Day 11: Continue R.V.

- A random variable takes a value in real line, $\mathbb{R}$. In order specify a RV, we need to specify $P[X \in A]$, for every subset of A.

This collection of probabilities is called the probability distribution of A. This would seem to be a huge task since there are lots of subsets of Real no's. Turns out however that we don't have to specify $P[X \in A]$ for all subsets A; Instead there is theorem due to Caratheodory that says:-

big course — It is enough to specify $P[X \in (-\infty, x)]$ for all x such $-\infty < x < \infty$. ie you only have to specify probability of these half infinite intervals of sets of type $(-\infty, x)$. Doing this uniquely determines the probabilities $P[X \in A]$ for all subsets $\underline{A}$

[De]finition: We call the function of x, given by $P[X \in (-\infty, x]] = P[X \le x]$, the cumulative distribution function of the R.V. X and we denote it by $F_X(x)$

Notes

1) $F_X(x)$ specifies the probability distribution for both discrete and continous R.V.'s

2) We define $P[X \in A]$ to be $P[\omega : X(\omega) \in A] = P[X^{-1}(A)]$
Thus, for example $P[X = x_0] = P[\omega \in S : X(\omega) = x_0]$

3) c.d.f's , $F_X(\cdot)$ have 3 basic properties

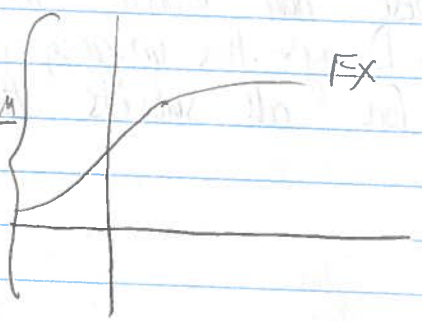
1) $F_X(x) \leq F_X(y)$ if $x \leq y$ i.e $F_X(x)$ ie $F_X(x)$ is non decreasing

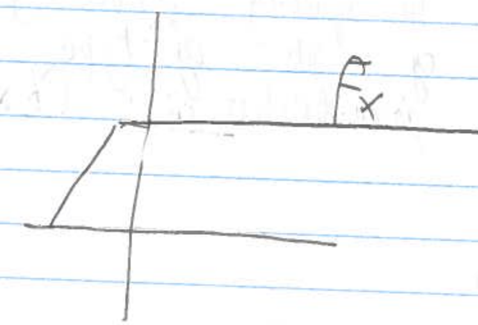
2) $F_X(+\infty) = \lim\limits_{X \to +\infty} F_X X = 1$ | reaching whole S here Prob = 1

$F_X(-\infty) = \lim\limits_{X \to -\infty} F_X X = 0$

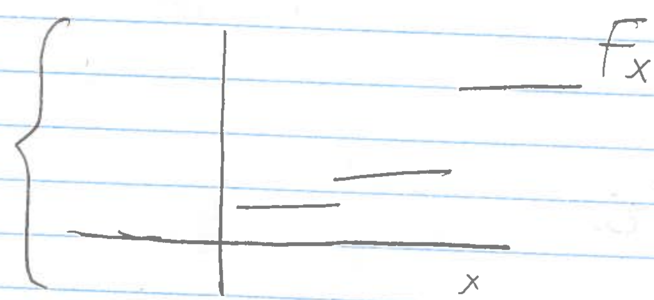
3) $F_X(x)$ is continous from the right , It need not be continous on the left Function with no jumps

Continuous





Discrete



0 We call R.V $\overset{X}{}$ continous if F_X is continous
for all $-\infty < x < \infty$

Otherwise X is said to be discrete.
We return to these 2 types later

Discrete RV.

We have seen CDF uniquely determines the Prob distribution of a R.V in bothe discrete and continous cases

- There is a second way to specify the prob distribution of a discrete random variable. This is through what is known as the probability mass function OR Prob function.

...ation: The real valued function of x that gives prob that ~~x~~ $P[X = x]$ for all x in the range of X, is called the prob function of X. We donate this function by: ~~p_X~~ $p_x(X)$ $(= P[X = x])$

...tes 1) $0 \leq p_x(X) \leq 1$

2) $\sum_{x: X = x} p_X(x) = 1$ (all little x in X)

3) The prob function uniquely determines the prob distribution of a discrete R.V. All we need to do is show we can get to C.D.F uniquely from the prob function $p_X(x)$

Proof =>

Proof of 3:

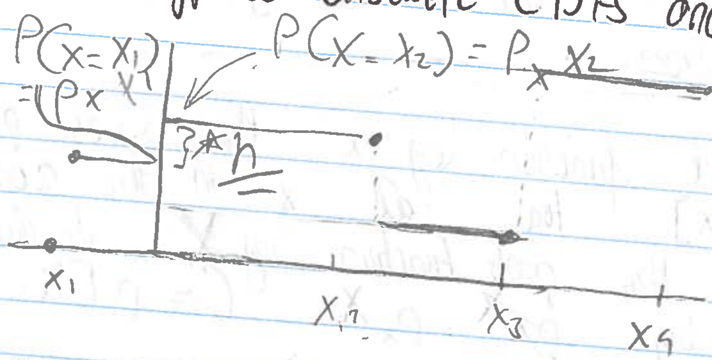
We have $F_X(x) = \sum_{y:\, y \le x} P_X(y)$ [Axiom three]

So $F_X(x)$ is obtained uniquely from the prob function. Conversely we can obtain the prob function $P_X(x)$ from $F_X(x)$, by differencing.

It is not difficult to show that $P_X(x) = F_X(x) - F_X(x-)$ very close to x

$$= F_X(x) - \lim_{y \to x} F_X(y)$$

Some typical discrete C.D.Fs and Prob functions



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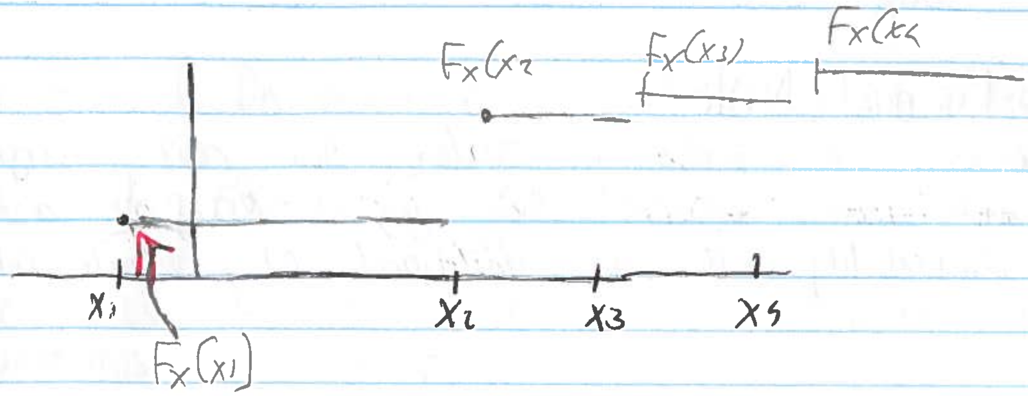
$P_X(x_2) - P_X(x_1)$

~~Cumulative Distribution~~


Fx(x2
Fx(x3)
Fx(x4
X1
X2
X3
X4
Fx(x1)