EE6129 Assignment II

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1 Problem 1

SOLUTION:

a)

From the description of the problem, it is a **8-QAM** modulate.

The modulated waveform function can be expanded as:

$$A_i \sin(\frac{2\pi}{T}t + \phi_j) = A_i \cos(\phi_j)\underline{\sin(\frac{2\pi}{T}t)} + A_i \sin(\phi_j)\underline{\cos(\frac{2\pi}{T}t)} \tag{1}$$

The underlined terms come from the basis functions. Since it's QAM modulation, two basis function should be orthonormal. Can be expressed as following:

$$\int_{t=0}^{T} \Psi_i(t)\Psi_j(t)\mathrm{d}t = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{2}$$

Using the underlined term in formula.1 we can get:

$$\begin{aligned} \int_{t=0}^{T} \sin^2(\frac{2\pi}{T}t)\mathrm{d}t = &\int_0^T \frac{1}{2}(1 - \cos(\frac{4\pi}{T}t)) \\ =&\frac{1}{2}(t - \frac{T}{4\pi}\sin(\frac{4\pi}{T}t))\Big|_0^T \\ =&\frac{T}{2} \end{aligned} \tag{3}$$

Similarly,

$$\int_{t=0}^{T} \cos^2(\frac{2\pi}{T}t)\mathrm{d}t = \frac{T}{2} \tag{4}$$

Moreover,

$$\int_{t=0}^{T} \sin(\frac{2\pi}{T}t)\cos(\frac{2\pi}{T}t)\mathrm{d}t = \left[\frac{T}{4\pi}\sin^2(\frac{2\pi}{T}t) + C\right]\Bigg|_0^T \\ =0 \tag{5}$$

In order to meet the condition in formula.2, the results of formula.3 and 4 must be 1, therefore, we add a constant in the underlined terms from the formula.1 without changing the result of formula.5.

We conclude that the basis function of this modulated signal is:

$$\Psi_1(t) = \sqrt{\frac{2}{T}}\sin(\frac{2\pi}{T}t)$$

$$\Psi_2(t) = \sqrt{\frac{2}{T}}\cos(\frac{2\pi}{T}t)$$

b)

As described in the problem, 1 of the 3 bits is carried in the amplitude A_i and the remaining 2 bits are carried in the phase ϕ_j. It can be considered as QPSK with 2 different amplitudes.

The constellation of 8-QAM is showed in figure.1[1].

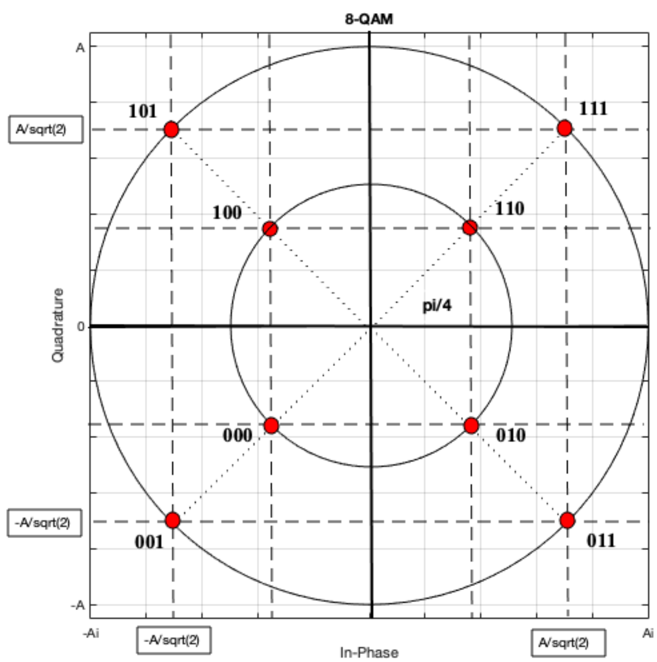

Figure 1: The constellation diagram of 8-QAM with initial phase $\frac{\pi}{4}$.

[1]This figure is edited based on the QPSK constellation diagram generated by MATLAB Communication Toolbox.

This modulation uses 2 possible amplitudes and 4 possible phases. In 8-QAM, the duration of a symbol is three times the duration of a bit (since each symbol carries 3 bits). There are both phase and amplitude changes for each symbol. For the constellation shown above, the eight output symbols might be:

$$\begin{aligned}
&A\cos(2\pi f_c t \pm \frac{\pi}{4}) \\
&A\cos(2\pi f_c t \pm \frac{3\pi}{4}) \\
&\frac{A}{2}\cos(2\pi f_c t \pm \frac{\pi}{4}) \\
&\frac{A}{2}\cos(2\pi f_c t \pm \frac{3\pi}{4})
\end{aligned} \tag{6}$$

Labeled using gray code where only one bit changes between adjacent coordinates. Gray code is used to minimize the number of bits that could be received in error. Besides, using $11X, 10X, 00X, 01X$ indicate phase in $\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4}$ respectively. Similarly, using $XX0, XX1$ represent amplitude in A, $\frac{A}{2}$. In this way, we get 8 kinds of combinations with different phases and amplitudes.

2 Problem 2

SOLUTION:

3 Additional Problem Set 1