

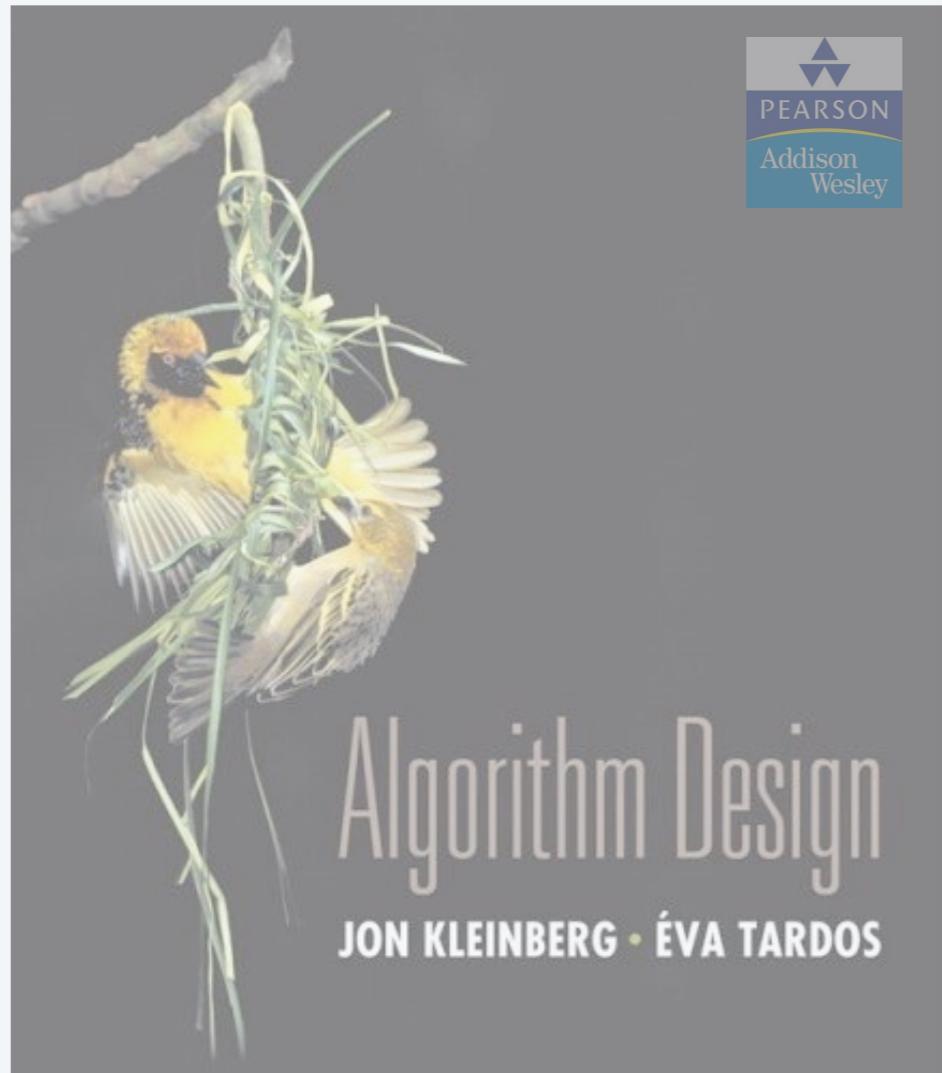
1. STABLE MATCHING

- ▶ *stable matching problem*
- ▶ *Gale–Shapley algorithm*
- ▶ *hospital optimality*
- ▶ *context*

Lecture slides by Kevin Wayne

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<http://www.cs.princeton.edu/~wayne/kleinberg-tardos>



1. STABLE MATCHING

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- ▶ *Gale–Shapley algorithm*
- ▶ *hospital optimality*
- ▶ *context*

SECTION 1.1

Matching med-school students to hospitals

Goal. Given a set of preferences among hospitals and med-school students, design a **self-reinforcing** admissions process.

Unstable pair. Hospital h and student s form an **unstable pair** if both:

- h prefers s to one of its admitted students.
- s prefers h to assigned hospital.

Stable assignment. Assignment with no unstable pairs.

