

# Multiscale Structural Analysis of Honeycomb Sandwich Structure Using Mechanics of Structure Genome

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## ABSTRACT

A new multiscale modeling approach based on mechanics of structure genome (MSG) has been proposed to analyze honeycomb sandwich structures. On the one hand, MSG can be used to homogenize a honeycomb core as a homogeneous solid to get effective elastic properties without various boundary conditions (BCs). The results are compared with those obtained by using finite element (FE) based numerical homogenization method and analytical formulas. Then a through-the-thickness constitutive modeling can be conducted by MSG to model the whole honeycomb sandwich structure as a laminate for equivalent plate properties (MSG two-step). On the other hand, MSG can directly give constitutive modeling of the whole honeycomb sandwich structure as an equivalent plate to obtain the equivalent plate properties (MSG direct). The numerical case shows that the equivalent plate properties provided by MSG direct are more accurate than traditional approach when compared with the direct numerical simulation (DNS) and much more efficient than the DNS.

## INTRODUCTION

Honeycomb sandwich structures are widely used in many fields of industry such as aerospace, automotive and construction. This type of structure has a low-density honeycomb cellular solid, i.e. the core, between two stiffer thin plates, i.e. the face sheets. As the height of the core increases, the moment of inertia of the plates also increases with slight increasing in weight because of its cellular structure, like I beam. This feature makes them efficient for resisting bending and buckling loads while being lightweight.

Many researches have been done to analyze honeycomb sandwich structures to provide guides for both design and manufacture. Theoretically, one can use three-dimensional (3D) detailed finite element analysis (FEA) to conduct the analysis. However, the computation is too expensive to represent the geometry of a core and the laminated composite face sheets since a fine mesh is needed to make the aspect ratio of elements small. Alternatively, one can model a honeycomb sandwich structure as an equivalent plate, which reduces the 3D problem to be a two-dimensional (2D) problem. There are generally two approaches to model a honeycomb sandwich structure as an equivalent plate.

The first approach considers a honeycomb sandwich structure as three or more layers. Usually, the core is treated as a homogeneous single layer while the top and bottom face sheets can be treated as one layer or multiple layers. Then the equivalent properties of the sandwich structure are obtained by using the classical laminated plate theory (CLPT) [1] or other higher order theories. To treat the core as one layer, there are many researches dealing with evaluating the effective elastic properties of the core. Gibson and Ashby's book [2] is the first and most widely used one to give the analytical formulas for predicting the full set of elastic properties of a honeycomb core which is made of isotropic material. Here, the honeycomb core is modeled as an orthotropic continuum and the effective elastic properties are the nine independent engineering constants ( $E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, \nu_{12}, \nu_{13}, \nu_{23}$ ). Other studies use finite element (FE) based numerical homogenization method to determine the effective elastic properties of honeycomb core structures. For instant, [3] applied a displacement-based homogenization technique to determine the elastic properties of foam-filled honeycomb cores by using shell elements. [4] makes use of solid elements for numerical homogenization of a honeycomb core unit cell. Although the FE-based numerical homogenization of the honeycomb core has been widely adopted in the literature, they also show a common weakness, i.e. various boundary conditions (BCs) are needed to be properly applied to the unit cell to perform homogenization. However, these BCs need to be carefully chosen and they may change if the geometry of the unit cell changes. Thus, it is not practical and efficient for industry applications.

The second approach directly models a honeycomb sandwich structure as an equivalent plate to obtain the equivalent plate properties, instead of considering it as several layers first. The related researches are not rich in the literature. [5] uses the homogenization theory based on the asymptotic expansion method to compute the effective properties (membrane and bending) of corrugated core sandwich panels. The corrugated core considered in it includes straight, hat type, triangular and curvilinear. In [6], an equivalent classical plate model of corrugated structures is derived using the variational asymptotic method. However, the above two works did not consider honeycomb core shape.

Recently, Yu [7] introduced mechanics of structure genome (MSG) for multiscale modeling, which represents a unified approach for constitutive modeling for all types of composite structures including beams, plates/shells, and 3D structures. The companion code SwiftComp<sup>TM</sup> provides an efficient and accurate approach for modeling composite materials and structures. Honeycomb sandwich structures can be modeled as plates/shells using MSG.

This paper will present a new way for multiscale modeling of honeycomb sandwich structures using MSG. On the one hand, MSG can be used to provide the effective elastic properties of a honeycomb core to replace the core as a homogeneous single layer as shown in the left part of Figure 1, which can then be used to model the whole structure as a laminate and this could also be done by MSG as shown in the right part of Figure 1. We will call this modeling approach MSG two-step. On the other hand, MSG could directly give constitutive modeling of the whole honeycomb sandwich structures to obtain the equivalent plate properties including  $A$ ,  $B$ , and  $D$  matrices, in-plane properties and flexural properties as illustrated in Figure 2. We will call this modeling approach MSG direct.

