Predicting customer profitability over time based on RFM time series

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Abstract: Predicting consumer profitability dynamically over time plays a vital role in today's customer-centric business. In this paper, we adopt a dynamic systems approach to address the dynamic prediction problem of customer profitability. Based on customer transaction records, RFM score-based time series are generated using cluster analysis. These time series are used to measure and describe customer profitability. Furthermore, multilayer feed-forward neural network models are trained to capture the dynamics of the evolving customer profitability. A set of real transactions from a UK-based online retailer is used in this study. Relevant experimental results have shown good performance of the proposed approach.

Keywords: dynamic prediction of consumer profitability; RFM-based customer segmentation; temporal data mining; *k*-means clustering; multilayer feed-forward neural networks.

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1 Introduction

Predicting consumer profitability dynamically over time plays a vital role in today's customer-centric business. Traditionally, businesses usually examine a customer's profitability at a given cut-off point in time in order to target some particular customers for marketing purposes. In reality, however, customer purchasing habits and preferences are heterogeneous and may fluctuate over time. Therefore, businesses would like to know how, based on each customer's historical purchases and profitability, his/her profitability is likely to evolve over time. The answers to this will have a direct impact on a business's marketing strategies and resource allocations.

In general, making such predictions requires considering several major aspects, including:

- Specify a metric to measure consumer profitability.
- Given consumer purchasing history records and the specified profitability metric, construct an appropriate dynamic model to describe the dynamics of customer profitability, and further to predict consumer profitability.

Many metrics have been introduced in a business context, including the Recency, Frequency, and Monetary (RFM) model, Customer Lifetime Value (CLV) model, and Customer Equity (CE) model (Hughes, 2012; Kumar and Reinartz, 2006). Each metric provides a unique perspective from which businesses can evaluate customer behaviour and customer profitability. Interestingly, some explicit or implicit relationships exist among different metrics. The selection of a certain metric is usually determined by what the business is concerned, and, sometimes, multiple metrics may be applied.

The major issues facing such predictions are usually modelling-related: considering consumer profitability as a time-varying process, which model is best-fitted to accurately capture the underlying dynamic characteristics of the evolving process? Various modelling techniques are available, for instances time series analysis (Tsay, 2010), dynamic Bayesian networks (Murphy, 2002), and Artificial Neural Networks (ANNs) (McNelis, 2005).

In this paper, we address the dynamic prediction problem of customer profitability by constructing a dynamic system model, and a multilayer feed-forward neural network (MFNN) is trained to capture the dynamics of evolving customer profitability. A set of transaction records from a UK-based online retailer is investigated. The RFM metrics are selected as customer profitability measures. In a given time period, customers are partitioned into groups by RFM values. Accordingly, each customer is assigned a unique RFM score to reflect collectively the customer purchasing behaviour and profitability. As such, a time series of customer profitability values with specific time period tags is formed and utilised to train and test the neural network model. The performance of the constructed MFNN model is further examined as an autonomous model, i.e. how it evolves as a dynamic feedback system given initial customer profitability.

The rest of this paper is organised as follows. Section 2 provides a literature review of the relevant work, focusing on approaches and models used for dynamic customer profitability prediction. Section 3 outlines the methodology of this study. A detailed discussion is given concerning the use of *k*-means clustering analysis to segment customers-based RFM values, and the target data set to be examined is presented. In Section 4, MFNN models are trained to predict customer profitability. The resultant models are also used as autonomous models. The performances of these models are evaluated in terms of prediction accuracy and confusion matrices. Section 5 summarises the essential findings of this study based on the experimental results. Finally, concluding remarks and outline of future work are given in Section 6.

2 Related work

Predicting customer profitability accurately over time based on historical purchasing records has been proven challenging and 'has not been entirely successful' (Rusta et al., 2011). This is mainly due to the fact that there is always a certain level of uncertainty associated with a customer's purchases (Malthouse and Blattberg, 2005), and the most profitable customer over one time period may become the least profitable over another time period. In addition, in the typical non-contractual settings in B2C or B2B environments, it is very common that the only information a firm holds about its customers is their purchases (and in some cases, their delivery and billing addresses, e.g. for online retailers). Firms hardly know if a customer's circumstances have changed over time, in terms of, for example, their job roles or salaries, etc. The lack of customers' demographics makes it even more difficult to predict customer profitability accurately and dynamically. Therefore, many probability theory and stochastic process-based models have been proposed in order to tackle the dynamic prediction problem of future customer profitability. Some previous work relating to this aspect is discussed below.

Markov Chain Models (MCMs) have seen a widespread use in estimating CLV and modelling customer relationships in general (Pfeifer and Carraway, 2000; Bozzetto et al., 2005; So and Thomas, 2011). In the research by Haa et al. (2002), each customer's profitability or 'state' at any given time t is represented by an MCM, and the associated state transition matrices are utilised to describe how RFM-based customer segments have evolved over time in a given customer base. This knowledge is further linked to dynamic customer profitability and appropriate marketing strategies.

