September lecture 5 or 4 IDK

A1. Questions will be from the book, but scanned for you.

Marble problem:-

Idea:- Set up a sample space S with equally likely outcomes. If we can do this, we can calculate ~~P(red)~~ →
P(red) = # of ways to get red / Total no of outcomes

From before we have an eg of another sample space that is suitable for this.

o S = {1, 2, ---- 10} where we agree that marbles numbered {1, 2 — 6} = red {7 — 10} = green

o Now for 10 outcomes, the numbers are equally likely ∎ cause unbiased and random draw.

Hence no of ways of red = 6 and total outcomes = 10
so P(red) = 6/10.

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o Professor did a gig of asking 32 birthdays and sees if 2 same.

No real reason

If you have 23 ppl in a room, the prob of having two people with same birthdays, it's about 50%

He'll prove later.

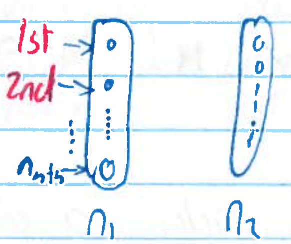
Note: Although theorem 5 is simple in principle, the counting can sometimes be difficult

Counting Rules :- These can make life easier

1) Multiplication rule of counting: [First Principle]

Suppose you have K-sets of $n_1, n_2 \dots n_K$ objects respectively. Then the number of ways you can choose one object from each set is given by $n_1, n_2 \dots n_K$

idea of Proof :- let $K=2$ [remember n_K are sets with objects]



The 1st object in **set 1** can be matched with each of n_2 objects in set 2

The 2nd obj in n_2 ways with each obj in **set 2**

Therefore total no of possible matches is $\underbrace{n_2 + n_2 + n_2 \cdots}_{n_1}$

$= n_1 \times n_2$ [on sizes of the sets
Just a better way of thinking]

2nd principle :- if a set has n distinct objects, then the no of ways to choose K objects from this set if we sample without replacement is given by: $n! / K!(n-K)! = \binom{n}{K}$

Note the order in which obj drawn considered unimportant.
So $[a, b, c] \equiv [b, c, a] \rightarrow$ same draw.

Third principle: if a set has n distinct objects, then the no of ways to select K of them, without Replacement if order is Imp:

$$n!/(n-K)! = {}^{n}P_{K}$$

So $[a,b,c] \neq [b,c,a]$, these are seperate

$\longleftrightarrow$

Jargon:

$\binom{n}{K}$ is called "n choose K"

${}^{n}P_{K}$ is called "n permutation K"

$\longleftrightarrow$

$n!$: 4th Principle The no of ways to order a set of n distinct objects is $n!$

Illustration of the concept above:- B-day Problem

Suppose there are n people in a room. What is the probability that atleast two people have the same B-Day.

Solution $\longrightarrow$

Solution Build up

- Set up a sample space S, with equally likely outcomes
We saw a sample space of such earlier in course
$S = \{\underbrace{(Jun1, Jun1 \ldots Jun1)}_{\omega_1}, (Feb1, Feb1, Feb1 \ldots) \ldots \underbrace{(Dec31, Dec31 \ldots)}_{\omega_N}\}$

- N = the no of possible outcomes

- We shall assume all N outcomes are equally likely

- Now let $\mathcal{E}$ be event = two people with same b-day.
Turns out; easier to find $P(\mathcal{E})$ by first finding $P(\mathcal{E}^c)$ [$\mathcal{E}$ complement]
where $P(\mathcal{E}) = 1 - P(\mathcal{E}^c)$

- $\mathcal{E}^c$: no two people have the same b-day. Therefore,
$$P(\mathcal{E}^c) = \frac{\#\text{ of ways no 2 people with same b-day}}{\text{total no of possible outcomes [N]}}$$

- so everyone has 365 choises, and these can be choosen. 365 times as in a sausage,

- So Denominator $[365 \cdot 365 \ldots\ldots 365] = 365^n$

Numinator

o As each person cant have same birthdays in $\mathcal{E}^c$ avoiding picking same birthdays, will have a decreasing order

1st person = 365 Choices
2nd " = 364 Choices
3rd " = 363 Choices
⋮
$[365-n+1]$

o Numinator:- $(365 \cdot 364 \cdot 363 \cdots\cdots (365-n+1))$

denominator:- $365 \cdot 365 \cdot 365 \cdots\cdots 365$ or 365^n

Numinator can be said to be ${}^{n}P_{k}$ aswell

$\therefore$ Finally $P(\mathcal{E}) = 1 - P(\mathcal{E}^c)$ where $P(\mathcal{E}^c)$ shown above.

