

COORDINATION OF A SUPPLY CHAIN WITH A LOSS-AVERSE RETAILER UNDER SUPPLY UNCERTAINTY AND MARKETING EFFORT

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ABSTRACT. This paper deals with a one-period two-stage supply chain, in which a loss-averse retailer facing stochastic demand orders products from a risk-neutral supplier subject to yield uncertainty. Marketing effort exerted by the retailer is employed to enhance the final market demand. We first establish a performance benchmark, and show that the wholesale price contract fails to coordinate the supply chain due to the effects of double marginalization and loss aversion. Then we propose a revenue-cost-sharing contract in order to achieve supply chain coordination. It is verified that a properly designed revenue-cost-sharing contract can achieve perfect coordination and a win-win outcome synchronously. Our results reveal that it is simple to implement and arbitrarily allocate the total channel profit between the retailer and the supplier. In addition, we examine the effect of the retailer's loss aversion degree on contract parameters and profit allocation, and we show that both the retailer and the supplier can benefit from marketing effort.

1. Introduction. Due to various reasons such as machine breakdown, production process risks, contamination of material, and unpredictable factors like weather and environment, random yield frequently occurs in the agricultural sector or in the chemical, food processing, bio-pharmaceuticals, semiconductors, electronic and mechanical manufacturing industries [10, 23, 24, 26]. That is, the same production

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input might result in different production output quantities [22]. Usually, the actual produced quantity is less than the planned one [9]. For example, the yield of automotive plastic bumpers typically does not exceed 75% [12]; In the Liquid Crystal Display manufacturing industry, it is quite common to get production yield rate of less than 50% [5]; In agriculture, fruit supply fluctuates because of weather conditions, diseases and so on [27]. As a consequence of random yield, the downstream firms always suffer from uncertain supply [32]. In addition, due to damages occurred during transferring, any transportation process is also a considerable source of supply uncertainty [34]. In general, supply uncertainty will affect the decisions of all supply chain members and result in lower system performance because of double marginalization [14, 30]. Therefore, an important issue for both the upstream supplier or manufacturer and the downstream retailer would be how to efficiently operate a supply chain in an environment with supply uncertainty.

It is well known that revenue-sharing contract has been proposed to align the incentives of supply chain members so as to achieve supply chain coordination and improve the system-wide efficiency in various industries [2]. Under a revenue-sharing contract, the retailer pays the supplier a wholesale price for each unit purchased, and offers a fixed percentage of the revenue the retailer gets from the market as well. Most existing researches on supply chain coordination with revenue-sharing contract tend to treat demand uncertainty rather carefully [2, 7, 8, 11, 31, 47]. For instance, Giannoccaro and Pontrandolfo [8] showed that revenue-sharing contract is effective to coordinate a three-echelon supply chain. Yao et al. [47] investigated a revenue-sharing contract for coordinating a supply chain comprising one manufacturer and two competing retailers. Govindan and Popiuc [11] showed that performance measures and total supply chain profits improve through coordination with revenue sharing contracts in both two- and three-echelon reverse supply chains. Liao et al. [31] showed that a revenue-sharing contract can achieve channel coordination both hotel and online travel agency (OTA) retailers in a dual-channel system. Furthermore, there are only a few studies on coordination issue with revenue-sharing contract in supply chain with random demand and supply. Hu et al. [19] proposed a revenue sharing policy with an order penalty and rebate contract to fully coordinate a supply chain with yield and demand uncertainty. Güler and Keski [13] showed that the randomness of the supply does not change the coordination ability of revenue-sharing contract but affects the values of contract parameters. He and Zhao [15] showed that a revenue sharing contract with an advance-purchase discount contract can efficiently coordinate a supply chain with uncertain demand and supply. Different from theirs, we will further take marketing effort and loss aversion into consideration and investigate the effect of both of them on optimal decisions and contract designs.

Marketing activities, such as advertising and sales promotion, are increasingly popular in various industries [35, 37, 48]. By which retailers can stimulate the customers' buying behavior. The impact of marketing effort on demand is even more pronounced in the industrial goods market, where demand relies more on how well the customers are informed and convinced through personal contact with sales people [33]. Generally, all marketing activities are costly, if there are no sufficient incentives, retailers will have no motivation to enhance marketing effort level, thus coordination in marketing effort between manufacturers/suppliers and retailers is crucial and significant. Cachon and Lariviere [2] demonstrated that revenue-sharing contract fails to coordinate a supply chain with marketing effort,

and the optimal effort level is lower than that of the integrated supply chain. Some of early works showed that supply chain with marketing activities can be coordinated effectively via cost sharing of marketing effort [16, 20, 28, 29, 38, 39]. In particular, Kunter [29] designed a cost and revenue-sharing contract to establish efficiency in a manufacturer-retailer supply chain, and showed that channel coordination is achievable via a revenue-sharing rate and marketing effort participation rates on both manufacturer and retailer level. However, little attention has been given to the scenario where both demand and supply are stochastic. Moreover, most of them engaged in coordination problems based on the assumption of risk neutrality. Our research will fill the gap and extend Kunter [29] by assuming that the supply is uncertain and the retailer is loss-averse.

Considerable evidences show that many managers' decisions frequently deviate from expected profit maximization [1, 3, 6, 17, 36, 42, 45, 49]. This type of decision-making behavior is identified as loss aversion, one of the key features in the Prospect Theory, which indicates that people are more sensitive to losses than to same-sized gains [25]. Since loss aversion is both intuitively appealing and well supported in finance, economics and marketing, supply chain management with loss aversion has gradually attracted increasing attention [41]. Xu et al. [43] explored the optimal option purchase of a loss-averse retailer under emergent replenishment. Furthermore, Xu et al. [46] studied the loss-averse newsvendor model with backordering. Wang and Webster [41] investigated channel coordination of a supply chain with a risk-neutral manufacturer and a loss-averse retailer under stochastic demand, and showed a distribution-free gain/loss-sharing-and-buyback contract that can achieve supply chain coordination and arbitrarily distribute the total expected profit between the manufacturer and the retailer. Chen and Xiao [4] developed three (re)ordering models of a supply chain consisting of one risk-neutral manufacturer and one loss-averse retailer, and designed a buyback-setup-cost-sharing mechanism to coordinate the supply chain for each policy. Huang et al. [21] addressed a vendor-managed inventory supply chain with a loss-averse manufacturer and a risk-neutral retailer under quality-dependency and marketing-dependency, and investigated coordination and profit allocation issues of the supply chain via a combined contract composed of option and cost-sharing. Hu et al. [18] studied channel coordination via revenue sharing contracts in two-echelon and three-echelon supply chains respectively. They derived supply chain coordination conditions considering the retailer's loss aversion behavior, and demonstrated that Pareto-improvement can be achieved under revenue sharing contract. It is noted that all the works mentioned above neglect supply uncertainty.

To the best of our knowledge, this paper is the first work that analytically models and characterizes the supply chain coordination problem with a loss-averse retailer in the context of supply uncertainty, marketing effort and revenue-sharing contract. The key research question asks whether a properly designed revenue-cost-sharing contract can perfectly coordinate the supply chain and achieve a win-win outcome, and under what conditions. The major contributions of this work can be summarized as follows. First, considering a supply chain consisting of a risk-neutral supplier and a loss-averse retailer in the presence of marketing effort and random supply and demand, we establish a performance benchmark by introducing the integrated case, and examine the decentralized supply chain with wholesale price contract. Second, we propose a revenue-cost-sharing contract and develop coordination models for the supply chain. We derive the retailer's optimal marketing and

ordering policies, and analyze the effect of both the loss aversion degree and the effort-cost coefficient on the optimal solutions and supply chain performance. We show that a properly designed revenue-cost-sharing contract is effective to coordinate the supply chain perfectly and achieve a win-win outcome. Last, we discuss how to set contract parameters and how to allocate the additional expected profit, as well as the impact of the loss aversion degree on contract parameters and profit allocation.

The remainder of this paper is organized as follows. Section 2 presents model notations and assumptions, and establishes a performance benchmark by introducing the integrated case. Section 3 develops supply chain models under wholesale price contract and revenue-cost-sharing contract, respectively. A series of numerical experiments are given in Section 4. We conclude our findings and highlight possible future work in Section 5.