More generally, a set of probabilistic models has been discussed for measuring future profitability of a customer in terms of net profit and net present value of profit (Rusta et al., 2011). The models include a joint probability model of gross profit, customer purchase incidence, and market contacts. These models, although complicated, remarkably outperform naïve models. Based on the simulation results, Rusta et al. (2011) also argue that 'predicting future customer profitability is not futile; it requires sophisticated modelling approach'.

The major advantage of the models examined above is that they represent a white-box approach to the problem and can be employed to build explicit relationships among various variables involved that are well grounded in probability and stochastic process theories. Therefore, these models are easy to understand and interpret. On the other hand, however, these models rely on sufficient independent and representative samples from a customer database to generate reliable estimates of various probability distributions as required, and they are usually computation-intensive. In addition, these models can only adapt to different stochastic processes to a limited extent.

3 Methodology and data set

In this paper, we adopt a dynamic system approach to the problem.

3.1 The dynamic model

Suppose in a given time period t the profitability of a customer is measured by a specific metric $\Theta(t)$. Without loss of generality, the model under consideration for dynamic prediction of customer profitability can be expressed as:

$$\Theta(t) = f\left(\Theta(t-1), \Theta(t-2), \ldots, \Theta(t-n)\right) \tag{1}$$

where $\Theta(t-i)$ represents the profitability measured in time period $t-i$, $i = 1, 2, \ldots n; 1 \le n \le t$. Suppose that an appropriate algorithm gives an estimate of the desired mapping f, denoted by $\hat{f}$. The estimated customer profitability in time period t can therefore be described as:

$$\hat{\Theta}(t) = \hat{f}\left(\Theta(t-1), \Theta(t-2), \ldots, \Theta(t-n)\right) \tag{2}$$

or

$$\hat{\Theta}(t) = \hat{f}\left(\hat{\Theta}(t-1), \hat{\Theta}(t-2), \ldots, \hat{\Theta}(t-n)\right) \tag{3}$$

Note that equation (2) represents an open-loop dynamic system model, and equation (3), in contrast, represents a closed-loop dynamic system model (i.e. a feedback dynamic system). The open-loop model always uses the actual customer profitability observed in a certain number of past time periods to predict the future profitability (n-step-ahead predictions, in this case), whereas the closed-loop model always uses the estimated customer profitability in a certain number of the past time periods made by the model itself to predict the future profitability, except for the first n consecutive time periods in which actual customer profitability values are utilised. In other words, the closed-loop model represents an autonomous system. To use such a system to predict future customer profitability, it requires an initial stimulus (state) or ‘push’ to the system, which is the actual customer profitability observed in the first n consecutive past time periods.

3.2 Profitability metric

In this research, RFM model is chosen to measure a customer’s profitability. In each time period t, the RFM values of a customer are calculated following the formulas shown in Table 1.

Table 1 Calculation of a customer’s RFM values

Purchased in time period t?	*RFM values*
Yes	$R(t)=0$ $F(t)=F(t-1)+$ no. of purchases $M(t)=M(t-1)+$ total spending
No	$R(t)=R(t-1)+1$ time period unit $F(t)=F(t-1)$ $M(t)=M(t-1)$

In order to transform a customer’s RFM values to a single-valued profitability measure $\Theta(t)$, customers are partitioned based on their RFM values by k-means cluster analysis. Each segment is further assigned a unique RFM score. Therefore, in any given time period t, customers in the same cluster (group) receive the same score.

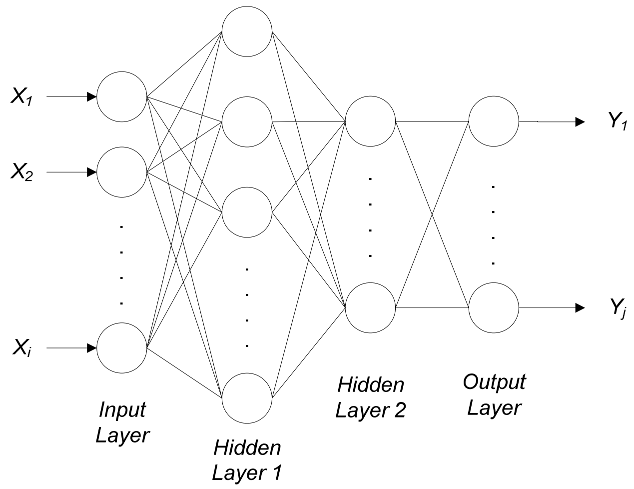
The role of this single-valued score is threefold: (1) reflects the three RFM values collectively; (2) represents a customer’s profitability in a given time period; and (3) indicates the segment membership of a customer at a specific time period. As such, an RFM score-based profitability time series of each customer is generated.

3.3 Model construction

In order to construct the required dynamic system model, MFNN is utilised. MFNN is a typical ANN paradigm, and, in theory, an MFNN with two hidden layers can be used as a universal approximator to approximate any continuous mappings to an arbitrary accuracy (Chen, 1993). Therefore, the network is a generally applicable model and is very flexible to use.

