4th October - Thursday

Thm Let x_1, x_2 be given by the recurrence $x_{n+1} = ax_n + bx_{n-1}$ with initial terms x_1 & x_2. Consider the characteristic equation.

$$\left[x^2 = ax + b\right]$$

ordered two recurrence : have to go back 2 steps

1) If two distinct roots $\Rightarrow x_n = cR_1^n + dR_2^n$ {two geometric series} (roots; seq using)
where c & d are determined by x_1 & x_2

2) if one repeated root, $x_n = (c + dn)R^n$ where c & d are determined by x_1 & x_2

$\longleftrightarrow$

Can be done in Proof induction

HW!

$\longleftrightarrow$

Example of Fibonacci

$$F_{n+1} = F_n + F_{n-1} \qquad F_1 = F_2 = 1$$

Characteristic Eq:

$$x^2 = x + 1 \longrightarrow x^2 - x - 1 = 0 \longrightarrow R_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$

$\rightarrow F_n = cR_1^n + dR_2^n$

Find c & d?

$x=1 = 1 = cR_1 + dR_2$
$x=2 = 1 = cR_1^2 + dR_2^2$

we end up with a System of equation

o eq 1 × R_1

$$\begin{cases} cR_1^2 + dR_1R_2 = R_1 \\ cR_1^2 + dR_2^2 = 1 \end{cases}$$

↳ $dR_1R_2 - dR_2^2 = R_1 - 1$

$\rightarrow d = \dfrac{R_1 - 1}{R_1R_2 - R_2^2}$ {Plug in Substitute}

$= \cancel{\ } - \dfrac{1}{\sqrt{5}}$

Similarly by plugging d in one of the eq $c = \dfrac{1}{\sqrt{5}}$ *

Class question: What happens if there is a 3 or 4 terms Recurrence.

→ For 3 $X_{n+2} = aX_{n+1} + bX_n + cX$

* cont from @ above

Final Ans = $F_n = \dfrac{1}{\sqrt{5}}\left(\dfrac{1+\sqrt{5}}{2}\right)^{n} - \dfrac{1}{\sqrt{5}}\left(\dfrac{1-\sqrt{5}}{2}\right)^{n}$

Example $a_{n+1} = 2a_n - a_{n-1}$ $a_1 = 0, a_2 = 1$

Characteristic eq: $x^2 = 2x - 1$ $R_{1,2} = 1, 1$

$a_n = (c + dn) \cdot 1^n = c + dn$ } Geometric step ⟲ you always guess

Solve for c & d

$$\begin{cases} c + d = 0 \\ c + 2d = 1 \end{cases} \Rightarrow \begin{cases} d = 1 \\ c = -1 \end{cases} \Rightarrow a_n = n - 1$$

$$\boxed{\begin{aligned} &a_{n+1} = \cancel{2a_n + a_n}\, 1 \\ &\rightarrow a_n + 1 - a_n \\ &= a_n - a_{n-1} \end{aligned}}$$

Propositional logic

- Truth tables
- Quantifiers

→ Statement or Preposition: An Assertion that is either true or false

Example P: $2^{n}-1$ is a prime
Q: for every int $n>1$, $2^n - 1$ is prime
R: There are infinitely many integers $n>1$ so that $2^n - 1$ is Prime

o $P = F$, $Q = F$, $R = ?$ } R is open Question, meaning Prime Quest. I.E it hasn't Been proven wrong but isn't true either

Logical Connectives (Similar to Sets)

• P, Q given statements $\begin{cases} P : 2^{n-1} \text{ is prime} \\ Q : 2^{s-1} \text{ is prime} \end{cases}$

$P \wedge Q$: P AND Q

$P \vee Q$: P OR Q → inclusive OR!

HW find formula for exclusive OR

$\neg P$: Not P

Truth tables

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

○ $P \Rightarrow Q$ [if P then Q]

Implication

$P \Leftrightarrow Q$ (P iff Q)

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical equivalences

Statements that have the same truth values

- $(P \leftrightarrow Q) = ((P \to Q)) \wedge (Q \Rightarrow P)$
- $\neg(P \wedge Q) = \neg P \vee \neg Q$
- $\neg(P \vee Q) = \neg P \wedge \neg Q$ } demorgans
- $(P \to Q) = (\neg Q \to \neg P)$ contrapositive

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg Q \to \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T