Anova

→ One Way

→ Two Way

→ Post Hoc Tests

Chi - Square

→ Association Tests

→ Goodness of fit Tests.

→ Tests of Variance [Chi Square Parametric]

Anova

Q. A retailer wants to understand having shelving height impacts on sales. That is, to do sales of a particular brand change significantly if they are placed at eye level, or at lower levels or higher levels?

→ One way to test this "hypothesis" - store the same product at different shelves and record sales for a fixed no of days at each height

→ Look at sales averages for each height and then run a test too see if any observed differences are statistically significant

Shelf 1	Shelf 2	Shelf 3	Shelf 4	Shelf 5
210.5	198.1	170.5	167.1	188.5
198.1	189	225.5	167.9	167.9
145.3	219.3	158	175.5	170.5
185.5	254.9	139.4	175	152
189.1	210.3	156.4	149.1	164.5
135.9	160.9	217.1	189.3	171.7
180	120.8	189.1	198.2	158.9
149.4	167.8	158.2	205	177.9
170.4	148.9	218.1	233.5	189.1
229	190.4	178.9	167.9	187.1
179.92	185.09	181.12	182.85	174.41
179.92	185.09	181.12	182.85	174.41

ANOVA → (Analysis of Variance

It uses variance to reach a conclusion about means

There are two variances that are calculated in an ANOVA

→ Within Group Variance
→ Between Group Variance

* SST → Sum Squares Total

* SSW → Sum Squares Within

* SSB → Sum Squares Between

Overall Variance	Total sum of squared differences between observation, the overall mean of all observations	Total sum of Squares (SST)
Within Group Variance	Sum of squared differences between each observation and the mean of the group it belongs to	Sum of squares within (SSW)
Between Group Variance	Sum of squared differences between each group mean and overall mean	Sum of Squares within (SSB)

How does an Anova Work?

It can be established (mathematically) that there are two independent ways of establishing the standard error of the mean (essentially a measure of variance)

Approach 1 – Use the sample variances to come up with an estimate of total variance – SSW

Approach 2 – Use comparison of group means – SSB.

If the group means are similar, then both methods of total variance will result in similar estimates.

If the group means different, then both methods will give different results.

ANOVA looks at a ratio of the two methods of estimating variance - if the ratio is similar, then the null hypothesis is unlikely to be rejected (ratio close to 1)

Another way of looking at ANOVA is

Any observation in an experiment can be broken down into

→ The overall mean

→ +(or -) How far the average of a group is from the overall mean - Between Group Variation

→ +(or -) How far an observation is from the average of the group - Within Group Variation

If the independent variable has no impact, then within group variation and between group variation should be similar with any small difference is due to randomness

If it is not same then there is some factor which is driving this difference.

Test stat for ANOVA $= \dfrac{MSB}{MSW}$

where

MSB (Mean Square Between)

$= \dfrac{\text{Sum of Squares Between (SSB)}}{\text{Degrees of Freedom Between (DFB)}}$

MSW (Mean Square Within)

$= \dfrac{\text{Sum of Squares Within (SSW)}}{\text{Degrees of Freedom Within (DFW)}}$

To run an ANOVA, we need to calculate:

$$SSB = \sum_{k=1}^{k} N_k (\overline{Y_k} - \overline{Y})^2$$

$\overline{Y_k} \Rightarrow$ Group mean $N \Rightarrow$ No of observations in each group

$\overline{Y} \Rightarrow$ Overall mean.

$DFB = \cancel{Dk}\ k-1$, $k \Rightarrow$ No of Groups.

$$SSW = \sum_{k} \sum_{I} (Y_{ik} - \overline{Y_k})^2$$

$\overline{Y_{ik}} \Rightarrow$ Individual Value

$DFW = n-k$, $n \Rightarrow$ no of observations (Total Samples over all groups)

The Test stat follows F- Distribution

→ It is continuous distribution

→ It depends on 2 degrees of freedom

(i) Numerator DF
(ii) Denominator DF

Any random variate of F-distribution can be charatenized as the ratio of two chi square distributions

$$\frac{U_1/d_1}{U_2/d_2}$$

where U_1 & U_2 ⇒ Chi Square distributions

d_1 & d_2 ⇒ DF

Null Hypothesis : All means are equal

Alt. Hypothesis : Alleast one pair of means are ~~equal~~ unequal

How do we calculate -

within group variations

Calculate the variance of each group, and then calculate an average across groups