2. Model formulation and Integrated case. Consider a two-stage supply chain consisting of a risk-neutral supplier and a loss-averse retailer in a single period. At the beginning of the selling season, the retailer orders products from the supplier to meet a stochastic demand. The supplier employs a make-to-order policy and experiences yield uncertainty, thus the retailer is subject to an uncertain supply risk. A stochastic proportional yield model is used to depict the yield uncertainty [9, 22]. For an order quantity Q , the actual yield quantity is YQ , where Y , defined on $[A, B]$ ($0 \leq A < B \leq 1$), is a nonnegative random variable characterized by probability density function $g(y)$ and cumulative distribution function $G(y)$ with mean μ . The stochastic demand faced by the retailer, denoted as $D(I, x)$, is sensitive to the marketing effort level I . That is, $D(I, x) = a(I) + x$. The term $a(I)$ is the effort-induced demand, and x , defined on $[l_d, u_d]$, is a random variable with probability density function $f(x)$ and cumulative distribution function $F(x)$. For sake of simplicity, $a(I)$ is assumed to be linear function and strictly increasing in I . The marketing-dependent stochastic demand with the additive form has been widely used in the existing literature [20, 38].

We let c be the marginal production cost per unit produced, p be the exogenous retail price per unit and w be the wholesale price of the supplier per unit, any unsold product is salvaged at value v per unit. In order to avoid the unreasonable cases, we assume $p > w > c$. In addition, Let $C(I)$ denote the cost for that the retailer implements marketing effort at level I , and $C(I)$ is assumed to be convex, increasing, second-order differential. The cost depends exclusively on the effort level, which satisfies $C(0) = 0$, $C'(I) > 0$ and $C''(I) > 0$ [16]. The cost function indicates that it becomes increasingly more expensive to acquire additional demand as the effort level increases. To simplify the presentation, we assume zero goodwill cost for both the supplier and the retailer [15], this simplification will not affect the distinct results of this paper. Also, we assume there is no information asymmetry between the supplier and the retailer.

In what follows, we investigate the integrated supply chain in order to establish a performance benchmark for the contractual arrangements to be examined later. We use the subscript T for the integrated case. In this case, the supplier owns a retailer and controls the whole channel, and the objective is to maximize the total expected profit by deciding the production quantity Q_T and the marketing effort level I_T . Without loss of generality, we assume that the supplier incurs a unit production cost c which is only contingent on and proportional to the realized yield, qualitatively identical solutions remain the same if the production cost is only contingent on and

proportional to the order quantity by replacing c with c/μ [15, 40]. Therefore, the integrated supply chain's profit, denoted as $\pi_{sc}(Q_T, I_T)$, is

$$\pi_{sc}(Q_T, I_T) = \begin{cases} (p-c)yQ_T - C(I_T) & D_T \geq yQ_T \\ (p-c)D_T - (c-v)(yQ_T - D_T) - C(I_T) & D_T < yQ_T. \end{cases} \quad (1)$$

Accordingly, the expected profit of the integrated entity can be rewritten as

$$\begin{aligned} E\pi_{sc}(Q_T, I_T) &= \int_A^B \int_{yQ_T - a(I_T)}^{u_d} (p-c)yQ_T - C(I_T) dF(x) dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_T - a(I_T)} (p-c)(a(I_T) + x) - C(I_T) dF(x) dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_T - a(I_T)} (c-v)(yQ_T - a(I_T) - x) dF(x) dG(y). \end{aligned} \quad (2)$$

We easily show that $E\pi_{sc}(Q_T, I_T)$ is a jointly concave function of Q_T and I_T , hence there exist optimal solutions that maximize the expected profit of the integrated entity.

Theorem 2.1. *In the integrated case, there exist unique optimal solutions (Q_T^*, I_T^*) , which can be jointly determined by*

$$\int_A^B F(yQ_T^* - a(I_T^*)) y dG(y) = \mu \frac{p-c}{p-v} \quad (3)$$

$$(p-v)a'(I_T^*) \int_A^B F(yQ_T^* - a(I_T^*)) dG(y) = C'(I_T^*). \quad (4)$$

Proof. Rearrange the integrated entity's expected profit function to be

$$\begin{aligned} E\pi_{sc}(Q_T, I_T) &= \int_A^B (p-c)yQ_T - C(I_T) dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_T - a(I_T)} (v-p)(yQ_T - a(I_T) - x) dF(x) dG(y). \end{aligned}$$

Differentiating with respect to Q_T and I_T gives

$$\begin{aligned} \frac{\partial E\pi_{sc}(Q_T, I_T)}{\partial Q_T} &= \int_A^B [(p-c) + (v-p)F(yQ_T - a(I_T))] y dG(y) \\ \frac{\partial^2 E\pi_{sc}(Q_T, I_T)}{\partial (Q_T)^2} &= \int_A^B (v-p)f(yQ_T - a(I_T)) y^2 dG(y) \\ \frac{\partial^2 E\pi_{sc}(Q_T, I_T)}{\partial Q_T \partial I_T} &= - \int_A^B (v-p)a'(I_T)f(yQ_T - a(I_T)) y dG(y) \\ \frac{E\pi_{sc}(Q_T, I_T)}{\partial I_T} &= \int_A^B -a'(I_T)(v-p)F(yQ_T - a(I_T)) - C'(I_T) dG(y) \\ \frac{\partial^2 E\pi_{sc}(Q_T, I_T)}{\partial (I_T)^2} &= \int_A^B (v-p)(a'(I_T))^2 f(yQ_T - a(I_T)) - C''(I_T) dG(y). \end{aligned}$$

Since $p > v$, $f(yQ_T - a(I_T)) > 0$ and $C''(I_T) > 0$, we have

$$\left| \frac{\frac{\partial^2 E\pi_{sc}(Q_T, I_T)}{\partial (Q_T)^2} \frac{\partial^2 E\pi_{sc}(Q_T, I_T)}{\partial Q_T \partial I_T}}{\frac{\partial^2 E\pi_{sc}(Q_T, I_T)}{\partial Q_T \partial I_T} \frac{\partial^2 E\pi_{sc}(Q_T, I_T)}{\partial (I_T)^2}} \right| = - \int_A^B (v-p)f(yQ_T - a(I_T)) C''(I_T) y^2 dG(y) > 0.$$

The Hessian matrix of $E\pi_{sc}(Q_T, I_T)$ is negative definite. Thus, $E\pi_{sc}(Q_T, I_T)$ is jointly concave in Q_T and I_T , there exist unique optimal solutions (Q_T^*, I_T^*) which jointly determined by $\partial E\pi_{sc}(Q_T, I_T)/\partial Q_T=0$ and $\partial E\pi_{sc}(Q_T, I_T)/\partial I_T=0$. Then, we get Eq.(3) and Eq.(4). \square

As mentioned by the existing literature [20, 41], coordination of the supply chain is achieved if there is a contract such that the joint profit in the decentralized system is equal to the system-wide profit in the centralized system, that is, the optimal solutions of the decentralized supply chain are equivalent to that of the integrated one.

3. Decentralized supply chain with contracts. In the decentralized supply chain, the loss-averse retailer and the risk-neutral supplier are independent firms, the retailer focuses only on its own objective and will decide an optimal order quantity and an optimal marketing effort level to maximize the expected utility. Following [1] and [17], we adopt multiple mental accounts and apply the utility function of prospect theory to upside and downside potentials of the retailer's order decision separately. For any given realized market demand, the upside potential is the income associated with selling products, and the downside potential is the loss associated with surplus products (i.e., the cost of overproduction). The utility function is given by.

$$U(A) = -\lambda[A]^- + [A]^+, \quad (5)$$

where A is the upside/downside potential of the retailer's decisions, $[A]^-$ is equal to the absolute value of A if A is negative and 0 otherwise, and $[A]^+$ is equal to A if A is positive and 0 otherwise. $\lambda(\lambda > 1)$ is the retailer's loss aversion coefficient. Higher value of λ corresponds to a higher level of the retailer's loss aversion. Especially, $\lambda = 1$ corresponds to the risk neutrality scenario.

Next, we will discuss how to achieve the most efficient supply chain performance by contractual arrangements.