The general topology of MFNN is depicted in Figure 1. In this study, the standard error backpropagation (BP) algorithm (Rumelhart et al., 1986) is employed to train the network to determine the network’s parameters (connection weights and biases). It should be noted that in general an MFNN represents a static system. In practice, however, the network can be used as a closed-loop system by linking the outputs of the nodes in the output layer to the nodes in the input layer. Hence, the network can be utilised to describe the two types of models expressed in equations (2) and (3).

Figure 1 Multilayer feed-forward neural network (MFNN) with two hidden layers



3.4 The data set

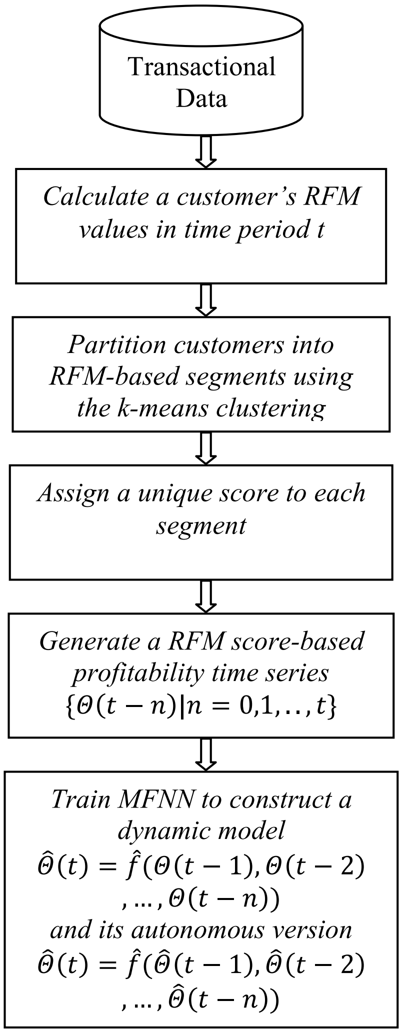
The data set used in this paper is from a UK-based and registered online retailer (Chen et al., 2012). There are 11 variables (attributes) in the data set as shown in Table 2, and it contains all the transactions occurring in years 2010 and 2011. In this study, only the transactions generated from 1 December 2010 to 30 November 2011 are explored. Over that particular period, there were 18,291 valid transactions in total, associated with some 4381 valid distinct UK postcodes. It should be noted that each individual consumer is identified by a distinct postcode.

Table 2 Variables in the customer transaction data set

Variable name	*Data type*	*Description; typical values and meanings*
Invoice	Nominal	Invoice number; a six-digit integral number uniquely assigned to each transaction
Stock code	Nominal	Product (item) code; a five-digit integral number uniquely assigned to each distinct product
Description	Nominal	Product (item) name; CARD I LOVE LONDON
Quantity	Numeric	The quantity of each product (item) per transaction
Price	Numeric	Product price per unit in sterling; £45.23
Invoice date	Numeric	The day and time when each transaction was generated; 31/05/2011 15:59
Address line 1	Nominal	Delivery address line 1; 103 Borough Road
Address line 2	Nominal	Delivery address line 2; Elephant and Castle
Address line 3	Nominal	Delivery address line 3; London
Postcode	Nominal	Delivery address postcode, mainly for consumers from the UK; SE1 0AA
Country	Nominal	Delivery address country; England

As a summary of this section, the entire simulation process for modelling customer profitability is illustrated in Figure 2.

Figure 2 The entire simulation process



4 Experiments and results

A target data set was created for the experiments in this study. Owing to the time series prediction problem under consideration, the target data set should have a fixed size containing the purchasing records of those customers who have made at least one purchase with the retailer at the beginning of the entire time period for the analysis. In this study, the entire time period considered was from December 2010 to November 2011, and the target data set comprises all purchases of some 751 customers who made at least one purchase in December 2010.

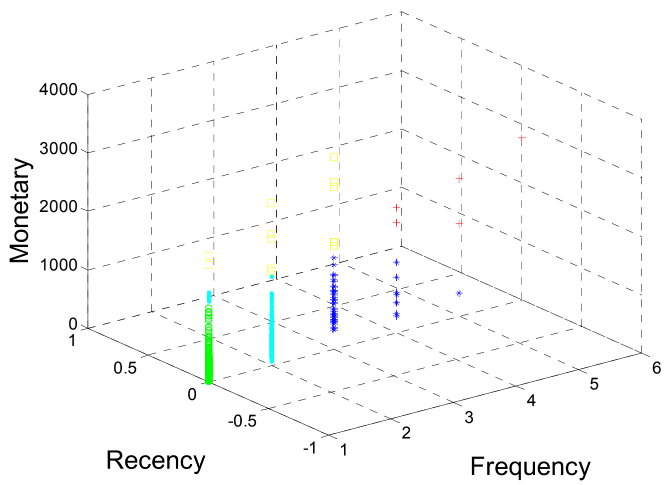
Each customer's purchases in the target data set were examined in order to determine a customer's RFM values. The cut-off point of time for calculating the RFM values was set to the end of each calendar month from December 2010 to November 2011. Subsequently, each customer had 12 sets of RFM values associated, each corresponding to a specific time period, that is December 2010, December 2010 to January 2011, December 2010 to February 2011, …, and December 2010 to November 2011.

