

HEAP

A heap is a data structure that stores a collection of objects (with keys), and has the following properties:

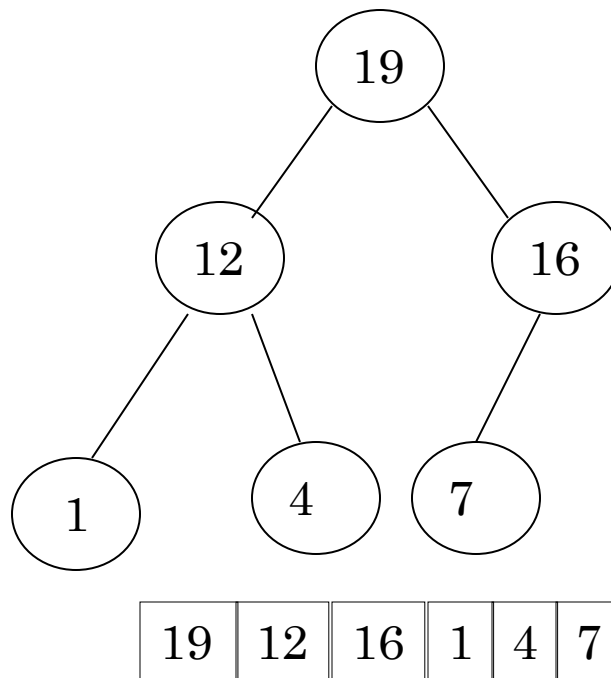
- Complete Binary tree
- Heap Order

It is implemented as an array where each node in the tree corresponds to an element of the array.



HEAP

- The binary heap data structure is an array that can be viewed as a complete binary tree. Each node of the binary tree corresponds to an element of the array. The array is completely filled on all levels except possibly lowest.



Array A



HEAP

- The root of the tree $A[1]$ and given index i of a node, the indices of its parent, left child and right child can be computed

PARENT (i)

return $\text{floor}(i/2)$

LEFT (i)

return $2i$

RIGHT (i)

return $2i + 1$



HEAP ORDER PROPERTY

- For every node v , other than the root, the key stored in v is greater or equal (smaller or equal for max heap) than the key stored in the parent of v .
- In this case the maximum value is stored in the root



DEFINITION

- Max Heap

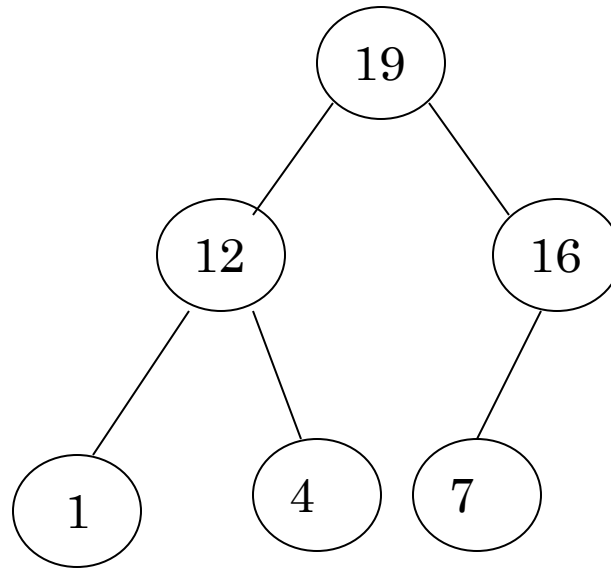
- Store data in ascending order
- Has property of
$$A[\text{Parent}(i)] \geq A[i]$$

- Min Heap

- Store data in descending order
- Has property of
$$A[\text{Parent}(i)] \leq A[i]$$