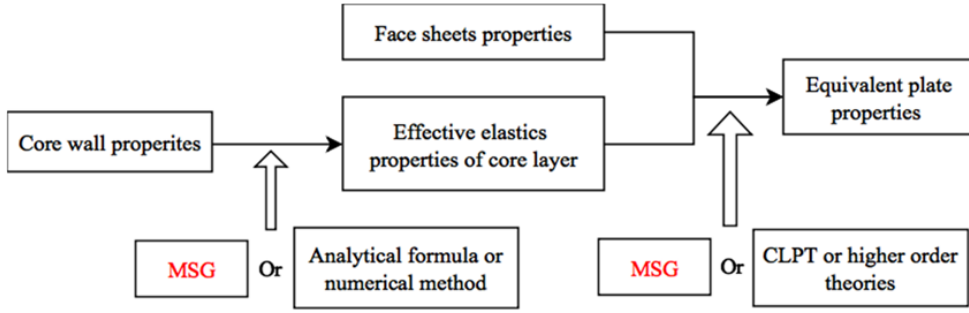


Figure 1 MSG two-step

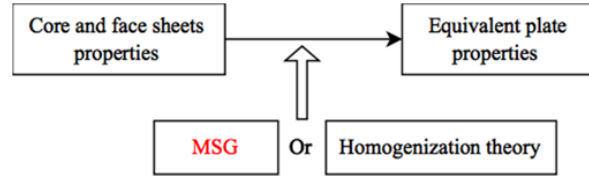


Figure 2 MSG direct

The paper is organized as follows: In the second section, we present a case study in literature to determine effective elastic properties of a honeycomb core by using MSG along with a comparison with the existing analytical and numerical models. In the third section, we research a case to determine the equivalent plate properties by two different ways of using MSG, which will be compared with traditional approach. In the fourth section, we will construct a numerical case for the macroscopic structural analysis, which will be compared with the MSG direct and traditional approach. The final section ends the paper with some concluding remarks.

## EFFECTIVE ELASTIC PROPERTIES OF A HONEYCOMB CORE

In this section, we will use MSG to obtain the effective elastic properties of a honeycomb core. There are two ways to achieve this using MSG. We can choose 2D or 3D structure genome (SG), defined as the smallest mathematical building block of a structure [7], based on the characteristics of the honeycomb core. Since the honeycomb features 2D heterogeneity, a 2D SG is sufficient for the analysis (see Figure 4). According to MSG, it can also be analyzed using a 3D SG (see Figure 5) to obtain the same results as using 2D SG. Although 3D SG is an unnecessary waste of computing resources, it is what has been commonly done in other FE-based numerical homogenization methods, also commonly called representative volume element (RVE) analysis.

The model is the same as in Catapano and Montemurro [4] for comparison. We also compare our results with those obtained by using analytical formulas of Burton and Noor [8]. Note, as many commercial honeycombs are made by expanding strip-glued sheets [2], the core has double thickness walls belonging to the ribbon direction (horizontal wall). The geometrical parameters of the honeycomb core are shown in Figure 3. Due to the symmetric feature of the core, only a quarter of the geometry is

shown.  $l_1$  is the length of the oblique wall,  $l_2$  represents the length of the horizontal wall,  $t_c$  is the thickness of the foil used to produce the core,  $\theta$  is the cell corrugation angle and  $h_c$  is the height of the core. The sizes of the core are given in TABLE I.

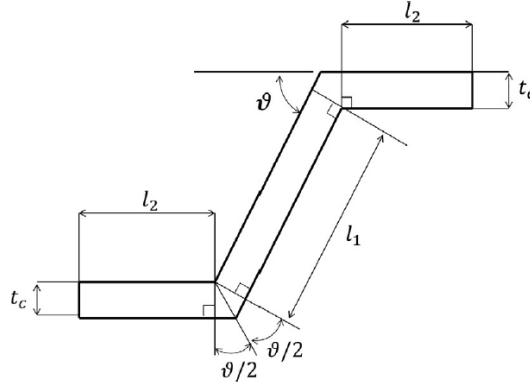


Figure 3 Geometrical parameters of the honeycomb core [4]

TABLE I SIZES OF THE CORE

$l_1$ (mm)	$l_2$ (mm)	$t_c$ (mm)	$\theta$ (deg)	$h_c$ (mm)
3.66	1.833	0.0635	60	20

The honeycomb core wall is made of aluminum alloy. The material properties of the aluminum alloy used for the honeycomb core are listed in TABLE II.