3.1. Wholesale price contract. We first consider a decentralized supply chain with wholesale price contract that has been widely used in practice because of its simplicity and lower administration cost. Under this contract, the retailer pays an exogenous wholesale price for each unit product. We use subscript W for wholesale price contract, and the utility function of the loss-averse retailer, denoted as $U_r(Q_W, I_W)$, is

$$U_r(Q_W, I_W) = \begin{cases} (p-w)yQ_W - C(I_W) & D_W \geq yQ_W \\ (p-w)D_W - \lambda(w-v)(yQ_W - D_W) - C(I_W) & D_W < yQ_W. \end{cases} \quad (6)$$

Then, the expected utility of the loss-averse retailer is calculated as

$$\begin{aligned} EU_r(Q_W, I_W) &= \int_A^B \int_{yQ_W - a(I_W)}^{u_d} (p-w)yQ_W - C(I_W) dF(x) dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_W - a(I_W)} (p-w)(a(I_W) + x) - C(I_W) dF(x) dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_W - a(I_W)} \lambda(w-v)(yQ_W - a(I_W) - x) dF(x) dG(y). \end{aligned} \quad (7)$$

Solving the loss-averse retailer's problem, we have the following proposition to reveal the optimal order strategy and the optimal marketing effort level.

Theorem 3.1. *Under the wholesale price contract, there exist unique optimal solutions (Q_W^*, I_W^*) which satisfy*

$$\int_A^B F(yQ_W^* - a(I_W^*))y dG(y) = \mu \frac{p - w}{p - w + \lambda(w - v)} \quad (8)$$

$$(p - w + \lambda(w - v))a'(I_W^*) \int_A^B F(yQ_W^* - a(I_W^*))dG(y) = C'(I_W^*). \quad (9)$$

Proof. Under the wholesale price contract, the expected utility of the loss-averse retailer is rewritten as

$$\begin{aligned} EU_r(Q_W, I_W) &= \int_A^B (p - w)yQ_W - C(I_W)dG(y) \\ &\quad - \int_A^B \int_{I_d}^{yQ_W - a(I_W)} (p - w + \lambda w - \lambda v)yQ_W dF(x)dG(y) \\ &\quad + \int_A^B \int_{I_d}^{yQ_W - a(I_W)} (p - w + \lambda w - \lambda v)(a(I_W) + x)dF(x)dG(y). \end{aligned}$$

Taking the first-order and second-order derivative of $EU_r(Q_W, I_W)$ with respect to Q_W and I_W , respectively. We have

$$\begin{aligned} \frac{\partial EU_r(Q_W, I_W)}{\partial Q_W} &= \int_A^B [(p - w) - (p - w + \lambda w - \lambda v)F(yQ_W - a(I_W))]y dG(y) \\ \frac{\partial^2 EU_r(Q_W, I_W)}{\partial (Q_W)^2} &= - \int_A^B (p - w + \lambda w - \lambda v)f(yQ_W - a(I_W))y^2 dG(y) \\ \frac{\partial^2 EU_r(Q_W, I_W)}{\partial Q_W \partial I_W} &= \int_A^B (p - w + \lambda w - \lambda v)a'(I_W)f(yQ_W - a(I_W))y dG(y) \\ \frac{\partial EU_r(Q_W, I_W)}{\partial I_W} &= \int_A^B a'(I_W)(p - w + \lambda w - \lambda v)F(yQ_W - a(I_W)) - C'(I_W)dG(y) \\ \frac{\partial^2 EU_r(Q_W, I_W)}{\partial (I_W)^2} &= \int_A^B [-(p - w + \lambda w - \lambda v)(a'(I_W))^2 f(yQ_W - a(I_W)) - C''(I_W)]dG(y). \end{aligned}$$

Next, we get

$$\begin{aligned} &\left| \begin{array}{cc} \frac{\partial^2 EU_r(Q_W, I_W)}{\partial (Q_W)^2} & \frac{\partial^2 EU_r(Q_W, I_W)}{\partial Q_W \partial I_W} \\ \frac{\partial^2 EU_r(Q_W, I_W)}{\partial Q_W \partial I_W} & \frac{\partial^2 EU_r(Q_W, I_W)}{\partial (I_W)^2} \end{array} \right| \\ &= \int_A^B (p - w + \lambda w - \lambda v)f(yQ_W - a(I_W))C''(I_W)y^2 dG(y) > 0. \end{aligned}$$

The Hessian matrix of $EU_r(Q_W, I_W)$ is negative definite. Thus, $EU_r(Q_W, I_W)$ is jointly concave in Q_W and I_W , there exist unique optimal solutions (Q_W^*, I_W^*) . Then, by the first-order optimality condition, we get Eq.(8) and Eq.(9). \square

Theorem 3.1 identifies that the existence and uniqueness of the optimal strategies of the retailer. Next, we explore the effect of some key factors on the retailer's optimal order policy.

Corollary 1. Q_W^* is strictly increasing in p and v while decreasing in w .

Proof. Using implicit function theorem, we have

$$\begin{aligned}\frac{\partial Q_W^*}{\partial p} &= -\frac{\partial^2 EU_r(Q_W, I_W)/\partial Q_W \partial p}{\partial^2 EU_r(Q_W, I_W)/\partial (Q_W)^2} \\ &= \frac{\int_A^B (1 - F(yQ_W - a(I_W)))y dG(y)}{\int_A^B (p - w + \lambda w - \lambda v)f(yQ_W - a(I_W))y^2 dG(y)} \\ \frac{\partial Q_W^*}{\partial v} &= -\frac{\partial^2 EU_r(Q_W, I_W)/\partial Q_W \partial v}{\partial^2 EU_r(Q_W, I_W)/\partial (Q_W)^2} \\ &= \frac{\int_A^B \lambda F(yQ_W - a(I_W))y dG(y)}{\int_A^B (p - w + \lambda w - \lambda v)f(yQ_W - a(I_W))y^2 dG(y)} \\ \frac{\partial Q_W^*}{\partial w} &= -\frac{\partial^2 EU_r(Q_W, I_W)/\partial Q_W \partial w}{\partial^2 EU_r(Q_W, I_W)/\partial (Q_W)^2} \\ &= \frac{-\int_A^B [1 + (\lambda - 1)F(yQ_W - a(I_W))]y dG(y)}{\int_A^B (p - w + \lambda w - \lambda v)f(yQ_W - a(I_W))y^2 dG(y)}.\end{aligned}$$

It is obvious that $\partial Q_W^*/\partial p > 0$, $\partial Q_W^*/\partial v > 0$ and $\partial Q_W^*/\partial w < 0$. Namely, Q_W^* is strictly increasing in p and v while decreasing in w . \square

Corollary 1 indeed coincides with the intuition in practice, as the retail price and the salvage value of unsold product increase, the profit obtained by the retailer from per unit product increases, this implies that the retailer has incentive to order more products. By the same way, with the increase of the wholesale price, the purchase cost paid by the retailer to the supplier increases. As a consequence, the retailer will obtain less profit from per unit product, and has no incentive to order more products.

Corollary 2. Q_W^* is strictly decreasing in λ .

Proof. Applying implicit function theorem, we get

$$\begin{aligned}\frac{\partial Q_W^*}{\partial \lambda} &= -\frac{\partial^2 EU_r(Q_W, I_W)/\partial Q_W \partial \lambda}{\partial^2 EU_r(Q_W, I_W)/\partial (Q_W)^2} \\ &= -\frac{\int_A^B (w - v)F(yQ_W - a(I_W))y dG(y)}{\int_A^B (p - w + \lambda w - \lambda v)f(yQ_W - a(I_W))y^2 dG(y)}\end{aligned}$$

It is obvious that $\partial Q_W^*/\partial \lambda < 0$. \square

Corollary 2 reveals the effect of changes in loss aversion degree on the optimal order quantity of the retailer. The more loss-averse the retailer is, the smaller the optimal order quantity of the retailer is. That is rational because the loss-averse retailer would like to avoid the risk of overstock by decreasing order quantity. In addition, Corollary 2 also implies that a loss-averse retailer's order quantity is not more than that of a risk-neutral retailer when other conditions maintain constant.