MAX HEAP EXAMPLE

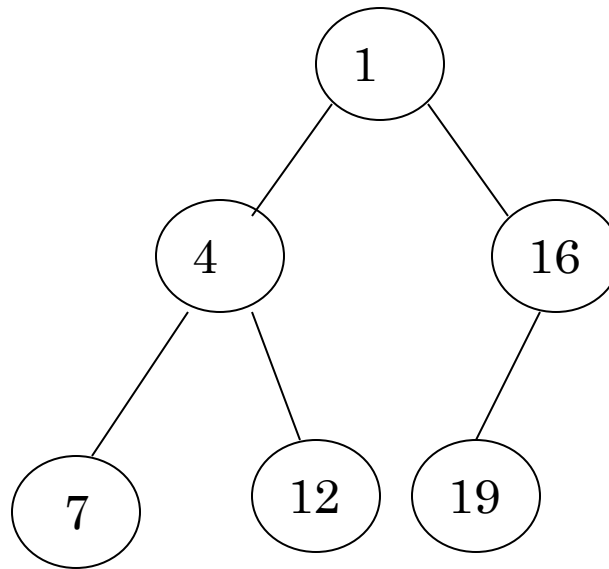


19	12	16	1	4	7
----	----	----	---	---	---

Array A



MIN HEAP EXAMPLE



1	4	16	7	12	19
---	---	----	---	----	----

Array A



INSERTION

○ Algorithm

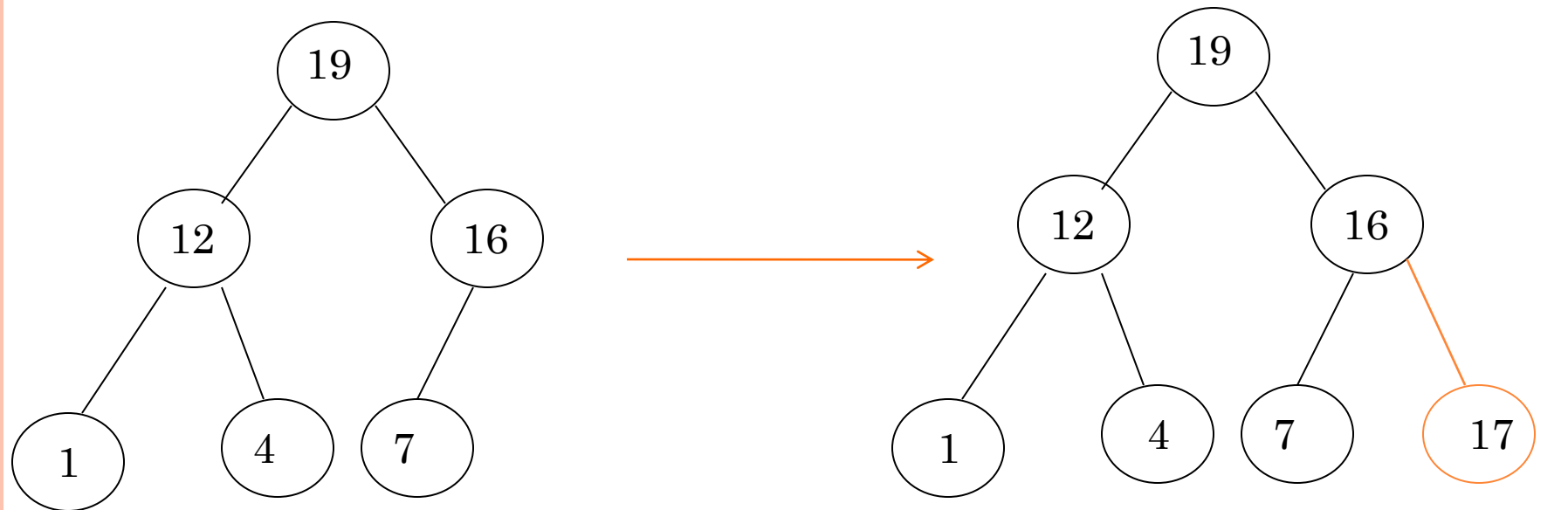
1. Add the new element to the next available position at the lowest level
2. Restore the max-heap property if violated
 - General strategy is percolate up (or bubble up): if the parent of the element is smaller than the element, then interchange the parent and child.

OR

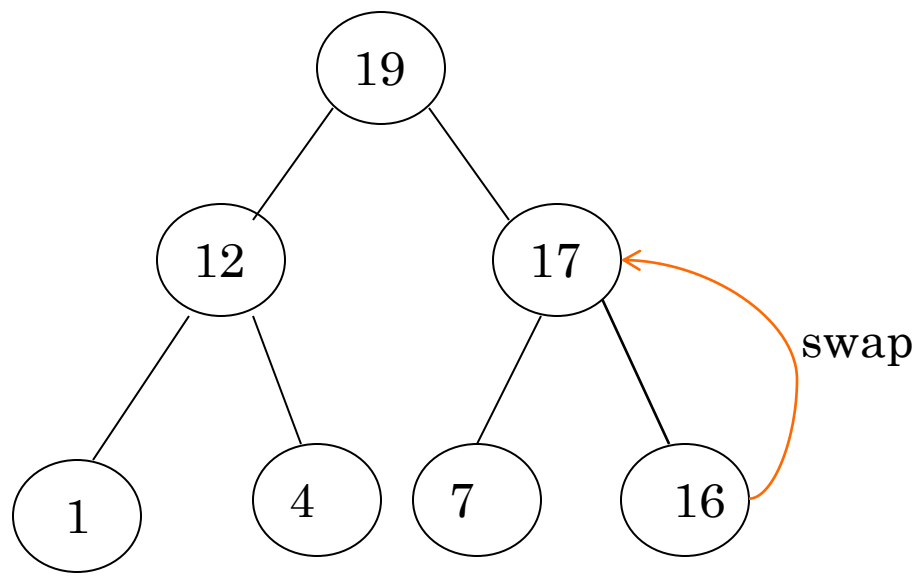
Restore the min-heap property if violated

- General strategy is percolate up (or bubble up): if the parent of the element is larger than the element, then interchange the parent and child.





Insert 17



Percolate up to maintain the heap property



DELETION

○ Delete max

- Copy the last number to the root (overwrite the maximum element stored there).
- Restore the max heap property by percolate down.

○ Delete min

- Copy the last number to the root (overwrite the minimum element stored there).
- Restore the min heap property by percolate down.