TABLE II MATERIAL PARAMETERS OF THE CORE

Elastic constant	Aluminum
$E$ (MPa)	70000
$\nu$	0.33
$\rho$ (kg/mm <sup>3</sup> )	$2.7 \times 10^{-6}$

The FE models for the 2D SG and 3D SG are shown in Figure 4 and Figure 5 respectively. Note, throughout this paper, the coordinate notation of  $x$ ,  $y$ , and  $z$  are the same as  $x_1$ ,  $x_2$ , and  $x_3$  respectively. The FE models are created in ANSYS by using APDL (ANSYS Parametric Design Language), in which all the material and geometric parameters are parameterized. We use 8-node plane element PLANE183 for 2D SG and 20-node solid element SOLID186 for 3D SG. The convergence studies show that the mesh pattern such as  $27 \times 2$  for the horizontal wall and  $54 \times 2$  for the oblique wall are suitable for homogenization by SwiftComp<sup>TM</sup>. The whole model of the 2D SG, meshed in this manner, consists of 3,356 nodes and 864 elements while 3D SG has 187,436 nodes and 34,560 elements. Note, both 2D SG and 3D SG have the same mesh pattern at the cross section of the core. There are 40 elements through the height of the core in 3D SG so the total element numbers of 3D SG are 40 times larger than that of 2D SG.

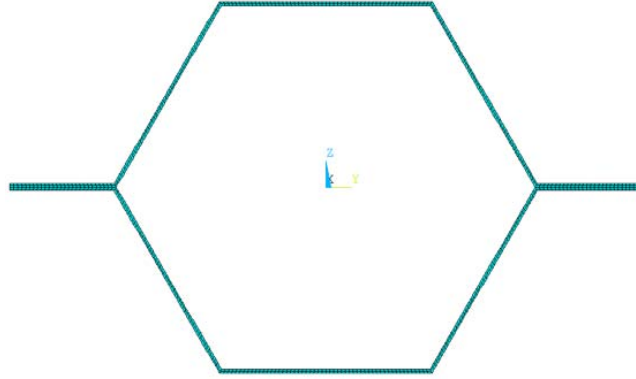


Figure 4 FE model of the 2D SG

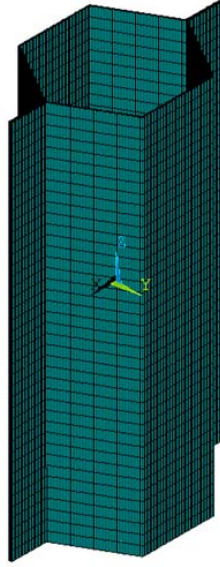


Figure 5 FE model of the 3D SG

Unlike what is usually done in the literature, where various BCs are imposed to the RVE to obtain stress and strain fields for the calculation of effective elastic properties, in this work no BCs are needed. This is very convenient if we want to study an optimization problem of the honeycomb core, in which we only need to change the material and geometric parameters without worrying about various BCs for each iteration.

The effective elastic properties of the honeycomb core and comparison are listed in TABLE III. MSG 2D obtained the exact same results as MSG 3D so only the results of MSG 2D are presented in the table. The relative differences between MSG 2D with [4] and [8] are presented in the column right beside [4] and [8] respectively. Note, Burton and Noor [8] only give lower bound (LB) and upper bound (UB) for  $G_{13}$ .

TABLE III EFFECTIVE ELASTIC PROPERTIES OF THE HONEYCOMB CORE

Properties	MSG 2D	Catapano and Montemurro [4]	Relative difference	Burton and Noor [8]	Relative difference
$E_1$ (MPa)	0.884	0.884	0.03%	0.840	5.30%
$E_2$ (MPa)	0.918	0.918	0.05%	0.839	9.42%
$E_3$ (MPa)	1812.381	1812.299	0.00%	1866.755	2.91%
$G_{12}$ (MPa)	0.565	0.640	11.76%	0.504	12.03%
$G_{23}$ (MPa)	262.647	262.981	0.13%	263.170	0.20%
$G_{13}$ (MPa)	384.505	390.833	1.62%	157.902 (LB) 438.617 (UB)	-
$\nu_{12}$	0.980	0.980	0.02%	1.000	1.95%
$\nu_{23}$	$0.167 \times 10^{-3}$	$0.167 \times 10^{-3}$	0.14%	$0.148 \times 10^{-3}$	12.71%
$\nu_{13}$	$0.161 \times 10^{-3}$	$0.161 \times 10^{-3}$	0.01%	$0.148 \times 10^{-3}$	8.46%
$\rho$ (kg/mm <sup>3</sup> )	$6.991 \times 10^{-8}$	$6.990 \times 10^{-8}$	0.01%	$7.200 \times 10^{-8}$	2.91%

