

Tutorial - 6

Solution 1:

Minimum spanning tree - Minimum height spanning tree is the subset of the edges of connected, edge-weighted, undirected graph that contains connects all the vertices together, without any cycle and with the minimum possible total edge weight.

Applications:

(i) consider 'n' stations are to be linked using a communication network and laying of communication link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.

(ii) suppose you want to construct highways or rail roads spanning several cities then we can use the concept of minimum spanning tree.

(iii) Designing LAN

(iv) Laying pipelines connecting offshore drilling sites refineries and consumer markets.

(v) Suppose you want to supply a set of houses with-

- Electric power

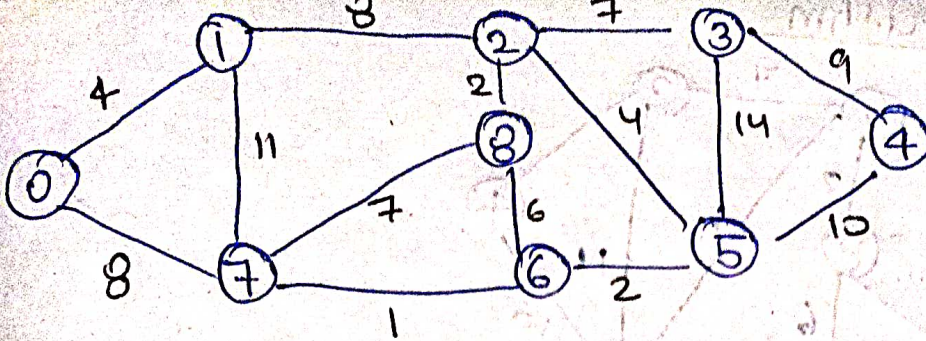
- water

- Telephone lines

- sewage lines

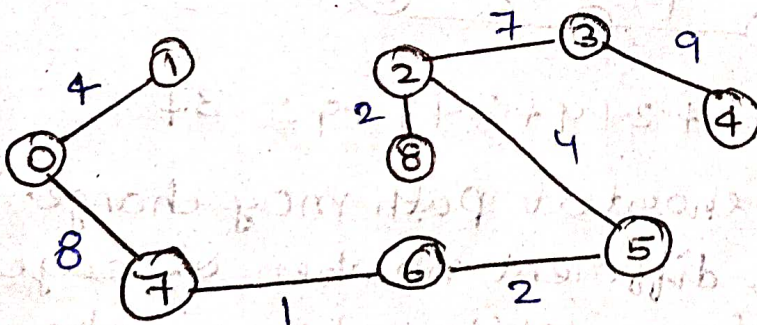
Solution 2:

	Time complexity	Space complexity
Prims algorithm	$O(E \log V)$	$O(V)$
Kruskals algorithm	$O(E \log E)$	$O(V)$
Dijkstra's algorithm	$O(V^2)$	$O(V^2)$
Bellman Ford's algorithm	$O(VE)$	$O(E)$



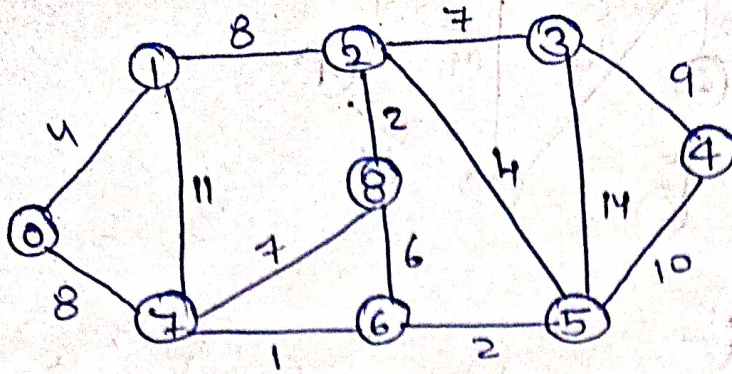
Kruskal's algorithm-

source (U)	destination (V)	weight (W)	U	V	W
6	7	1 ✓			
5	6	2 ✓			
2	8	2 ✓			
0	1	4 ✓			
2	5	4 ✓			
6	8	6 ✗			
2	3	7 ✓			
7	8	7 ✗			
0	7	8 ✓			
1	2	8 ✗			
4	3	9 ✓			
4	5	10 ✗			
1	7	11 ✗			
3	5	14 ✗			



$$\text{Weight} = 1 + 2 + 2 + 4 + 4 + 7 + 8 + 9 = 37$$

Prim's algorithm.

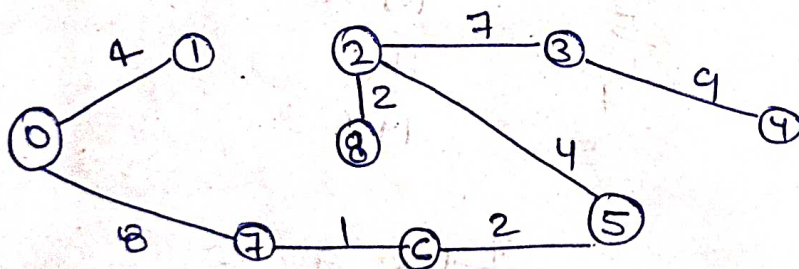


weight:

0	1	2	3	4	5	6	7	8
0	∞	∞	∞	∞	∞	∞	∞	∞
	4						8	
		8				11		7
	11		7		4			2
		14	14	10	2			6
			7	9				

Parent:

0	1	2	3	4	5	6	7	8
-	-	-	-	-	-	-	-	-
	0	1				7	0	

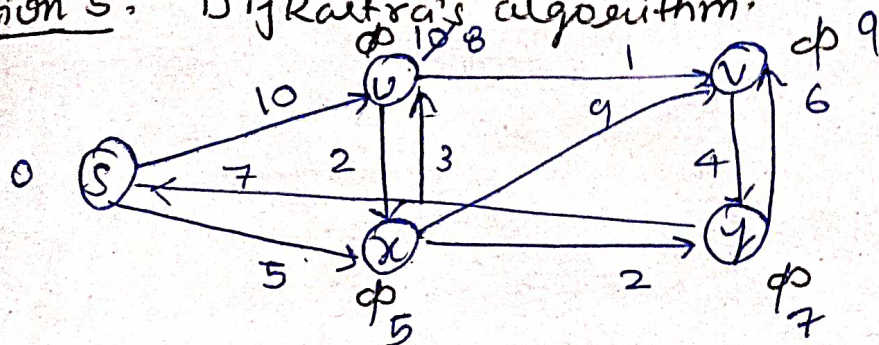


$$\text{Weight} = 4 + 8 + 1 + 2 + 4 + 2 + 7 + 9 = 37$$

Solution 4: The shortest path may change. The reason is there may be different number of edges in different paths from 's' to 't'. eg - shortest path be of weight 15 and has edge 5 edges. Let there be another path with 2 edges & total weight 25. The weight of the shortest path increased by 5 $\sqrt{10}$ and becomes 25 + 20 so the shortest path changes to the other path with weight ≈ 45 .

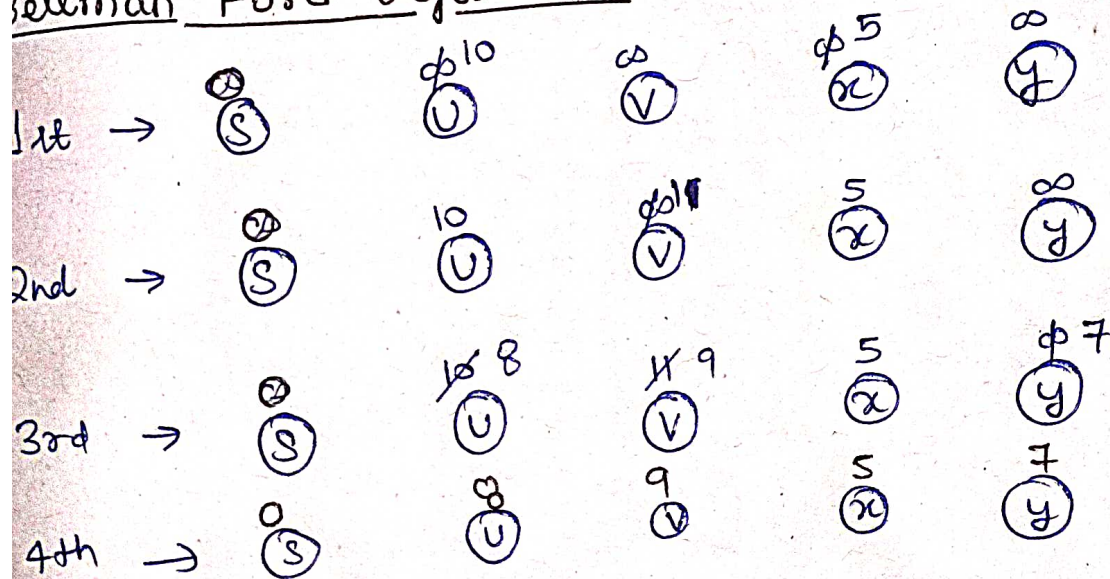
If we multiply all edges weight by 10, the shortest path doesn't change. The reason is simple, weights of all edges from 's' to 't' get multiplied by some amount, no. of edge on a path doesn't matter. It is like changing units of weights.

Question 5: Dijkstra's algorithm.



node	shortest distance from source node
u	8
v	9
x	5
y	7

Bellman Ford algorithm-



Final graph

