

# Ch.1: Non-ideal behaviour

2018 - 2019 I.(a) :

1. (a) The impedance of  $a-a'$  in Figure 1 shows an equivalent circuit of an inductor.  $L_s$  and  $R_s$  are the equivalent series inductance and equivalent series resistance, respectively; and  $C$  is the parasitic capacitance. The quality factor  $Q$  of the inductor is much larger than 10.

- (i) What is the self-resonant frequency of the inductor? Calculate the  $Q$  of the inductor at its self-resonant frequency. (6 Marks)

- (ii) Sketch the impedance magnitude versus frequency of the inductor with its self-resonant frequency and the corresponding impedance value indicated. (4 Marks)

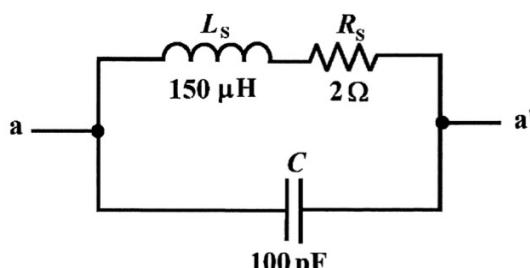


Figure 1

Solution :

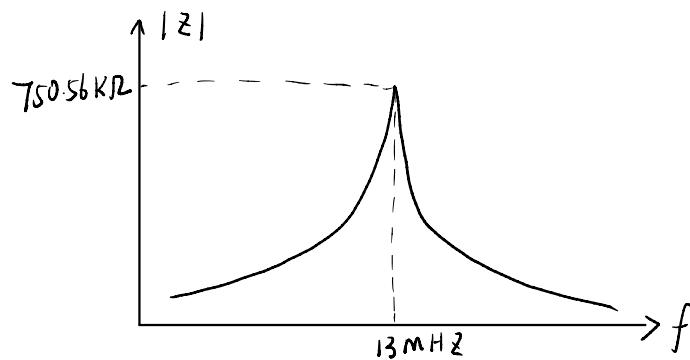
(i) Since  $Q > 10$ ,  $L_p \approx L_s$ ,  $R_p \approx Q^2 R_s$

$$\text{The SRF is } f_o = \frac{1}{2\pi\sqrt{L_p C}} = \frac{1}{2\pi\sqrt{L_s C}} = \frac{1}{2\pi\sqrt{150 \times 10^{-6} \times 100 \times 10^{-12}}} = 1.3 \text{ MHz}$$

$$\text{The Q factor is } \frac{\omega L_s}{R_s} = \frac{2\pi f_o L_s}{R_s} = \frac{2\pi \times 1.3 \times 10^6 \times 150 \times 10^{-6}}{2 \Omega} = 612.6$$

(ii) SRF is 1.3 MHz

@ 1.3 MHz, the impedance is  $R_p = Q^2 R_s = 750.56 \text{ k}\Omega$



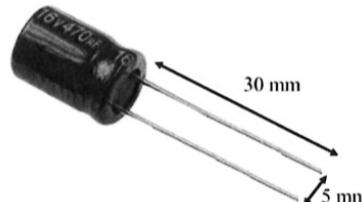
1. (a) Figure 1 shows a  $470 \mu\text{F}$  electrolytic capacitor with two copper leads. The diameter of the lead is 0.5 mm, the length of each lead is 30 mm and the centre-to-centre separation between leads is 5 mm. The capacitor will be used for low frequency applications where skin effect of the leads can be ignored. Conductivity and permeability of copper are  $5.82 \times 10^7 \text{ S/m}$  and  $4\pi \times 10^{-7} \text{ H/m}$ , respectively.

- (i) If the parasitic effects are mainly contributed by the leads, draw the equivalent circuit model of the capacitor and indicate the values of the respective parasitic elements clearly.

(6 Marks)

- (ii) Sketch the impedance frequency response of the capacitor with its self-resonant frequency and the corresponding impedance value indicated.