- Natural and desirable condition.
- Individual self-interest prevents any hospital–student side deal.



Stable matching problem: input

Input. A set of n hospitals H and a set of n students S .

- Each hospital $h \in H$ ranks students.
- Each student $s \in S$ ranks hospitals.

one student per hospital (for now)

	favorite ↓	least favorite ↓	
	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

	favorite ↓	least favorite ↓	
	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

Perfect matching

Def. A **matching** M is a set of ordered pairs $h-s$ with $h \in H$ and $s \in S$ s.t.

- Each hospital $h \in H$ appears in at most one pair of M .
- Each student $s \in S$ appears in at most one pair of M .

Def. A matching M is **perfect** if $|M| = |H| = |S| = n$.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a perfect matching $M = \{ A-Z, B-Y, C-X \}$

Unstable pair

Def. Given a perfect matching M , hospital h and student s form an **unstable pair** if both:

- h prefers s to matched student.
- s prefers h to matched hospital.

Key point. An unstable pair $h-s$ could each improve by joint action.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

A-Y is an unstable pair for matching $M = \{ A-Z, B-Y, C-X \}$

Stable matching: quiz 1



Which pair is unstable in the matching { A-X, B-Z, C-Y } ?

- A. A-Y.
- B. B-X.
- C. B-Z.
- D. None of the above.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

Stable matching problem

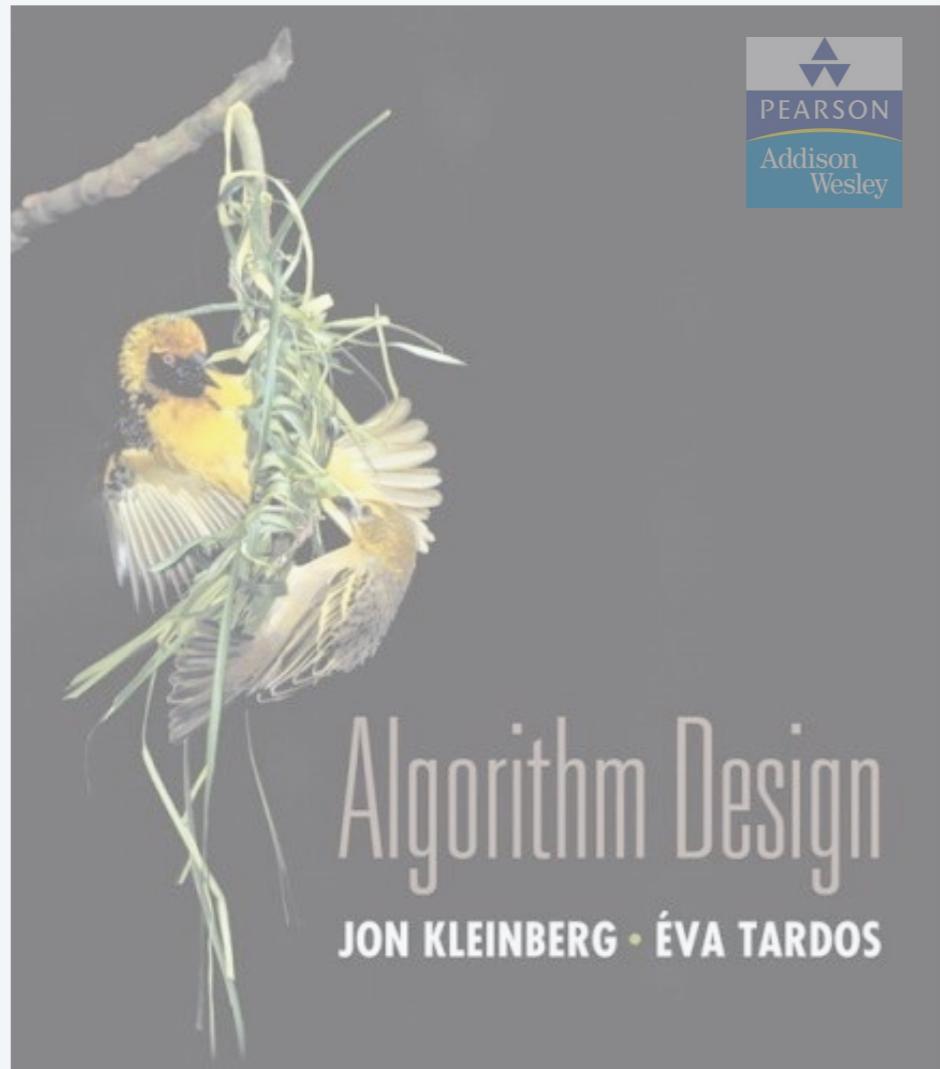
Def. A **stable matching** is a perfect matching with no unstable pairs.