HEAP SORT

A sorting algorithm that works by first organizing the data to be sorted into a special type of binary tree called a heap



PROCEDURES ON HEAP

- Heapify
- Build Heap
- Heap Sort



HEAPIFY

- Heapify picks the largest child key and compare it to the parent key. If parent key is larger than heapify quits, otherwise it swaps the parent key with the largest child key. So that the parent is now becomes larger than its children.

Heapify(A, i)

```
{  
    l  $\leftarrow$  left(i)  
    r  $\leftarrow$  right(i)  
    if l  $\leq$  heapsize[A] and A[l] > A[i]  
        then largest  $\leftarrow$  l  
        else largest  $\leftarrow$  i  
    if r  $\leq$  heapsize[A] and A[r] > A[largest]  
        then largest  $\leftarrow$  r  
    if largest  $\neq$  i  
        then swap A[i]  $\leftrightarrow$  A[largest]  
        Heapify(A, largest)  
}
```



BUILD HEAP

- We can use the procedure 'Heapify' in a bottom-up fashion to convert an array $A[1 \dots n]$ into a heap. Since the elements in the subarray $A[n/2 + 1 \dots n]$ are all leaves, the procedure BUILD_HEAP goes through the remaining nodes of the tree and runs 'Heapify' on each one. The bottom-up order of processing node guarantees that the subtree rooted at children are heap before 'Heapify' is run at their parent.

Buildheap(A)

```
{  
    heapsize[A] ← length[A]  
    for i ← |length[A]/2 //down to 1  
        do Heapify(A, i)  
}
```



HEAP SORT ALGORITHM

- The heap sort algorithm starts by using procedure BUILD-HEAP to build a heap on the input array $A[1 \dots n]$. Since the maximum element of the array stored at the root $A[1]$, it can be put into its correct final position by exchanging it with $A[n]$ (the last element in A). If we now discard node n from the heap than the remaining elements can be made into heap. Note that the new element at the root may violate the heap property. All that is needed to restore the heap property.

Heapsort(A)

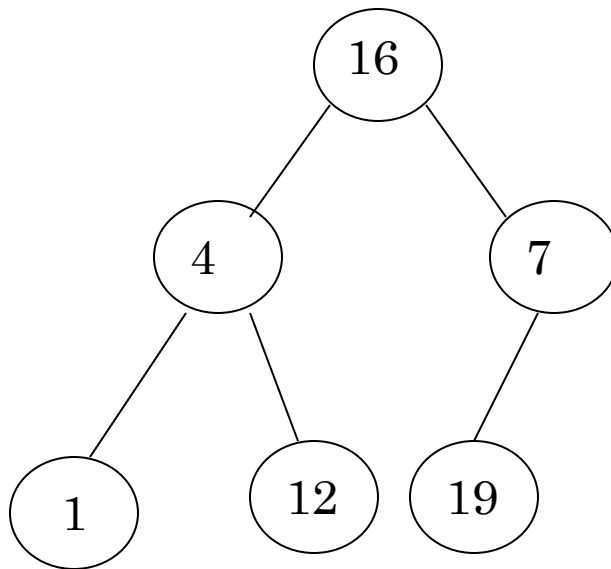
```
{  
    Buildheap(A)  
    for i ← length[A] //down to 2  
        do swap  $A[1] \leftrightarrow A[i]$   
        heapsize[A] ← heapsize[A] - 1  
        Heapify(A, 1)  
}
```