So far, it is obvious that the optimal strategies of the decentralized supply chain under the wholesale price contract are not equivalent to that of the integrated

one. Also, the joint profit in the decentralized system is not equal to the system-wide profit in the centralized system, thus coordination of supply chain cannot be achieved. This can be explained by two aspects: double marginalization caused by the uncertainties of supply and demand, and the loss aversion behavior of the retailer. Therefore, a coordination contract should be able to eliminate the effect of both double marginalization and loss aversion.

3.2. Revenue-cost-sharing contract. In order to encourage the loss-averse retailer to order more products and spend more on marketing so as to coordinate the supply chain, a revenue-cost-sharing contract is introduced in this subsection. With the revenue-cost-sharing contract, the maximizing objective of the retailer is aligned with that of the whole supply chain during decision making. The revenue-cost-sharing contract is characterized by three parameters, namely wholesale price w , revenue-sharing coefficient φ and cost-sharing coefficient θ . Under which the retailer earns φ portion of the revenue of per unit product and the other $(1 - \varphi)$ portion of the per unit revenue belongs to the supplier, meanwhile, the supplier needs to agree to bear a fraction θ of marketing cost of the retailer as well.

In the following, we discuss how to select appropriate contract parameters to achieve coordination of the supply chain and then to allocate the system-wide profit under the revenue-cost-sharing contract.

We use subscript R for the revenue-cost-sharing contract, and the loss-averse retailer's utility, denoted as $U_r(Q_R, I_R)$, is

$$U_r(Q_R, I_R) = \begin{cases} (\varphi p - w)yQ_R - \theta C(I_R) & D_R \geq yQ_R \\ (\varphi p - w)D_R - \lambda(w - \varphi v)(yQ_R - D_R) - \theta C(I_R) & D_R < yQ_R. \end{cases} \quad (10)$$

Then, the expected utility of the retailer is given by

$$\begin{aligned} EU_r(Q_R, I_R) &= \int_A^B \int_{yQ_R - a(I_R)}^{u_d} (\varphi p - w)yQ_R - \theta C(I_R) dF(x) dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_R - a(I_R)} (\varphi p - w)D_R - \theta C(I_R) dF(x) dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_R - a(I_R)} \lambda(w - \varphi v)(yQ_R - D_R) dF(x) dG(y). \end{aligned} \quad (11)$$

We can easily show that $EU_r(Q_R, I_R)$ is jointly concave in Q_R and I_R , and derive the optimal decisions of the retailer as follows.

Theorem 3.2. *Under the revenue-cost-sharing contract, there exist unique optimal solutions (Q_R^*, I_R^*) which satisfy*

$$\int_A^B F(yQ_R^* - a(I_R^*)) y dG(y) = \mu \frac{\varphi p - w}{\varphi p - w + \lambda(w - \varphi v)} \quad (12)$$

$$(\varphi p - w + \lambda(w - \varphi v)) a'(I_R^*) \int_A^B F(yQ_R^* - a(I_R^*)) dG(y) = \theta C'(I_R^*). \quad (13)$$

Proof. From Eq.(11), we get

$$EU_r(Q_R, I_R) = \int_A^B (\varphi p - w)yQ_R - \theta C(I_R) dG(y)$$

$$\begin{aligned}
& - \int_A^B \int_{l_d}^{yQ_R - a(I_R)} (\varphi p - w + \lambda w - \lambda \varphi v) y Q_R dF(x) dG(y) \\
& + \int_A^B \int_{l_d}^{yQ_R - a(I_R)} (\varphi p - w + \lambda w - \lambda \varphi v) (a(I_R) + x) dF(x) dG(y).
\end{aligned}$$

Taking the first-order and second-order derivative of $EU_r(Q_R, I_R)$ with respect to Q_R and I_R , respectively. We have

$$\begin{aligned}
\frac{\partial EU_r(Q_R, I_R)}{\partial Q_R} &= \int_A^B [(\varphi p - w) - (\varphi p - w + \lambda w - \lambda \varphi v) F(yQ_R - a(I_R))] y dG(y) \\
\frac{\partial^2 EU_r(Q_R, I_R)}{\partial (Q_R)^2} &= - \int_A^B (\varphi p - w + \lambda w - \lambda \varphi v) f(yQ_R - a(I_R)) y^2 dG(y) \\
\frac{\partial^2 EU_r(Q_R, I_R)}{\partial Q_R \partial I_R} &= \int_A^B (\varphi p - w + \lambda w - \lambda \varphi v) a'(I_R) f(yQ_R - a(I_R)) y dG(y) \\
\frac{\partial EU_r(Q_R, I_R)}{\partial I_R} &= \int_A^B a'(I_R) (\varphi p - w + \lambda w - \lambda \varphi v) F(yQ_R - a(I_R)) - \theta C'(I_R) dG(y) \\
\frac{\partial^2 EU_r(Q_R, I_R)}{\partial (I_R)^2} &= - \int_A^B (\varphi p - w + \lambda w - \lambda \varphi v) (a'(I_R))^2 f(yQ_R - a(I_R)) dG(y) \\
&\quad - \int_A^B \theta C''(I_R) dG(y).
\end{aligned}$$

Then, we have

$$\begin{aligned}
& \left| \begin{array}{cc} \frac{\partial^2 EU_r(Q_R, I_R)}{\partial (Q_R)^2} & \frac{\partial^2 EU_r(Q_R, I_R)}{\partial Q_R \partial I_R} \\ \frac{\partial^2 EU_r(Q_R, I_R)}{\partial Q_R \partial I_R} & \frac{\partial^2 EU_r(Q_R, I_R)}{\partial (I_R)^2} \end{array} \right| \\
&= \int_A^B (\varphi p - w + \lambda w - \lambda \varphi v) f(yQ_R - a(I_R)) \theta C''(I_R) y^2 dG(y) > 0.
\end{aligned}$$

The Hessian matrix of $EU_r(Q_R, I_R)$ is negative definite. Thus, $EU_r(Q_R, I_R)$ is jointly concave in Q_R and I_R , there exist unique optimal solutions (Q_R^*, I_R^*) , this indicates that the first-order condition works, thus we get Eq.(12) and Eq.(13). \square

Although the cost-sharing coefficient would affect the order strategy of the retailer in intuition, as we can see from Theorem 3.2, it is important that the retailer's order quantity is independent of the cost-sharing coefficient when the marketing effort has been exerted. We can also see from Theorem 3.2 that the optimal solutions of the retailer rely on the revenue sharing coefficient, which yields the following corollary.

Corollary 3. Q_R^* is strictly increasing in φ .

Proof. Using implicit function theorem, we have

$$\begin{aligned}
\frac{\partial Q_R^*}{\partial \varphi} &= - \frac{\partial^2 EU(Q_R, I_R) / \partial Q_R \partial \varphi}{\partial^2 EU(Q_R, I_R) / \partial (Q_R)^2} \\
&= \frac{\int_A^B [p - (p - \lambda v) F(yQ_R - a(I_R))] y dG(y)}{\int_A^B (\varphi p - w + \lambda w - \lambda \varphi v) f(yQ_R - a(I_R)) y^2 dG(y)}
\end{aligned}$$

It is obvious that $\partial Q_R^* / \partial \varphi > 0$, namely, Q_R^* is strictly increasing in φ . \square

Corollary 3 shows that, the larger the revenue-sharing coefficient is, the more the retailer orders. This is intelligible and rational in practice, if the retailer can earn more from per unit product, she will have incentive to order more products.

As stated previously, the supply chain is coordinated if $Q_R^* = Q_T^*$ and $I_R^* = I_T^*$, thus we have the following proposition.