Stable matching problem. Given the preference lists of n hospitals and n students, find a stable matching (if one exists).

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a stable matching $M = \{ A-X, B-Y, C-Z \}$



1. STABLE MATCHING

- ▶ *stable matching problem*
- ▶ **Gale–Shapley algorithm**
- ▶ *hospital optimality*
- ▶ *context*

SECTION 1.1

Gale–Shapley deferred acceptance algorithm

An intuitive method that **guarantees** to find a stable matching.



GALE–SHAPLEY (*preference lists for hospitals and students*)

INITIALIZE M to empty matching.

WHILE (some hospital h is unmatched and hasn't proposed to every student)

$s \leftarrow$ first student on h 's list to whom h has not yet proposed.

IF (s is unmatched)

Add $h-s$ to matching M .

ELSE IF (s prefers h to current partner h')

Replace $h'-s$ with $h-s$ in matching M .

ELSE

s rejects h .

RETURN stable matching M .

Proof of correctness: termination

Observation 1. Hospitals propose to students in decreasing order of preference.

Observation 2. Once a student is matched, the student never becomes unmatched; only “trades up.”

Claim. Algorithm terminates after at most n^2 iterations of WHILE loop.

Pf. Each time through the WHILE loop, a hospital proposes to a new student. Thus, there are at most n^2 possible proposals. ▀

	1 st	2 nd	3 rd	4 th	5 th		1 st	2 nd	3 rd	4 th	5 th	
A	V	W	X	Y	Z		V	B	C	D	E	A
B	W	X	Y	V	Z		W	C	D	E	A	B
C	X	Y	V	W	Z		X	D	E	A	B	C
D	Y	V	W	X	Z		Y	E	A	B	C	D
E	V	W	X	Y	Z		Z	A	B	C	D	E

$n(n-1) + 1$ proposals

Proof of correctness: perfect matching

Claim. Gale–Shapley outputs a matching.

Pf.

- Hospital proposes only if unmatched. \Rightarrow matched to ≤ 1 student
- Student keeps only best hospital. \Rightarrow matched to ≤ 1 hospital

Claim. In Gale–Shapley matching, all hospitals get matched.

Pf. [by contradiction]

- Suppose, for sake of contradiction, that some hospital $h \in H$ is unmatched upon termination of Gale–Shapley algorithm.
- Then some student, say $s \in S$, is unmatched upon termination.
- By Observation 2, s was never proposed to.
- But, h proposes to every student, since h ends up unmatched. \diamond

Claim. In Gale–Shapley matching, all students get matched.

Pf. [by counting]

- By previous claim, all n hospitals get matched.
- Thus, all n students get matched. ■

Proof of correctness: stability

Claim. In Gale–Shapley matching M^* , there are no unstable pairs.

Pf. Consider any pair $h-s$ that is not in M^* .

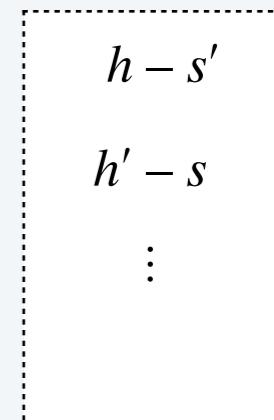
- Case 1: h never proposed to s .

$\Rightarrow h$ prefers its Gale–Shapley partner s' to s . ← hospitals propose in decreasing order of preference
 $\Rightarrow h-s$ is not unstable.

- Case 2: h proposed to s .