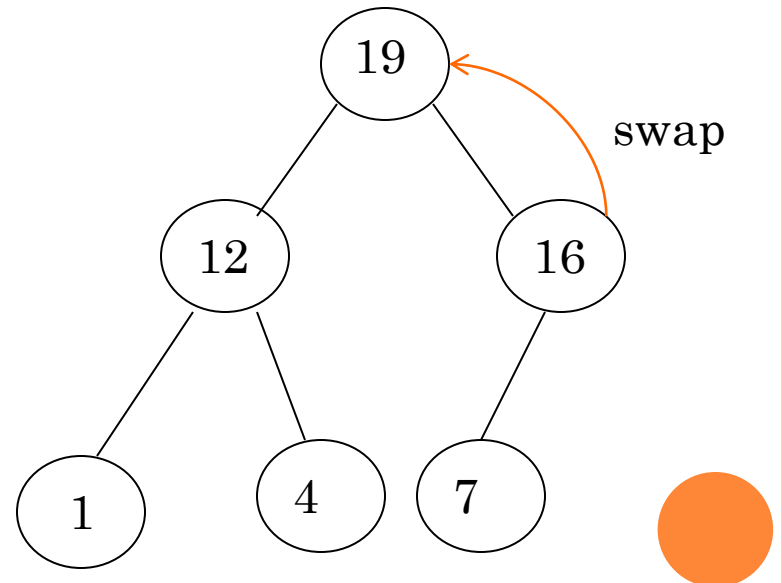
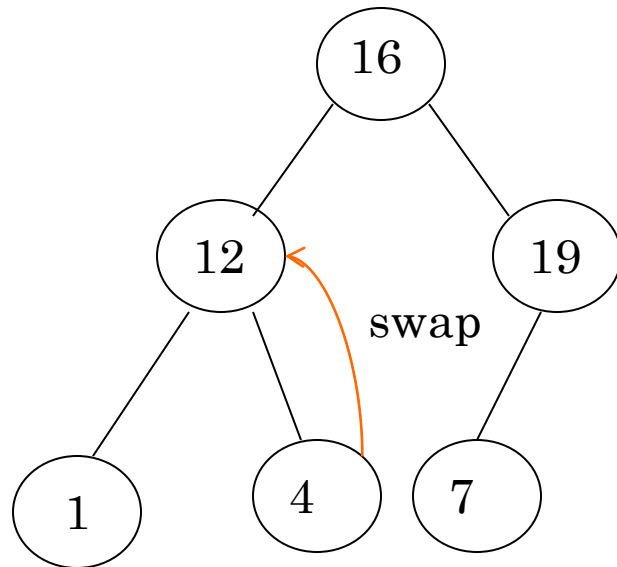
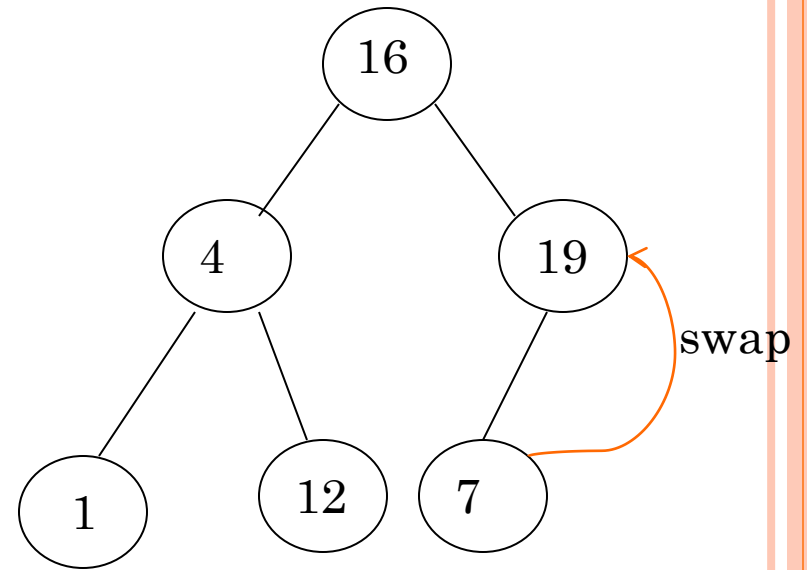
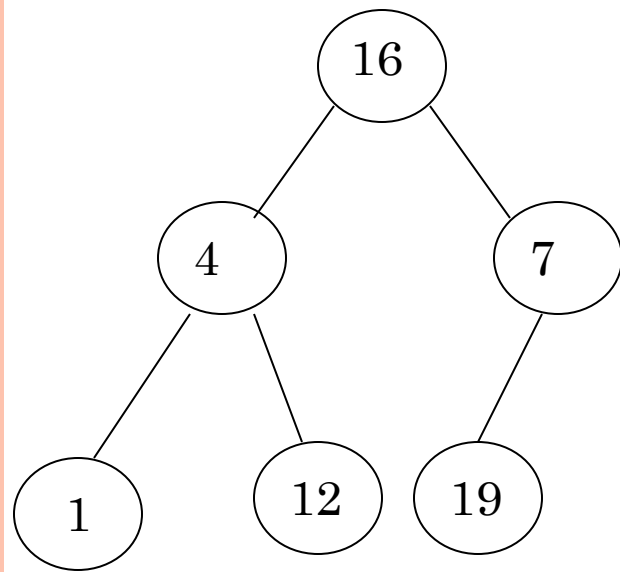


Example: Convert the following array to a heap

16	4	7	1	12	19
----	---	---	---	----	----

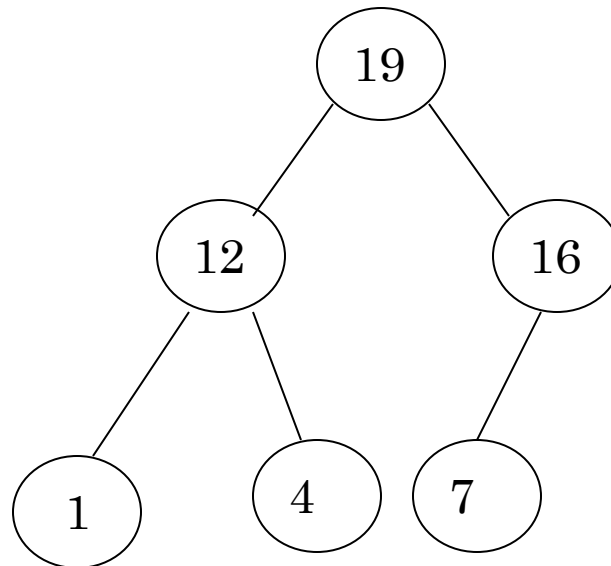
Picture **the array as a complete binary tree:**



