Homework

Next due on June 23, 24:00

Homework 1

1. Show that in the double-exponential distribution, $f(x;\theta) = 1/2\exp\{-|x-\theta|\}$, the score for θ at a given θ^* is $s(x;\theta^*) = sg(x-\theta^*)$ where
$$sg(u) = \begin{cases} 1 & \text{if } u \geq 0 \\ -1 & \text{if } u < 0 \end{cases}$$
and calculate the information for θ at θ^*.

2. Calculate the mean square error of Hodges estimator. Show that Hodges estimator is not regular.

3. Weibull translation model.

   Suppose that $\theta = (\alpha, \beta, \gamma) \in \Theta \equiv \{\theta : \alpha > 0, \beta > 0, \gamma \in R\}$ and
$$p(x;\theta) = \frac{\beta}{\alpha}\left(\frac{x-\gamma}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}\right\} I_{(\gamma,\infty)}(x)$$
   the Weibull translation model. This model is not regular, but the restricted model $\mathbf{P} = \{P_\theta : \theta \in \Theta_0\}$, where $\Theta_0 \equiv \{\theta : \alpha > 0, \beta > 2, \gamma \in R\} \subset \Theta$, is regular.

4. Show that an asymptotically linear estimator has a unique influence function.

Homework 2

1. Suppose that in a parametric model the index θ is $\theta = (\beta^{\mathrm{T}}, \eta^{\mathrm{T}})^{\mathrm{T}}$. Where β and η are variationally independent. Show that the efficient Influence function for β is $\phi_{\text{eff}}(X) = \text{var}\{S_{\beta,\text{eff}}\}^{-1} S_{\beta,\text{eff}}$.

2. Compute the C-R bound for $\beta = E(X)$ under the lognormal parametric submodel $\mathcal{F}_{sub} = \{\log N(\mu, \sigma^2) : \mu \in \mathcal{R}, \sigma > 0\}$. Why it is not least favorable for the mean functional in the nonparametric model?

3. Provide an example of an estimator that is regular for a parameter $\beta(\theta)$ in a regular parametric model $\mathcal{F} = \{f(x;\theta); \theta \in \Theta\}$ which does not converge to a mean zero normal random distribution.

4. Consider estimation of $\theta = (\mu, \sigma^2)$ in the normal model $\mathcal{F} = N(\mu, \sigma^2) : \mu \in \mathcal{R}, \sigma > 0$. Compute the C-R bound for estimating θ, the efficient influence function. How does the estimation of μ impact the estimation of σ^2?

Homework 3

1. Prove results 5.1 and 5.5 in slides 5.

2. Consider the partial linear regression model

$$E(Y \mid X, Z) = \beta^{\mathrm{T}} X + \eta(Z),$$

- Describe the parameter space and likelihood of this model;
- Derive the score for β and the nuisance tangent space of this model;
- Characterize the efficient score and efficiency bound;
- Describe the efficient estimation of β.

Homework 4

1. Prove the result on page 55 in slides 5.

2. Let X be a fully observed covariate, Y an outcome with missing values and R the missingness indicator with $R = 1$ for observed values and $R = 0$ for missing values. Suppose the missingness is at random, i.e., $R \perp\!\!\!\perp Y \mid X$. Let $\mathcal{F}^{\text{fix}}$ denote the semiparametric model for the joint distribution $f(X, Y, R)$ such that the outcome model $E(Y \mid X, R = 1)$ is known and $\mathcal{F}^{\text{para}}$ the semiparametric model that $E(Y \mid X, R = 1) = E(Y \mid X, R = 1; \gamma)$ follows a parametric model. Derive the efficiency bounds for estimating the outcome mean $\mu = E(Y)$ in these two semiparametric models. What is the difference between these two bounds? What is the difference from the efficiency bound for the nonparametric model?

3. Suppose the missingness is not at random, i.e., $R \not\perp\!\!\!\perp Y \mid X$. Suppose the propensity score is $f(R = 1 \mid X, Y) = \text{expit}\{\alpha(X) + \gamma(X, Y)\}$ with $\gamma(X, Y = 0) = 0$. Consider the semiparametric model where $\gamma(X, Y)$ is known and the remaining part of the joint distribution is unrestricted, then derive the tangent space and the efficiency bound for estimating the outcome mean $\mu = E(Y)$.

Homework 5

1. Prove the second inequality on page 20 in slides 8.

2. Prove the Generalized Minkowski inequality on page 28 in slides 8.