19 September 2017 Lecture 5

- Discussed our fish in lake problem again

  - The idea of $\dfrac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$

- Reasoning: denominator = { The number of ways to draw a set of n objects (here only +ve ints) From the integers $1, 2 \ldots n$, is by Defination $\binom{N}{n}$

Note [Do not work out your combinatorial coefficients in exams or assignments, won't lose marks]

- Reasoning: numerator = { Use the sausage rule, 2-sausages. One Sausage has objects, sets of integers that come from the a integers corresponding to tagged fish $(1, 2 \cdots a)$. The other sausage consists of sets $(n-x)$ integers that come from $(N-a)$ integers of untagged fish

Consider it as the set available for selection shrinking, limiting you choice for count.

- We need to count number of ways of drawing obj from Sausage 1 and obj from Sausage 2

- By the multiplication rule of counting, it's just $n_1 \times n_2$ where n_1, n_2 are objects in their respective sausages

- $n_1 = \binom{a}{x}$ $\qquad n_2 = \binom{N-a}{n-x}$

so $\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}$ } ways of choosing pairs between sets like this $= \binom{a}{x}\binom{N-a}{n-x}$.

$\longleftrightarrow$

Cont with fish = Order-unorder dilemma:

- Or vs non-ordered doesn't matter, cause if you add more for "order is imp" in denominator, you will end up adding equally more to numerator as well

- So order vs unordered = same in essence

$\longleftrightarrow$

- New Topic : Conditional Probability : In Prob and Stats, Conditional Prob is fundemental and very useful.

- We begin with the following eg, to justify the notion of conditional probability:

Example: Same box with 6 red and 4 green marbles. Now draw two marbles without replacement. What's the probability that 2nd Marble drawn is red, knowing that 1st drawn Marble was Red.

- It is clear on intiuative grounds that once you know that first marble is red, you end up with new S: Sample Space with 9 equally likely outcomes (for 2nd draw) such that there are 5 reds and still 4 greens

- Hence the second draw has probability $5/9$
  - Prob of getting red on first draw $6/10$ (note $\neq 5/9$)
  - Point being we have changed initial Prob of $6/10$ by including new info, that first draw is already a red, so that options leaves sample space

- We formalize the idea of conditional prob with a definition

Def :- (cond. Prob) : Let B be a given event, while A is another event, such that prob A is positive. Then we define the conditional prob of B given A to be

$$P(B \cap A) / P(A) \Rightarrow P[B \mid A] \quad \text{where} \quad P(A) > 0$$

- We call it "P of B given A"
- Thus by definition only $P[B|A] = \frac{P[B \cap A]}{P[A]}$

e [For the prob to be meaningful, R.H.S need to be well defined.]

Notes of Prof :-

1) Similarly $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B) > 0$

2.) Conditional Probability satisfy three axioms of probability
Note that, in what follows, we fix the event upon which we condition and we mess with the events whose conditional Prob's we want

Thus:

Axiom 1: $P(B|A) \geq 0$ for all events

leorem
r Matter

Answer: $P(B|A) = \frac{P(B \cap A)}{P(A)} \geq 0,$

Since $P(A) > 0$ and $P(B \cap A) \geq 0$
By Axiom1 of original axiom.

Theorem 8 (or Axiom): $P(S|A) = 1$

Answer $P(S|A) = P(A \cap S)/P(A)$
$= P(A)/P(A) = 1$

Theorem 9: Let B_1, B_2 be disjoint events, i.e. $B_1 \cap B_2 = \emptyset$

Then $P\left[\bigcup_{i=1}^{\infty} B_i \middle| A\right] = \sum_{i=1}^{\infty} P(B_i|A)$

Answers

$$P\left[\bigcup_{i=1}^{\infty} B_i \middle| A\right] = P\left[\bigcup_{i=1}^{\infty} B_i \cap A\right] / P[A]$$

$$= P\left[\bigcup_{i=1}^{\infty} (B_i \cap A)\right] / P(A)$$

$$= \sum_{i=1}^{\infty} P[B_i \cap A] / P(A)$$

[Axiom 3 and since B is Disjoint

$B_i \cap A$'s are Disjoint too

$$= \sum_{i=1}^{\infty} P[B_i|A]$$

Ans

IMP Note: if A_1 and A_2 are disjoint it is not true that $P[B \mid A_1 \cup A_2] = P[B \mid A_1] + P[B \mid A_2]$.
You'd be messing with conditionals

Also: $P[B \mid A^c] \neq 1 - P[B \mid A]$

But: $P[B^c \mid A] = 1 - P[B \mid A]$

DO NOT FUCK WITH CONDITIONING