Theorem 3.3. *Coordination of the supply chain can be achieved with any revenue-cost-sharing contract in the following set M :*

$$M = \left\{ (w, \varphi, \theta) : \varphi = \frac{w[(p-c)\lambda + c - v]}{(p-c)\lambda v + p(c-v)}, \theta = \frac{w\lambda(p-v)}{(p-c)\lambda v + p(c-v)} \right\} \quad (14)$$

Proof. According to the above analysis, coordination of the supply chain is achievable only when $Q_R^* = Q_T^*$ and $I_R^* = I_T^*$, which implies that $(\varphi p - w)/(\varphi p - w + \lambda(w - \varphi v)) = (p - c)/(p - v)$ and $(\varphi p - w + \lambda(w - \varphi v))/\theta = p - v$. Rearranging, we obtain

$$\begin{aligned} \varphi &= \frac{w[(p-c)\lambda + c - v]}{(p-c)\lambda v + p(c-v)} \\ \theta &= \frac{w\lambda(p-v)}{(p-c)\lambda v + p(c-v)}. \end{aligned}$$

□

Theorem 3.3 characterizes the specific condition under which coordination of the supply chain is reachable, namely, under any revenue-cost-sharing contract in the set M , the optimal solutions of the decentralized supply chain are equivalent to that of the integrated one, and the joint profit in the decentralized system is equal to the system-wide optimal profit in the centralized system as well. We can see from Theorem 3.3 that, when the retailer does not exert marketing effort, a pure revenue sharing contract can coordinate the supply chain with a loss-averse retailer under random demand and supply. Further, we have the following corollaries under coordination with the revenue-cost-sharing contract.

Corollary 4. *Under any revenue-cost-sharing contract in the set M , there always exists $\partial\varphi/\partial w > 0$, $\partial\varphi/\partial\lambda > 0$, $\partial\theta/\partial w > 0$ and $\partial\theta/\partial\lambda > 0$.*

Proof. Under any revenue-cost-sharing contract in the set M , taking the first-order derivative of φ with respect to w and λ , it follows that

$$\begin{aligned} \frac{\partial\varphi}{\partial w} &= \frac{(p-c)\lambda + c - v}{(p-c)\lambda v + p(c-v)} > 0 \\ \frac{\partial\varphi}{\partial\lambda} &= \frac{w(p-c)(c-v)(p-v)}{[(p-c)\lambda v + p(c-v)]^2} > 0. \end{aligned}$$

Similarly, differentiating θ with respect to w and λ gives

$$\begin{aligned} \frac{\partial\theta}{\partial w} &= \frac{(p-v)\lambda}{p(c-v) + (p-c)\lambda v} > 0 \\ \frac{\partial\theta}{\partial\lambda} &= \frac{pw(p-v)(c-v)}{[p(c-v) + (p-c)\lambda v]^2} > 0. \end{aligned}$$

□

Corollary 4 shows that, the percentage of the revenue the retailer shares from per unit product is increasing in both the wholesale price and the loss aversion degree under coordination. It is intelligible that, the retailer needs to pay more purchase cost for per unit product as the wholesale price increases, thus the supplier has to allow the retailer to keep more revenue of per unit product to coordinate the supply chain. Meanwhile, if the retailer becomes more loss-averse, the optimal order quantity decreases, in order to induce the retailer to purchase more products so that coordination of the supply chain is achievable, the supplier has to allow the retailer to share more revenue of per unit product as well. That is, given that other conditions remain unchanged, the percentage of the revenue of per unit product the loss-averse retailer shares is more than that a risk-neutral retailer shares. In addition, it is interesting that, in order to coordinate the supply chain, the retailer needs to share more cost of marketing effort as the wholesale price increases. Also, the retailer needs to bear more cost of marketing effort so as to achieve coordination of the supply chain when the loss aversion degree increases.

Corollary 5. *Under any revenue-cost-sharing contract in the set M , there always exists $w < c$ and $\partial w / \partial \lambda < 0$.*

Proof. Under any revenue-cost-sharing contract in the set M , we get

$$w = \frac{\varphi[(p-c)\lambda v + p(c-v)]}{(p-c)\lambda + c - v}.$$

Then, have

$$\begin{aligned} w - c &= \frac{\varphi[(p-c)\lambda v + p(c-v)]}{(p-c)\lambda + c - v} - c \\ &= \frac{(p-c)\lambda(\varphi v - c) + (c-v)(\varphi p - c)}{(p-c)\lambda + c - v}. \end{aligned}$$

Since $\varphi < 1$, then we obtain

$$\frac{(p-c)\lambda(\varphi v - c) + (c-v)(\varphi p - c)}{(p-c)\lambda + c - v} \leq \frac{(p-c)\lambda(\varphi v - c) + (c-v)(p - c)}{(p-c)\lambda + c - v}.$$

Since $p - c > 0$, we get

$$\frac{(p-c)\lambda(\varphi v - c) + (c-v)(p - c)}{(p-c)\lambda + c - v} \leq \frac{(p-c)[\lambda(v - c) + (c - v)]}{(p-c)\lambda + c - v}.$$

Therefore, we have

$$w - c \leq \frac{(p-c)[\lambda(v - c) + (c - v)]}{(p-c)\lambda + c - v} < 0.$$

That is, $w < c$.

Further, taking the first-order derivative of w with respect to λ , it follows that

$$\frac{\partial w}{\partial \lambda} = \frac{\varphi(p-c)(c-v)(v-p)}{[(p-c)\lambda + c - v]^2} < 0.$$

□

As we can see from Corollary 5, under coordination with the revenue-cost-sharing contract, the wholesale price paid by the retailer to the supplier is lower than the production cost since the supplier could share the retail revenue for each product successfully sold and the salvage revenue for each product unsold, this is consist with that in the case of risk neutrality. Furthermore, Corollary 5 also implies that

the wholesale price is decreasing in the loss aversion degree under coordination, that is, the wholesale price paid by a loss-averse retailer is less than that paid by a risk-neutral retailer.

Let $EU_r(Q_R^*, I_R^*)$ and $E\pi_r(Q_R^*, I_R^*)$ denote the optimal expected utility and the optimal expected profit of the retailer under coordination with the revenue-cost-sharing contract, respectively. Then we have Corollary 6.

Corollary 6. *Under any revenue-cost-sharing contract in the set M , there always exists $\partial EU_r(Q_R^*, I_R^*)/\partial \lambda > 0$, $\partial E\pi_r(Q_R^*, I_R^*)/\partial \lambda > 0$, $\partial EU_r(Q_R^*, I_R^*)/\partial w > 0$ and $\partial E\pi_r(Q_R^*, I_R^*)/\partial w > 0$.*

Proof. Under any revenue-cost-sharing contract in the set M , the optimal expected utility of the loss-averse retailer is

$$\begin{aligned} EU_r(Q_R^*, I_R^*) &= \int_A^B (\varphi p - w)yQ_R^* - \theta C(I_R^*)dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_R^* - a(I_R^*)} (\varphi p - w + \lambda w - \lambda \varphi v)yQ_R^* dF(x)dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_R^* - a(I_R^*)} (\varphi p - w + \lambda w - \lambda \varphi v)(a(I_R^*) + x)dF(x)dG(y) \end{aligned}$$

Then, we get

$$\begin{aligned} EU_r(Q_R^*, I_R^*) &= \int_A^B \frac{w\lambda(p-v)(p-c)}{(p-c)\lambda v + p(c-v)}yQ_T^* - \frac{(p-v)w\lambda}{(p-c)\lambda v + p(c-v)}C(I_T^*)dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_T^* - a(I_T^*)} \frac{w\lambda(p-v)^2}{(p-c)\lambda v + p(c-v)}yQ_T^* dF(x)dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_T^* - a(I_T^*)} \frac{w\lambda(p-v)^2}{(p-c)\lambda v + p(c-v)}(a(I_T^*) + x)dF(x)dG(y) \\ &= \frac{(p-v)w\lambda}{(p-c)\lambda v + p(c-v)}E\pi_{sc}(Q_T^*, I_T^*). \end{aligned}$$

Differentiating $EU_r(Q_R^*, I_R^*)$ with respect to w and λ gives

$$\frac{\partial EU(Q_R^*, I_R^*)}{\partial w} = \frac{(p-v)\lambda}{(p-c)\lambda v + p(c-v)}E\pi_{sc}(Q_T^*, I_T^*) > 0$$

$$\frac{\partial EU(Q_R^*, I_R^*)}{\partial \lambda} = \frac{(p-v)(c-v)w}{[(p-c)\lambda v + p(c-v)]^2}E\pi_{sc}(Q_T^*, I_T^*) > 0.$$