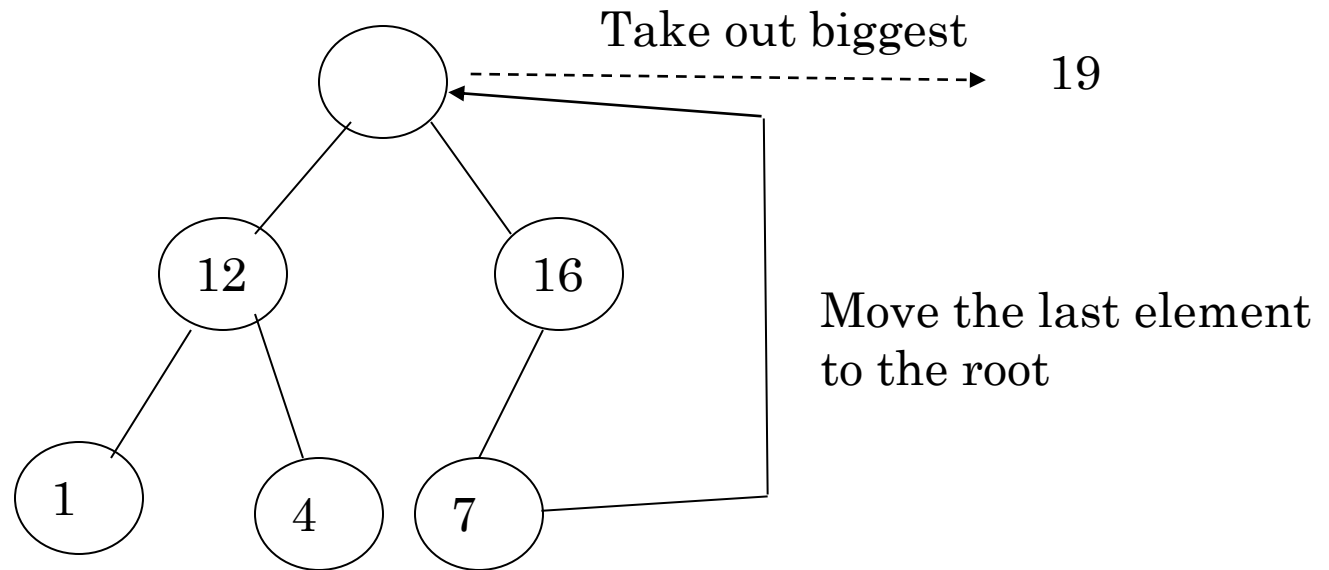


HEAP SORT

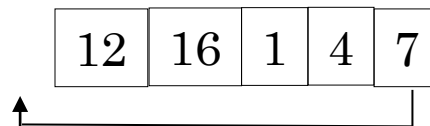
- The heapsort algorithm consists of two phases:
 - build a heap from an arbitrary array
 - use the heap to sort the data
- To sort the elements in the **decreasing order**, use a **min heap**
- To sort the elements in the **increasing order**, use a **max heap**



EXAMPLE OF HEAP SORT



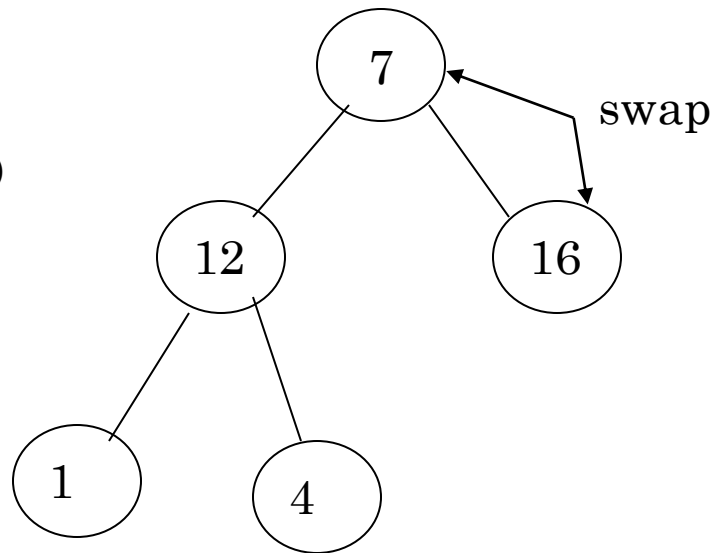
Array A



Sorted:



HEAPIFY()



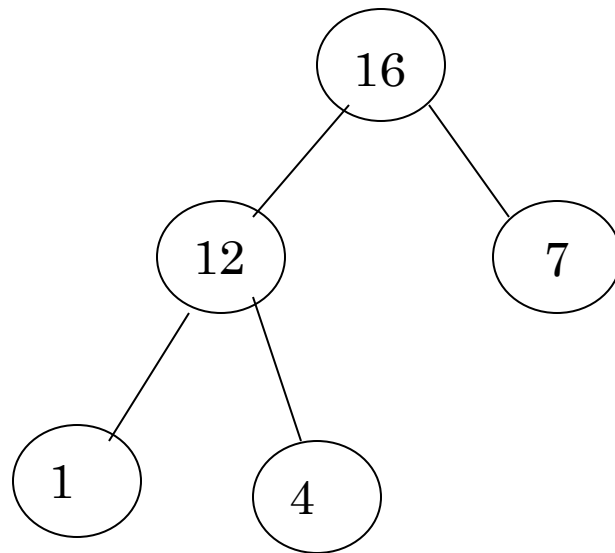
Array A

7	12	16	1	4
---	----	----	---	---

Sorted:

