Problem Set #5

Observational Techniques of Modern Astrophysics — PHYS 641

Fall 2018

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This write-up isn't as fancy as the others I've submitted. It is more just a vessel for some useful images for this assignment.

The code script for this problem is located in "./hw5.py". We also have an additional script called "./hw5_plots.py" that contains all the code for generating the plots that we present in this report. Note that "./hw5_plots.py" is imported by "./hw5.py".

1 The Initial Parameters and Data

To start our MCMC fit, we get initial values for the 6 basic cosmological parameters from the WMAP9 Cosmology papers. More specifically, our values come from Table 3 in the paper at this link. The resulting values are: $\Omega_b h^2 = 0.02264$, $\Omega_c h^2 = 0.1138$, $\tau = 0.089$, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $n_s = 0.972$, $A_s = 2.41 \cdot 10^{-9}$. In addition, we set our initial stepsizes according to the errors of the values given in the same WMAP9 paper table: $\sigma_{\Omega_b h^2} = 0.0005$, $\sigma_{\Omega_c h^2} = 0.0045$, $\sigma_\tau = 0.014$, $\sigma_{H_0} = 2.2\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\sigma_{n_s} = 0.013$, $\sigma_{A_s} = 0.1 \cdot 10^{-9}$. The resulting input power spectrum data as well as the predicted power spectrum from the initial cosmological parameters chosen from the WMAP9 Cosmology papers are shown in Figure 1. We can see that the predicted power spectrum approximately matches the input data.

2 Finding 1D Curvatures

Next, we find 1D curvatures for each of the parameters while keeping the other parameters fixed. Plots of the parameter value versus the χ^2 are shown in Figure 2. We then fit each of these curvatures with a parabola of definition:

$$y = a\,(x-c)^2 + b = ax^2 - 2acx + \left(ac^2 + b\right) \tag{1}$$

We can get error estimates from these fits by taking $\sigma = 1/\sqrt{a}$. We can derive why this is a good estimate by deconstructing the equation for χ^2:

$$\chi^2 = \sum_i \frac{(x-\mu_i)^2}{\sigma_i^2} = \frac{(x-\mu)^2}{\sigma^2} + \chi^2_{\text{other}} \tag{2}$$

where x is the value of the parameter we are currently considering and χ^2_{other} is the chi-squared term due to all of the other parameters except for the parameter whose curvature we are currently

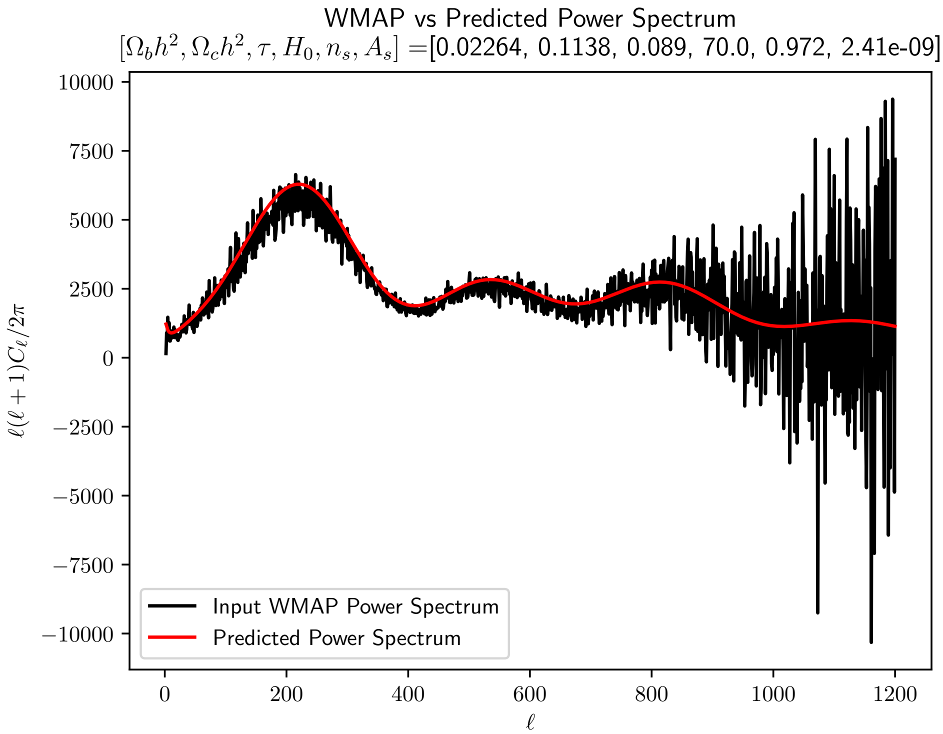

Figure 1: A plot of the input power spectrum data and the predicted power spectrum from the initial cosmological parameters chosen from the WMAP9 Cosmology papers.

