University of Iowa | Homework 10 | CS 3330
Department of CS | Fall 2024

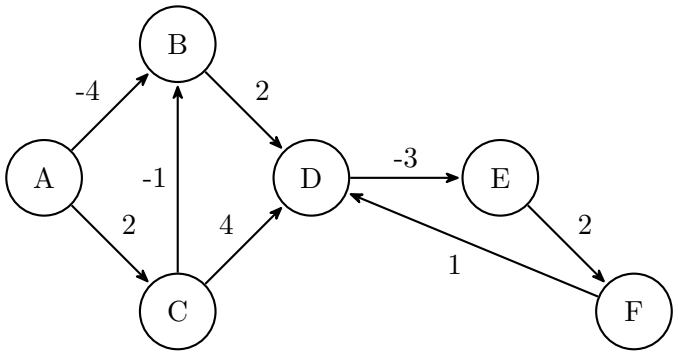
Assigned: Friday 11:59 PM, November 22, 2024
Due: Friday 11:59 PM, December 6, 2024
Reading: Kleinberg and Tardos, Chapters 6, Slides of Week 11 and 12

1. **[A Human Compiler]**: A computer scientist must understand how algorithms work. Compile each of the following algorithms.

   (a) **[10 points]** Execute the "Hirschberg's algorithm" to find the edit distance and an optimal alignment, between the words "Plain" and "Plan", assuming the cost of gap is $\delta = 2$ and the cost of misalignment is $\alpha = 1$. (The order of calls matter!)

	Input Strings	Output
1st Call	$\{p, l, a, i, n\}$ $\{p, l, a, n\}$	alignment:$\{p \leftrightarrow p, l \leftrightarrow l, a \leftrightarrow a, i \leftrightarrow \_, n \leftrightarrow n\}$

   (b) **[10 points]** Execute the "Bellman-Ford-Moore algorithm" on the following graph, showing the vectors $d[v]$ and $successor[v]$ at the beginning of each iteration. The destination node is $t = F$.



	$d([A],[B],[C],[D],[E],[F])$	$successor([A],[B],[C],[D],[E],[F])$
$i = 1$	$(\infty, \infty, \infty, \infty, \infty, 0)$	$(null, null, null, null, null, null)$

2. **[The Teacher 3]**: In the game "Teacher 3", the protagonist, Geralt of Oblivia, battles demons. We are nearing the completion of the game, with only one boss fight remaining.

   Before facing the final boss, Geralt must confront n waves of monsters. Before each wave, he can purchase health potions to prepare for the upcoming fight. Any potions not used in one wave can be saved for subsequent waves. Monsters drop coins, which Geralt can use to buy more potions.

   After researching online, we have gathered information about what to expect during each fight. In wave i, Geralt needs d_i health potions to survive the wave. The cost of a potion before wave i is denoted by c_i, and during wave i, the monsters will drop p_i coins in total.

   Our objective is to acquire enough potions before each wave to ensure Geralt's survival while minimizing the total amount of coins spent on the potions. The cost of potions before each wave depends on whether you play the game in the easy setting or the hard setting. Suppose that before the first wave, Geralt has no health potions, and he has P coins to purchase some. Note that if P is insufficient, it might be impossible for him to survive all the waves.

number of health potions needed to pass wave i	d_i
coins needed to buy a potion before wave i	c_i
coins dropped by monsters during wave i	p_i
initial coins in pocket	P
initial health potions in backpack	0

(a) **[Easy Mode]**: In this scenario, the cost of a health potion decreases after each wave, i.e., $c_1 > c_2 > \cdots > c_n$.

i. **[5 points]** What is the best strategy to ensure Geralt's survival while optimizing the total remaining coin balance at the end of all waves? The complexity of your algorithm should be linear.

ii. **[5 points]** Argue that your algorithm is optimal.

(b) **[Hard Mode]**: In this scenario, the cost of a health potion does not follow a particular pattern.

i. **[5 points]** To solve the problem, we have to first pick a problem structure. It's important to note that your decision before each wave depends on three factors: the amount of money you have, the quantity of health potions you have, and the number of waves that have passed. Define a suitable OPT for the problem, taking into consideration these three factors.

ii. **[5 points]** What is the goal of the problem, in terms of OPT?

iii. **[15 points]** Write down the Bellman equation for OPT. Describe your equation.

iv. **[5 points]** What is the complexity of your algorithm? Justify your answer.

3. **[Cyberjunk]**: The new car released by the multi-billionaire Elon Tusk's company, named Cyberjunk, has shown great promise and has become extremely popular on Tooth-Social. This marvelous piece of technology can drive many miles on a single charge and tow multiple cars simultaneously! However, the price of the car is a bit too steep for a university professor's salary, so I need your help to optimize my travel plan to Los Angeles at the end of the semester.

Suppose that on the route from Iowa City to Los Angeles, there are charging stations located at $x_1 < x_2 < \cdots < x_n$, where the cost of charging at station i located at x_i is c_i dollars per unit. The car can take up to C units of battery charge. We start with a fully charged battery. The battery usage for traveling from location x_i to location x_j is given by $x_j - x_i$.

I need your assistance to minimize my costs and perhaps even make a little profit!

(a) **[10 points]** Suppose my goal is to reach Los Angeles while minimizing my spending on recharging the car. After each stop, I either fully charge the car or proceed to the next station. Formulate the problem by defining the appropriate OPT and write down a recursive relation for OPT. What is the time and space complexity of your algorithm?

(b) **[15 points]** Following the same setup as the previous part, this time I want the flexibility to charge the car as much as needed after each stop. What are the optimal actions after each stop? Formulate the problem by defining the appropriate OPT and write down a recursive relation for OPT. What is the time and space complexity of your algorithm?

(c) **[15 points]** Since maintaining the car is expensive, I'm considering offering a ride to a hitchhiker after each stop. Adding a passenger will double the battery consumption! A hitchhiker at station i will drop off at station $i + 1$ and is willing to pay p_i for the ride. Formulate the problem by defining the appropriate OPT and write down a recursive relation for OPT. What is the time and space complexity of your algorithm?