# Incentivizing Exploration by Heterogeneous Users

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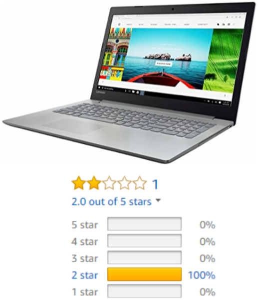
# Motivation

Amazon wants users to ***explore***
Each customer only wants to buy one good item





# Previous Work

**Without Money Transfer:**

- Implementing the "Wisdom of the Crowd", Kremer et al. 2014;
- Bayesian incentive-compatible bandit exploration, Mansour et al. 2015;
- $\cdots$

**With Money Transfer**

- Incentivizing exploration, Frazier et al. 2014;
- Incentivizing exploration with heterogeneous value of money, Han et al. 2015;
- $\cdots$

# Heterogeneity presents a new challenge

**Customers prefer different kinds of items**

**Amazon doesn't know which item each user prefers**

# Heterogeneity Provides Free Exploration

- In the classical MAB: cumulative regret is $O(\log(T))$
- In incentizing exploration with heterogeneous users: we show, with assumptions, cumulative regret is $O(1)$
- Key insight: Heterogeneity provides free exploration
- Our contribution: First algorithm and analysis for incentivizing exploration when users have heterogeneous preferences over arms

# Problem Setting

**$N$ arms**

- Each arm has an unknown feature vector $u_i \in R^d$
- Pulling arm $i$ gives $u_i$ perturbed by independent sub-Gaussian noise
- The agents and principal observe averages of each arm's past pulls $\hat{u}_{i,t}$

**Myopic Agents**

- Agents arrive sequentially
- Agent $t$ has linear preferences with weight vector $\theta_t \in R^d$ drawn from known distribution $F$
- Without incentives, agent $t$ would choose the arm maximizing $\theta_t \cdot \hat{u}_{i,t}$.

# Problem Setting

**Agents' behavior**

- Principal chooses payment $c_{t,i}$ for arm $i$ at time $t$
- Agent $\theta_t$ pulls arm $i_t = \arg\max_i \{\theta_t \cdot \hat{u}_{i,t} + c_{t,i}\}$

**Principal's goal**

- Regret $r_t = (\max_i \theta_t \cdot u_i) - \theta_t \cdot u_{i_t}$ and payment $c_t = c_{t,i_t}$
- Incentivize to minimize the cumulative regret while making a small cumulative payment

# Key Assumptions

- (**Every arm is someone's best**) We use $p$ to denote the minimum (over all arms) fraction of users that prefer any particular arm.
- (**Not too many near-ties**) Let $q(z)$ be the cumulative distribution function of those agents whose utility difference between their best and second best arm is less than or equal to $z$, then there exists a $\hat{z} > 0$, $L$ such that $q(z) \leq L \cdot z$ for all $z \leq \hat{z}$.
- (**Compact Support**) $\theta$ has a compact support set contained in $[0, D]^d$.

# Main Result

**Theorem 1**

Our policy achieves:

expected cumulative regret $O(Ne^{2/p} + LN \log^3(T))$,

using expected cumulative payments of $O(N^2 e^{2/p})$.

Special case: When agent preferences are discrete, i.e. $L = 0$, regret and payment are bounded by constants in $T$.

# Notations

**Phase**

- Phase $s$ starts when each arm has been pulled at least $s$ times.

**Payment-eligible**

- Arm $i$ has been pulled at most $s$ times up to time $t$;
- The conditional probability of pulling arm $i$ is less than $\frac{1}{\log(s)}$ given the current estimates $\hat{u}_{i,t}$.

# Algorithm

Set the current phase number $s = 1$. {Each arm is pulled once initially "for free."}

**for** time steps $t = 1, 2, 3, \ldots$ **do**

  **if** $m_{t,i} \geq s + 1$ for all arms $i$ **then**

    Increment the phase $s = s + 1$.

  **if** there is a payment-eligible arm $i$ **then**

    Let $i$ be an arbitrary payment-eligible arm.

    Offer payment $c_{t,i} = \max_{\theta, i'} \theta \cdot (\hat{\mu}_{t,i'} - \hat{\mu}_{t,i})$ for pulling arm $i$ (and payment 0 for all other arms).

  **else**

    Let agent $t$ play myopically, i.e., offer payments 0 for all arms.

# Payment Analysis

**Key technical lemma**: an adaptive concentration inequality (Zhao et al. 2016);

**Early Phases**: bound by $N$;

**Later Phases**: exponentially unlikely as the phases advances;

# Regret Analysis

**When principal incentivizes:** similar to the payment proof;

**When agents pull myopically:** We define a phase-dependent cutoff $\gamma(s(t))$ to further distinguish agents based on the regret.

- $r(t) \geq \gamma(s(t))$:
  - ⋆ this happens with exponentially decreasing probability;
  - ⋆ since $\theta_t$ has a compact support, the maximum regret is bounded by a constant;
- $r(t) \leq \gamma(s(t))$:
  - ⋆ not so many agents have near-ties preferences;
  - ⋆ the maximum regret is bounded above by $\gamma(s(t))$;

# Question?

Thanks for your time!