Math 323 - lectues 2 · Mathematical Defination

Random Exp:- Don't know what the outcome would be till the observation

Simple eg:- Toss a coin twice, and Observe outcomes. Before you toss a coin the pairs of out comes are unknown. (uncertain)

another eg:- Take 60 individuals who go surgery for a disorder; Before we observe them times to recovery, these times are uncertain

or Toss a coin, till first head. This no of tosses are uncertain Before you carry out the experiment

Defination:- The set of all Possible outcomes of an experiment is called Sample Space or S

Note:- The sample space is in eye of holder. i.e, it may depend on how you define outcomes.

eg of Note:- Marble eg; 6 Red, 4 green & draw a marble at random.

(One Way to define) S; consider two possible outcomes where R = Red g = green; then we could define S_1 to be $S_1 = \{R, G\}$

Here $\{\omega_1 = 1, \omega_2 = 2 \ldots \omega_{10} = 10\} \to$ Outcomes

(Second way) S is to be given numbering between 1 to 10. We could agree that the numbers (1 → 6) are red and (7 → 10) green.

S_2 then is {1, 2 ---- 10} and one positive integer (1 → 10) is outcome.

Note :- S_2 is a more convient sample space and we prefer convience for easier answer.

Example 2 Suppose there are n people in ~~a~~ n room. We will ask all n little shits when their birthdays are. The set of all possible outcomes, S for this experiment can be defined as {(Jan 1st, Jan 1st ---- Jan 1st), (Dec 31 ... — Dec 31)}
(ω_1) (ω_n)

Here the outcomes are n-dimensional vectors corresponding to possible assignments

Note CONCENTRATE; on the outcomes, what can happen rather than the most optimal possibility

This Q had finite outcomes

Ex 3 (Infinite outcomes): Toss a coin until you observe the 1st head. let N denote trial no at which 1st head occurs

1) head on first toss 1
2) " on second toss 2
⋮ ⋮
n n

so $S = \{1, 2, 3 \cdots n\}$
$\omega_1, \omega_2 \cdots \omega_n$

Ex 4 Toss a coin twice, S defined to be $\{H_1, H_2\}$, $\{H_1, T_2\}$, $\{T_1, H_2\}$, $\{T_1, T_2\} = S$

where each outcome $= \{\omega_1, \omega_2, \omega_3, \omega_4\}$

Ex 5 The following examples are uncountable (note all interval eg {1–2} irrational are uncountable)

cont: Observe the exact depth that a dam will be on Jun 1. Here there sample space is $S = [0, 20]$, if we assume max = 20.
The point here is that depth can be any real number (eg 2.512…) b/w 0 to 20, hence uncountable.

Like the lord knows how many no's b/w 0 – 20 really.

Note: Nothing in real life that are continous

Axioms of Prob

Now for the 3 axioms of Probability* of Fuch you A.N. Kolmogorov

30. We shall call any subset of sample space, an event, so $\mathcal{E} \subset S$, then we will call $\mathcal{E}$ as events

STOP Thinking about Non-events, every subset → event. even an empty set $\{\emptyset\}$.

$\longleftrightarrow$

Let S be a sample space. A set function P (one that gives a real number to every event in S), is called a probability measure*.

If ~~ϕ~~ P satisfies the following axioms:

1) For every event $\mathcal{E} \subset S$, $P(\mathcal{E}) \geq 0$ Non-Negative
2) $P(S) = 1$
3) let $\mathcal{E}_1, \mathcal{E}_2 \ldots$ be possibily infinite sequence of disjoint events (no points in common) → $(\mathcal{E}_i \cap \mathcal{E}_j = \emptyset)$
Then $P(\mathcal{E}_1 \cup \mathcal{E}_2 \ldots) = \sum_{i=1} P(\mathcal{E}_i)$

These are assumptions not proven tales

$\longleftrightarrow$

NOTE; We shall say "P of E" or "P-E" as probability of an event.

《Boiiii》

> As a frequenst be able to discribe the outcome
> if you flip coin, yes probability is 50/50 at pre (random), at post → answer is either (0, 1)

Randy Jonson needs to stop hitting pigeons :'(

[Idea is once an event is done, its done, thats now the sample space]

- We shall provide several theorems following the axioms that prove essental for WORD Problems

Theorems 1) if ε is an Event, $P(\varepsilon^c) = 1 - P(\varepsilon)$

Proof: we have $P(\varepsilon \cup \varepsilon^c) = 1 = P(S)$
(Axiom 1/2)
On the other hand $P(\varepsilon \cup \varepsilon^c) = P(\varepsilon) + P(\varepsilon^c)$
(Axiom 3) $\therefore$ $1 = P(\varepsilon) + P(\varepsilon)^c$