Day 14 : 17 October

Application of poisson:-

Suppose that sections of textile of length 1cm have a flaw in them with probability 0.01. If 1000 such sections are examined what is the ~~assumption~~ approx probability that atleast 50 will be flawed

Solution:-

Let x be the no of flaws in the 1000, sections
Then $X \sim Bin(1000, 0.1)$ [Reasonably] therefore exactly,

$$P[X \geq 50] = \sum_{x=50}^{1000} \binom{1000}{x} (0.01)^x (1-0.01)^{1000-x}$$

$$\text{Also} = 1 - P[X < 50] = 1 - P[X \leq 49]$$

$$= 1 - \sum_{x=0}^{49} \binom{1000}{x} (0.01)^x (1-0.01)^{(1000-x)}$$

Note compliment of $P[X \leq 50] \neq 1 - P[X \leq 50]$

but $1 - P[X < 50]$

will lose marks.

Since n is large (1000) and p is small (.01) we could use the poisson distribution approx to the binomial, by setting $\lambda = np$

$= 1000 \times .01 = 10$

$\therefore P[X \geq 50] = \sum_{x=50}^{\infty} \frac{10^x e^{-10}}{x!}$

$$= 1 - \sum_{x=0}^{49} \frac{10^x e^{-10}}{x!}$$

Next Distribution: hypergeometric Distribution

- The R.V x is said to have hyper geometric distribution with the parameters N, a & n if:

$$P[X=x] = \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}, \quad \text{for all } x = 0, 1 \cdots\cdots \min(a,n)$$

and $a \leq N$, $n \leq N$

N = total size   n sample sub space

Remember fish in lake problem!! Similar..

- Formalization of pick with replacement. ie Sample without replacing., from a box containing N objects of which a are type 1 and N- a are type 2
- Sample size is n and you want the probability of observing x #'s of type 1 in the sample

Self Note: If N is very large and a not to small, it's similar to binomial distribution

[When doing sampling without replacement, first check if hyper geo distribution. If N too big then think of binomial]

The Geometric distribution:

The R.V x is Geometric distribution with parameter p if

$$P[X = x] = (1-P)^{x-1}\, p \quad \text{for} \quad x = 1, 2, 3 \cdots$$

Q How does the geometric distribution arise?

Claim:

X gives trial at which the 1st success occurs in a sequence of independant bernoulli trials with constant success of prob of success p

Proving the claim:

- $\{X = x\}$ iff $\{$ the first $x-1$ trial results in failure, & $\cap$ trial no x results in success $\}$

- $\therefore$ $P[X = x]$

$$= P[F_1 \cap F_2 \cap F_3 \text{---} F_{x-1} \cap S_x]$$

as these are independant

$$= P[F_1]\, P[F_2]\, P[F_3] \text{---} P[S]$$

$P(F) = [1-P]$

so $$= \underbrace{(1-P)(1-P) \cdots (1-P)}_{X-1} \underbrace{P}_{X}$$

$$= (1-P)^{x-1}\, P$$

- Remember Binomial gives no# of success in those fixed trials

- Geometric is the trial no till the first trial
  How many trials it takes till first S

The negative Binomial

- X has a negative Binomials Distribution with parameters p & k if

$$P[X = x] = \binom{x-1}{k-1} p^k (1-p)^{x-k} \quad \text{for } x = k, k+1, \ldots$$

p is prob of success on the k^{th} try

- How it arises:-

- The negative Binomial distribution arises as trial at which the k^{th} success occurs in a sequence of independent Bernoulli trials with a constant prob of success, p

- We know for success at k^n, we know for $x-1$ we have had $k-1$ successes

$$P[X = x] = P[k-1 \text{ successes in } x-1 \text{ trials} \cap \text{success on trial } x]$$

$$= P[k-1 \text{ successes in } x-1 \text{ trials}] \cdot P[\text{success trial on } x]$$

These are indep

$$= \binom{x-1}{k-1} p^{k-1} (1-p)^{x-1-(k-1)} \cdot p$$

$$= \binom{x-1}{k-1} p^{k} (1-p)^{x-k}$$

Ans

New Concept : Mathematical expectation

- The probability distribution of R.V ~~X~~ tells the whole story ie all the information about ~~x~~ X is contained in the prob. distribution.

- We may wish to summarize some features of this distribution
- For example we may want to say where center of this distribution lies or how spread out this distribution is.

To this end, we introduce expect value or expectation of a discrete Random variable.

Defination

Let X be a discrete Random variable, then the expected value of X is defined as

$$E(X) = \sum_{\left[\substack{x:\ \text{is in} \\ \text{Range of} \\ X}\right]} x\, P[X=x] = \sum_{\substack{x:\ \text{in} \\ \text{Range} \\ \text{of } X}} x\, P_x(X)$$

Notes for $E(X)$

1) We call $E(X)$, the expected value of X or mean of X.

2) The notation often used for $E(X)$ is μ or μ_x for mean of x.

3) The expected value can be tought of as weighted Avg of the values of X.

Ex if X has a discrete uniform distribution on $a_1, a_2, \ldots, a_N$

$$E(X) = \sum_{i=1}^{N} \underset{\text{"}X\text{"}}{a_i} \; P[\underset{x}{X = a_i}]$$

$$= 1/N \sum_{i=1}^{N} a_i \qquad = \text{Avg of } a_i$$