September 28 - Thursday =

To find $P(A \cap B)$, need know

a) A & B independant and use $P(A) \cdot P(B)$

b) $A \cap B = \phi$ in which case $P(A \cap B) = 0$

or c) that you know $P(B|A)$, use $P(A \cap B) = P(B|A) \cdot P(A)$

d) know $P(A)$, $P(B)$ and $P(A \cup B)$

$\longleftrightarrow$

5) If A and B are independant, then A^c and B^c are independant

Proof must show $P(A^c \cap B^c) = P(A^c) P(B^c)$

- The L.H.S = $P((A \cup B)^c)$ = (Demorgan)
  $= (1 - P(A \cup B))$ (Thm 1)
  $= (1 - P(A) - P(B) + P(A \cap B))$
  $= 1 - P(A) - P(B) + P(A) P(B)$

- The R.H.S $[1 - P(A)][1 - P(B)]$

  $= 1 - P(A) - P(B) + P(A) P(B)$

LHS = RHS $\square$ Proven

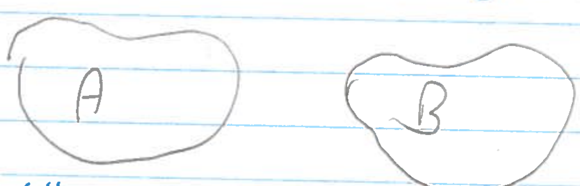
o More generally if A, B and C are independant, then
- $A \cup B$ and C are independant
- $A^c \cap B^c$ and C " "
- $A \cap C$ and B " "
- $C \cup B^c$ and C " "

o Even more generally, if you have non-overlapping collections of events, say $A_1, A_2 \ldots A_m$ and $B_1, B_2 \ldots B_n$, then the independance of the collection $A_1, A_2 \ldots A_m$ & $B_1, B_2 \ldots B_n$ implies the independance of any thing you do to the A's and any thing you do to the B's
As long as the collections are independant without overlap.

$\longleftrightarrow$

Note: Independance and disjointness are two different concept. Disjointness is entirely a set property, Independance is a probablistic property that depends on how probabilitys assigned to events.

Intut Events that look like this generaly not independant



We have following theorem hence:-

Theorem: Let A & B be disjoint then A & B will be independant only if either $P(A)$ or $P(B) = 0$

Proof:- if $A \cap B = \phi$ then $P(A \cap B) = P(\phi) = 0$

other hand if A and B are to be independant, then $P(A \cap B)$ must $= P(A)P(B)$ i.e we must have $P(A)\,P(B) = 0$

$\Rightarrow$ either $P(A)$ or $P(B) = 0$

Eg Cant be female + have prostate cancer, but are dependant

$$P(A|B) = P(A) \quad \left.\begin{array}{l}\text{cant be female and} \\ \text{have prostate cancer} \\ \text{but people can} \\ \text{have prostate cancer}\end{array}\right.$$

$\Downarrow$ $\Downarrow$

0 not 0

So Disjoint is Just a set theory ie Apartness of A & B.

Example ↓

Suppose that in a very large city, 20% of people have a genetic mutation. A sample of 10 people is drawn from this city. Whats the prob that 3 will have the mutation. Whats the prob that atleast 1 will have the mutation.

Solution ⇒

Solution:-

Note: In this prob, sample without replacement, therefore Red flag! The outcomes for 10 trials are strictly dependant. Reason, every time you remove a person, you are changing the population (ie Box) which you will use for your next observation.

Here however 10 is so small relative to the very large size of the city, we get very little info (taking people of a undefined sample size changes box / sample size very little). Therefore we can assume that the 10 outcomes are roughly independant

Actual solution:-

Begin with a specific outcome in which there are 3 with the genetic defect (Say the first three from the sample)

Thus consider
$$P(D_1 \cap D_2 \cap D_3 \cap D_4^c \cap D_5^c \cdots \cap D_{10}^c)$$

Now this is equal to:

$$P(D_1)P(D_2)P(D_3)P(D_4)P(D_5)^c \cdots P(D_{10})^c$$

[By Independence]

$$= 0.2^3 \cdot (1-0.2)^7$$

You want this but with all possible ways of such outcomes. Hence it is union of all the outcomes (as shown before) in which 3 have defect. Call these outcomes as A_i (ie A_i is ith collection of 10 in which 3 have defect)

The event that 3 have defect is $\bigcup_i A_i$ over all such collections. (Doesn't have to be first three here)

Now note that A_is are Disjoint.
Hence probability $P(3) = \sum_{i=1}^{?} P(A_i)$

It is easily seen probability A_i

$$\therefore P(A_i) = (.2)^3(1-0.2)^7 \text{ for all } i$$

Finally since there are $\binom{10}{3}$ ways to place the three defects among the 10.

$$\therefore P(3) = \binom{10}{3}(.2)^3(.8)^7$$