Question continue : September 26

a) $S = BC \cup B^c$ - disjoint

∴ by the law of total Prob

$$P(Pos) = P(Pos \mid BC)\,P(BC) + P(Pos \mid BC^c)\,P(BC^c)$$

A, B_1, B_2

so replacing with (A, B)

- $P(A) = P(A \mid B_1)\,P(B_1) + P(A \mid B_2)\,P(B_2)$

// we know
$P(Pos \mid BC) = .95$
$P(Neg \mid BC^c) = .95$
$P(BC) = 5/1000$

∴ $P(Pos)$ or $P(A) = 0.95(5/1000) + (1-0.95)(1-5/1000)$

ii // Here we want $P(BC \mid Pos)$, which is a reversal of $P(Pos \mid BC)$ (given above)

we have
$$P(BC \mid Pos) = \frac{P(Pos \mid BC)\,P(BC)}{P(Pos \mid BC)\,P(BC) + P(Pos \mid BC^c)\,P(BC^c)}$$

$$= \frac{0.95 * 5/1000}{.95 * 5/1000 + (1-.95) \times (1 - 5/1000)}$$

NOTE: In word problems, conditional prob is suggested if you can resonably use the word "given" to describe the probability.
Prof

Notes on Bayes theorem:-

1) In a diagnostic test such that above (example $P(Pos \mid Diesese)$ is called the sensitivity of the test.
$P(Neg \mid Diesease^c)$ is called specificity of the test.

2) $P(Diasese \mid Positive)$ = positive predictive value of the test.
- Depends on not only the sensitivity and specifity of test, but also on prevalance of the diseases.

Further Jargon:-

- In the description of Baye's theorem, the probabilites $B_1 B_2 \cdots B_m$ (ie $P(B_1), P(B_2) \cdots P(B_m)$) are called prior probabilities, since they are found in absence of current data.
- Thus we have prior probabilities of breast cancer before we see the result of the test.
- The probabilites $P(B_k \mid A)$ are called posterior prob. Think A as representative of current data.
- In this light, Bayes Theorem tells us how to update prior prob → to → posterior

Statistical Independance:-

- Sometimes, knowledge that an event A has occured, does not effect our calculations of the probability that B will occur.

- For example in the marble problem, if we draw 2 marbles with replacement than $P(G_2 | R_1)$ is just $P(G_2)$; since knowing that drawing ~~drawing~~ red marble at first doesn't affect drawing green

- This idea translates into an immediate defination (2)

Defination 1:- Two events A/B are said to be independent if prob $\{P(B|A) = P(B)$ or $P(A|B) = P(A)\}$

Notes:-

1) Its hard to extend this natural defination to more than two events.
   - We therefore give a second defination that is easily extended to arbitary set of events: $A_1, A_2, A_3 \ldots A_n$

Defination 2:- The events A & B are independant if
$$P(A \cap B) = P(A)\,P(B)$$

Theorem to show the Def 1-2 are equal

- Two events are independant according to Def 1 $\Leftrightarrow$ they are independant according to Def 2

Proof

1) Let A & B be independant by ~~Def~~ Def 1
Then $P(B|A) = P(B)$. But $P(B|A) = P(A \cap B)/P(A)$

$\therefore P(B) = P(A \cap B)/P(A) \Rightarrow \underbrace{P(A \cap B) = P(B)P(A)}_{\text{Def 2}}$

$\longleftrightarrow$

2) We can now extend defination of Independance to more than 2 events.
DeF:- The events $A_1, A_2 \cdots A_n$ are mutually independant if

$$P\left(\bigcap_{j=1}^{k} A_{ij}\right) = \prod_{j=1}^{k} P(A_{ij})$$

symbol for product

For all subsets $A_{i_1}, A_{i_2}, \cdots A_{i_k}$ of All $A_1 \cdots A_n$ for all k

Remember k are subsets.

3 We say $A_1, A_2 \ldots A_n$ are pairwise independant if $P(A_{ij} \cap A_{ix}) = P(A_{ij})\,P(A_{ix})$ for all pairs.

- It is possible to show that pairwise independance doesn't imply mutual independance

4) Very IMP: Independance arise in two ways

1) We have data and we want to check if certain events are independant
   Eg: Is recovery after abdominal surgery independant of the temp of operating room.

   So Checking independance

ites
Assi 1

2) (Most Imp way) if we can assume independance, base on our knowledge of basic science of the problem, then, calculations of probability involving intersections are rendered much easier

- All we need to know is $P(A_1)\,P(A_2) \ldots P(A_n)$

Example, when you toss a fair coin twice, it's reasonable to assume, that the outcomes on the two tosses are independant. Because we can't think of any reasonable way in which they might be dependant.

Then we can easily compute $P(H_1 \cap T_2)$ $P(H_1)\,P(T)_2$ $= 1/2 \cdot 1/2 = 1/4$, & all other outcomes are also easily found to have prob $1/4$. If you required to find $P(A \cap B)$

[illegible]