The results of MSG 2D generally agree with those by [4]. Particularly, concerning the evaluation of the Young's modulus, Poisson's ratio and density, the relative differences are less than 0.14% for all the quantities. For shear modulus, there are some differences. For  $G_{23}$ , the difference is low: 0.13%. However, the difference is 1.62% for  $G_{13}$  and 11.76% for  $G_{12}$ . The possible explanation for the difference may be in the use of BCs in [4]. [4] uses uniform displacement constraints applied to the boundary of the FE model of the RVE for BCs as given in TABLE IV and TABLE V. As mentioned in [9], the first three load cases in TABLE IV correspond to periodic boundary conditions (PBCs). However, the fourth, fifth and sixth load cases in TABLE V are not PBCs. It is pointed out in [9], the BCs must be enforced by using coupling constraint equations (CE) because of the lack of symmetry of the loads for the fourth, fifth and sixth load cases. Since [4] still uses uniform displacement constraint as BCs for the fourth, fifth and sixth load cases as shown in TABLE V, the results of shear modulus calculated by [4] are not the same as the results calculated by PBCs. Nevertheless, it is noted that PBCs are the best BCs to use for the RVE analysis [10] and SwiftComp<sup>TM</sup> consistently maintains excellent agreement with results by PBCs as examples given in [11].

TABLE IV BCs FOR THE FE MODEL OF THE RVE IN [4]: 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> STATIC ANALYSIS

	1 <sup>st</sup> load case			2 <sup>nd</sup> load case			3 <sup>rd</sup> load case		
Nodes	$U_1$	$U_2$	$U_3$	$U_1$	$U_2$	$U_3$	$U_1$	$U_2$	$U_3$
$x_1 = 0$	0	Free	Free	0	Free	Free	0	Free	Free
$x_1 = a_1$	$u$	Free	Free	0	Free	Free	0	Free	Free
$x_2 = 0$	Free	0	Free	Free	0	Free	Free	0	Free
$x_2 = a_2$	Free	0	Free	Free	$u$	Free	Free	0	Free
$x_3 = 0$	Free	Free	0	Free	Free	0	Free	Free	0
$x_3 = a_3$	Free	Free	0	Free	Free	0	Free	Free	$u$

TABLE V BCs FOR THE FE MODEL OF THE RVE IN [4]: 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup> STATIC ANALYSIS

	4 <sup>th</sup> load case	5 <sup>th</sup> load case	6 <sup>th</sup> load case
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Nodes	$U_1$	$U_2$	$U_3$	$U_1$	$U_2$	$U_3$	$U_1$	$U_2$	$U_3$
$x_1 = 0$	0	Free	Free	Free	0	0	Free	0	0
$x_1 = a_1$	0	Free	Free	Free	0	0	Free	0	0
$x_2 = 0$	0	Free	0	Free	0	Free	0	Free	0
$x_2 = a_2$	0	Free	0	Free	0	Free	$u$	0	0
$x_3 = 0$	0	0	Free	0	0	Free	Free	Free	0
$x_3 = a_3$	0	$u$	0	$u$	0	0	Free	Free	0

Finally, both MSG 2D and [4] are slightly different from the results of analytically formulas by [8].  $G_{13}$  by MSG 2D is within the LB and UB of [8]. For  $G_{23}$ , the difference between MSG 2D and [8] is low: about 0.2%. The main reason for the difference of other components are in the use of shell-based theories as discussed in [4].

## EQUIVALENT PLATE PROPERTIES OF A HONEYCOMB SANDWICH STRUCTURE

In this section, we will use MSG to obtain the equivalent plate properties of a honeycomb sandwich structure. As shown in Figure 1 and Figure 2, there are two approaches to fulfill this purpose. On the one hand, we can use MSG two-step and the effective elastic properties of the core have been calculated in the previous section. On the other hand, we can use MSG direct to directly get the equivalent plate properties.

The material properties and the geometry of the core are the same as the previous section. The material of the face sheets is Carbon/Epoxy. The material properties (orthotropic) and the geometry of the face sheets are listed in TABLE VI [12].

TABLE VI THE MATERIAL PROPERTIES AND THE GEOMETRY OF THE FACE SHEETS

$E_1 = 181000 \text{ MPa}$	$G_{12} = 7170 \text{ MPa}$	$\nu_{12} = 0.28$	$\rho = 1.58 \times 10^{-6} \text{ kg/mm}$
$E_2 = 10300 \text{ MPa}$	$G_{13} = 7170 \text{ MPa}$	$\nu_{13} = 0.28$	$t = 0.125 \text{ mm}$
$E_3 = 10300 \text{ MPa}$	$G_{23} = 3456.37 \text{ MPa}$	$\nu_{23} = 0.49$	[45/-45]

For MSG two-step, the effective elastic properties of the core have been obtained by MSG 2D as listed in TABLE III. We continue to use MSG to conduct a through-the-thickness constitutive modeling of the whole sandwich structure as a laminate. Here we use 1D SG as shown in Figure 6. Then by MSG-based plate theory, we obtain the equivalent plate properties including  $A$ ,  $B$ ,  $D$  matrices, in-plane properties and flexural properties.