$\Rightarrow s$ rejected h (either right away or later)
 $\Rightarrow s$ prefers Gale–Shapley partner h' to h .
 $\Rightarrow h-s$ is not unstable.

↑ students only trade up



- In either case, the pair $h-s$ is not unstable. ▀

Gale–Shapley matching M^*

Summary

Stable matching problem. Given n hospitals and n students, and their preference lists, find a stable matching if one exists.

Theorem. [Gale–Shapley 1962] The Gale–Shapley algorithm guarantees to find a stable matching for **any** problem instance.

COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE

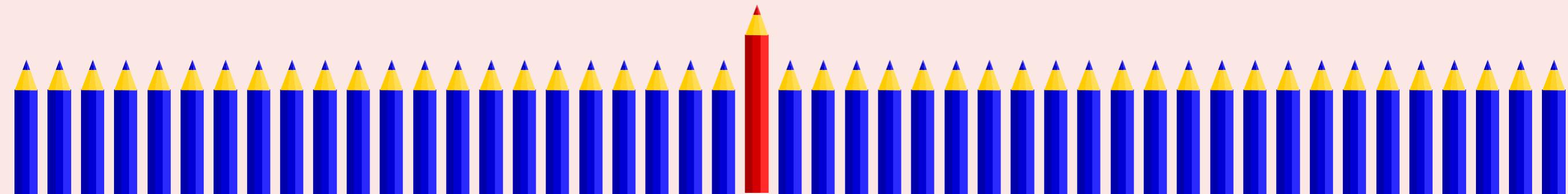
D. GALE* AND L. S. SHAPLEY, Brown University and the RAND Corporation

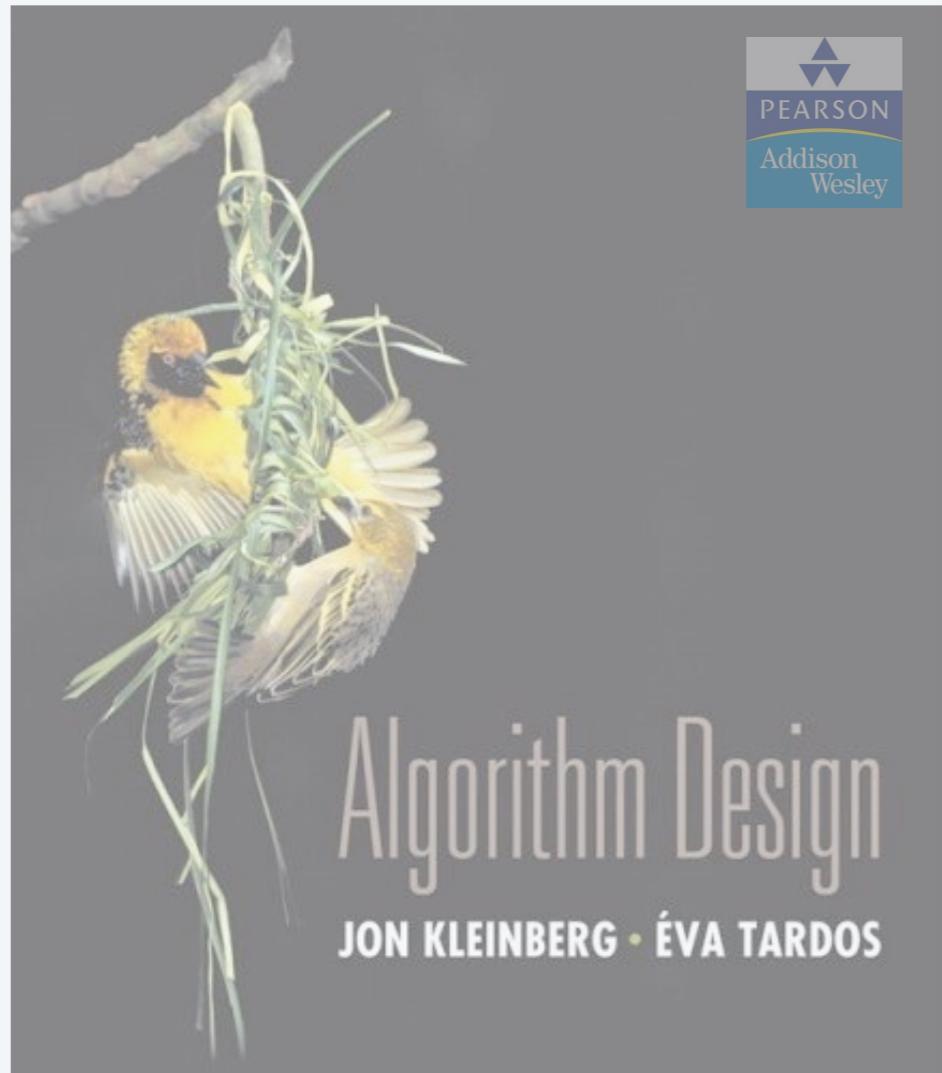
1. Introduction. The problem with which we shall be concerned relates to the following typical situation: A college is considering a set of n applicants of which it can admit a quota of only q . Having evaluated their qualifications, the admissions office must decide which ones to admit. The procedure of offering admission only to the q best-qualified applicants will not generally be satisfactory, for it cannot be assumed that all who are offered admission will accept. Accordingly, in order for a college to receive q acceptances, it will generally have to offer to admit more than q applicants. The problem of determining how many and which ones to admit requires some rather involved guesswork. It may not be known (a) whether a given applicant has also applied elsewhere; if this is known it may not be known (b) how he ranks the colleges to which he has applied; even if this is known it will not be known (c) which of the other colleges will offer to admit him. A result of all this uncertainty is that colleges can expect only that the entering class will come reasonably close in numbers to the desired quota, and be reasonably close to the attainable optimum in quality.



