## 第一题

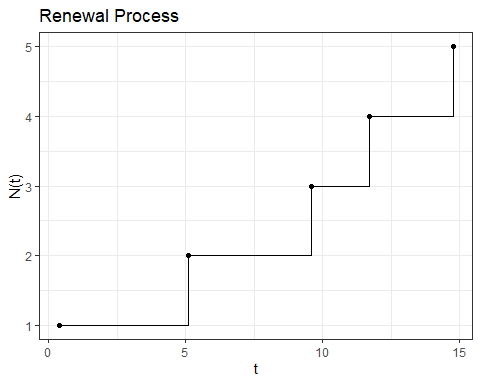
#后续代码不再加载包  
library(purrr)  
library(ggplot2)  
library(dplyr)

##   
## 载入程辑包：'dplyr'

## The following objects are masked from 'package:stats':  
##   
## filter, lag

## The following objects are masked from 'package:base':  
##   
## intersect, setdiff, setequal, union

library(tidyr)  
set.seed(1)  
X\_n <- rgamma(5,shape = 1,scale = 2.5)  
T\_n <- cumsum(X\_n)  
t <- c(0,3,7,10,12)  
N\_t <- map(t,function(t) T\_n<=t) %>%  
 map\_vec(sum)  
ggplot(data.frame(时刻=T\_n,次数=1:5)) +  
 geom\_step(aes(x=时刻,y=次数)) +  
 geom\_point(aes(x=时刻,y=次数)) +  
 xlab("t") +  
 ylab("N(t)") +  
 labs(title = "Renewal Process") +  
 theme\_bw()



result\_out <- function(order,name,q\_num,answers,answers\_names){  
 split\_1 <- paste(rep("=",25),collapse = "")  
 split\_2 <- paste(rep("-",20),collapse = "")  
 cat(split\_1,paste0(order,"th Simulation"),split\_1,"\n")  
 cat("Name:",name,"\n")  
 cat("\n")  
 for (i in 1:q\_num){  
 cat(split\_2,paste0("Question ",i),split\_2,"\n")  
 if (is.data.frame(answers[[i]])){  
 print(round(answers[[i]],1))  
 }else{  
 cat(answers\_names[[i]],": ",round(answers[[i]],1),"\n")  
 }  
 cat("\n")  
 }  
}  
options(digits = 2)  
result\_out(1,"孙浩杰",3,list(X\_n,T\_n,N\_t),c("Xn","Tn","N(t)"))

## ========================= 1th Simulation =========================   
## Name: 孙浩杰   
##   
## -------------------- Question 1 --------------------   
## Xn : 0.4 4.7 4.5 2.1 3.1   
##   
## -------------------- Question 2 --------------------   
## Tn : 0.4 5.1 9.6 12 15   
##   
## -------------------- Question 3 --------------------   
## N(t) : 0 1 2 3 4

## 第二题

由Poisson分布的可加性知，,即，则

F\_n <- function(t,n,lambda=2){  
 t <- floor(t);lambda <- n\*lambda  
 exp(-lambda)\*sum(lambda^(0:t)/factorial(0:t))  
}  
F\_n\_out <- map\_vec(c(3,6,9,12),F\_n,n=5)

利用得到分布列

P\_n <- function(t,n){  
 F\_n(t,n)-F\_n(t,n+1)  
}  
#分布列  
P\_data <- data.frame(t=rep(c(3,6,9,12),each=11),  
 n=rep(0:10,4)) %>%  
 rowwise() %>%  
 mutate(p=P\_n(t,n)) %>%  
 ungroup() %>%  
 pivot\_wider(names\_from = t,names\_glue = "P(N({t})=n)",values\_from = p)  
#更新函数  
M <- function(t){  
 data.frame(t=rep(t,each=101),#由于n过大时，分布函数趋近于0，考虑这一部分意义不大，故只取到100  
 n=0:100) %>%  
 rowwise() %>%  
 mutate(p=P\_n(t,n)) %>%  
 ungroup() %>%  
 summarise(M=sum(n\*p)) %>%  
 .$M  
}  
M\_t <- map\_vec(c(3,6,9,12),M)  
result\_out(3,"孙浩杰",3,list(F\_n\_out,P\_data,M\_t),c("Fn","分布列","M(t)"))

## ========================= 3th Simulation =========================   
## Name: 孙浩杰   
##   
## -------------------- Question 1 --------------------   
## Fn : 0 0.1 0.5 0.8   
##   
## -------------------- Question 2 --------------------   
## # A tibble: 11 × 5  
## n `P(N(3)=n)` `P(N(6)=n)` `P(N(9)=n)` `P(N(12)=n)`  
## <dbl> <dbl> <dbl> <dbl> <dbl>  
## 1 0 0.1 0 0 0   
## 2 1 0.4 0.1 0 0   
## 3 2 0.3 0.3 0.1 0   
## 4 3 0.1 0.3 0.2 0.1  
## 5 4 0 0.2 0.3 0.1  
## 6 5 0 0.1 0.2 0.2  
## 7 6 0 0 0.1 0.2  
## 8 7 0 0 0.1 0.2  
## 9 8 0 0 0 0.1  
## 10 9 0 0 0 0.1  
## 11 10 0 0 0 0   
##   
## -------------------- Question 3 --------------------   
## M(t) : 1.5 3 4.5 6

## 第三题

#沿用第二题的函数  
#Feller初等更新定理  
a1 <- map\_vec(c(0.5,1,10,100),M)/c(0.5,1,10,100)  
#Blackwell更新定理  
a2 <- map\_vec(c(2,4,6,8),function(a) M(0.5+a)-M(0.5))  
a3 <- map\_vec(c(2,4,6,8),function(a) M(100+a)-M(100))  
special\_out <- function(){  
 split\_1 <- paste(rep("=",25),collapse = "")  
 split\_2 <- paste(rep("-",20),collapse = "")  
 cat(split\_1,"3th Simulation",split\_1,"\n")  
 cat("Name:","孙浩杰","\n")  
 cat("\n")  
 cat(split\_2,"Question 1",split\_2,"\n")  
 cat("M(t)/t: ",a1,"\n")  
 cat("1/mu",0.5,"\n")  
 cat("\n")  
 cat(split\_2,"Question 2",split\_2,"\n")  
 cat("M(t+a)-M(t) at t=0.5: ",a2,"\n")  
 cat("M(t+a)-M(t) at t=100: ",a3,"\n")  
 cat("a/mu",1,2,3,4)  
}  
special\_out()

## ========================= 3th Simulation =========================   
## Name: 孙浩杰   
##   
## -------------------- Question 1 --------------------   
## M(t)/t: 0.31 0.52 0.5 0.5   
## 1/mu 0.5   
##   
## -------------------- Question 2 --------------------   
## M(t+a)-M(t) at t=0.5: 0.84 1.8 2.8 3.8   
## M(t+a)-M(t) at t=100: 1 2 3 4   
## a/mu 1 2 3 4