Further, the retailer's optimal expected profit is

$$\begin{aligned} E\pi_r(Q_R^*, I_R^*) &= \int_A^B \int_{yQ_R^* - a(I_R^*)}^{u_d} (\varphi p - w)yQ_R^* - \theta C(I_R^*)dF(x)dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_R^* - a(I_R^*)} (\varphi p - w)D_R^* - \theta C(I_R^*)dF(x)dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_R^* - a(I_R^*)} (w - \varphi v)(yQ_R^* - D_R^*)dF(x)dG(y). \end{aligned}$$

Then, we get

$$\begin{aligned} E\pi_r(Q_R^*, I_R^*) &= \int_A^B \frac{w\lambda(p-v)(p-c)}{(p-c)\lambda v + p(c-v)} yQ_T^* - \frac{(p-v)w\lambda}{(p-c)\lambda v + p(c-v)} C(I_T^*) dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_T^* - a(I_T^*)} \frac{w[(p-c)\lambda + c-v](p-v)}{(p-c)\lambda v + p(c-v)} yQ_T^* dF(x) dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_T^* - a(I_T^*)} \frac{w[(p-c)\lambda + c-v](p-v)}{(p-c)\lambda v + p(c-v)} (a(I_T^*) + x) dF(x) dG(y) \end{aligned}$$

Since $(p-v)w\lambda > w[(p-c)\lambda + c-v]$, we have

$$\begin{aligned} E\pi_r(Q_R^*, I_R^*) &> \int_A^B \frac{w\lambda(p-v)(p-c)}{(p-c)\lambda v + p(c-v)} yQ_T^* - \frac{(p-v)w\lambda}{(p-c)\lambda v + p(c-v)} C(I_T^*) dG(y) \\ &\quad - \int_A^B \int_{l_d}^{yQ_T^* - a(I_T^*)} \frac{(p-v)w\lambda(p-v)}{(p-c)\lambda v + p(c-v)} yQ_T^* dF(x) dG(y) \\ &\quad + \int_A^B \int_{l_d}^{yQ_T^* - a(I_T^*)} \frac{(p-v)w\lambda(p-v)}{(p-c)\lambda v + p(c-v)} (a(I_T^*) + x) dF(x) dG(y) \end{aligned}$$

Namely, $E\pi_r(Q_R^*, I_R^*) > \frac{(p-v)w\lambda}{(p-c)\lambda v + p(c-v)} E\pi_{sc}(Q_T^*, I_T^*)$. Further, we get

$$\begin{aligned} \frac{\partial E\pi_r(Q_R^*, I_R^*)}{\partial w} &> \frac{(p-v)\lambda}{(p-c)\lambda v + p(c-v)} E\pi_{sc}(Q_T^*, I_T^*) > 0 \\ \frac{\partial E\pi_r(Q_R^*, I_R^*)}{\partial \lambda} &> \frac{(p-v)(c-v)w}{[(p-c)\lambda v + p(c-v)]^2} E\pi_{sc}(Q_T^*, I_T^*) > 0. \end{aligned}$$

□

Under coordination with the revenue-cost-sharing contract, Corollary 6 reveals that the effect of the loss aversion degree and the wholesale price on both the expected utility and the expected profit of the retailer. As the loss aversion degree increases, both the expected utility and the expected profit of the retailer increase. It is reasonable that, when the wholesale price is given, as demonstrated in Corollary 4, the percentage of the revenue of per unit product a loss-averse retailer shares is more than that a risk-neutral retailer shares, and is increasing in the loss aversion degree, which will lead to more profit for the retailer. Additionally, if the revenue sharing proportion between the retailer and the supplier is given, the wholesale price paid by a loss-averse retailer is less than that paid by a risk-neutral retailer, and is decreasing in the loss aversion degree, which will lead to more profit for the retailer as well. In sum, compared with a risk-neutral retailer, a loss-averse retailer will benefit from its own loss aversion behavior under coordination with the revenue-cost-sharing contract.

We can also see from Corollary 6 that, both the expected utility and the expected profit of the retailer are increasing in the wholesale price under any revenue-cost-sharing contract in the set M . Since the sum of the retailer's and the supplier's expected profits under coordination is a constant, which is equal to the expected profit in the integrated supply chain, it follows from Corollary 6 that the supplier's expected profit is decreasing in the wholesale price under coordination with the revenue-cost-sharing contract. This indicates that the profit of the whole supply chain can be allocated arbitrarily between the supplier and the retailer only by changing the wholesale price when other exogenous parameters are given.

Thus, both coordination of the supply chain and Pareto-improvement can be achieved synchronously by the proposed revenue-cost-sharing contract, that is, the revenue-cost-sharing contract can eliminate the effect of both double marginalization and loss aversion, and neither of the supplier and the retailer is becoming worse off under appropriate contract parameters. This implies that the revenue-cost-sharing contract is applicable in practice.

4. Numerical analysis. In this section, we perform several numerical experiments to illustrate our analytical results obtained in the previous sections and gain additional managerial insights. All numerical values are set based on the assumptions of all parameters in this paper. For convenience, the stochastic variables X and Y are assumed to follow uniform distribution with $F(x) \sim U(500, 1500)$ and $F(y) \sim U(0.6, 1)$. Similar to [20] and [38], we assume $a(I) = I$ and $C(I) = kI^2$, where k may be interpreted as the costliness of marketing effort. The base parameters in numerical experiments are as follows: $c = 10$, $w = 18$, $p = 30$, $v = 5$, $\lambda = 2$ and $k = 0.02$.

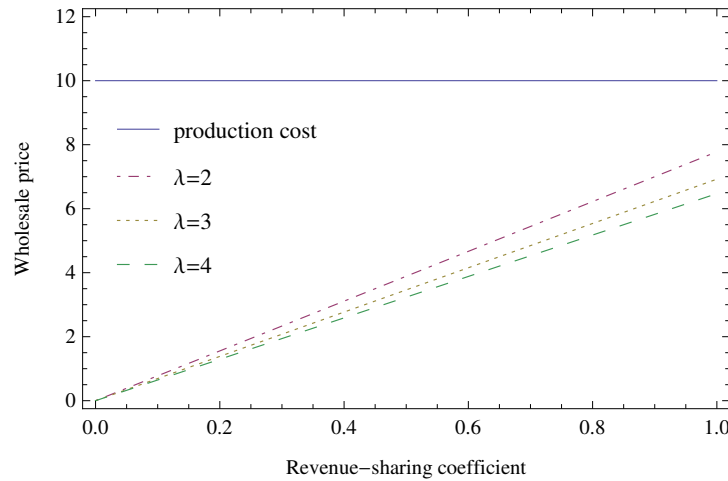


FIGURE 1. Wholesale price with respect to revenue-sharing coefficient under different loss aversion degrees.

4.1. Contract analysis and profit allocation. First of all, we examine how to set contract parameters and how to allocate the additional expected profit under coordination with the revenue-cost-sharing contract. As proved in previous section, Fig.1 shows that the wholesale price under supply chain coordination is not more than the production cost of per unit product, and the wholesale price correlates positively with the revenue-sharing coefficient. That is, the higher the revenue-sharing coefficient is, the larger the wholesale price is. Fig.2 shows that the cost-sharing coefficient also correlates positively with the wholesale price, the higher the wholesale price is, the larger the cost-sharing coefficient is. When the wholesale price remains unchanged, we can see from Fig.1 and Fig.2 that, the more loss-averse the retailer becomes, the larger both the revenue-sharing coefficient and the cost-sharing coefficient are. This indicates that the loss aversion degree of the retailer is a significant factor for designing coordination contracts.

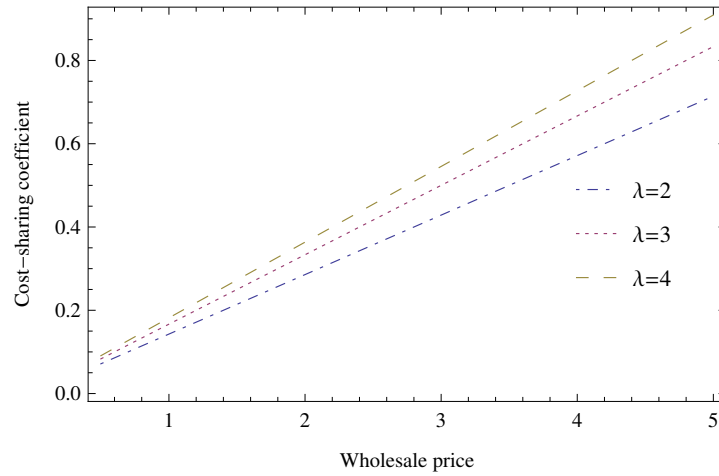


FIGURE 2. Cost-sharing coefficient with respect to wholesale price under different loss aversion degrees.