Based on the RFM values at the end of each time period, the customers in question were segmented into five groups (clusters) by the *k*-means clustering algorithm. Each cluster contains a group of customers who had similar RFM values. Moreover, these RFM values were aggregated collectively to determine a unique single-valued RFM score for all the customers in the same cluster. Table 3 gives the rules applied to determine an RFM score for customers in the same cluster at the end of any given time period. Figure 3 depicts the RFM values of all the 751 customers over different time periods and their corresponding RFM scores.

Table 3 Rules of assigning an RFM score

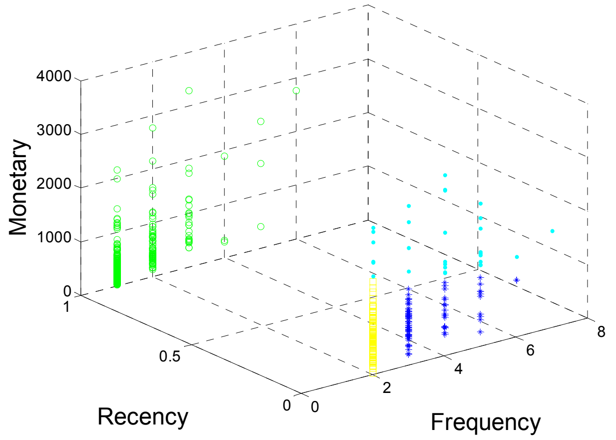
Cluster number	*RFM score*	*Customer profitability*
1	1	High
2, 3, or 4	2	Medium
5	3	Low

Figure 3 Customer segmentations in different time periods (see online version for colours)

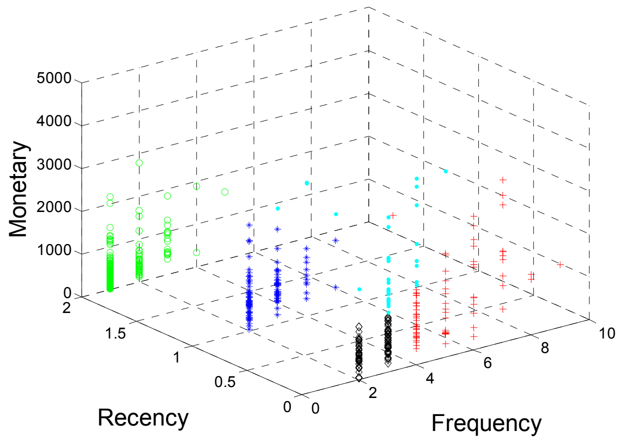


(a) December 2010

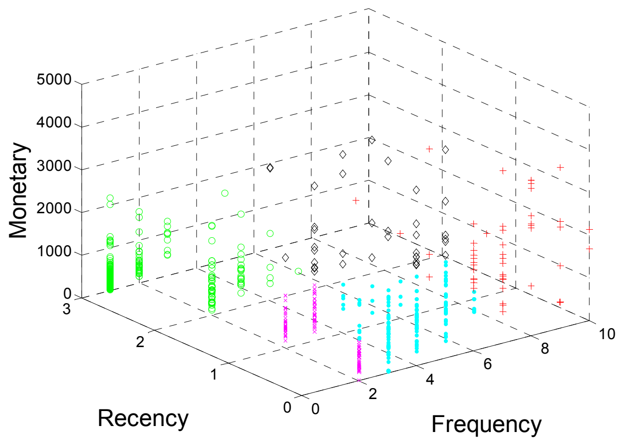
Figure 3 Customer segmentations in different time periods (see online version for colours) (continued)