considering. Then we have:

$$\chi^2 = \frac{(x-\mu)^2}{\sigma^2} + \chi^2_{\text{other}} = \frac{x^2}{\sigma^2} - \frac{2\mu x}{\sigma^2} + \frac{\mu^2}{\sigma^2} + \chi^2_{\text{other}} = ax^2 - 2acx + \left(ac^2 + b\right) \tag{3}$$

So, by taking $a = 1/\sigma^2$, we can solve for our errors estimate as: $\sigma = 1/\sqrt{a}$. By completing a quadratic fit for each 1D curvature, the resulting errors for each parameter are: $\sigma_{\Omega_b h^2} = 0.0002$, $\sigma_{\Omega_c h^2} = 0.0008$, $\sigma_\tau = 0.0017$, $\sigma_{H_0} = 0.6070\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\sigma_{n_s} = 0.0047$, $\sigma_{A_s} = 8.06 \cdot 10^{-12}$.

3 Estimating the Covariance Matrix

Using the errors obtained from the 1D curvatures, we then simulated a very short chain of only 500 iterations and then used this chain to estimate the covariance matrix. For the particulars of how we did this, see the `generate_cov_matrix()` function. Plots of the chains for each parameter, the acceptance rate, the correlation functions, and the power spectra are shown in Figure 3 (zoom in to see the details). Clearly this chain has not converged, but we can use it to get the following

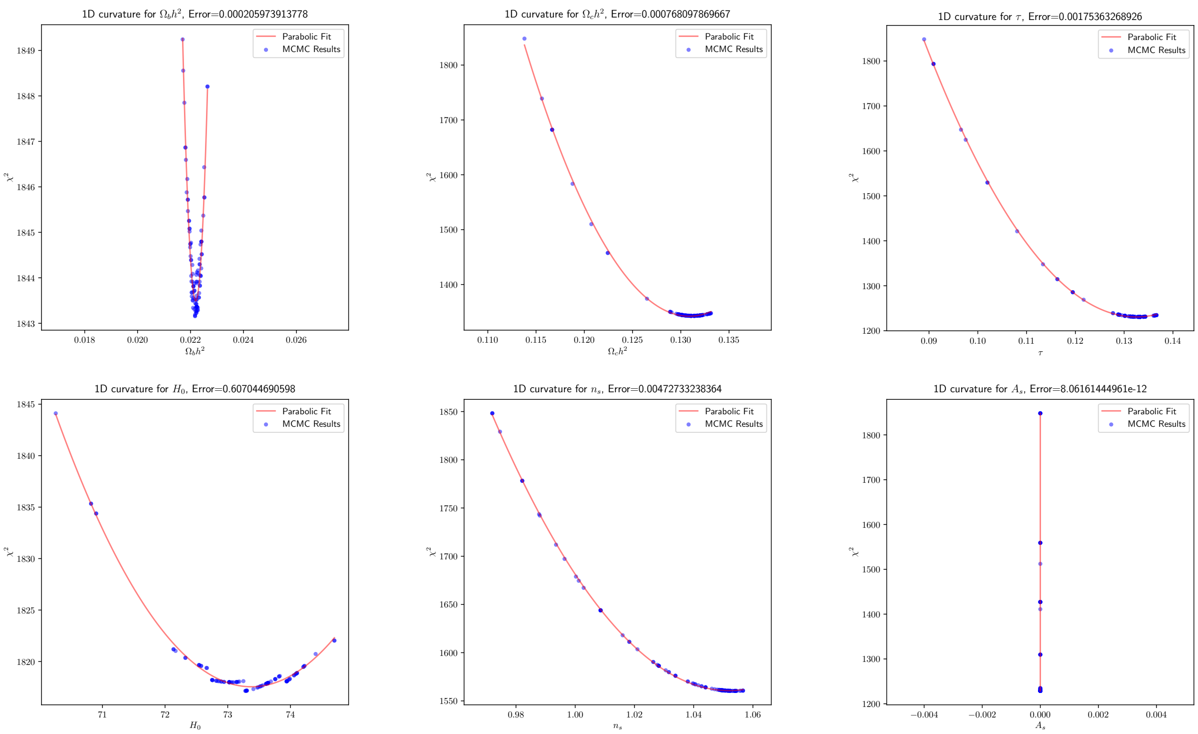

Figure 2: Plots of the 1D curvatures and resulting quadratic fits for each parameter.