Do all executions of Gale-Shapley lead to the same stable matching?

- A. No, because the algorithm is nondeterministic.
- B. No, because an instance can have several stable matchings.
- C. Yes, because each instance has a unique stable matching.
- D. Yes, even though an instance can have several stable matchings and the algorithm is nondeterministic.





1. STABLE MATCHING

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- ▶ *Gale–Shapley algorithm*
- ▶ *hospital optimality*
- ▶ *context*

SECTION 1.1

Understanding the solution

For a given problem instance, there may be several stable matchings.

	1 st	2 nd	3 rd
A	X	Y	Z
B	Y	X	Z
C	X	Y	Z

	1 st	2 nd	3 rd
X	B	A	C
Y	A	B	C
Z	A	B	C

an instance with two stable matchings: $S = \{ A-X, B-Y, C-Z \}$ and $S' = \{ A-Y, B-X, C-Z \}$

Understanding the solution

Def. Student s is a **valid partner** for hospital h if there exists any stable matching in which h and s are matched.

Ex.

- Both X and Y are valid partners for A.
- Both X and Y are valid partners for B.
- Z is the only valid partner for C.

	1 st	2 nd	3 rd
A	X	Y	Z
B	Y	X	Z
C	X	Y	Z

	1 st	2 nd	3 rd
X	B	A	C
Y	A	B	C
Z	A	B	C

an instance with two stable matchings: $S = \{ A-X, B-Y, C-Z \}$ and $S' = \{ A-Y, B-X, C-Z \}$

Stable matching: quiz 3



Who is the best valid partner for W in the following instance?

A.

6 stable matchings

{ A-W, B-X, C-Y, D-Z }

B.

{ A-X, B-W, C-Y, D-Z }

C.

{ A-X, B-Y, C-W, D-Z }

D.

{ A-Z, B-W, C-Y, D-X }

{ A-Z, B-Y, C-W, D-X }

{ A-Y, B-Z, C-W, D-X }

	1 st	2 nd	3 rd	4 th
A	Y	Z	X	W
B	Z	Y	W	X
C	W	Y	X	Z
D	X	Z	W	Y

	1 st	2 nd	3 rd	4 th
W	D	A	B	C
X	C	B	A	D
Y	C	B	A	D
Z	D	A	B	C

Understanding the solution

Def. Student s is a **valid partner** for hospital h if there exists any stable matching in which h and s are matched.

Hospital-optimal assignment. Each hospital receives best valid partner.

- Is it a perfect matching?
- Is it stable?