(b) December 2010 – January 2011

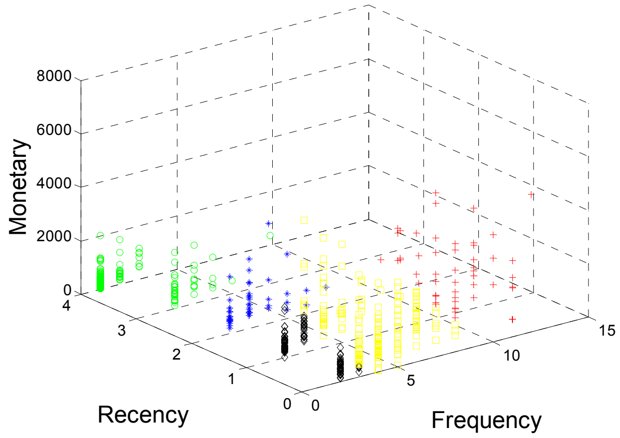


(c) December 2010 – February 2011

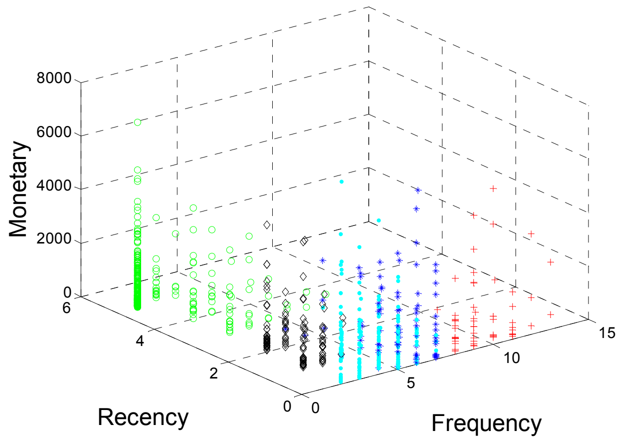


(d) December 2010 – March 2011

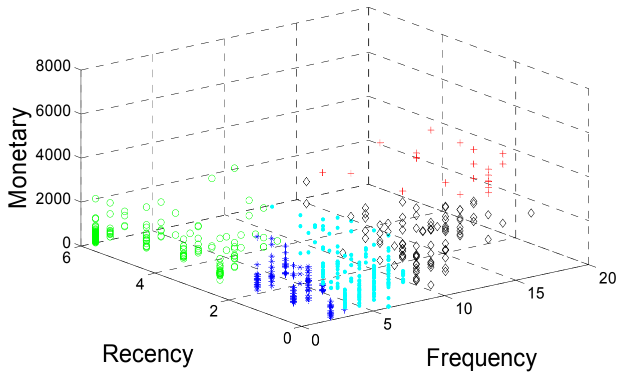
Figure 3 Customer segmentations in different time periods (see online version for colours) (continued)



(e) December 2010 – April 2011

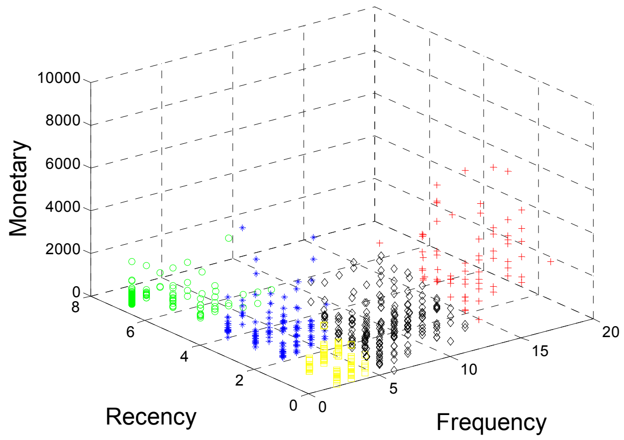


(f) December 2010 – May 2011

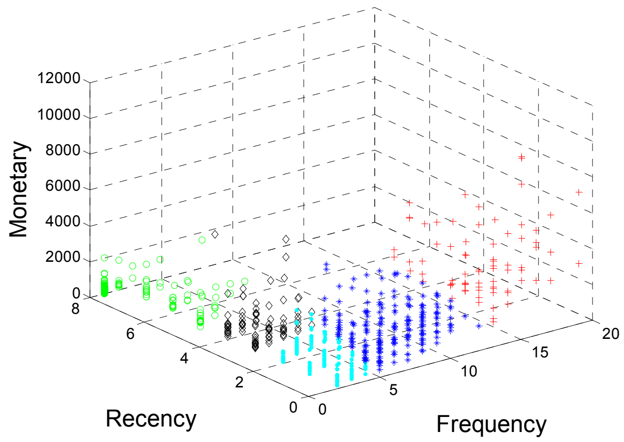


(g) December 2010 – June 2011

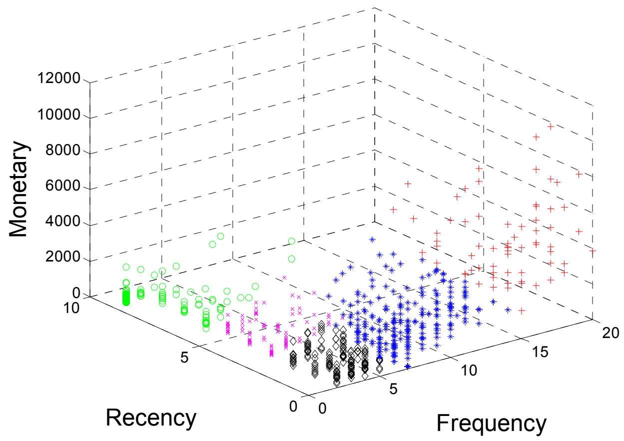
Figure 3 Customer segmentations in different time periods (see online version for colours) (continued)



(h) December 2010 – July 2011

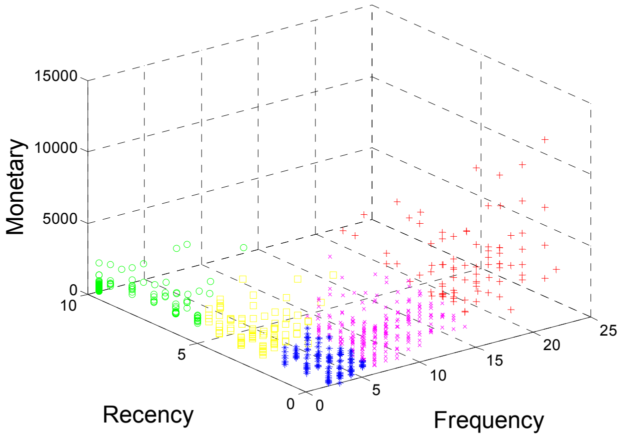


(i) December 2010 – August 2011

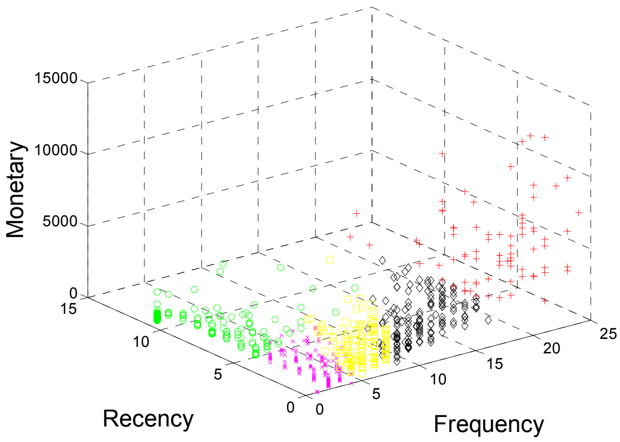


(j) December 2010 – September 2011

Figure 3 Customer segmentations in different time periods (see online version for colours) (continued)



(k) December 2010 – October 2011



(l) December 2010 – November 2011

As such, each customer was associated with 12 RFM scores over the entire time period for the analysis, and these scores form an RFM score-based profitability time series for an individual customer.

Figure 4 shows several typical time series of RFM scores found in the experiments. It is evident that usually a customer's profitability varies over time – in some cases very considerably, and the dynamic evolution of the scores is diverse. On the other hand, however, as shown in Table 4 and Figure 5, the number of customers in different profitability groups has become stable over time.

Figure 4 Typical time series of RFM scores

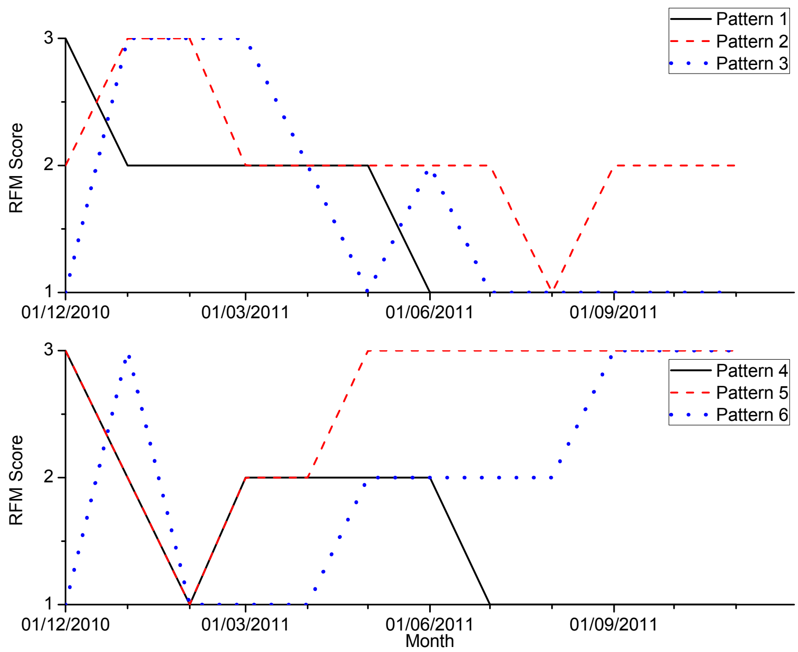


Table 4 The number of customers in different profitability groups over time

Time period (from December 2010)	*Score 1*	*Score 2*	*Score 3*
December 2010	5	186	560
January 2011	0	231	520
February 2011	76	257	418
March 2011	48	303	400
April 2011	51	376	324
May 2011	58	384	309
June 2011	22	415	314
July 2011	72	438	241
August 2011	75	436	240
September 2011	68	464	219
October 2011	86	459	206
November 2011	80	464	207