reasonable approximation of the covariance matrix:

$$\begin{bmatrix} 2.9\text{e}{-07} & -1.3\text{e}{-06} & 1.9\text{e}{-06} & 1.0\text{e}{-03} & 7.3\text{e}{-06} & 5.0\text{e}{-16} \\ -1.3\text{e}{-06} & 1.5\text{e}{-05} & -2.1\text{e}{-05} & -8.3\text{e}{-03} & -4.9\text{e}{-05} & -8.8\text{e}{-15} \\ 1.9\text{e}{-06} & -2.1\text{e}{-05} & 5.2\text{e}{-05} & 1.2\text{e}{-02} & 6.7\text{e}{-05} & 8.3\text{e}{-15} \\ 1.0\text{e}{-03} & -8.3\text{e}{-03} & 1.2\text{e}{-02} & 5.6 & 3.3\text{e}{-02} & 5.9\text{e}{-12} \\ 7.3\text{e}{-06} & -4.9\text{e}{-05} & 6.7\text{e}{-05} & 3.4\text{e}{-02} & 2.5\text{e}{-04} & 6.0\text{e}{-14} \\ 5.0\text{e}{-16} & -8.8\text{e}{-15} & 8.3\text{e}{-15} & 5.9\text{e}{-12} & 6.0\text{e}{-14} & 3.8\text{e}{-22} \end{bmatrix} \tag{4}$$

with matching correlation matrix:

$$\begin{bmatrix} 1 & -0.64 & 0.51 & 0.83 & 0.84 & 0.047 \\ -0.64 & 1 & -0.77 & -0.90 & -0.80 & -0.12 \\ 0.51 & -0.77 & 1 & 0.71 & 0.58 & 0.059 \\ 0.83 & -0.90 & 0.71 & 1 & 0.89 & 0.13 \\ 0.84 & -0.80 & 0.58 & 0.89 & 1 & 0.19 \\ 0.047 & -0.12 & 0.059 & 0.13 & 0.19 & 1 \end{bmatrix} \tag{5}$$

with the axes of each matrix labelled in the following order: $[\Omega_b h^2, \Omega_c h^2, \tau, H_0, n_s, A_s]$.

4 Running A Longer Chain

Next, we used the measured covariance matrix to derive correlated steps sizes (see the `get_step()` function) and run an even longer chain of 15000 iterations. Plots of the chains for each parameter, the acceptance rate, the correlation functions, and the power spectra are shown in Figure 4 (zoom