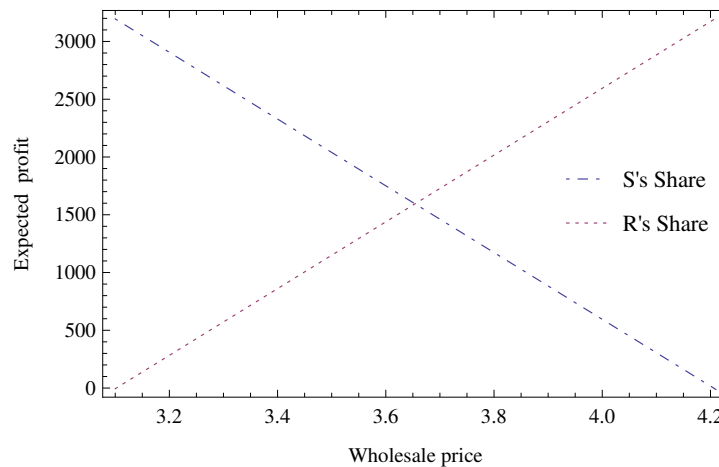


FIGURE 3. Allocation of additional expected profit under coordination.

In the decentralized supply chain with wholesale price contract, the optimal marketing effort level is $I_W^*=112.80$, the optimal order quantity is $Q_W^*=1137.05$. Accordingly, the retailer's expected profit is $E\pi_r(Q_W^*, I_W^*)=8962.58$, and the supplier's expected profit is $E\pi_s(Q_W^*, I_W^*)=7277.10$. Then, the supply chain's total expected profit is $E\pi_r(Q_W^*, I_W^*) + E\pi_s(Q_W^*, I_W^*) = 16239.68$. Under coordination with revenue-sharing contract, the optimal marketing effort level and the optimal order quantity of the retailer are equivalent to that of the integrated entity, and the supply chain's total expected profit is equal to the system-wide profit in the centralized system. Thus, we have $I_T^*=192.38$, $Q_T^*=1827.41$ and $E\pi_{sc}(Q_T^*, I_T^*)=19428.93$. Compared with the wholesale price contract, the revenue-cost-sharing contract can increase the total expected profit of the supply chain by $E\pi_{sc}(Q_T^*, I_T^*) - E\pi_r(Q_R^*, I_R^*) - E\pi_s(Q_R^*, I_R^*)=3189.25$. Fig. 3 illustrates each party's

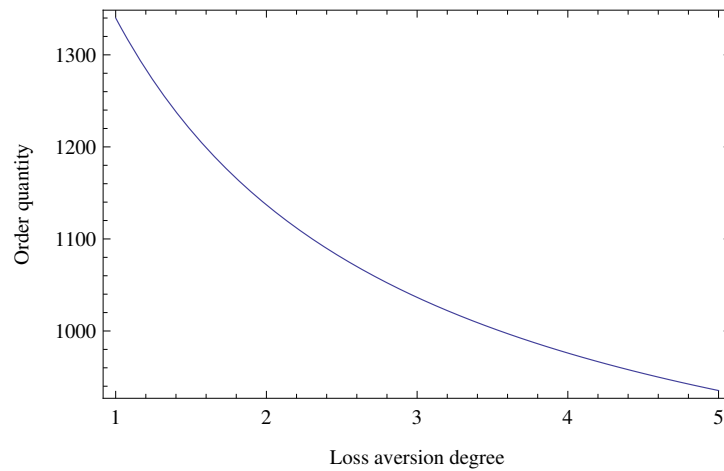


FIGURE 4. Effect of loss aversion on order quantity.

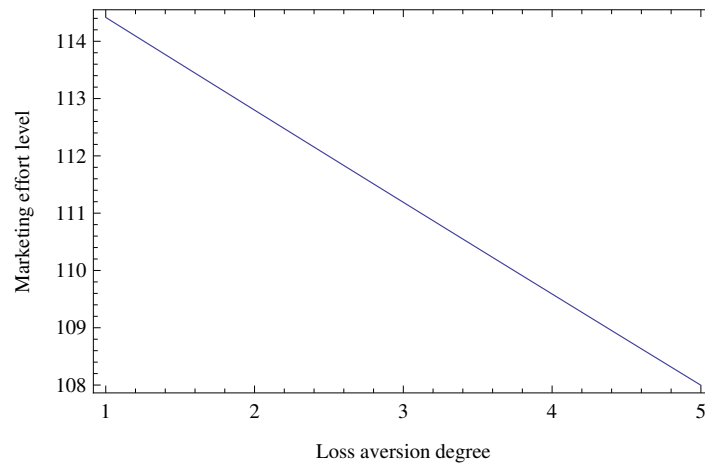


FIGURE 5. Effect of loss aversion on marketing effort level.

share on additional expected profit under coordination with the revenue-sharing contract. The additional expected profit obtained by the retailer is increasing in the wholesale price while the additional expected profit obtained by the supplier is decreasing in the wholesale price. Clearly, the retailer's additional expected profit increases from 0 to 3189.25 when the wholesale price increases from 3.10 to 4.21, and the converse is applicable for the supplier. In the region $w \in (3.10, 4.21)$, both the supplier and the retailer can obtain more profit than they obtain under a wholesale price contract. This indicates that the revenue-sharing contract is profitable.

4.2. Effects of loss aversion. In what follows, we examine the impact of changes of loss aversion degree on the optimal order quantity, marketing effort level and expected profit in the decentralized supply chain with wholesale price contract. Fig.4 and Fig.5 show that both the optimal order quantity and the optimal marketing effort level are decreasing in the loss aversion degree, that is, the more loss-averse the

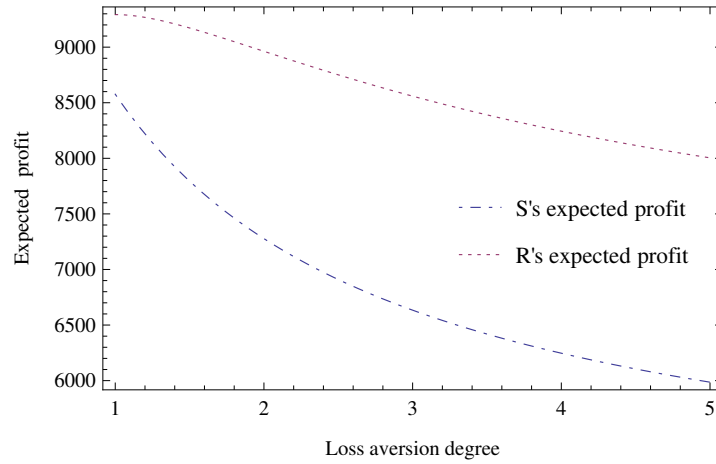


FIGURE 6. Effect of loss aversion on expected profit.

