University of Iowa	<u>Homework 4</u>	CS 3330
Department of CS	Fall 2024	

Assigned: Friday 11:59 PM, September 20, 2024
Due: Friday 11:59 PM, September 27, 2024
Reading: Kleinberg and Tardos, Chapters 4.1-4.2, Slides of Week 4

1. [**A Human Compiler**] A computer scientist must understand how algorithms work. The following algorithm minimizes the maximum lateness of n jobs, where job j requires t_j units of processing time and is due at time d_j. Complete the algorithm for the given example and fill in the blanks.

Data: n, $t_1, t_2, \dots, t_n$, $d_1, d_2, \dots, d_n$
Result: Intervals $[s_1, f_1], [s_2, f_2], \dots, [s_n, f_n]$
Sort jobs by due dates and renumber so that $d_1 \leq d_2 \leq \cdots \leq d_n$;
$t \leftarrow 0$;
for $j = 1$ **to** n **do**
  Assign job j to interval $[t, t + t_j]$;
  $s_j \leftarrow t$;
  $f_j \leftarrow t + t_j$;
  $t \leftarrow t + t_j$;
end
return Intervals $[s_1, f_1], [s_2, f_2], \dots, [s_n, f_n]$;

(a) [**10 points**] Suppose you have the following 4 jobs with their respective processing times and due dates:

Job	Processing Time(t_j)	Due Date(d_j)
1	3	9
2	2	8
3	4	15
4	1	6

Now, fill out the following table:

Iteration (j)	Selected Job	Start Time(s_j)	Finish Time(f_j)	t after iteration
1				
2				
3				
4				

2. [**Ambitious Game Development**]: An *ambitious* game studio named *Electronic Darts* plans to develop m games $\{G_1, G_2, \dots, G_m\}$. The development of game G_i requires h_i hours.

(a) [**5 points**] Assuming the company decides to release the games sequentially, starting with G_1 and ending with G_m, what is the average release time of the games? (Hint: it is **not** simply $(1/n) \sum_i h_i$.)

(b) **[10 points]** If the company aims to minimize the average release time, what is the optimal strategy? The company can start developing a new game only if no other game is currently in development.

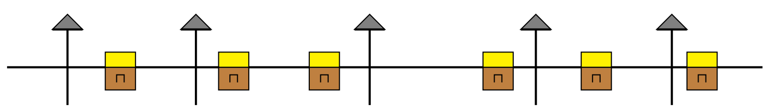
(c) **[15 points]** Suppose the company can only start the development of a game after its official announcement, which occurs at time t_i for game i. The company is allowed to temporarily halt the development of one game and resume it later. How should the company manage the development process to minimize the average release time?

3. **[Greedy Treasure Hunters]**: In the distant land of "Treasure Valley", a group of explorers is tasked with collecting all the treasures scattered across the valley. The valley is a long, narrow stretch of land where treasures are scattered at various points along its length.

   Digging up treasures can take days to complete, so the explorers need to establish stations. After a station is built, an explorer is assigned to search a 3-mile radius around the station to find all the treasures. These stations may have overlapping coverage areas.

   (a) **[10 points]** Establishing stations is costly. Suppose you are given a treasure map that marks the exact locations of the treasures. Develop a linear-time algorithm, in terms of the number of treasures, that minimizes the number of stations required to cover all the treasures.

   (b) **[15 points]** Despite your optimal suggestion, the explorers decided to follow their own instincts and constructed the stations without consulting you! After setting up the stations, they discovered that some of them overlap in coverage. The explorers soon realize that their overlapping efforts are unnecessary and costly. They now want to minimize the number of explorers while still ensuring that all treasures are collected. Develop a linear-time algorithm, in terms of the number of treasures and the number of stations already established, that determines which stations should be staffed by explorers and which can be left vacant.



4. **[Space Battlecruiser]**: Your mission is to prepare a fleet of n defense ships to protect the Battlecruiser. Each ship requires two stages of preparation: hull construction followed by targeting system calibration.

   For the i'th ship, hull construction takes c_i seconds, and calibrating the targeting system takes an additional a_i seconds. A ship is combat-ready and can be deployed to defend the Battlecruiser only when both hull construction and targeting system calibration are completed.

   (a) [10 points] How long will it take to assemble the entire fleet? What is the wait time for ship i before it is combat-ready, assuming it is constructed after the first $i-1$ ships?

   (b) [10 points] You are tasked with organizing the assembly process to minimize the average wait time before each ship is ready to defend the Battlecruiser. What is the optimal scheduling algorithm to achieve this objective? Provide a justification for your answer.

   (c) [15 points] A new calibration technology has arrived that automatically calibrates the targeting system immediately after hull construction. It still takes a_i seconds to calibrate the targeting system for ship i, but this process no longer requires human labor and can be performed simultaneously across multiple ships. What is the optimal scheduling algorithm that minimizes the average time before each ship is ready to defend the Battlecruiser using this new technology?