University of Iowa	Homework 2	CS 3330
Department of CS	Fall 2024	

Assigned: Friday 11:59 PM, September 6, 2024
Due: Friday 11:59 PM, September 13, 2024
Reading: Kleinberg and Tardos, Chapter 2.1-2.2, Chapter 2.4, Slides of Week 2

1. [**O-notation, Ω-notation, Θ-notation**]
   (a) [5 points] Is $3^{n+1} = O(3^n)$? Is $3^{3n} = O(3^n)$? Justify your answer.
   (b) [5 points] Is $n^3 = O(n^{2+\cos n})$? Is $n^3 = \Omega(n^{2+\cos n})$? Justify your answer.
   (c) [10 points] Suppose that $g(n) = \frac{n^5+n^{3+sinn}+1}{n^2+n+1}$. Which of the following are correct?

   $A.\ g(n) = \Omega(n^2)$ $\quad B.\ g(n) = \Theta(n^3)$ $\quad C. g(n) = O(n^4)$ $\quad D.$All

   Justify your answer.

2. [**Worst Case Analysis**] What is the worst case running time of the following sudo codes, in θ-notation? Suppose that all arithmetic operations (including simple multiplication) take a constant amount of time. Justify your answer.
   (a) [10 points]

Data: Array arr of length n
Result: Scalar val
$val \leftarrow 0$;
for $i \leftarrow 1$ **to** n^2 **do**
  $j \leftarrow 1$;
  while $j \leq i^2$ **do**
    $val \leftarrow val + arr[i] \times j^2$;
    $j \leftarrow j + 1$;
  end
end

   (b) [10 points]

Data: Array arr of length n
Result: Scalar val
$val \leftarrow 0$;
for $i \leftarrow 1$ **to** n **do**
  $j \leftarrow 1$;
  for $j \leftarrow 1$ **to** $\ln(i)$ **do**
    $val \leftarrow val + j \times (arr[i])^n$;
  end
end

   [Hint: $\ln(n!) = n \ln n - n + O(\ln n)$]

(c) [10 points]

Data: None
Result: Scalar val
$val \leftarrow 0$;
for $i \leftarrow 1$ **to** n **do**
  $j \leftarrow 1$;
  while $j \leq i$ **do**
    $val \leftarrow val + j \times i$;
    $j \leftarrow 3 \times j$
  end
end

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3. **[Fill the Blank]** Replace "____" with the best symbol (the most informative) from O, Ω, Θ. If none of these apply, write "None".

(a) [5 points] $\binom{n}{k} = \_\_\_\_(n^k)$

(b) [5 points] $n^{2+\sin n} = \_\_\_\_(n^2)$

(c) [5 points] $\log(n^{5+4\sin n}) = \_\_\_\_(\log n)$

(d) [5 points] $n^3 + 2n^2 + 5 = \_\_\_\_(n^3)$

(e) [5 points] $2^n = \_\_\_\_(n^3)$

(f) [5 points] $n \log n + n = \_\_\_\_(n \log n)$

(g) [5 points] $\frac{\log n}{n^2} = \_\_\_\_(1)$

(h) [5 points] $k^n = \_\_\_\_(2^n)$, where $k > 2$

(i) [5 points] $\sqrt{n} = \_\_\_\_(\log n)$

(j) [5 points] $\log^2 n = \_\_\_\_(n)$