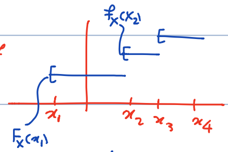
Math 323
Oct 10th, 2017
Recall:




A discrete r.v X has a cdf that is non-decreasing, tends to $\begin{cases} 0 & \text{as } x \to -\infty \\ +1 & \text{as } x \to +\infty \end{cases}$ is continuous from the right and is a step function with jumps at the values $x_1, x_2, \ldots$ which have positive probability.

Note: The size of the jump of the cdf as $x_i$ is $P[X = x_i]$.

Diagramatically we have thus for

The probability distribution: $\{P(A) : A \subseteq \mathbb{R}\}$

$\updownarrow$

The C.D.F.: $F_X(x) = P[X \le x]$ $\forall x$

Discrete: The prob. (mass) func. $P_X(x) = P[X = x]$ for $x$ in range of $X$

Continuous: ? later

Example: Suppose that if a flight is cancelled the airline loses \$5000. If the flight leaves more than ½ hr it loses 2000 if it leaves ontime it makes a profit of \$10000 and if it leaves $\le \frac{1}{2}$ hr late, it also makes \$10000.

Suppose that the prob. of these events are:

$P[\underset{\text{cancelled}}{C}] = 0.05$, $P[> \tfrac{1}{2} \text{ hr late}] = 0.1$, $P[\text{ontime}] = 0.7$

Q1) Find the Prob. func. of the random variable that represents the gain on a flight.

Q2) Find the C.D.F of this r.v.

Let $\omega_1, \omega_2, \omega_3$ and $\omega_4$ be the 4 possible outcomes for each flight. Let $X$ be the gain for each flight.

Clearly $X(\omega_1) = -\$5000$, $P[\omega_1] = 0.05$

$X(\omega_2) = -\$2000$, $P[\omega_2] = 0.1$

$X(\omega_3) = 10{,}000$, $P[\omega_3] = 0.7$

$= X(\omega_4) = 10{,}000$, $P[\omega_4] = 1 - \sum_{i=1}^{3} P(\omega_i) = 0.15$

We need $P[X = -5000] = P[\omega : X(\omega) = -5000] = P[\omega_1] = 0.05 = P_X(-5000)$

$P[X = -2000] = P[\omega_2] = 0.1$

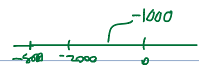
disjoint by def.

$P[X = +10000] = P[\omega_3 \cup \omega_4] = P[\omega_3] + P[\omega_4] = 0.7 + 0.15 = \boxed{0.85}$

To find the cdf $F_X(x)$ we need to give the prob. $P[X \le x]$ for all $-\infty < x < +\infty$

To begin:

$$F_X(x) = \begin{cases} 0 & \text{for } X < -5000 \\ \underbrace{P[X=-5000]}_{P[X\le x] \text{ for } -5000 \le x < -2000} = \boxed{0.05} & \text{for } -5000 \le x \le -2000 \\ P[X=-5000]+P[X=-2000] = 0.05+0.1=0.15 & \text{for } -2000 \le x < 10000 \\ 0+0.05+0.1+0.85=1 & \text{for } 10000 \le x < \infty \end{cases}$$




Since $P[X \le x] = P[-\infty < X < -5000 \;\cup\; -5000 \le X < -2000 \;\cup\; -2000 \le X \le x] = P[-\infty < X < -5000] + P[-5000 \le X < -2000]$ $+ P[-2000 \le X \le x] = 0 + 0.1 + 0.5 = 0.15$

<u>Some named distributions (r.v.s):</u>

The following prob. distributions occurs so frequently:

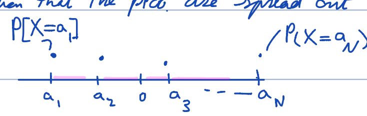
<u>Discrete Uniform Distribution:</u>

Def: The r.v $X$ is said to have a discrete uniform distrib. on the numbers $a_1, a_2, \ldots, a_N$ if

$$P_X(a_i) = P[X=a_i] = \frac{1}{N} \quad \text{for all } 1 \le i \le N$$

75 Notes: 1) The name comes from the observation that the prob. are spread out equally (uniformly) among $\forall a_i$'s.

2) The prob. func. looks like this




3) The discrete uniform dist. is often used to model <u>complete randomness</u> in a discrete setting.

4) We usually do not specify discrete random variables through cdf.s

<u>The Bernolli Distribution:</u>

The r.v $X$ has a discrete Bernolli dist. with parameter $0 \le p \le 1$, if $P[X=1]=p$ and $P[X=0]=1-p=q$

Or more compactly $P_X(x) = P[X=x] = p^x(1-p)^{1-x}$ for $x=0,1$