19





Array A

16	12	7	1	4
----	----	---	---	---

Sorted:

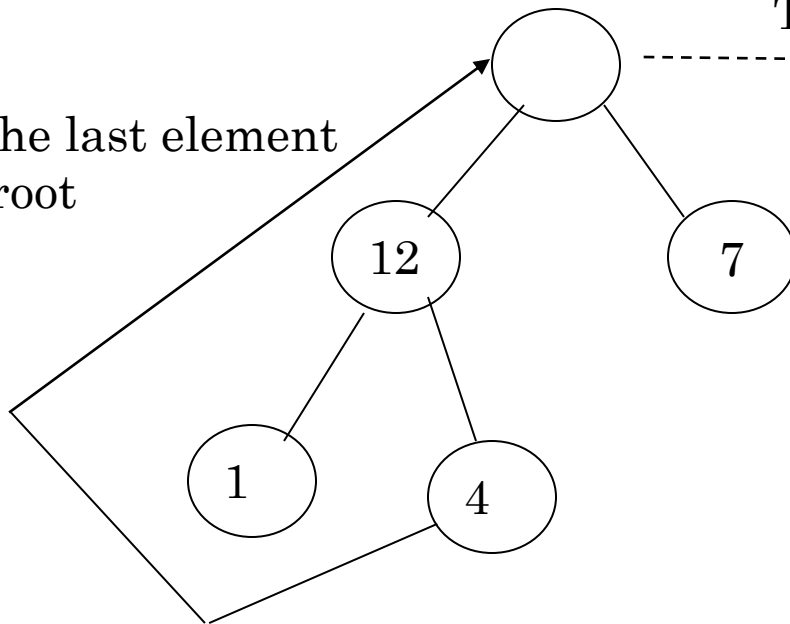
19



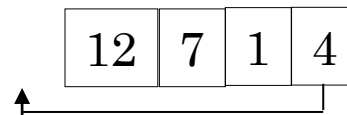
Take out biggest

16

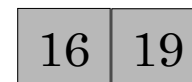
Move the last element
to the root

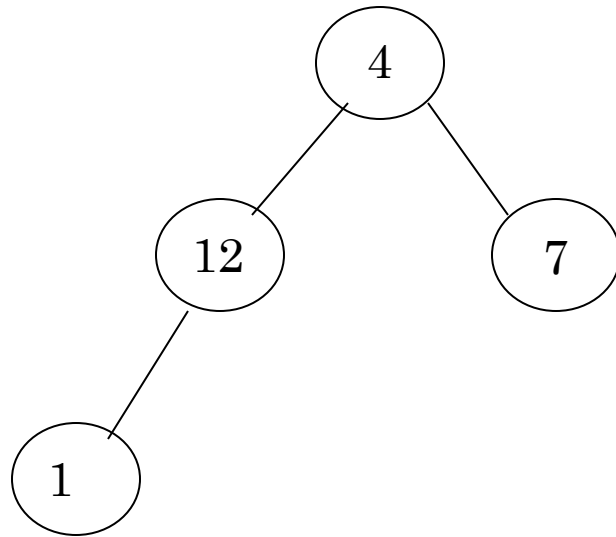


Array A



Sorted:





Array A

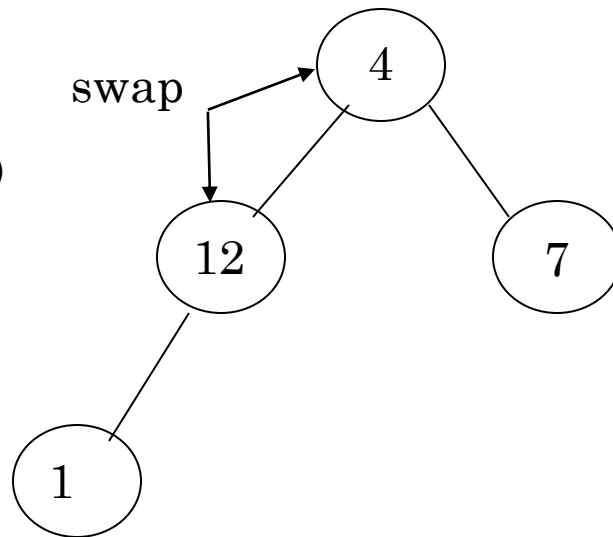
4	12	7	1
---	----	---	---

Sorted:

16	19
----	----



HEAPIFY()