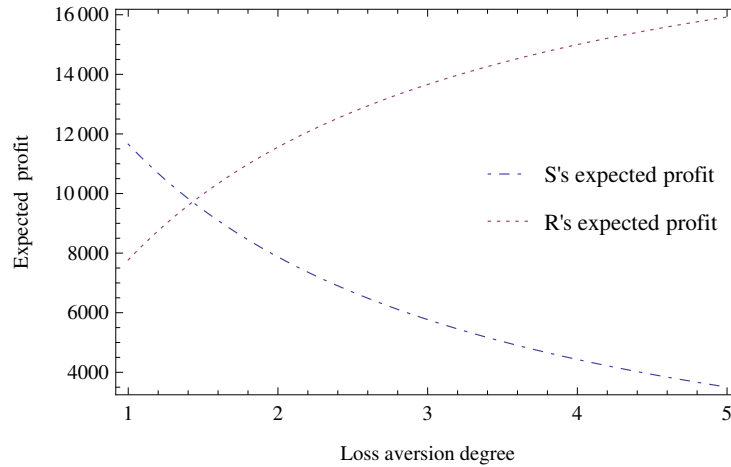
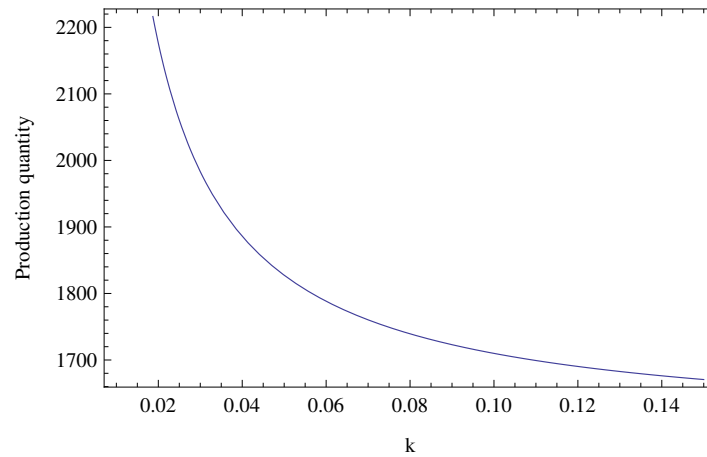
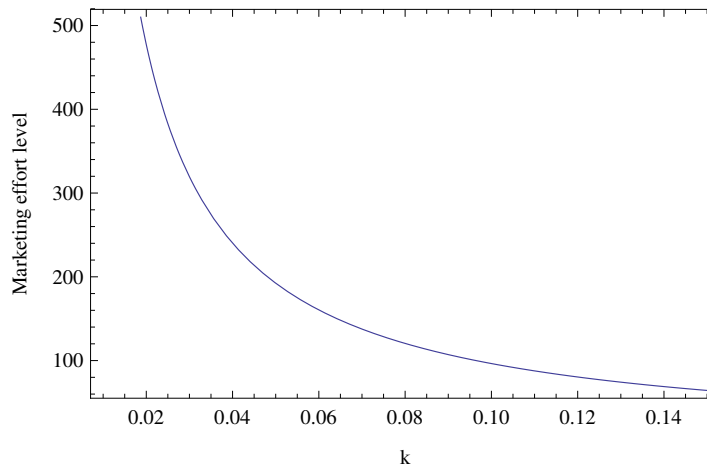


FIGURE 7. Effect of loss aversion on profit allocation.

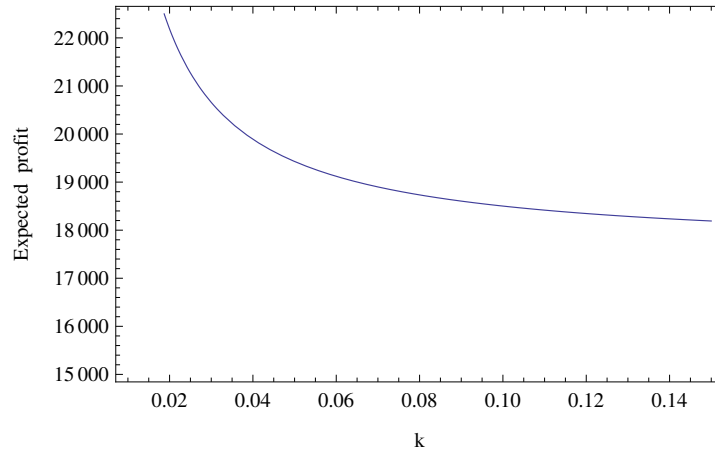
retailer is, the lower both the optimal order quantity and the optimal marketing effort level are. In the decentralized system with wholesale price contract, Fig.6 shows that the expected profits of both the retailer and the supplier are decreasing in the loss aversion degree. We know that the optimal expected profit in the centralized system is independent on the loss aversion degree, thus the difference in expected profits between the decentralized system and the centralized system increases as the loss aversion degree increases. This implies that it is more beneficial for the total supply chain to offer a coordinating contract when the retailer becomes more loss-averse. In particular, both the retailer and the supplier can achieve a win-win outcome by setting appropriate contract parameters.

Fig.7 depicts the impact of changes of loss aversion degree on profit allocation under coordination with the revenue-sharing contract. Given the wholesale price $w = 4$, the expected profit of the supplier is decreasing in the loss aversion degree

FIGURE 8. Production quantity with respect to k .FIGURE 9. Marketing effort level with respect to k .

while the retailer's expected profit is increasing in the loss aversion degree. That is, the retailer may gain more profit when it becomes more loss-averse.

4.3. Effects of effort cost. In this subsection, we explore the influence of the cost coefficient of marketing effort on the optimal decisions and expected profit in the centralized system. Fig.8 and Fig.9 show that, in the integrated supply chain, both the optimal production quantity and the optimal marketing effort level decrease as the effort cost coefficient increases. Accordingly, the expected profit of the integrated entity is decreasing in the effort-cost coefficient, which is shown in Fig.10. That is, the smaller the effort-cost coefficient is, the more the expected profit of the integrated entity is. This reveals that coordinating the supply chain will lead to more additional profit when the cost of marketing effort is lower.

FIGURE 10. Expected profit with respect to k .

5. Conclusions. Nowadays, many industries are moving towards complex supply chains and facing increasingly uncertain demand as well as supply. In the meantime, decision-making behaviors of managers always deviate from the assumption of risk neutrality and are consistent with loss aversion. In this paper, we study a single-period two-echelon supply chain composed of a risk-neutral supplier and a loss-averse retailer, in which the retailer purchases products from the contracting supplier to satisfy a stochastic market demand, where the supplier's production is subject to random yield and the market demand is influenced by marketing effort of the retailer. We first introduce the integrated case to establish a performance benchmark, and then examine the decentralized supply chain with wholesale price contract. We identify that the wholesale price contract remains ineffective for achieving supply chain coordination due to the effects of both double marginalization and loss aversion. In order to achieve supply chain coordination, a revenue-cost-sharing contract is proposed. We derive the closed-form solutions, and verify that the properly designed revenue-cost-sharing contract can coordinate the supply chain perfectly and achieve a win-win outcome for both the supplier and the retailer. Under the revenue-cost-sharing contract, we discuss how to set contract parameters and allocate the additional expected profit, as well as the effects of both the loss aversion degree and the effort-cost coefficient on the expected profits of channel members.

We summarize other main results as follows. Analytically, we show that the optimal order quantity is decreasing in the loss aversion degree, that is, the more loss-averse the retailer is, the less the retailer orders. Under coordination with the revenue-cost-sharing contract, the wholesale price paid by the retailer to the supplier is lower than the production cost, and is decreasing in the loss aversion degree. We also show that both the revenue-sharing coefficient and the cost-sharing coefficient correlate positively with the wholesale price, meanwhile, the more loss-averse the retailer becomes, the larger both the revenue-sharing coefficient and the cost-sharing coefficient are. In the premise of coordination, the expected profit of the retailer is increasing in the wholesale price while the supplier's expected profit is decreasing in the wholesale price. This indicates that, compared with the wholesale price contract, there always exist some revenue-cost-sharing contracts with appropriate parameters such that neither of the retailer and the supplier becomes worse off.

Given the wholesale price, the expected profit of the retailer increases as the loss aversion degree increases, and the supplier's expected profit decreases as the loss aversion degree increases. Through numerical experimentation, we show that it is more beneficial for the total supply chain to offer a coordinating contract when the retailer becomes more loss-averse. We also show the influence of the effort-cost coefficient on the integrated entity's expected profit, and our results reveal that coordinating the supply chain will lead to more additional profit when the cost of marketing effort is lower.

There are several interesting directions for future research. It is interesting to investigate how to coordinate the supply chain with asymmetric information by contractual arrangements, where the supply uncertainty is not fully known by the retailer and the demand uncertainty is not fully known by the supplier. Furthermore, due to supply uncertainty, we may consider that the retailer may purchase products from another reliable source such as spot market after observing the supplier's status. Additionally, following Xu and Meng [44], it is also interesting to explore how to coordinate a supply chain with a supplier and a retailer in terms of profit concession under supply and demand uncertainty.

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