Between Group Variation

Calculate the average of the square variations of each population mean from the mean for all the data (Grand Mean)

Within Group Variance

1. Calculate the Mean for each Group.

2. Subtract each sample mean from every score in that group.

3. Square the difference

4. Add up all the squared Differences.

The SSW (Sum of Squares Within) can be written as

$$SSW = \sum_{k} \sum_{I} (y_{ik} - \overline{y_k})^2$$

$y_{ik} \Rightarrow$ each observation
$y_k \Rightarrow$ Group mean
$I \Rightarrow$ No of observations in Group
$k \Rightarrow$ No of Groups

* ANOVA works even if sample size of each group is different

Between Group Variance

1. Calculate a Grand Mean for all observations across all groups

2. Substract ~~each~~ Grand mean from each Group mean

3. Square the differences

4. Multiply each squared ~~mean~~ score by sample size

5. Add them up

The SSB (Sum of Squares Between) can be written as

$$SSB = \sum_{k=1}^{k} N_k (\overline{Y_k} - \overline{Y})$$

Where $\overline{Y}$ = Grand Mean

$\overline{Y_k}$ = Group mean

N = No of observations in each Group.

* How to find Deg Critical stat value from Distribution Table ??

→ We need Nom DF, Denom DF and alpha value

→ In ANOVA Distribution Table, Col name (value) indicates the Nom DF and Row name (value) indicates Denom DF

→ Bold Numbers are for alpha = 0.01
Pale/light Numbers are for alpha = 0.05

Total Variation = Within Group Variation + Between Group Variation

ANOVA Assumptions

1. The populations from which the samples were obtained must be normally or approximatelly normally distributed

2. The samples must be independent

3. All variances of the populations must be equal

An ANOVA is used when dependent variable (outcome) is continuous and independent variables are discrete.

In Retail e.g. –

DV ⇒ Sales

IV ⇒ Shelf Height

ANOVA (2 Factors Influencing outcome)

→ Let's say we are interested in understanding the impact of both shelf level as well as aisle placement on sales for Brand A

→ That is, not only the height of the product placed, but also other brands/categories that the product is placed in are hypothesized to have an impact on Brand A sales

→ If there are three different aisles, we have 3 x 5 different placements for Brand A

A Two-way ANOVA is useful when we desire to compare the effect of multiple levels of two factors and we have multiple observations at each level.

There are 3 Null Hypothesis that can be Tested in Two-way ANOVA

1) The population means of the first factor are equal. (like One-way ANOVA for row factor)

2) The population means of the second factor are equal (like one-way ANOVA for column factor)

3) No interaction between 2 factors (combinations of first factor & second factor have ~~to~~ no impact on DV) (Interaction Hypothesis)

(similar to performing a test for independence with contingency tables)

Output Interpretation

Look at the Interaction p-value first

→ If significant, implies impact of factor 1 depends on levels of factor 2

→ May not matter if the individual impacts are significant

If Interaction p-value is Not Significant

→ Re run ANOVA dropping the Interaction Term

9. If there is a difference between calories burned based on the streching before exercise and weights during exercise?

Pre strech	Ankle Weights	Energy
No strech	No Weights	106.9
No strech	No Weights	84
No strech	No Weights	97.5
No strech	No Weights	97.1
No strech	No Weights	99.5
No strech	Weights	100.2
No strech	Weights	101
No strech	Weights	118.5
No strech	Weights	104.5
No strech	Weights	111.2
strech	No Weights	82.8
strech	No Weights	80.4
strech	No Weights	95.6
strech	No Weights	82
strech	No Weights	83.2
strech	Weights	89.1
strech	Weights	106.4
strech	Weights	98.3
strech	Weights	89.2
strech	Weights	104.6

Factors (IV) — Pre strech & Weights

Two levels in each factor : Yes / No.

2 Way ANOVA : With Replication

Multiple observations for same combination of factors

→ We have to re-arrange the data before feeding into Data Analysis

	No Weights	Weights
No strech		
Strech		

→ In this case, interaction is not statistically significant but individual factors are signifcant

→ But, we will not know exactly which factors is significant

→ To know that we have to Run Post hoc Tests

Types of Post Hoc Tests:

- LSD Tests
- Tukey Tests
- Scheffe Tests

http://pages.uoregon.edu/stevensj/posthoc.pdf