With the established time series of customers' RFM scores, four models – 2 one-step-ahead and 2 two-step-ahead models – were built using MFNN, respectively. These models can be expressed as:

$$\hat{\Theta}(t) = \hat{f}\left(\Theta(t-1)\right) \tag{4}$$

$$\hat{\Theta}(t) = \hat{f}\left(\hat{\Theta}(t-1)\right) \tag{5}$$

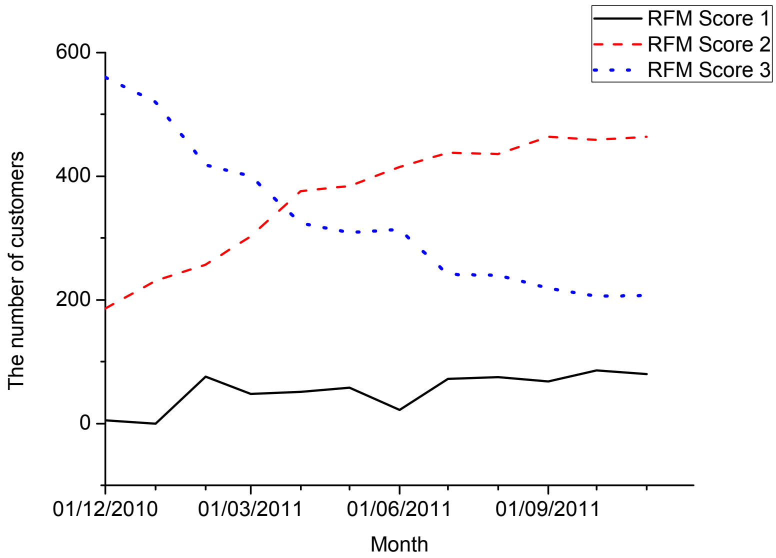
and

$$\hat{\Theta}(t) = \hat{f}\left(\Theta(t-1), \Theta(t-2)\right) \tag{6}$$

$$\hat{\Theta}(t) = \hat{f}\left(\hat{\Theta}(t-1), \hat{\Theta}(t-2)\right) \tag{7}$$

Note that equations (5) and (7) represent autonomous systems.

Figure 5 The change of the number of customers having different RFM scores



A set of orthogonal vectors with three components has been used to represent the RFM scores for modelling purpose, that is $(1\ 0\ 0)^T$, $(0\ 1\ 0)^T$, and $(0\ 0\ 1)^T$ donate the RFM scores 1, 2, and 3, respectively.

The topology of the neural networks to be trained was set up as follows. For models (4) and (5), a network with 3-input-node, 3-output-node, and a single hidden layer of three hidden nodes was used. For models (6) and (7), a network with 6-input-node, 3-output-node, and a single hidden layer of six hidden nodes was used.

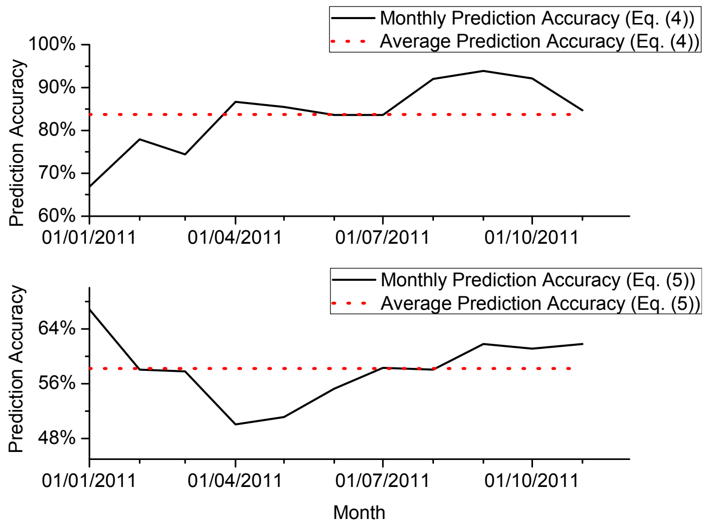
The ratio of the total number of samples for training and the total of number of samples for testing was set to 65%:35%. Therefore, the RFM score-based time series of some 65% of the customers were randomly selected from the target data set and used to train the networks, and the remaining was used to test the trained networks. The total number of iterations for the training was set to 1000.

The prediction performance of each model is shown and summarised in Figure 6 and Table 5 with the relevant confusion matrices given in Table 6, where the value at the *i*th row and the *j*th column of the matrix represents the total number of customers from the *i*th group that have been predicted as from the *j*th group $(i, j = 1,2,3)$ over all the time periods for prediction. Note that for models (4) and (5), there are 11 time periods; and for models (6) and (7), there are 10 time periods.

