Lecture 3: 12 September

Recall the three axioms of Kolmogorov

A1 $P(E) \geq 0$

A2 $P(S) = 1$

A3: $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$ if $E_i \cap E_j = \emptyset$

Disjoint sets

Note The idea of many proofs is to try to write a union of events as a disjoint union so that you can use Axiom 3

- Theorem 1: $P(E^c) = 1 - P(E)$
- Theorem 2: $P(\emptyset) = 0$

T2: may seem obvious but need formal proof using the axioms

so Pf: $P(\emptyset) = 0$

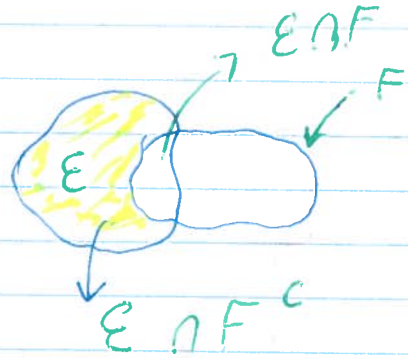
- $P(E \cup \emptyset) = P(E)$ on the one hand. On the other hand since $\emptyset \cap E = \emptyset$, we have by axiom 3, that $P(E \cup \emptyset) = P(E) + P(\emptyset)$

$\therefore P(\emptyset) = 0$

- Theorem 3: $P(E \cap F^c) = P(E) - P(E \cap F)$

Proof →

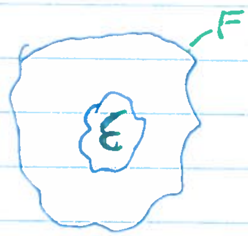
Proof: exercise



involving in terms of axiom 3 <Always the approachs

Theorems: $\underbrace{\mathcal{E} \subset F}_{\text{subset}} \Rightarrow P(\mathcal{E}) \leq P(F)$

Proof



Write F as $F = \mathcal{E} \cup (\mathcal{E}^c \cap F)$ Disjoint

$\therefore$ By Axiom 3:

$$P(F) = P(\mathcal{E}) + \overbrace{P(\mathcal{E}^c \cap F)}^{\geq 0 \text{ by } A1}$$

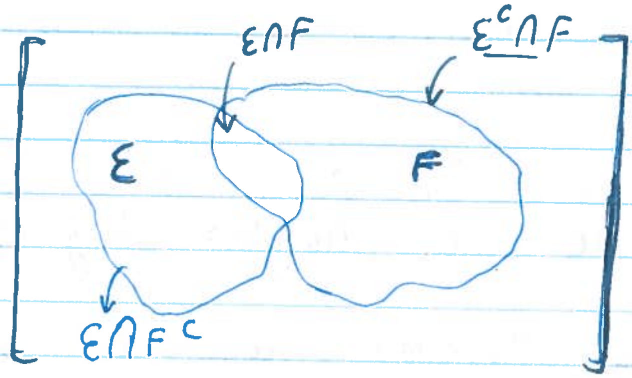
$\therefore$ $P(F) \geq P(\mathcal{E})$

Corollary: If $\mathcal{E}$ is any event, then $0 \leq P(\mathcal{E}) \leq 1$
[Reason: $\mathcal{E} \subset S$ and by A2, $P(S) = 1$]

Don't be STUPID, do not write negative Probability or $P(\mathcal{E}) > 1$

Theorem 5: For any two events, E and F, $P(E \cap F) = \emptyset$, then we get $P(E \cup F) = P(E) + P(F)$, which is consistent with Axiom 3.

Proof: Picture



note the arrows represent that section of venn diag ONLY

Note that $E \cup F = \underbrace{(E \cap F^c) \cup (E \cap F) \cup (E^c \cap F)}_{\text{all disjoint}}$

By axiom 3:

$$P(E \cup F) = P(E \cap F^c) + P(E \cap F) + P(E^c \cap F)$$

using Theorem 3 (T3)

$$= \underbrace{P(E) - P(E \cap F)}_{T3} + P(E \cap F) + \underbrace{P(F) - P(E \cap F)}_{T3}$$

$$= P(E) + P(F) - P(E \cap F)$$

$\longleftrightarrow$

Note Before doing a word Problem

1- START by defining given information as simple as possible (ie) Keep events Simple

2- Construct any more complex events who P are required by set operations ie Union – intersection – complements

3- atleast of 'or' "either or" is union where 'AND' → intersections

Word Problems

Ex 1 Suppose it is known that 20% of People and that 1% of people will get blood cancer.

Note : these two are independant from each other, it ISN'T that
in me 1% of the 20% develop this

Cont :- Suppose that the probability that some one will either smoke or develop lung cancer is .205

1) Find the propotion who will smoke AND develop Blood cancer

2) what is probability that someone does not smoke and develop blood cancer

3) " " " that someone smoke but NOT develop Blood cancer

4/ neither smoke nor get cancer.

Solution

let A = smokes
B = Gets Cancer (R.I.P)

1) we want $P(A \cap B)$ Thm 5 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.20 + 0.01 - 0.205 = 0.05$

ii) $P(A^c \cap B)$ where A^c is {does not smoke)

Thm 3 $= P(B) - P(A \cap B) = .01 - 0.005$

$= 0.005$ Ans

iii) $P(A \cap B^c) = P(A) - P(A \cap B)$

$= 0.20 - 0.005$

$= 0.195$

iv) neither smoke nor cancer; so $P(A^c \cap B^c)$

$= P(A \cup B)^c = 1 - P(A \cup B)$

$=$

We are in a position to provide a powerful tool for calculating probability by counting; if the setting is ~~the~~ right

Theorem 6 let S be a finite sample space ~~s~~, suppose that all outcomes are equally probable (IMP assumption)

Let ~~A~~ E be any Event in S, then probability of E, $P(E)$ such

$$P(E) = \frac{\text{no of outcomes in } E}{N}$$

or

$$\frac{\text{no of ways in } E \text{ can occur}}{\text{all outcomes}}$$

Proof:- First $E = \bigcup_{i;\, w_i \in E} w_i$,

- All Garbage Lingo
- Do not worry

Second $w_i \cap w_j = \phi$ $i \neq j$, by defination

$\therefore$ $P(E) = P[\bigcup_{i;\, w_i \in E} w_i] = \sum_{i;\, w_i \in E} P(w_i)$

In Particular $P(S) = \sum_{i;\, w_i \in S} P(w_i)$

$\parallel$

Axiom 2

if $P(w_i) = c$ for all i (assumed) then we have $1 = \sum_{i:\, w_i \in S} c = N\, c$ (since there were N points in S)

$\therefore$ $c = P(w_i) = 1/N$ for all i

Now $P(E) = \sum_{i;\, w_i \in E} P(w_i) = \sum_{i;\, w_i \in E} 1/N$

$= \frac{1}{N}$. No of outcomes in E

Marble Problem revisited

- Start by defining your sample space S, to have 10 equally likely outcomes
- We have seen that after numbering marbles $\{1 ---- 10\}$ with say 1st 6 numbers = red
  we have hence $S = \{1, 2 ---- 10\}$

to be continued