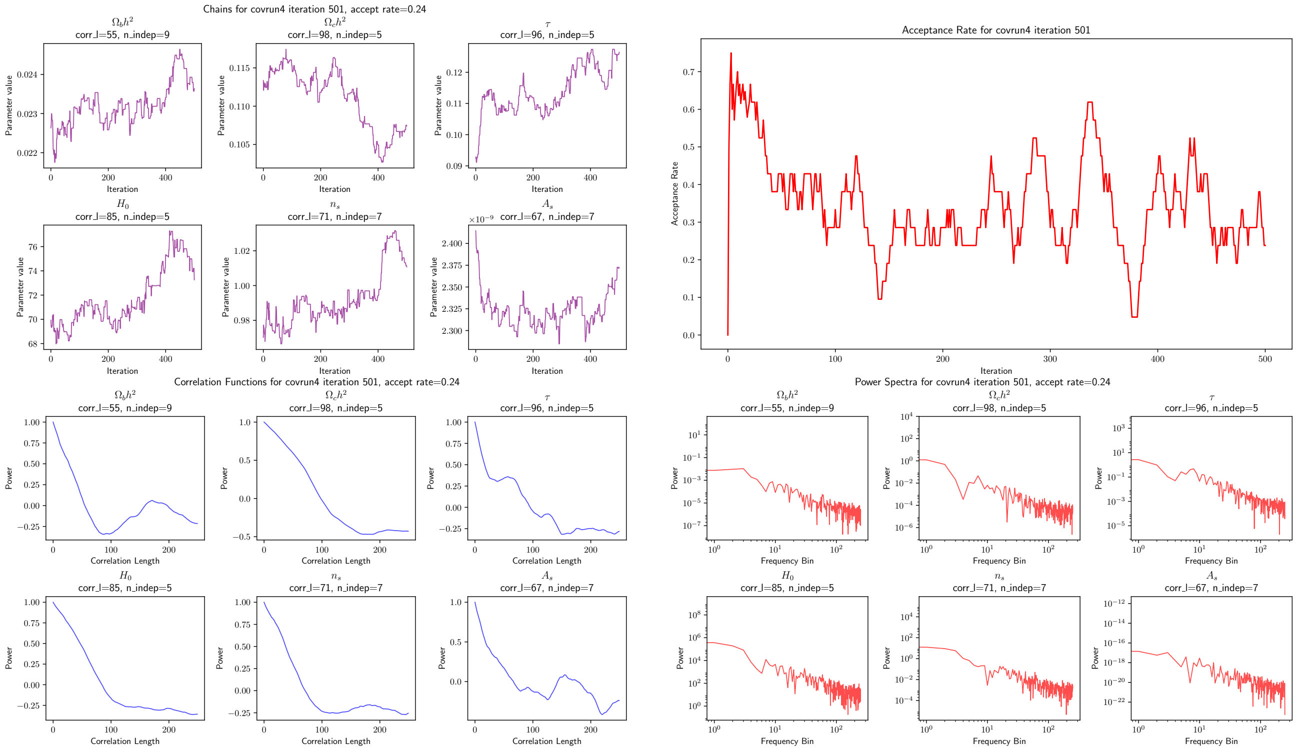

Figure 3: Diagnostic plots for the MCMC chain used to estimate the parameter covariance matrix. Clearly this chain has not converged.

in to see the details). We tried several different scalings of the stepsizes for this chain, but we ultimately couldn't get our chain to converge.

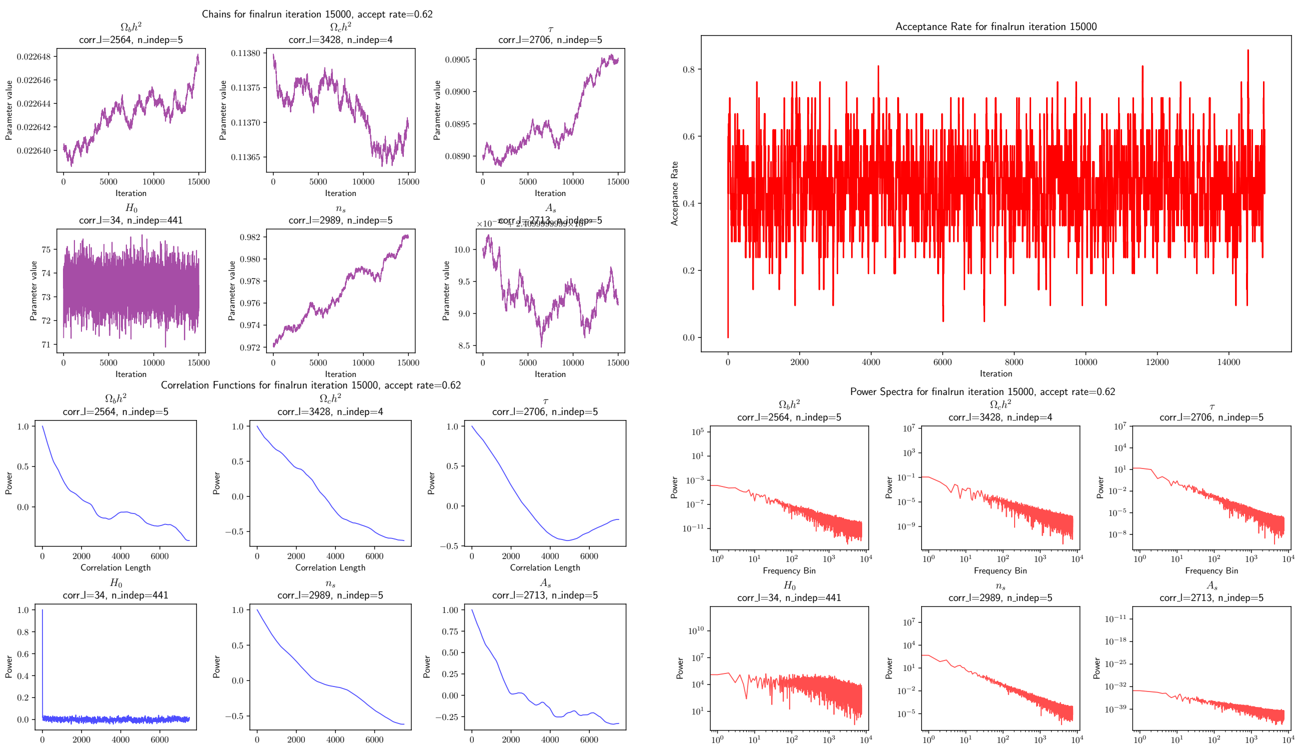

Figure 4: Diagnostic plots for the final MCMC chain.