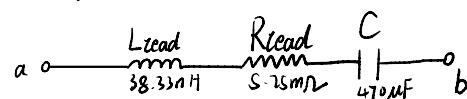
(4 Marks)

**Figure 1**

Note: Question No. 1 continues on page 2

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(i) The equivalent circuit is

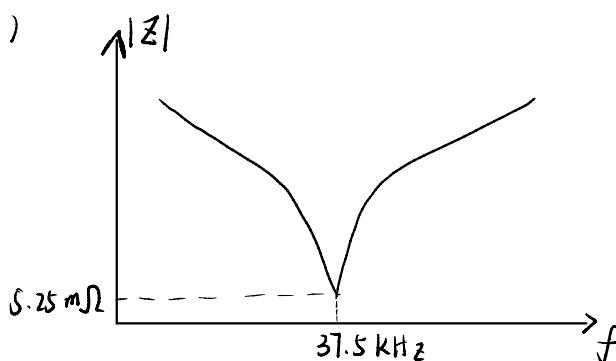


The skin effect doesn't exist,

$$L_{\text{lead}} = 4 \times 10^{-7} \times l \times \ln\left(\frac{D-r}{0.778r}\right) \text{ H} = 4 \times 10^{-7} \times 30 \times 10^{-3} \ln\left(\frac{5 - 0.25}{0.778 \times 0.25}\right) = 38.33 \text{ nH}$$

$$R_{\text{lead}} = 2 \times \left(\frac{l}{\sigma \pi r^2}\right) = 2 \times \frac{30 \times 10^{-3}}{5.82 \times 10^7 \times \pi \times (0.25 \times 10^{-3})^2} \\ \approx 5.25 \text{ m}\Omega$$

(iii)



$$\text{The SRF is : } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= 37.5 \text{ kHz}$$

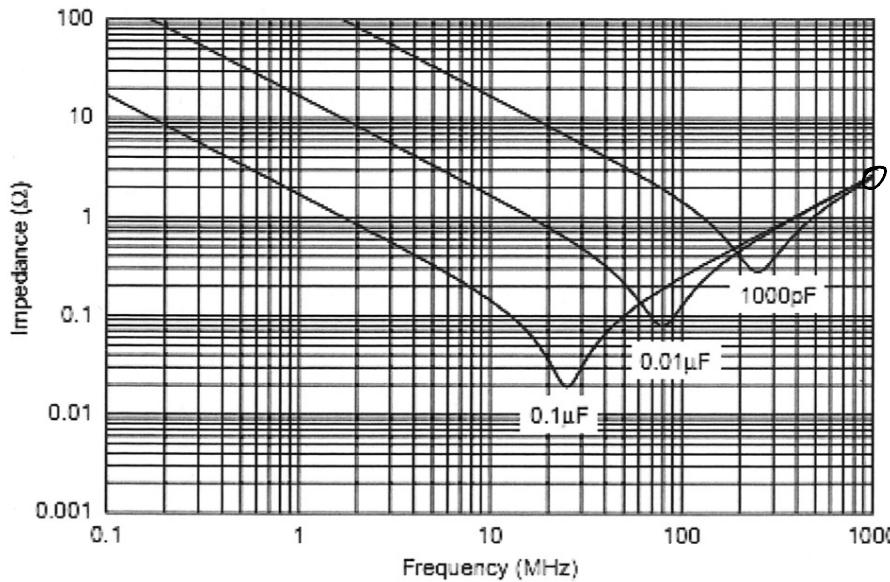
$$\Rightarrow \delta = 3.41 \times 10^{-4} \text{ m}$$

$$r = 2.5 \times 10^{-4} \text{ m}$$

 $\delta > r$ , skin effect doesn't occur.

1. (a) Figure 1 shows the impedance magnitude versus frequency of three capacitors with the same packaging technology in a manufacturer's catalogue. Based on these frequency response curves, draw the equivalent circuit models of  $0.1 \mu\text{F}$ ,  $0.01 \mu\text{F}$  and  $1000 \text{ pF}$  capacitors in terms of RLC series circuits. Indicate clearly the values of R, L and C in the equivalent circuit models.