Claim. All executions of Gale–Shapley yield **hospital-optimal** assignment.

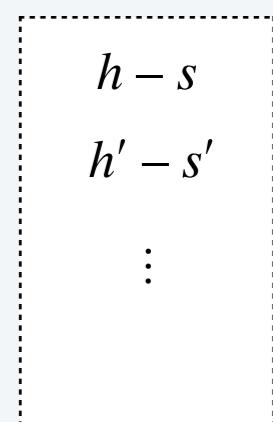
Corollary. Hospital-optimal assignment is a stable matching!

Hospital optimality

Claim. Gale–Shapley matching M^* is hospital-optimal.

Pf. [by contradiction]

- Suppose a hospital is matched with student other than best valid partner.
- Hospitals propose in decreasing order of preference.
 \Rightarrow some hospital is rejected by a valid partner during Gale–Shapley
- Let h be first such hospital, and let s be the first valid partner that rejects h .
- Let M be a stable matching where h and s are matched.
- When s rejects h in Gale–Shapley, s forms (or re-affirms) commitment to a hospital, say h' .
 \Rightarrow **s prefers h' to h .**
- Let s' be partner of h' in M .
- h' had not been rejected by any valid partner (including s') at the point when h is rejected by s . \leftarrow because this is the first rejection by a valid partner
- Thus, h' had not yet proposed to s' when h' proposed to s .
 \Rightarrow **h' prefers s to s' .**
- Thus, $h'-s$ is unstable in M , a contradiction. ■



stable matching M

Student pessimality

Q. Does hospital-optimality come at the expense of the students?

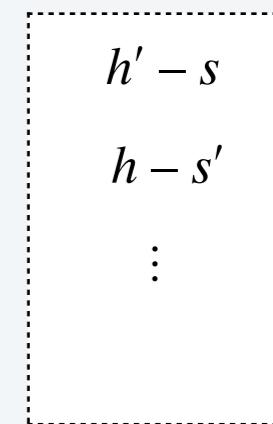
A. Yes.

Student-pessimal assignment. Each student receives worst valid partner.

Claim. Gale–Shapley finds **student-pessimal** stable matching M^* .

Pf. [by contradiction]

- Suppose $h-s$ matched in M^* but h is not the worst valid partner for s .
- There exists stable matching M in which s is paired with a hospital, say h' , whom s prefers less than h .
 $\Rightarrow s$ prefers h to h' .
- Let s' be the partner of h in M .
- By hospital-optimality, s is the best valid partner for h .
 $\Rightarrow h$ prefers s to s' .
- Thus, $h-s$ is an unstable pair in M , a contradiction. ▀