Figure 6 FE model of the MSG two-step

For MSG direct, we use 3D SG as the whole honeycomb sandwich structure features 3D heterogeneity. The FE model of the 3D SG is shown in Figure 7. We use the SOLID186 element for the 3D SG. The whole model of the 3D SG consists of 67,838 nodes and 10,880 elements. After homogenization, we obtained the  $A$ ,  $B$ ,  $D$  matrices, in-plane properties and flexural properties.

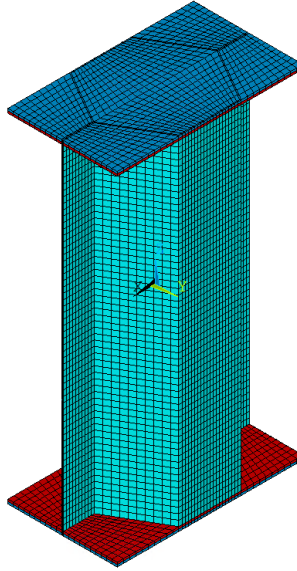


Figure 7 FE model of the MSG direct

To compare the results, we also use the effective elastic properties by [4] and CLPT to obtain the equivalent plate properties. The detailed procedure of using CLPT to obtain equivalent plate properties can be found in [13]. We will call this approach by traditional approach.

The results of MSG direct, MSG two-step and traditional approach are compared and listed in TABLE VII and TABLE VIII. Note, we only compare in-plane properties



and flexural properties since they are derived from  $A$  and  $D$  matrices. Also,  $B$  matrix is zero for all the approaches. The relative differences between MSG direct with MSG two-step and traditional approach are presented in the column right beside MSG two-step and traditional approach respectively.

TABLE VII COMPARISON OF IN-PLANE PROPERTIES

In-plane properties	MSG direct	MSG two-step	Relative difference	Traditional approach	Relative difference
$E_1^I$ (MPa)	663.592	626.555	5.91%	626.555	5.91%
$E_2^I$ (MPa)	665.735	631.133	5.48%	631.133	5.48%
$G_{12}^I$ (MPa)	1145.493	1136.914	0.75%	1136.987	0.75%
$\nu_{12}^I$	0.793	0.793	0.09%	0.793	0.09%

TABLE VIII COMPARISON OF FLEXURAL PROPERTIES

Flexural properties	MSG direct	MSG two-step	Relative difference	Traditional approach	Relative difference
$E_1^f$ (MPa)	1906.296	1805.548	5.58%	1805.548	5.58%
$E_2^f$ (MPa)	1899.276	1810.468	4.91%	1810.468	4.91%
$G_{12}^f$ (MPa)	3351.383	3326.769	0.74%	3326.838	0.74%
$\nu_{12}^f$	0.759	0.764	0.64%	0.764	0.64%

We can see that the results of MSG two-step are almost the same as traditional approach. This is due to the fact that MSG-based plate theory can reproduce the plate properties computed by CLPT when the structure is made of homogeneous layers. The only difference is the effective elastic properties obtained from the previous section. Since, in the previous section, the results obtained by MSG 2D are almost the same as [4], the results of equivalent plate properties are very close.

We also notice that there are some slight differences between MSG direct and MSG two-step. For instant, the relative difference of the in-plane properties between MSG direct and MSG two-step are 5.91%, 5.48%, 0.75% and 0.09% for  $E_1^I$ ,  $E_2^I$ ,  $G_{12}^I$  and  $\nu_{12}^I$  respectively. The relative difference of the flexural properties between MSG direct and MSG two-step are 5.58%, 4.91%, 0.74% and 0.64% for  $E_1^f$ ,  $E_2^f$ ,  $G_{12}^f$  and  $\nu_{12}^f$  respectively. As noted in [5], homogenization of the whole structure in this case is more appropriate than the homogenization of the core into an equivalent medium separately then of the equivalent characteristics of the plate. To further demonstrate the accuracy of homogenization of the whole structure in comparison to homogenization of the core separately then of the laminate, the accuracy and efficiency of MSG direct will be presented by a numerical case study in the next section.

## CASE STUDY

### Model description

In this section, we study a numerical case to verify the accuracy and efficiency of MSG direct. A reference model has been created for the DNS. The DNS has the same core as in the second section and the same face sheets as in the third section. The whole structure is the repetition of the 3D SG of Figure 7 16 times in  $x_1$  direction and 26 times in  $x_2$  direction as shown in Figure 8. Then length of the whole structure in  $x_1$ ,  $x_2$  and  $x_3$  direction are 177.73 mm, 170.05 mm and 20.5 mm respectively. It is clamped at  $x_1 = 0$ . A uniform pressure of 0.01 MPa is applied at the top surface along negative  $x_3$  direction.

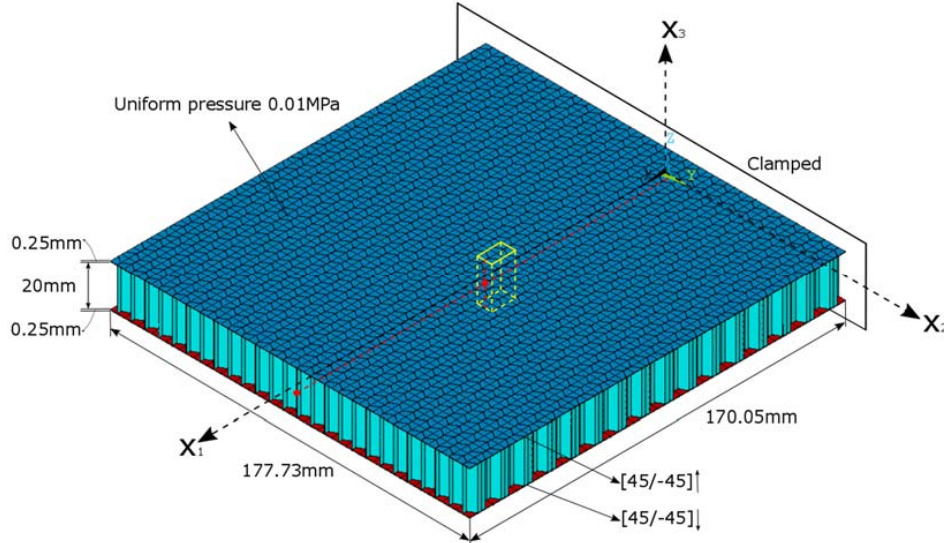


Figure 8 Geometry and BCs of the DNS

Using MSG direct, the original problem is decoupled, as shown in Figure 9, into a constitutive modeling over a 3D SG (upper left) and a structural analysis of a plate (bottom left). The plate model in this case has the same length, width and BCs as the DNS. We use 8-node shell element SHELL281 in ANSYS for the plate model. We input  $A$ ,  $B$  and  $D$  matrices calculated from MSG direct in the previous section to SHELL281 element for analysis. It is noted that in ANSYS, we need to provide not only  $A$ ,  $B$ , and  $D$  matrices but also transverse shear stiffness matrix  $H$  for the SHELL281 element. Without loss of generality, we input 7500 for both  $H_{11}$  and  $H_{22}$ , 0 for  $H_{12}$ .

We also use the traditional approach to solve this problem. In this case, the core is first homogenized as an equivalent layer. Here we use the results of [4] for this layer. Then the core is combined with face sheets for analysis. We also use SHELL281 element in ANSYS for this approach. It is noted that ANSYS can automatically calculate  $A$ ,  $B$ ,  $D$ , and  $H$  matrices for plate analysis and we only need to provide the material properties and geometric information for the plate.

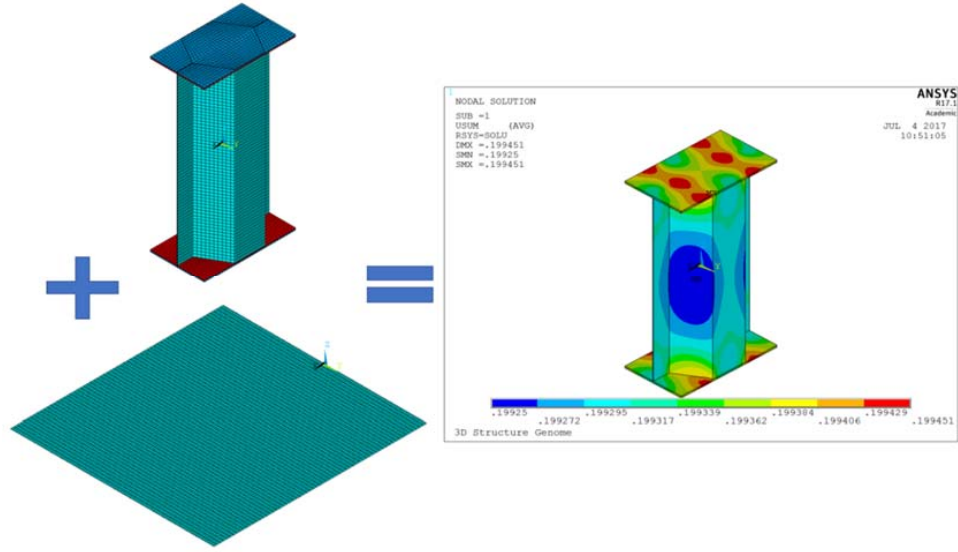


Figure 9 Using MSG direct to decouple the original problem

## Results and Discussion

We first compare the displacement results at tip (170.05, 3.27, 0) and middle (88.86, 3.27, 0) points of the structure. The corresponding points are marked by red dots in Figure 8. The results are summarized in TABLE IX. The relative differences between the DNS with MSG direct and traditional approach are presented in the column right beside MSG direct and traditional approach respectively. The relative differences between the DNS with MSG direct at top and middles points are 0.49% and 0.27% respectively while the relative differences between the DNS with traditional approach are 3.12% and 2.40% respectively. As we can see, the displacement results of MSG direct are more accuracy than traditional approach at these two points.

TABLE IX COMPARISON OF DISPLACEMENT AT TIP AND MIDDLE POINTS

Displacement	DNS	MSG direct	Relative difference	Traditional approach	Relative difference
$u_I$ at tip	-0.5632	-0.5604	0.49%	-0.5779	3.12%
$u_I$ at middle	-0.1998	-0.1992	0.27%	-0.2040	2.40%

Next, we compare the displacement results along the path ( $x_I$ , 0, 0) as shown in Figure 10 (The path is along the dashed red line parallel to  $x_I$  in Figure 8). Since there are some voids in the DNS, the results of the DNS are not continuous. As we can see from the figure, both MSG direct and traditional approach agree well with the DNS. As  $x_I$  becomes larger, the relative differences between the DNS with MSG direct and traditional approach become larger. However, the line of MSG direct is closer to the line of DNS than the line of traditional approach.

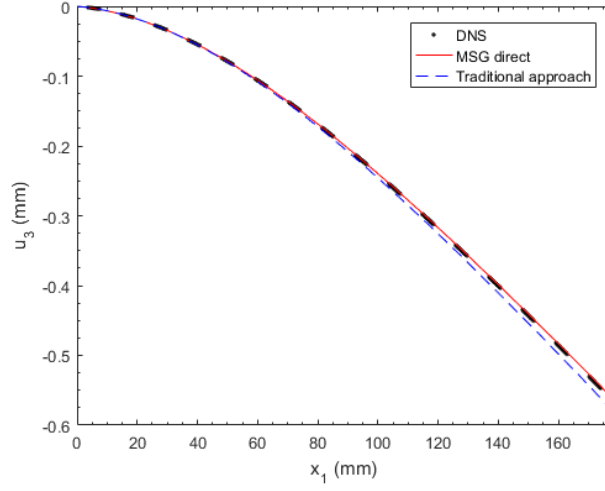


Figure 10 Comparison of displacement along  $(x_1, 0, 0)$

Finally we compare the stress components  $\sigma_{11}, \sigma_{22}, \sigma_{33}$ , along the path  $(88.86, 3.27, x_3)$  in DNS (The path is along the dashed red line parallel to  $x_3$  in Figure 8). In this case, the local field of 3D SG corresponds to the unit cell marked by yellow lines in Figure 8.

The results are shown from Figure 11 to Figure 13. As we can see, MSG direct agrees very well with the DNS for  $\sigma_{11}$  and  $\sigma_{22}$ . For  $\sigma_{33}$ , MSG direct differs a little with the DNS but it is still very close to the DNS. However, traditional approach are not accurate for all of the stress components. For  $\sigma_{11}$  and  $\sigma_{22}$ , traditional approach can only capture the stress results at face sheets while it gives constant zero at the core. For  $\sigma_{33}$ , it only gives a linear line decreasing from 0 to -0.01 MPa (pressure at the top surface).

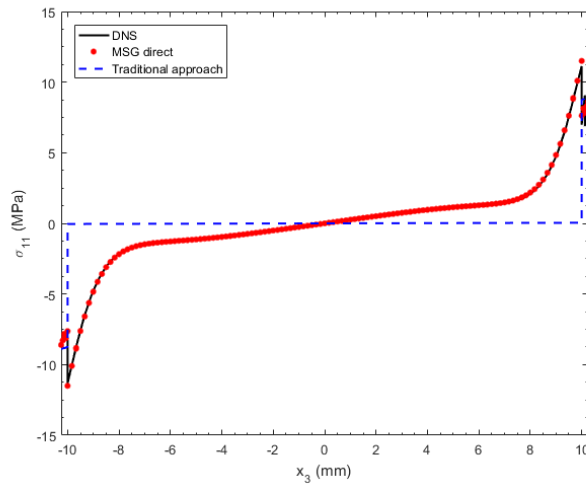


Figure 11 Comparison of  $\sigma_{11}$  along  $(88.86, 3.27, x_3)$

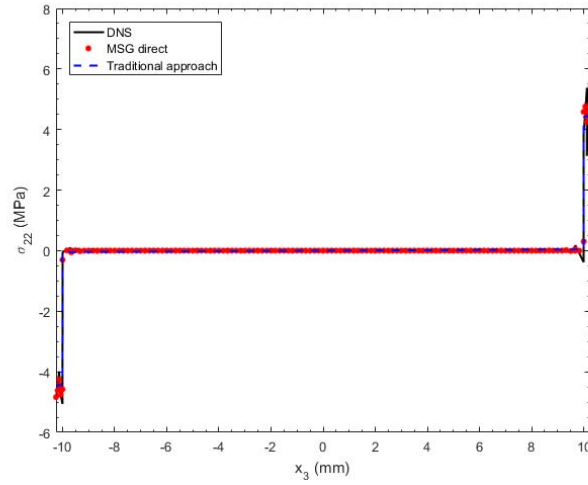


Figure 12 Comparison of  $\sigma_{22}$  along  $(88.86, 3.27, x_3)$

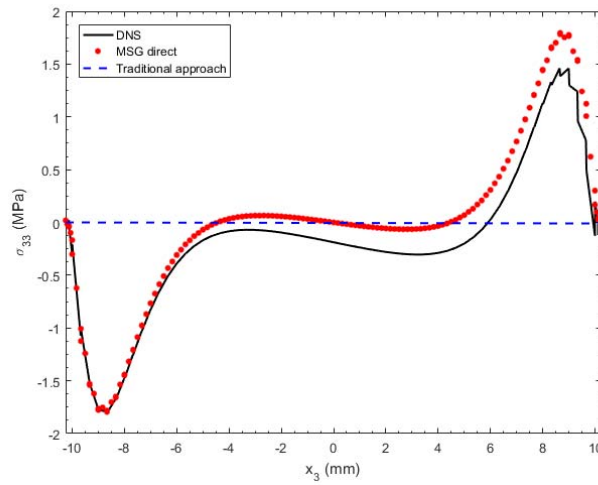


Figure 13 Comparison of  $\sigma_{33}$  along  $(88.86, 3.27, x_3)$

To verify the efficiency of MSG, we will compare the mesh information and calculation time. The comparison of the finite element mesh information is summarized in TABLE X. As we can see, both MSG direct and traditional approach use much less elements and nodes than DNS. The total number of elements and nodes for MSG direct plate analysis and traditional approach are the same. The only difference is the elements and nodes created for the 3D SG.

TABLE X COMPARISON OF FINITE ELEMENT MESH INFORMATION

Method		Elements	Nodes
DNS		4,526,080	26,166,328
MSG Plate	3D SG	10,880	67,838
	Plate analysis	6,656	20,305
Traditional approach		6,656	20,305

The comparison of the calculation time is given in TABLE XI. Both MSG direct and traditional approach need much less calculation time than the DNS. The DNS needs 28 cores and more than 1 days for calculation while MSG direct needs only 2 cores and less than 2 minutes to finish calculation. MSG direct is 7,939 times more efficient than DNS.

TABLE XI COMPARISON OF CALCULATION TIME

Method		CPU	Time	
DNS		28	17 h 19 min 37 sec	
MSG direct	SG homogenization	2	25 sec	1 min 50 sec
	Plate analysis		33 sec	
	SG dehomogenization		52 sec	
Traditional approach		2	39 sec	

## CONCLUSION

In conclusion, this paper presents a new way for multiscale modeling of honeycomb sandwich structures by using MSG. There are three main contributions summarized as follow:

First, we showed that MSG can be used to homogenize a honeycomb core as a homogeneous solid. Two different ways, 2D SG or 3D SG, can be applied to achieve it. No various BCs are needed. The effective elastic properties calculated by MSG agree well with FE-based numerical homogenization approach in literature if real PBCs are used.

Second, on the one hand, we presented that MSG can be used to first homogenize the core and then the whole structure (MSG two-step). On the other hand, we also use MSG to directly homogenize the whole honeycomb sandwich structure as an equivalent plate (MSG direct). For the results of equivalent plate properties, the MSG two-step are very close to traditional approach while MSG direct are slightly different from both MSG two-step and traditional approach.

Third, we conducted a numerical case study to verify the accuracy and efficiency of MSG direct. For accuracy, the MSG direct is more accurate than traditional approach for displacement results and much more accurate than traditional approach for stress

results. Also, MSG direct agrees very well with DNS for all the quantities while traditional approach cannot capture the variation of stress results. In terms of efficiency, MSG direct is 7,939 times more efficient than DNS and uses much less total number of nodes and elements.

In summary, MSG is a powerful and versatile new way for multiscale modeling of honeycomb sandwich structure. It can not only homogenize the core into a homogeneous 3D solid then of the equivalent properties of the plate but also directly homogenize the sandwich structure to obtain the equivalent plate properties. The equivalent plate properties provided by MSG direct are more accurate than traditional approach and much more efficient than DNS.

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