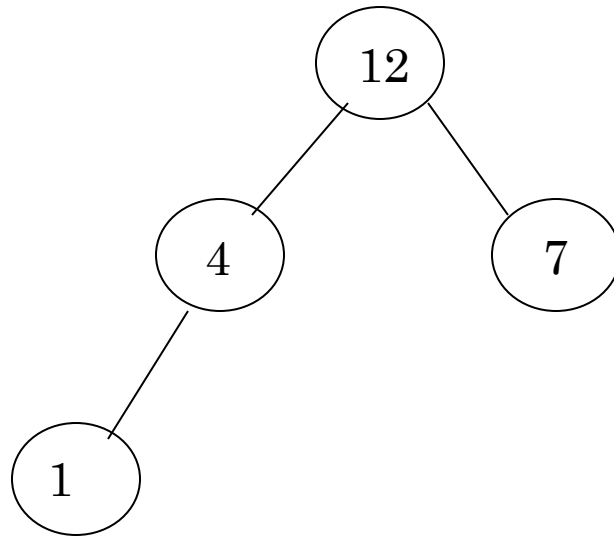
Array A

4	12	7	1
---	----	---	---

Sorted:

16	19
----	----





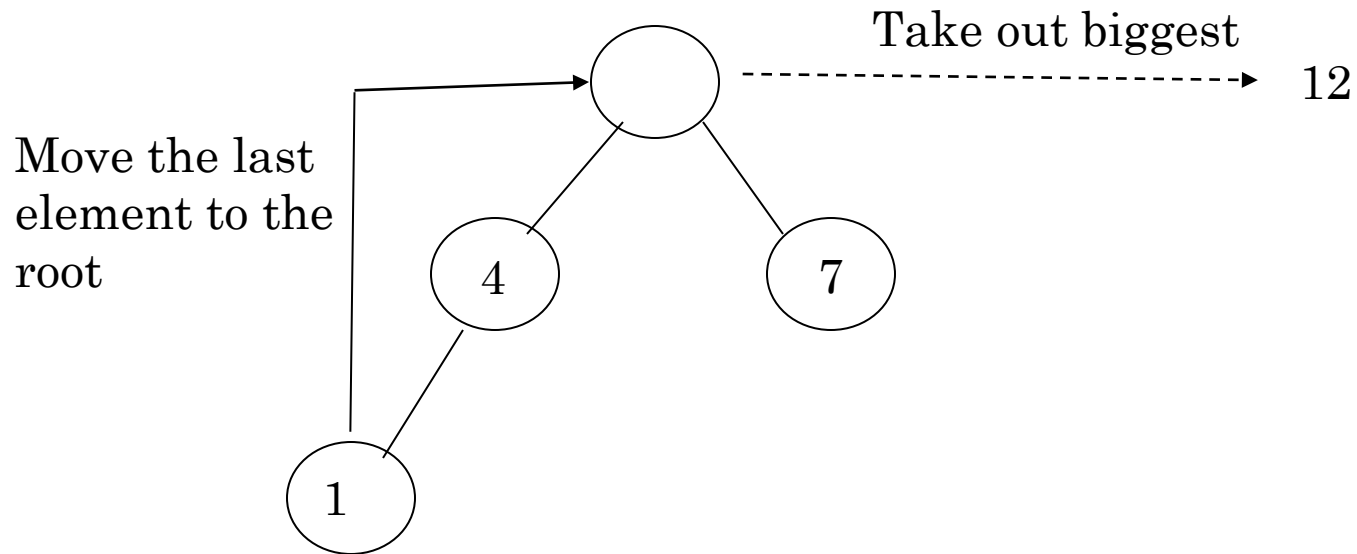
Array A

12	4	7	1
----	---	---	---

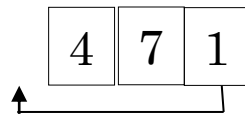
Sorted:

16	19
----	----

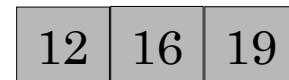


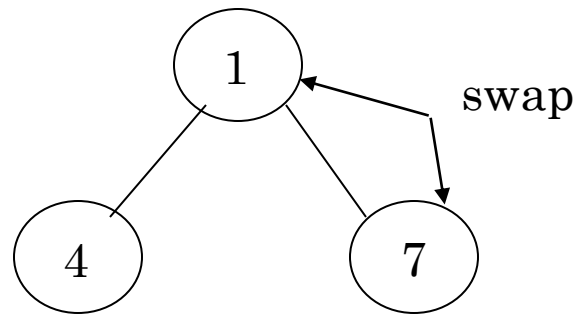


Array A



Sorted:





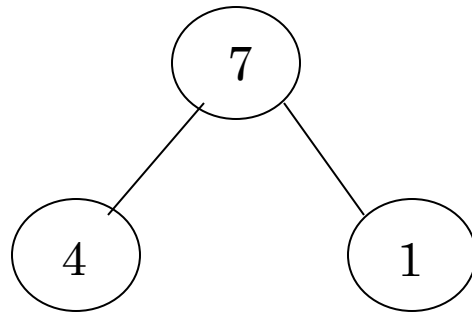
Array A

1	4	7
---	---	---

Sorted:

