Incentivizing Exploration by Heterogeneous Users

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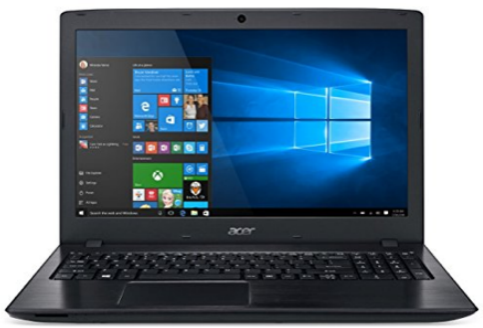
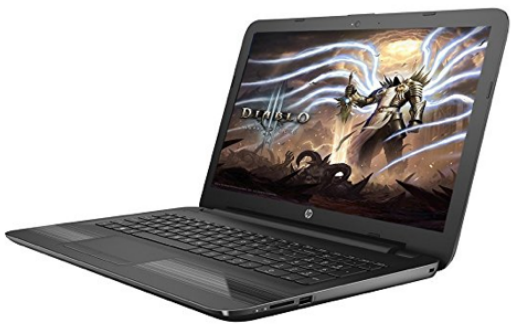
Motivation

Amazon wants users to ***explore***
Each customer only wants to buy one good item



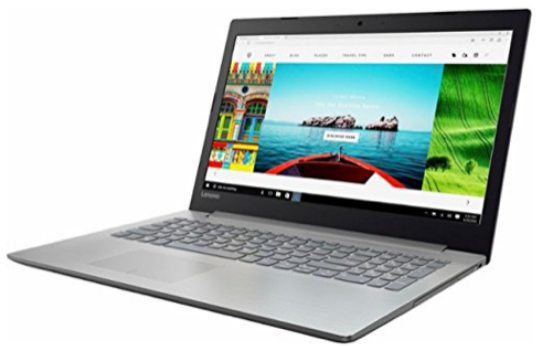
2,202
3.7 out of 5 stars

5 star	51%
4 star	16%
3 star	7%
2 star	6%
1 star	20%



508
3.6 out of 5 stars

5 star	49%
4 star	15%
3 star	8%
2 star	6%
1 star	22%



1
2.0 out of 5 stars

5 star	0%
4 star	0%
3 star	0%
2 star	100%
1 star	0%

Previous Work

Without Money Transfer:

- Implementing the âĂIJWisdom of the CrowdâĂİ, Kremer et al. 2014;
- Bayesian incentive-compatible bandit exploration, Mansour et al. 2015;
- ...

With Money Transfer

- Incentivizing exploration, Frazier et al. 2014;
- Incentivizing exploration with heterogeneous value of money, Han et al. 2015;
- ...

Heterogeneity Provides Free Exploration

- In the classical MAB: cumulative regret is $O(\log(T))$
- In incentizing exploration with heterogeneous users: we show, with assumptions, cumulative regret is $O(1)$
- Key insight: Heterogeneity provides free exploration
- Our contribution: First algorithm and analysis for incentivizing exploration when users have heterogeneous preferences over arms

Problem Setting

N arms

- Each arm has an unknown feature vector $u_i \in R^d$
- Pulling arm i gives u_i perturbed by independent sub-Gaussian noise
- The agents and principal observe averages of each arm's past pulls $\hat{u}_{i,t}$

Myopic Agents

- Agents arrive sequentially
- Agent t has linear preferences with weight vector $\theta_t \in R^d$ drawn from known distribution F
- Without incentives, agent t would choose the arm maximizing $\theta_t \cdot \hat{u}_{i,t}$.

Problem Setting

Agents' behavior

- Principal chooses payment $c_{t,i}$ for arm i at time t
- Agent θ_t pulls arm $i_t = \arg\max_i \{\theta_t \cdot \hat{u}_{i,t} + c_{t,i}\}$

Principal's goal

- Regret $r_t = (\max_i \theta_t \cdot u_i) - \theta_t \cdot u_{i_t}$ and payment $c_t = c_{t,i_t}$
- Incentivize to minimize the cumulative regret while making a small cumulative payment

Key Assumptions

- (**Every arm is someone's best**) We use p to denote the minimum (over all arms) fraction of users that prefer any particular arm.
- (**Not too many near-ties**) Let $q(z)$ be the cumulative distribution function of those agents whose utility difference between their best and second best arm is less than or equal to z, then there exists a $\hat{z} > 0$, L such that $q(z) \leq L \cdot z$ for all $z \leq \hat{z}$.
- (**Compact Support**) θ has a compact support set contained in $[0, D]^d$.

Main Result

Theorem 1

Our policy achieves:

expected cumulative regret $O(Ne^{2/p} + LN\log^3(T))$,

using expected cumulative payments of $O(N^2e^{2/p})$.

Special case: When agent preferences are discrete, $L = 0$ and regret and payment are bounded by constants in T.

Notations

Phase

- Phase s starts when each arm has been pulled at least s times.

Payment-eligible

- Arm i has been pulled at most s times up to time t;
- The conditional probability of pulling arm i is less than $\frac{1}{\log(s)}$ given the current estimates $\hat{u}_{i,t}$.

Algorithm

Set the current phase number $s = 1$. {Each arm is pulled once initially "for free."}
for time steps $t = 1, 2, 3, \ldots$ **do**
  if $m_{t,i} \geq s + 1$ for all arms i **then**
    Increment the phase $s = s + 1$.
  if there is a payment-eligible arm i **then**
    Let i be an arbitrary payment-eligible arm.
    Offer payment $c_{t,i} = \max_{\theta,i'} \theta \cdot (\hat{\mu}_{t,i'} - \hat{\mu}_{t,i})$ for pulling arm i (and payment 0 for all other arms).
  else
    Let agent t play myopically, i.e., offer payments 0 for all arms.

Payment Analysis

Key technical lemma: an adaptive concentration inequality (Zhao et al. 2016);

Early Phases: bound by N;

Later Phases: exponentially unlikely as the phases advances;

Regret Analysis

When principal incentivizes: similar to the payment proof;

When agents pull myopically: We define a phase-dependent cutoff $\gamma(s(t))$ to further distinguish agents based on the regret.

- $r(t) \geq \gamma(s(t))$:
  - ⋆ this happens with exponentially decreasing probability;
  - ⋆ since θ_t has a compact support, the maximum regret is bounded by a constant;
- $r(t) \leq \gamma(s(t))$:
  - ⋆ not so many agents have near-ties preferences;
  - ⋆ the maximum regret is bounded above by $\gamma(s(t))$;

Question?

Thanks for your time!