

Let $X = (10 \ 10 \ 10)$

Translate point 20 along positive Y axis

$$x' = 10 * 1$$

$$y' = 10 * 30$$

$$z' = 10 * 1$$

$$X' = \begin{bmatrix} 10 \\ 30 \\ 10 \end{bmatrix}$$

Rotate point around Z axis 30 deg

$$x' = \cos(30) * 10 - \sin(30) * 10$$

$$y' = \sin(30) * 10 + \cos(30) * 10$$

$$z' = 10 * 1$$

$$X' = \begin{bmatrix} -6.33 \\ 30.98 \\ 10 \end{bmatrix}$$

Translate point 20 along positive Y axis

$$x' = \cos(30) * 10 + \sin(30) * 10$$

$$y' = 10 * 1$$

$$z' = -\sin(30) * 10 + \cos(30) * 10$$

$$X' = \begin{bmatrix} 3.66 \\ 10 \\ 3.66 \end{bmatrix}$$

Let $X = (10 \ 10 \ 10 \ 1)$

Translate point 20 along positive Y axis

$$X' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 10 \\ 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} 10 \\ 30 \\ 10 \\ 1 \end{bmatrix}$$

$$X'_{euc} = \begin{bmatrix} 10 \\ 30 \\ 10 \end{bmatrix}$$

Rotate point around Z axis 30 deg

$$X' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos(30) & -\sin(30) & 0 & 0 \\ \sin(30) & \cos(30) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 10 \\ 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} 3.66 \\ 13.66 \\ 10 \\ 1 \end{bmatrix}$$

$$X'_{euc} = \begin{bmatrix} 3.66 \\ 13.66 \\ 10 \end{bmatrix}$$

Rotate point around Y axis -10 deg

$$X' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos(30) & 0 & \sin(30) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(30) & 0 & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 10 \\ 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} 3.66 \\ 10 \\ 3.66 \\ 1 \end{bmatrix}$$

$$X'_{euc} = \begin{bmatrix} 3.66 \\ 10 \\ 3.66 \end{bmatrix}$$

Performing the same operations to transform the coordinate frame

Translate point 20 along positive Y axis

$$X' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 10 \\ 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} 10 \\ 30 \\ 10 \\ 1 \end{bmatrix}$$

$$X'_{euc} = \begin{bmatrix} 10 \\ 30 \\ 10 \end{bmatrix}$$

Rotate point around Z axis 30 deg

$$X' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos(30) & -\sin(30) & 0 & 0 \\ \sin(30) & \cos(30) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 30 \\ 10 \\ 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} -6.33 \\ 30.98 \\ 10 \\ 1 \end{bmatrix}$$

$$X'_{euc} = \begin{bmatrix} -6.33 \\ 30.98 \\ 10 \end{bmatrix}$$

Rotate point around Y axis -10 deg

$$X' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos(30) & 0 & \sin(30) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(30) & 0 & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -6.33 \\ 30.98 \\ 10 \\ 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} 10.01 \\ 30.98 \\ 11.83 \\ 1 \end{bmatrix}$$

$$X'_{euc} = \begin{bmatrix} 10.01 \\ 30.98 \\ 11.83 \end{bmatrix}$$