12	16	19
----	----	----





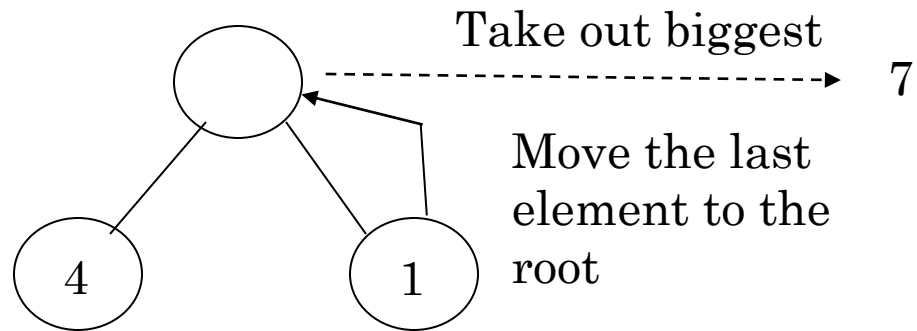
Array A

7	4	1
---	---	---

Sorted:

12	16	19
----	----	----





Array A

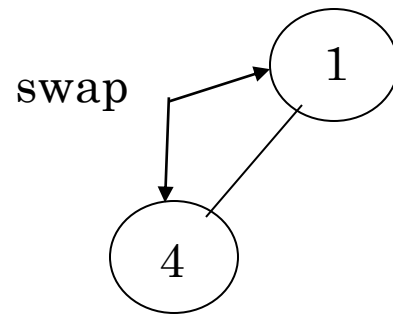
1	4
---	---

Sorted:

7	12	16	19
---	----	----	----



HEAPIFY()



Array A

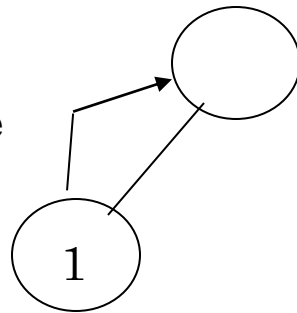
4	1
---	---

Sorted:

7	12	16	19
---	----	----	----



Move the last
element to the
root



Take out biggest

4

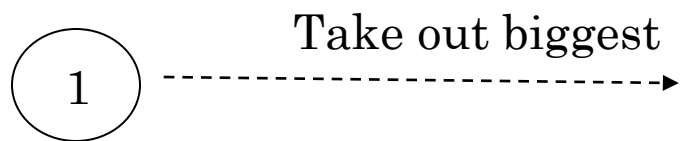
Array A

1

Sorted:

4	7	12	16	19
---	---	----	----	----





Array A

Sorted:

1	4	7	12	16	19
---	---	---	----	----	----



Sorted:

1	4	7	12	16	19
---	---	---	----	----	----



TIME ANALYSIS

- Build Heap Algorithm will run in $O(n)$ time
- There are $n-1$ calls to Heapify each call requires $O(\log n)$ time
- Heap sort program combine Build Heap program and Heapify, therefore it has the running time of $O(n \log n)$ time
- Total time complexity: $O(n \log n)$



COMPARISON WITH QUICK SORT AND MERGE SORT

- Quick sort is typically somewhat faster, due to better cache behavior and other factors, but the worst-case running time for quick sort is $O(n^2)$, which is unacceptable for large data sets and can be deliberately triggered given enough knowledge of the implementation, creating a security risk.
- The quick sort algorithm also requires $\Omega(\log n)$ extra storage space, making it not a strictly in-place algorithm. This typically does not pose a problem except on the smallest embedded systems, or on systems where memory allocation is highly restricted. Constant space (in-place) variants of quick sort are possible to construct, but are rarely used in practice due to their extra complexity.



COMPARISON WITH QUICK SORT AND MERGE SORT (CONT)

- Thus, because of the $O(n \log n)$ upper bound on heap sort's running time and constant upper bound on its auxiliary storage, embedded systems with real-time constraints or systems concerned with security often use heap sort.
- Heap sort also competes with merge sort, which has the same time bounds, but requires $\Omega(n)$ auxiliary space, whereas heap sort requires only a constant amount. Heap sort also typically runs more quickly in practice. However, merge sort is simpler to understand than heap sort, is a stable sort, parallelizes better, and can be easily adapted to operate on linked lists and very large lists stored on slow-to-access media such as disk storage or network attached storage. Heap sort shares none of these benefits; in particular, it relies strongly on random access.



POSSIBLE APPLICATION

- When we want to know the task that carry the highest priority given a large number of things to do
- Interval scheduling, when we have a lists of certain task with start and finish times and we want to do as many tasks as possible
- Sorting a list of elements that needs and efficient sorting algorithm



CONCLUSION

- The primary advantage of the heap sort is its efficiency. The execution time efficiency of the heap sort is $O(n \log n)$. The memory efficiency of the heap sort, unlike the other $n \log n$ sorts, is constant, $O(1)$, because the heap sort algorithm is not recursive.
- The heap sort algorithm has two major steps. The first major step involves transforming the complete tree into a heap. The second major step is to perform the actual sort by extracting the largest element from the root and transforming the remaining tree into a heap.



REFERENCE

- Deitel, P.J. and Deitel, H.M. (2008) “C++ How to Program”. 6th ed. Upper Saddle River, New Jersey, Pearson Education, Inc.
- Carrano, Frank M. (2007) “Data Abstraction and problem solving with C++: walls and mirrors”. 5th ed. Upper Saddle River, New Jersey, Pearson Education, Inc.

