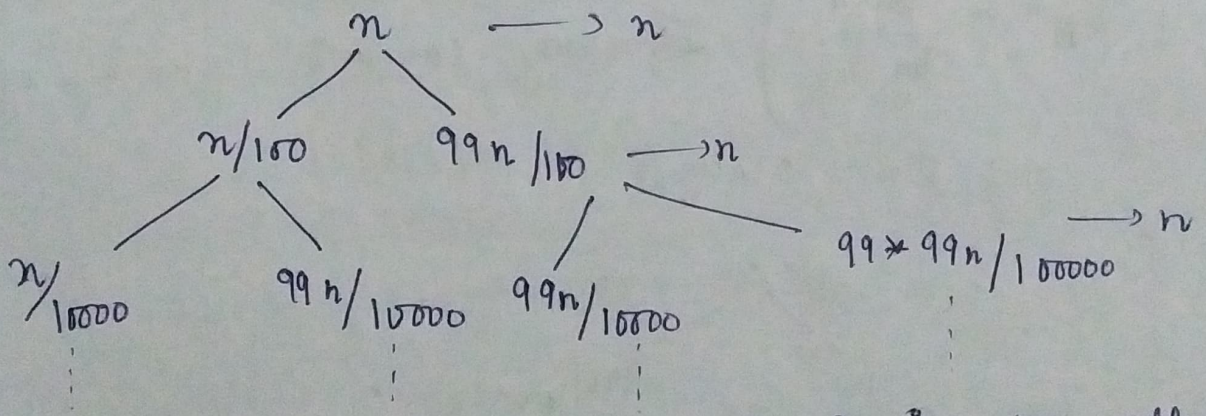
∴ There are total $\log_k(\log(n))$ many iterations and each iteration takes constant amount of time to run,

∴ Total times complexity $= O(\log(\log n))$.

Ans7:- The running time when in quick sort when the partition is putting 99% of elements on one side and 1% elements on another in each repetition

$$T(n) = T\left(\frac{99n}{100}\right) + T\left(\frac{n}{100}\right) + cn$$

Recursion tree of the above equation is,



We can see that initially, the cost is n for all levels, This will follow untill the left most branch of the tree reaches its base case (size 1) because the left most branch has least elements in each division, so it'll finish first.

The rightmost branch will reach its base case at last because it has maximum no. of elements in each division.

At level i, the rightmost node has $n * \left(\frac{99}{100}\right)^i$ elements,

for the last level,

$$n * \left(\frac{99}{100}\right)^i = 1$$

$$\rightarrow i = \frac{\log_{100} n}{\log_{100} n} \quad \log_{\frac{100}{99}} n$$

So, there are total $\left(\log_{\frac{100}{99}} n\right) + 1$ levels

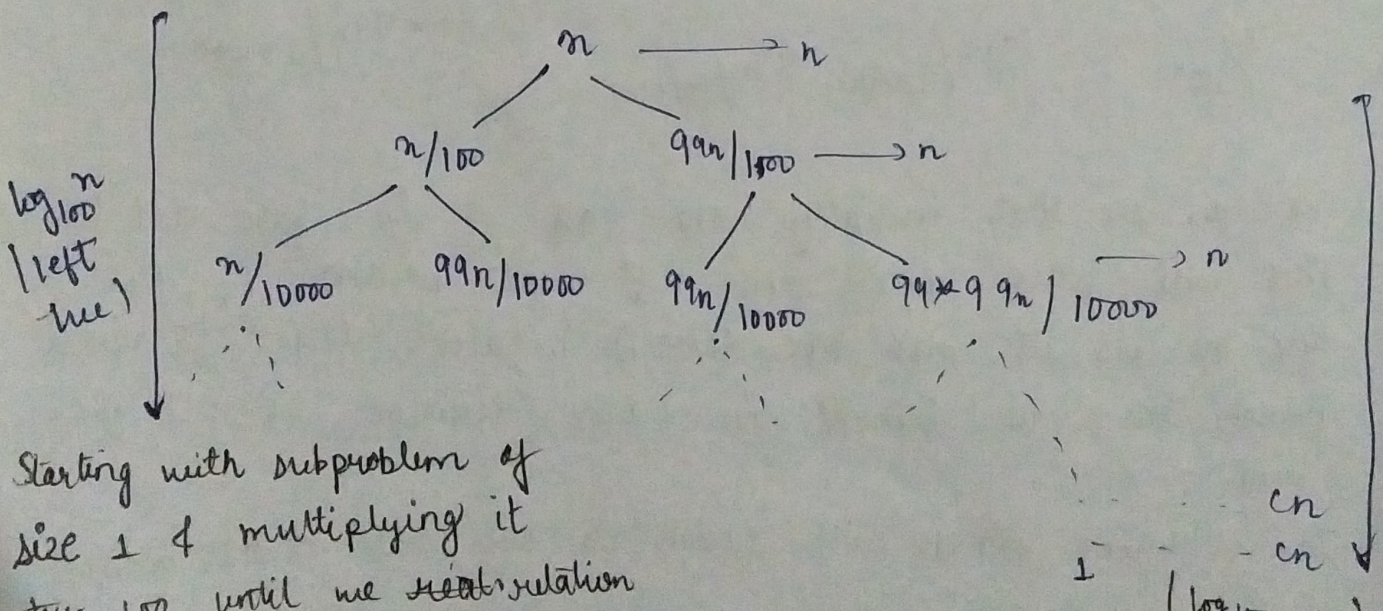
This,

$$T(n) = \left(\frac{cn + cn + \cdots \text{(} \log_{\frac{100}{99}} n + 1 \text{ times)}}{\log_{\frac{100}{99}} n + 1 \text{ times}}\right) \leq \left(\log_{\frac{100}{99}} n + 1\right) cn$$

$$= O\left(n \log_{\frac{100}{99}} n\right)$$

$\left(\log_{\frac{100}{99}} n = \frac{\log_2 n}{\log_2 \frac{100}{99}}\right)$ Ignoring constant term by $\log_2 \frac{100}{99}$

$\Rightarrow T(n) = O(n \log n)$



Starting with subproblem of size 1 & multiplying it by 100 until we reach relation

$100^x = n$

$\rightarrow x = \log_{100} n$

Right child is $\frac{99}{100}$ of size of nodes above it. Each parent is $\frac{100}{99}$ times the size of right child.

$\left(\frac{100}{99}\right)^x = n$

Ans18:- a) Increasing order of rate of growth —

$100, \log(\log n), \log n, \sqrt{n}, n, \log(n!), n\log n, n^2, 2^n,$

$4^n, n!, 2^{2^n}$

b) $1 < \log(\log n) < \sqrt{\log(n)} < \log(n) < \log 2n < 2\log(n) <$

$n < 2n < 4n < \log(n!) < n\log n < n^2 < n! < 2(2^n)$

c) $96 < \log_8 n < \log_2 n < 5n < \log(n!) < n\log_6(n) <$

$n\log_2 n < 8n^2 < 7n^3 < n! < 8^{2n}$