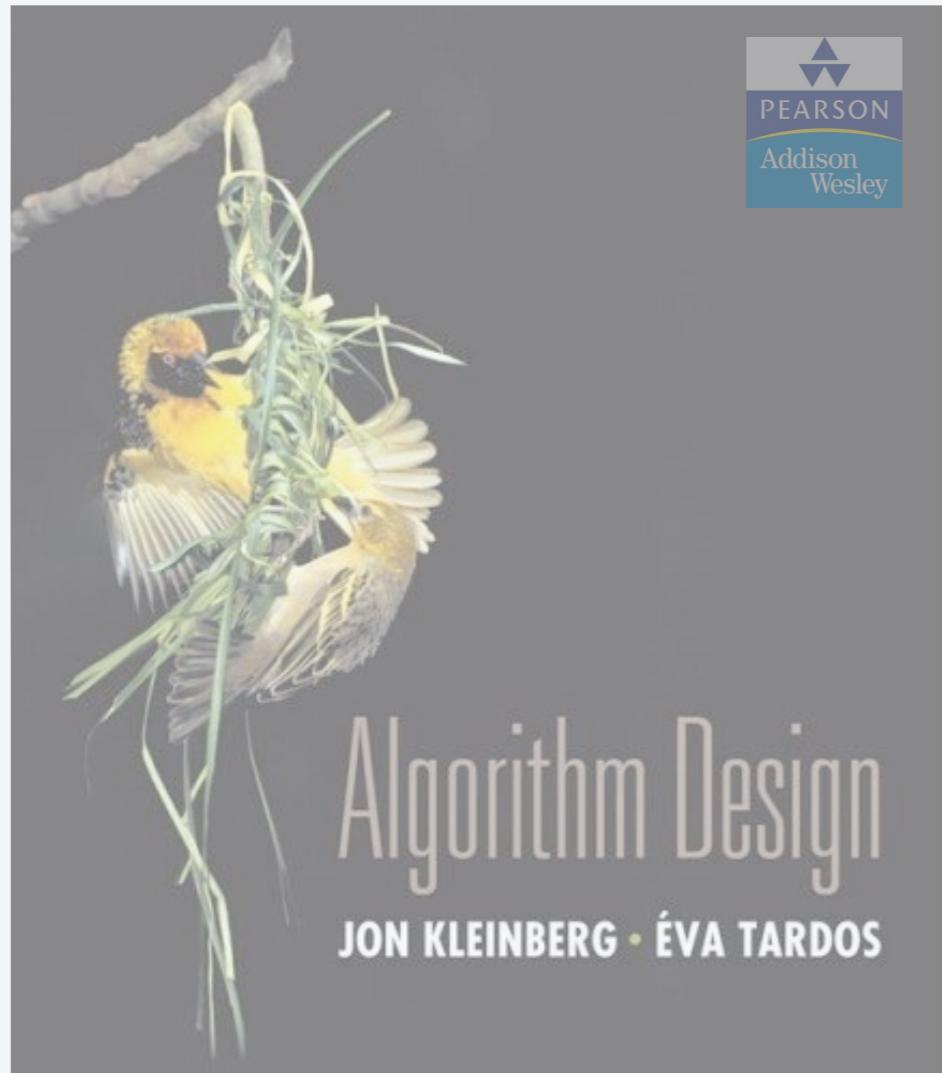




Suppose each agent knows the preference lists of every other agent before the hospital propose-and-reject algorithm is executed.

Which is true?

- A. No hospital can improve by falsifying its preference list.
- B. No student can improve by falsifying their preference list.
- C. Both A and B.
- D. Neither A nor B.



SECTION 1.1

1. STABLE MATCHING

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- ▶ ***context***

Historical context

National resident matching program (NRMP).

- Centralized clearinghouse to match med-school students to hospitals.
- Began in 1952 to fix unraveling of offer dates.
- Originally used the “Boston Pool” algorithm.
- Algorithm overhauled in 1998.
 - med-school student optimal
 - deals with various side constraints
(e.g., allow couples to match together)