(10 Marks)

**Figure 1**

(a) From the graph we can get

Capacitor	SRF	R
$0.1 \mu\text{F}$	$25 \text{ MHz}$	$0.02 \Omega$
$0.01 \mu\text{F}$	$80 \text{ MHz}$	$0.08 \Omega$
$1000 \text{ pF}$	$250 \text{ MHz}$	$0.3 \Omega$

The ESLs of these three capacitor are the same, since at  $1000 \text{ MHz}$ , the three lines have the same impedance.  $\text{ESL} \approx \frac{2.5 \Omega}{2\pi \times 10^3 \times 10^6 \text{ Hz}} = 0.4 \text{ nH}$

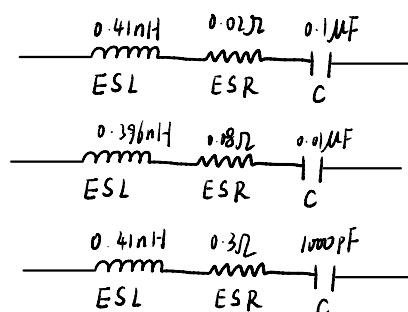
$$\text{Since } SRF = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{(2\pi f_0)^2 C}$$

$$C = 0.1 \mu\text{F}, L = 0.4 \text{ nH}$$

$$C = 0.01 \mu\text{F}, L = 0.396 \text{ nH}$$

$$C = 1000 \text{ pF}, L = 0.41 \text{ nH}$$

The equivalent circuit is



1. Fig. 1 shows a  $0.1 \mu\text{F}$  ceramic capacitor with two parallel copper leads. The diameter of the copper lead is  $0.5 \text{ mm}$  and the length of each lead is  $25 \text{ mm}$ . The centre-to-centre separation between leads is  $4 \text{ mm}$ . In your calculation, consider only the inductance and resistance of the leads. The capacitance of the leads is negligible and can be ignored. Electrical properties of copper:  $\sigma = 5.82 \times 10^7 \text{ S/m}$  and  $\mu = 4\pi \times 10^{-7} \text{ H/m}$ .

- (a) Draw the equivalent circuit of the capacitor between points "a" and "b" in terms of resistance (R), inductance (L) and capacitance (C).

(5 Marks)

- (a) Determine the self-resonant frequency of the capacitor.

(10 Marks)

- (a) Sketch the magnitude of the impedance versus frequency and indicate the value of SRF and the value of the impedance at SRF.

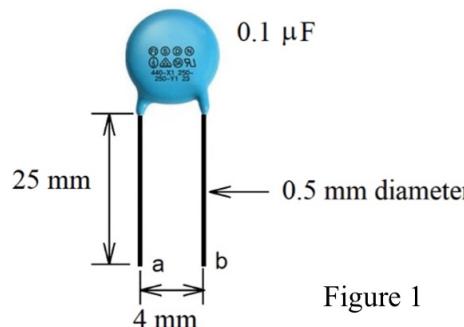


Figure 1

(10 Marks)

(a) The equivalent circuit is,



(b) Assuming that SRF is high enough that skin effect exists

$$\begin{aligned} L_{\text{lead}} &= L_{\text{HF}} = 4 \times 10^{-7} \times l \times \ln\left(\frac{D-t}{r}\right) \\ &= 4 \times 10^{-7} \times 25 \times 10^{-3} \times \ln\left(\frac{4 - 0.25}{0.25}\right) \\ &= 27.08 \text{ nH} \end{aligned}$$

$$\text{Hence SRF is } f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{27.08 \times 10^{-9} \times 0.1 \times 10^{-6}}} = 3.06 \text{ MHz}$$

$$\text{At SRF, the skin depth is } S = \frac{1}{\sqrt{\mu_0 f_0 \sigma}} = \frac{1}{\sqrt{3.06 \times 10^6 \times 5.82 \times 10^7}} = 3.77 \times 10^{-5} \text{ m}$$

$r = 0.25 \times 10^{-3} \text{ m}$ ,  $S < r$ , So the assumption is valid.

$$R_{\text{lead}} = 2 \times \frac{l}{2\pi r \sigma} = 2 \times \frac{25 \times 10^{-3}}{2\pi \times 0.25 \times 10^{-3} \times 5.82 \times 10^7} = 14.5 \text{ m}\Omega$$

