# UNION, INTERSECTION & Diff

UNION — $A \cup B$ or $A + B = \{x : x \in A$ or ~~x~~ $x \in B\}$

intersection — $A \cap B$ or $A \& B = \{x : x \in A$ and $x \in B$

difference $A - B$ $= \{x : x \in A$ and $x \notin B\}$

11 September 2018

## De Morgans law

☐ $\overline{A \cup B} = \bar{A} \cap \bar{B}$

☐ $\overline{A \cap B} = \bar{A} \cup \bar{B}$

→ $\bar{A} = U / A$

example: $\overline{A \backslash B} \backslash A$

$\hookrightarrow \overline{A \cap \bar{B}} \cap \bar{A}$

$$= \underline{\underline{(\bar{A} \cup B) \cap \bar{A} = \bar{A}}} \quad \square$$

More Ex: Simplify $[(\overline{B \cap A}) \cap B] \cup A$

$$= [(\bar{B} \cup \bar{A}) \cap B] \cup A$$

$$= [\underbrace{(\bar{B} \cap B)}_{\emptyset} \cup (\bar{A} \cap B)] \cup A$$

$$= (\bar{A} \cap B) \cup A = (\bar{A} \cup A) \cap (A \cup B)$$

$$= (A \cup B)$$

Def: A Partition of a set $\mathcal{U}$ is a collection of Non-empty Subsets of $\mathcal{U}$ so that

i) the Union of the subsets is all of $\mathcal{U}$

ii) the subsets are mutually disjoint No Repeats

⇒ that is any two distinct subsets have empty intersections

ormal defination:-

↳ $\{A_i : i \in I\}$ Partition

if • $\phi \neq A_i \subseteq U$

↳ $\bigcup_{i \in I} A_i = U$ ①

• $A_i \cap A_j = \phi$ for $i \neq j$ ②

⟷

xample of Partition

⇒ $\{A, \bar{A}\}$ partition of $U$ Provided $A \neq \Phi$ and $A \neq U$ $(A \subsetneq U)$

[A proper subset of $U$]

example. Let $A$ be a set, $|A| = n$.

$$P_k(A) = \{X : X \subseteq A, |X| = k\}$$

↳ $\{k = 0 \ldots\ldots, n\}$

Then $\{P_0(A), \cdots P_n(A)\}$ Partition of $P(A)$

Example : Pol = set of polynomials with real coefficents

○ $Pol_K$ = the set of polynomials having a degree $K$

↳ $\{k = 0, 1, 2 \ldots\ldots\}$

⟷

Example : $A_0 = \{n \in \mathbb{Z}$ : $n$ has remainder 0 upon division by 3$\}$

$A_1 = \{n \in \mathbb{Z} : n = 0\} \to A_1 = \{$ — // — $\}$ 1

↳ $A_2 = \{$ — // — $\}$ 2

— // —

$\{A_0, A_1, A_2\}$ partitions of $\mathbb{Z}$

↳ Remainders $\{0, 1, 2\}$ in other words

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Foot NOTE not from class but simple paritition example

taking a set = $\{1, 2, 3\}$ partitions =

1) $\{\{1\}, \{2\}, \{3\}\}$

2) $\{\{1, 2\}, \{3\}\}$

3) $\{\{1, 3\}, \{2\}\}$

4) $\{\{1\}, \{2, 3\}\}$

5) $\{\{1, 2, 3\}\}$

# Relations

**ef**

A relation on a set A is a Subset $\overset{\text{Relation}}{R} \subseteq \underbrace{A \times A}_{\text{Cart product}}$ □

**Examples**

$A = \{1,2,3,4\}$

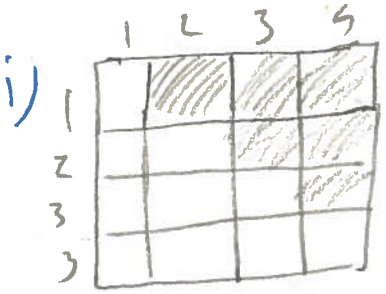
$R = \{(1,2),(1,3),(1,4),(2,3),(2,4),$
$(3,4)\}$

so $aRb$ means $a<b$

**Side Note**

Relationships can be shown Visually.

i)



ii)

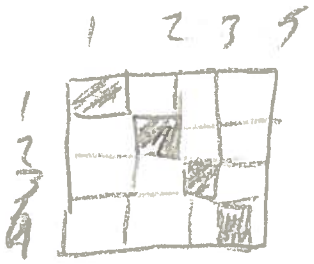
**so ex 2**

$A = \{1,2,3,4\}$

$R = \{(1,1),(2,2),(3,3),$
$(4,4)\}$

so $aRb$ means $a=b$

Visually



**Example 3** On $P(A)$, inclusion is a relation

$X \subseteq Y$

**Example 4** On $\mathbb{R}$, (strict) order is a relation

$a<b$, $a \leq b$

**Example 5** ON $\mathbb{Z}$, divisibility is a relation

$(a|b \rightsquigarrow \underline{2|4}$ but $2\nmid 5)$

As you can see 3,4,5 are more complex
so visually can't show relation...

**Def**

A relation R on a set A is **reflexive** if $aRa$ for all $a \in A$, **Symetric** if $aRb$ then $bRa$

**anti-symetric**: if $aRb$ and $bRa$ [If this]
then $a=b$ [then this]

hence anti-sym

**transitive** if $aRb$ and $bRc$
then $aRc$

R is An **equilance** if { Reflexive
Symettric
transitive

R is **ORDER** if { Reflexive
anti symetric
transitive

Examples

**Example** The relation of equalility is an equivalenc
and an ORDER

**Example** On $\mathbb{R}$, $xRy$ if $|x-y|<1$

↳ Reflexive $xRx$ means $|x-x|<1$

↳ Symettric if $\underbrace{|x-y|<1}_{xRy}$ then $\underbrace{|y-x|<1}_{yRx}$

↳ transitive

so $1R1.5$, $1.5R2.1$

but $1 \not R 2.1$

**Example** on $\mathbb{R}$

| | $a<b$ | $a\leq b$ |
|---|---|---|
| Reflexive | No | Yes |
| Symetric | No | NO |
| antisymetric | Yes ★ | Yes |
| transitive | Yes | Yes |
| | | ORDER |

★ if $a<b$ & $b<a$
then $a=b$

**Ex** ON $P(A)$ inclusion is an Order

$$(x \subseteq y)$$

## Equivalence & Partition

for equivalance Relation ~~of~~ on $A$

**Def** Given an element $a \in A$, the equivalene class of $a$ is the subset

$$[a] = \{x \in A : x R a\}$$

**Remark** . Such set is non-empty: $[a]$ non-empty

END Class

**Theorem** $\{\underbrace{[a] : a \in A}_{\text{set of equivalen class}}\}$ for partition of $A$