hospitals began making offers earlier and earlier, up to 2 years in advance

stable matching no longer guaranteed to exist

The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design

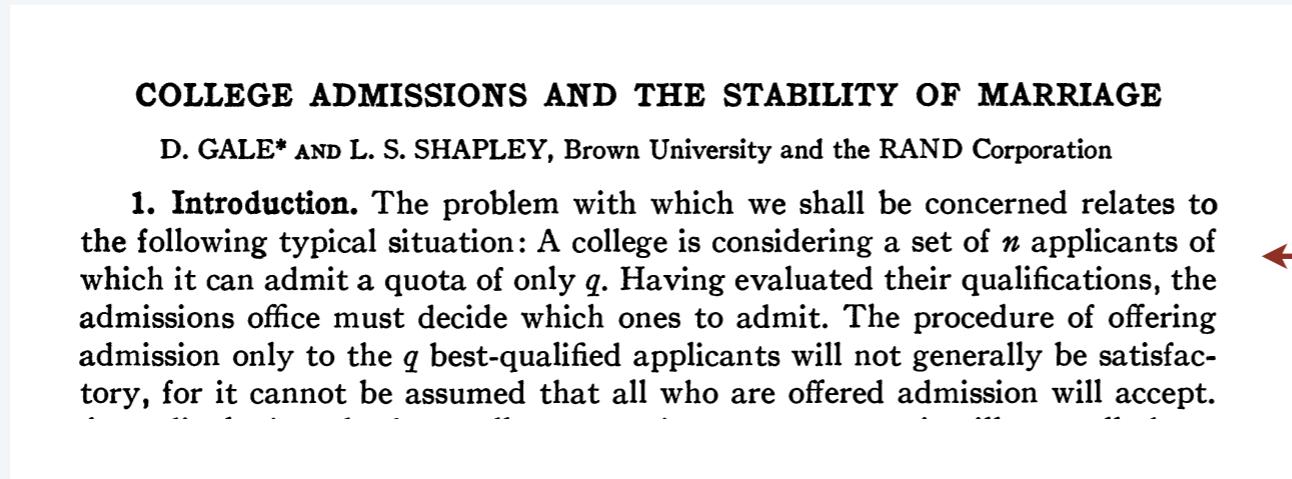
By ALVIN E. ROTH AND ELLIOTT PERANSON*

We report on the design of the new clearinghouse adopted by the National Resident Matching Program, which annually fills approximately 20,000 jobs for new physicians. Because the market has complementarities between applicants and between positions, the theory of simple matching markets does not apply directly. However, computational experiments show the theory provides good approximations. Furthermore, the set of stable matchings, and the opportunities for strategic manipulation, are surprisingly small. A new kind of “core convergence” result explains this; that each applicant interviews only a small fraction of available positions is important. We also describe engineering aspects of the design process. (JEL C78, B41, J44)



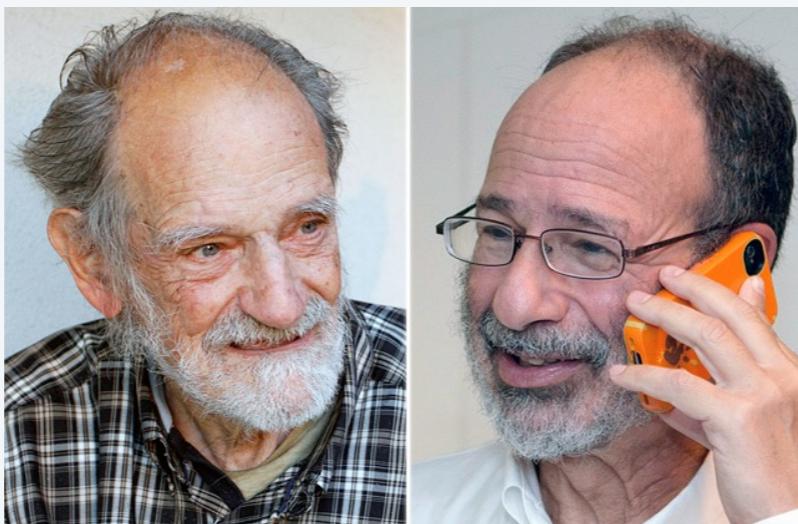
2012 Nobel Prize in Economics

[Lloyd Shapley](#). Stable matching theory and Gale–Shapley algorithm.



original applications:
college admissions and
opposite-sex marriage

[Alvin Roth](#). Applied Gale–Shapley to matching med-school students with hospitals, students with schools, and organ donors with patients.



Lloyd Shapley

Alvin Roth



Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Atlanta proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Wayne

Wayne accepts
(since previously unmatched)

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Yolanda

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Yolanda
Yolanda accepts
(since previously unmatched)

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Chicago proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Chicago proposes to Wayne

**Wayne accepts
(and renounces Atlanta)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Atlanta proposes to Val

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Val

Val accepts
(since previously unmatched)

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Val

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Val

**Val rejects
(since she prefers Atlanta)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Yolanda

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Yolanda

**Yolanda accepts
(and renounces Boston)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Wayne

**Wayne rejects
(since he prefers Chicago)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Val

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Val

**Val rejects
(since she prefers Atlanta)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Xavier

**Xavier accepts
(since previously unmatched)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

El Paso proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Wayne

**Wayne rejects
(since he prefers Chicago)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

El Paso proposes to Yolanda

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Yolanda

**Yolanda accepts
(and renounces Detroit)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Xavier

**Xavier rejects
(since he prefers Boston)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Wayne

**Wayne rejects
(since he prefers Chicago)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Zeus

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Zeus

**Zeus accepts
(since previously unmatched)**

Gale-Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

**STOP
(stable matching)**

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta