Day 13 :-

Personal NOTE :-

[Choosing a number at random, you can't assign a denominator ~~such that~~ — such that you assign a constant probability to an undefined set, infinite set of no's]

The Bernolli Distribution

$P[x=1] = p$

$P[x=0] = 1-p = q$

or more compactly, $P_{(x)}(X) = P[X=x] = p^{x}(1-p)^{1-x}$:-

For $x=0,1$

Do Always give RAnge Will lose Marks if you dont

The Bernolli distribution can be used to describe R.V.S that can take on two values — for eg on/off switch.

Where these outcomes occur at random

These distributions are used a building blocks for more complex variables

The Binomial Distribution [Built with Bernolli distribution]

lets write down the formula first:-

Definition: The R.Vs X has a Binomial Distribution with Parameters n & p, if;

Formula: $$P[X = x] = \binom{n}{x} p^x (1-p)^{n-x}, \quad \text{For } x = 0, 1 \cdots n$$

Also: We write $X \sim Bin(n, p)$ to mean X has a binomial distribution with parameter n & p.
$X \sim Bernolli(p)$

Notes:

1) Does $P_X(x)$ sum to 1?

$$\sum_{x=0}^{n} P_X(x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = {}^{*}(p + 1 - p)^n \quad \text{[Binomial Theorem]}$$

with "a" = p
and "b" = $1-p$

* $1^n = 1$

2) The important thing about Binomial distribution is that it arises in The Binomial set up

Binomial Setup:

i) ~~We~~ we have n independent Bernoulli trials (A Bernoulli trial is one that can result in exactly one of two possible outcomes) Eg one toss either H or T

We call outcomes of these trials Success or failure; these are just labels though

ii) The probability of success on trial i is p (and of failure, $1-p = q$) For all trials $i = 1, 2 \dots n$.

let X be no of successes in these n trials

Theorem. $P[X = x] = \binom{n}{x} p^x (1-p)^x$ for $x = 1, 2 \dots n$

so number of success arises when in n independent ~~the~~ trials $\longleftrightarrow$

Proof: We've done it already

When you are concentrating on a specific sequence
$\binom{n}{x}$

Details $\longrightarrow$

Take any particular configuration, with x successes and $n-x$ failures. say $\{S_1 \cap S_2 \cap S_3 \cdots \cap f_{x+1} \cdots f_n\}$

Then $P[S_1 \cap S_2 \cap \cdots \cap S_x \cap f_{x+1} \cap \cdots \cap f_n]$

~~Pute~~ * $= P[S_1]\, P[S_2] \cdots P[S_x]\, P[F_{x+1}] \cdots P[F_n]$

Note:- S = success F = failure

Contd.- $p^x (1-p)^{n-x}$

o $P[X=x] = P[\bigcup_{\text{all config}} \{\text{configs with } x \text{ success and } n-x \text{ failures}\}]$

o $= \sum_{\text{all configs}} P[\text{configs}] = \binom{n}{x} p^x (1-p)^{n-x}$

o Since all configs have prob $p^x (1-p)^{n-x}$ and there are $\binom{n}{x}$ such configs

Example :-

Suppose that patients under going a certain treatment can survive more than 5 years or $\leq$ 5 years. Suppose further that the probation of people that survive more than 5 years is .80

- If 30 patients are to recieve this treatment, what is the probability that atleast 3 will survive more than 5 years.

Solution:-

[✓ is just checking off conditions]

- ~~Reco~~ Recognize the binomial setup. One of two possible outcomes for each patient ✓. What happens to patients occurs independantly (outcomes Independant) ✓. Prob of success ~~and~~ is constant from patient to patient ✓

- Let success = event surveval > 5 years
  Failure = event survival ~~doesn't~~ ~~to~~ ~~...~~ $\leq$ 5 years

Let X = no of patients that survive more than 5 years

Since $X \sim Bin(30, .80)$, we have
$n = 30$, $p = .80$

$$P[X \geq 3] = \sum_{x=3}^{30} \binom{30}{x} (.80)^x (1-.8)^{30-x}$$

From 3 to 30 (can leave ans here)

You can also write as

$$1 - P[X < 3]$$

$$= 1 - \left[\sum_{x=0}^{2} \binom{30}{x} (.8)^{x} (1 - 0.8)^{30-x} \right]$$

Note: Do not automatically jump to Binomial distribution when you see two possible types of outcome

Ask yourself if trials independent

The Poisson Distribution (we write $X \sim$ Poisson)

Def: The r.v X is said to have a poisson distribution with parameter $\lambda > 0$, if

$$P[X = x] = P_X(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}, \text{ for } x = 0, 1 \cdots n$$

Notes:- $P_X(x) \geq 0 \quad \forall \; x = 0, 1, \ldots\ldots$

and $$\sum_{x=0}^{\infty} P_X(x) = \sum_{x=0}^{\infty} \frac{\lambda^{x} e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^{x}}{x!} = e^{-\lambda} \cdot e^{+\lambda}$$

by def of e^{λ}

VIP

2) The poisson distribution arises as an approximation to the Binomial distribution, we have.

~~We have~~

Theorem: Let $X \sim Bin(n, p)$ then

$$P[X=x] \rightarrow \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{for } x = 0, 1, 2$$

as $n \rightarrow \infty$ and $p \rightarrow 0$, such that

$np = \lambda$, is contant.

Proof: [we will use Poisson distribution in Binomial setup when n is large, p is small]

eg $n = 5000$ $p = 0.1$ is fair game

$$P[X=x] = \binom{n}{x} p^x (1-p)^{n-x} \quad \text{For } x = 0, 1, 2 \cdots n$$

$$= \left(n! / x!\,(n-x)! \right) p^x (1-p)^n (1-p)^{-x}$$

Take x! out

$$= \frac{1}{x!} \cdot \left[\frac{n(n-1) \cdots 3 \cdot 2 \cdot 1}{(n-x)(n-x-1) \cdots 3 \cdot 2 \cdot 1} \right] \cdot p^x (1-p)^n (1-p)^{-x}$$

$$= \frac{1}{x!} \cdot n(n-1) \cdots (n-x+1) \cdot \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$= \frac{1}{x!} \cdot \frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n-x+1}{n} \cdot \lambda^x \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$\underbrace{\frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n-x+1}{n}}_{\text{These } n\text{'s come from } n^x}$

now let $n \to \infty$ for fixed x

(at ∞)

$\left(\frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n+1-x}{n} \to 1\right)$

$\left(1 - \frac{\lambda}{n}\right) \to 1$

$\left(1 \pm \frac{\lambda}{n}\right)^n \to e^{\pm \lambda}$

$$\frac{1}{x!} \cdot (1) \cdot (1) \cdot \lambda^x e^{-\lambda}$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0, 1 \ldots n$$

Proven $\square$