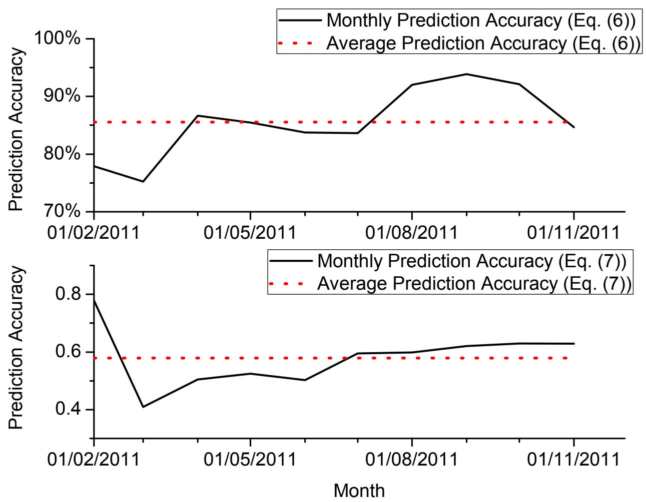
Table 5 Simulation results

Model	*Misclassification rate (%)*		
	Min	*Average*	*Max*
$\hat{\Theta}(t)=\hat{f}\left(\Theta(t-1)\right)$	6.13	16.25	33.16
$\hat{\Theta}(t)=\hat{f}\left(\hat{\Theta}(t-1)\right)$	33.16	41.80	49.93
$\hat{\Theta}(t)=\hat{f}\left(\Theta(t-1),\Theta(t-2)\right)$	6.13	14.46	24.77
$\hat{\Theta}(t)=\hat{f}\left(\hat{\Theta}(t-1),\hat{\Theta}(t-2)\right)$	22.10	42.09	59.12

Figure 6 The comparison of monthly prediction accuracy with average prediction accuracy



(a) Equations (4) and (5)



(b) Equations (6) and (7)

Table 6 Confusion matrices

Model	*Confusion matrix*
$\hat{\Theta}(t)=\hat{f}\left(\Theta(t-1)\right)$	$\begin{pmatrix} 424 & 199 & 13 \\ 132 & 3426 & 669 \\ 5 & 324 & 3069 \end{pmatrix}$
$\hat{\Theta}(t)=\hat{f}\left(\hat{\Theta}(t-1)\right)$	$\begin{pmatrix} 3 & 602 & 31 \\ 1 & 3691 & 535 \\ 6 & 2278 & 1114 \end{pmatrix}$
Model	*Confusion matrix*
$\hat{\Theta}(t)=\hat{f}\left(\Theta(t-1),\Theta(t-2)\right)$	$\begin{pmatrix} 421 & 202 & 13 \\ 121 & 3350 & 525 \\ 1 & 224 & 2653 \end{pmatrix}$
$\hat{\Theta}(t)=\hat{f}\left(\hat{\Theta}(t-1),\hat{\Theta}(t-2)\right)$	$\begin{pmatrix} 318 & 306 & 12 \\ 293 & 3613 & 90 \\ 53 & 2407 & 418 \end{pmatrix}$

5 Discussion

Based on the experiment results presented in the previous section, we have the following remarks.

Remark 1: RFM values-based customer profitability is predictable, although the time varying process of the metric has a certain level of diversity and uncertainty. This predictability indicates that the purchasing habits and preferences of the customers are stable over time to a reasonable extent. As shown in Figures 3 and 5, with more transaction records accumulated over time, more stable and clearer RFM-based customer segments have formed, and the number of customers in different profitability groups has become more stable as well. The existence and persistence of different customer profitability groups make prediction possible at a very acceptable accuracy level. In particular, for the open-loop models, the average misclassification rate was very low.

It should also be noted that the organisational customers of the firm are an important attributing factor to this.

Remark 2: MFNNs have exhibited stable and good performance in the prediction. MFNNs are very suitable for modelling a dynamic process like RFM-based customer profitability measures. The main strengths of MFNNs are that they are flexible and do not require any additional prior statistical knowledge of the dynamic process under consideration. In particular, due to the universal mapping capability, MFNNs are a good option for dealing with the diversity and uncertainty in the prediction of RFM-based customer profitability.

Remark 3: Using more historical customer transaction records can make the prediction of customer profitability more accurate and reliable. As shown in Tables 5 and 6 and Figure 6, the two-step-ahead prediction had a better performance than the one-step-ahead prediction. If more time steps to be considered, for example three-step-ahead or four-step-ahead predictions, the prediction accuracy can be expected to be further improved.

Remark 4: Using autonomous models in the prediction is less reliable and may cause severe prediction errors compared to open-loop models. As shown in Table 5, on average the performances of the autonomous models were poor, although they were better than random predictions. However, autonomous models require less computation resources. If more time steps are involved, the performance of autonomous models can be improved.

6 Conclusion and further research

In this paper, we have presented a novel approach for predicting customer profitability over time. The main idea of the approach is to construct a set of RFM score-based time series to represent the dynamic processes of customer profitability. These RFM score-based time series are further used to train MFNNs to predict each customer's profitability. It has been shown that the RMF score-based customer profitability is predicable, and especially if open-loop models are used, the prediction accuracy can be very high, even though only one-step-ahead prediction model is considered. These results also indicate that a RFM score-based time series reflects some intrinsic characteristics in relation to customer purchasing preferences and habits.

The proposed approach is promising and deserves further research, particularly in the following areas:

- Investigating which products customers in different profitability groups have purchased in order to explore the possibility of predicting which products a customer may purchase over next time period.
- How the length of time period for calculating the RFM values would affect the predictability of customer profitability.
- Alternative models for predicting an RFM score-based time series, for example genetic algorithm and dynamic Bayesian networks.

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