$\longleftrightarrow$

The Sausage Idea:-

Example 2 : Fish in the lake example

- Suppose that a lake has N fish of which A are tagged & N-A are untagged. We select a sample of size n with replacement

- What is the probability that x of the sample will be tagged

Simple terms some tagged fish, some untagged, find probability that a random sample will be tagged

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Solution:- Want to use counting; therefore set up a sample space with equal outcomes

- Start by numbering the fish from 1 to N.
  - lets agree the numbers $\{1, 2 \ldots A\}$ correspond to tagged fish

  - Its perhaps reasonable to assume all unordered samples of size n (ie sets of n numbers from no's $1, 2 \ldots N$) are equally likely. Simple enough.

- Let ε = Event that x out of n are tagged

By theorem 6, $P(\varepsilon) = \dfrac{\text{\# of ways of x tagged out of n}}{\text{Total outcomes}}$

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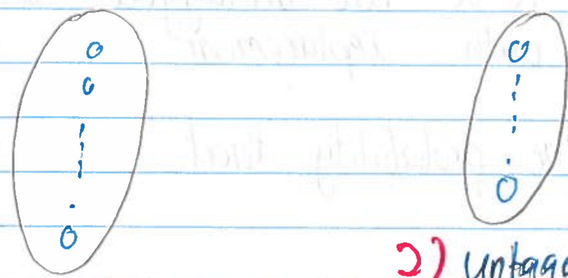
Denominator:- How many sets of size n can we make
so $\binom{N}{n}$ - 2nd principle

Numerator :-



1) Tagged n_1

2) untagged n_2

- each object in Sausage 1 is x integers drawn from A
  example if $x = 2$
  then each object will be like : 1, 2
  1, 3
  3, 4

- we want to know the no of time of matching of x from 1'st set to 2nd set

1st Set $n_1 = \binom{a}{x}$ — first choices

2nd $n_2 = \binom{n-a}{n-x}$ — remains

answer $\binom{a}{x}\binom{n-a}{n-x}$

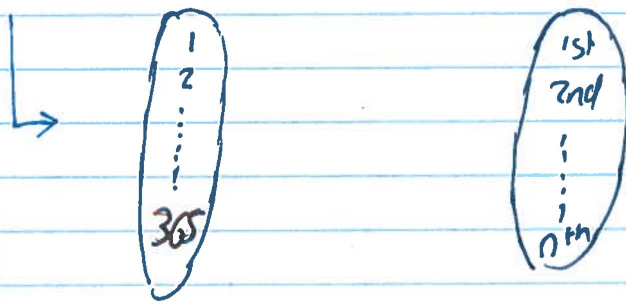
END

Extra Page:- Expanding on the whole sausage deal

- Its called the multiplication rule or m Rule

- Consider you have two independant sets, m and n
$m = \{a_1, a_2 \cdots a_m\}$ objects and $n = \{b_1, b_2, \cdots b_n\}$ objects, you can form a tuple $(a_1, b_1), \cdots (a_m, b_n)$ by $m \times n$ [not multiplication]

- now lets say m = any day in 365 possible Days and n = any one of the child selecting there own day of Birth



$|m| = 365$ $|N| = N$

take 1st kid, her can choose any 365 days

2nd kid any 365 day

⋮

n^{th} kid any 365 Days

so denominator $= \underbrace{365}_{1st} \cdot \underbrace{365}_{2nd} \cdots\cdots \underbrace{365}_{n^{th}}$

$= (365)^n$